

Module 5 : Signal Distortion on Optical Fibers - Dispersion

Lecture : Signal Distortion on Optical Fibers - Dispersion

Objectives

In this lecture you will learn the following

- Material Dispersion
- Waveguide Dispersion
- b-V Diagram
- Inter-Modal Dispersion
- Polarization Mode Dispersion (PMD)
- Dispersion is a phenomenon related to the variation in velocity of different frequencies (wavelengths) or different modes. The velocity of different frequencies can be different due to intrinsic properties of the medium or due to dispersive nature of the bound structure like the optical fiber.

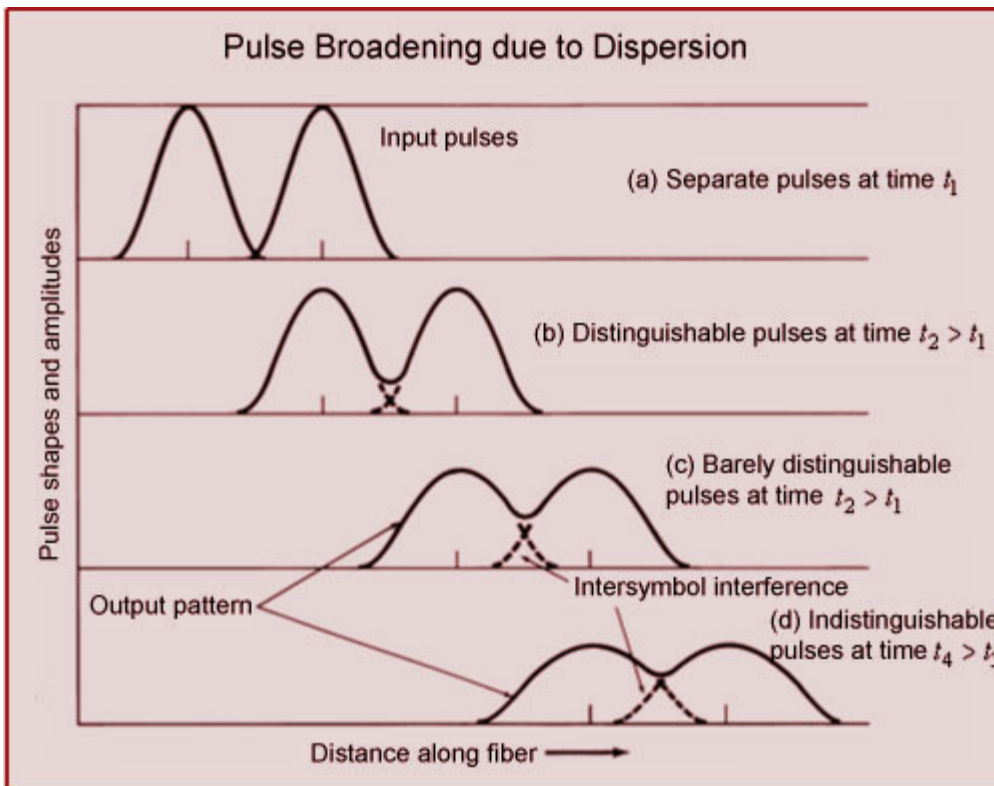
- Due to dispersion when the optical pulses travel along the fiber they broaden as shown in
- Fig. As the pulses travel on the fiber due to broadening, slowly they start overlapping with each other. Consequently, the AC content of the signal reduces and the DC content of the signal increases. After certain distance the pulses barely remain distinguishable. If the signal propagates further the pulses lose their identity and the data is lost.

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NOTE:

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It is not the data rate but the Data rate-Distance product which is governed by the dispersion.



We can have general formulation for the dispersion as follows

- (1) First get the group velocity of the pulse.
- (2) Then we find the group delay which is pulse delay per unit distance.
- (3) Find delay over the spectral width and pulse broadening.
- (4) Get dispersion, pulse broadening per unit distance per unit spectral width of the source.

Velocity Group
$$v_g = \frac{\partial \omega}{\partial \beta} = 2\pi c \frac{\partial}{\partial \beta} (1/\lambda)$$

Group Delay per unit length
$$t_g = 1/v_g$$

Pulse Broadening
$$\tau = \frac{dt_g}{d\lambda} \sigma_\lambda = - \frac{\sigma_\lambda}{2\pi c} \left\{ 2\lambda \frac{d\beta}{d\lambda} + \lambda^2 \frac{d^2\beta}{d\lambda^2} \right\}$$

Dispersion
$$D = \frac{dt_g}{d\lambda} = - \frac{1}{2\pi c} \left\{ 2\lambda \frac{d\beta}{d\lambda} + \lambda^2 \frac{d^2\beta}{d\lambda^2} \right\}$$

= Pulse broadening per unit distance per unit spectral width (ps/Km/nm)

Where β is the propagation constant, ω is the frequency of the signal and λ is corresponding free-space wavelength, and c is the velocity of light in vacuum.

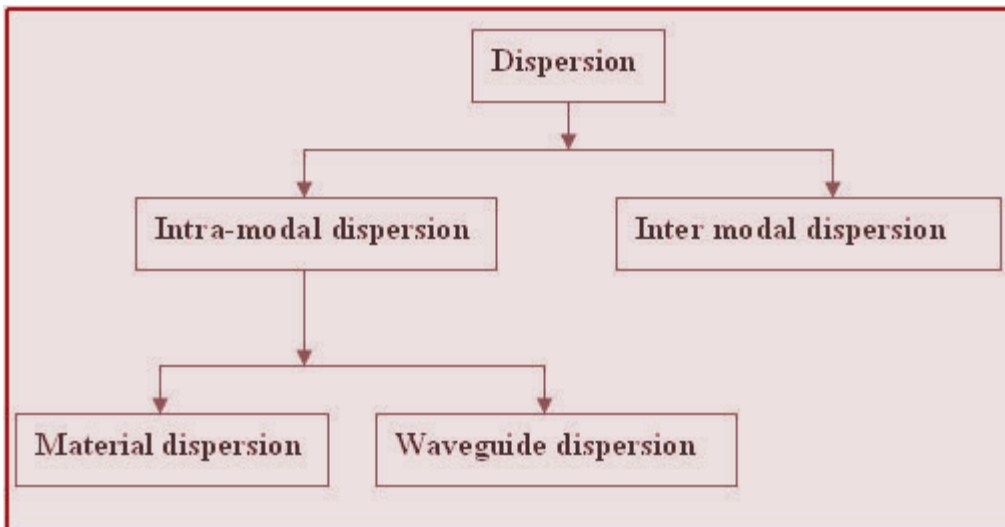
- In an optical fiber we find two types of dispersions

(1) **Intra-modal or Chromatic dispersion** (present in single mode and multimode optical fiber). This dispersion is due to finite bandwidth of the signal.

The Intra-modal dispersion consists of two parts:

- (a) **Material dispersion:** This is due to intrinsic properties of the material, glass.
- (b) **Waveguide dispersion:** This is due to dispersive nature of the bound medium. In a bound medium like the optical fiber, the velocity is a function of frequency.

(2) **Inter-modal dispersion** (present in only multimode optical fiber). This dispersion is due to change in velocity from one mode to other.



- The dispersion is a rather weak phenomenon. What that means is the pulses slowly broaden as they travel along the optical fiber.
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- It is therefore appropriate to investigate one type of dispersion at a time. That is when we investigate waveguide dispersion we assume the fiber material to be non dispersive, and when we investigate material dispersion, we assume the medium to be unbound.
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Material Dispersion

- The material dispersion is due to intrinsic property of the material. Glass is a dispersive medium. We can recall from our high school physics that glass has different refractive index for different colors. Since the velocity of light is

$$v = \frac{\text{Velocity of light in vacuum}}{\text{Refractive index}}$$

- Different colors (wavelengths) have different velocity in glass.

- As mentioned above while calculating the material we assume that there is no waveguide dispersion, that is fiber has infinite radius. The medium then is an unbound medium with refractive index $n(\lambda)$, which is a function of wavelength, λ .

Since the medium is unbound, the propagation constant of light is

$$\beta = \frac{\omega n(\lambda)}{c}$$

Substituting for β in above general expression for dispersion, we get material dispersion as

$$D_{mat} = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2}$$

- The material dispersion therefore is proportional to second derivative of the refractive index with respect to the wavelength.
- Since the second derivative denotes the curvature of the function, **the material dispersion is proportional to the curvature of the refractive index function.**

Note:

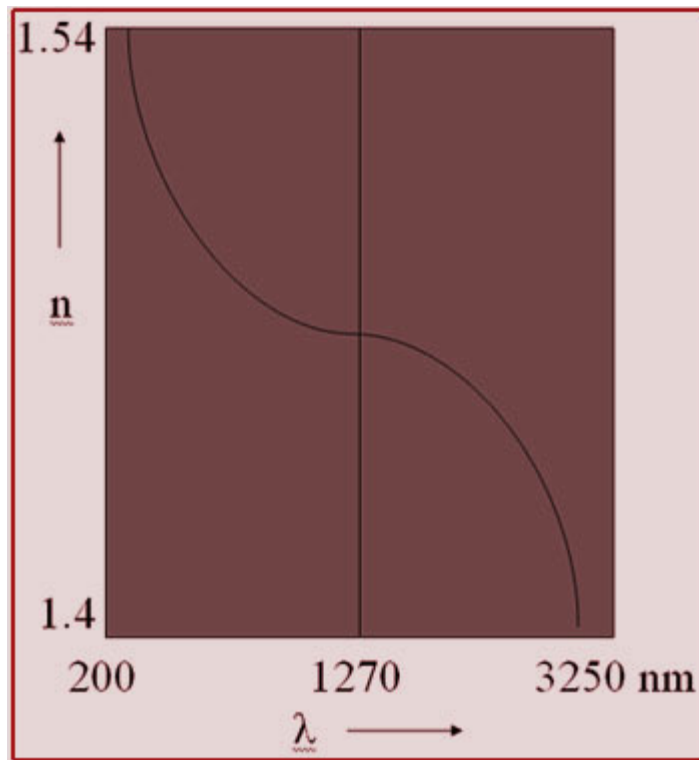
If the refractive index varies linearly with λ , the curvature is zero and therefore the dispersion is zero.

- At this point it is worthwhile to look at the variation of refractive index of glass as a function of frequency.
- The refractive of glass is generally described by Sellmeier relation.

$$n(\lambda) = n_1 + \sum_i \frac{S_i \lambda^i}{\lambda_i}$$

Where S_i and λ_i are material dependent constants. For silica glass their values are given in Table.

Following Fig. shows the refractive index variation for glass.



- Note from the Fig. the following things:

- (1) The refractive index for glass does not vary linearly.
- (2) The curvature of the function is positive below 1270nm and it is negative above 1270nm.
- (3) The curvature of the function is zero at 1270nm .

The material dispersion is zero at 1270nm .

Waveguide Dispersion

The waveguide dispersion is due to the bound structure of the optical fiber. While

- analyzing waveguide dispersion we assume that the material dispersion is absent, that is the refractive indices of the core and the cladding are independent of wavelength.
- For analyzing waveguide dispersion we need $\beta-\omega$ relation. As we have seen in lecture 3 and 4, that the whole exercise of analyzing modal propagation was to obtain the phase constant β for a particular mode at a particular frequency. There is no analytical expression which can describe $\beta-\omega$ relation. This relation is rather expressed in the form a numerical plot.

- Generally we define two normalized parameters, **V-number** and b which can substitute ω and β respectively. These parameters are characteristic parameters of a fiber.

optical fiber

$$\text{V-number of an optical fiber} \quad V = \frac{\omega a}{c} \sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} (NA)$$

- The V-number is a comprehensive parameter of an optical fiber as it takes into account the size of the optical fiber, a , Numerical aperture (NA), and the wavelength of operation.
- For a given fiber, a and NA are constant and the V-number is proportional to frequency. The V-number therefore is also referred to as the **Normalized frequency**.
- As we have earlier, the propagation constant β has bound,

$$\beta_2 < \beta < \beta_1$$

- Where $\beta_1 = \frac{\omega n_1}{c}$ is the propagation constant in medium having refractive index n_1 , and $\beta_2 = \frac{\omega n_2}{c}$ is the propagation constant in medium having refractive index n_2 .

- The bound on β is independent of the frequency of operation, and the mode. One can therefore define a quantity called the **Normalized Propagation Constant**, b as

Normalized Propagation constant

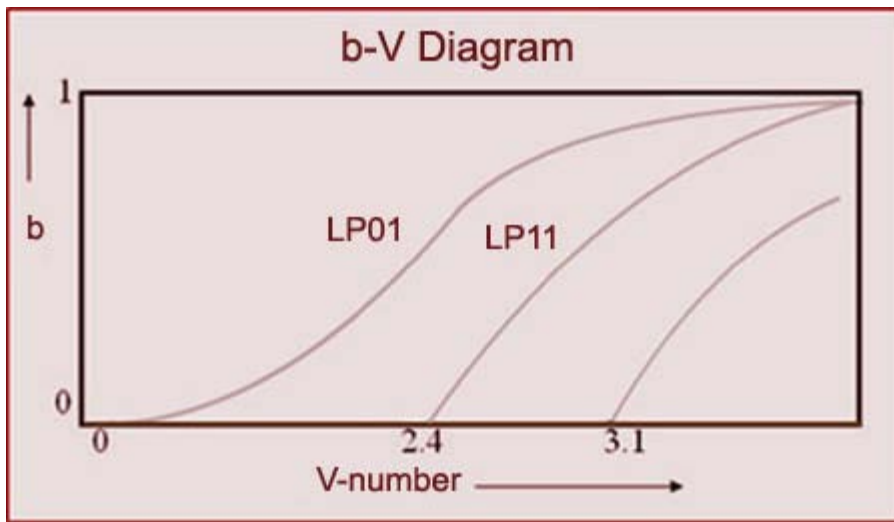
$$b = \frac{n_{\text{eff}}^2 - n_2^2}{n_1^2 - n_2^2} \approx \frac{n_{\text{eff}} - n_2}{n_1 - n_2}$$

Effective index of propagation

$$n_{\text{eff}} = \frac{\beta}{(2\pi / \lambda)}$$

$0 < b < 1$

- The $\beta - \omega$ relation can be equivalently described by b-V relation. The b-V diagram of a step index optical fiber for different modes is shown in the following.



As can be seen clearly, the b-V diagram is not linear and consequently, there is dispersion.

- The waveguide dispersion due to non-linear b-V diagram is obtained as

$$D_{wg} = -\frac{n_2 \Delta}{c \lambda} V \frac{d^2(bV)}{dV^2}$$

- From the b-V diagram we can calculate $V \frac{d^2(bV)}{dV^2}$, and subsequently the waveguide dispersion.
- The $V \frac{d^2(bV)}{dV^2}$ peaks around $V=1.2$ for the lowest order LP_{01} mode. Consequently, in the single mode optical fiber the dispersion is maximum around $V = 1.2$.
- For higher V-number the dispersion is low. However, as pointed out earlier, for the fiber to be single mode, $V < 2.4$.
- **The single mode optical fibers therefore have V-number very close to but less than 2.4 so that they have lowest possible waveguide dispersion.**
- The total chromatic dispersion on a single mode fiber is approximately

$$D = D_{mat} + D_{wg}$$

The figure shows the plot of material, waveguide and total dispersion.

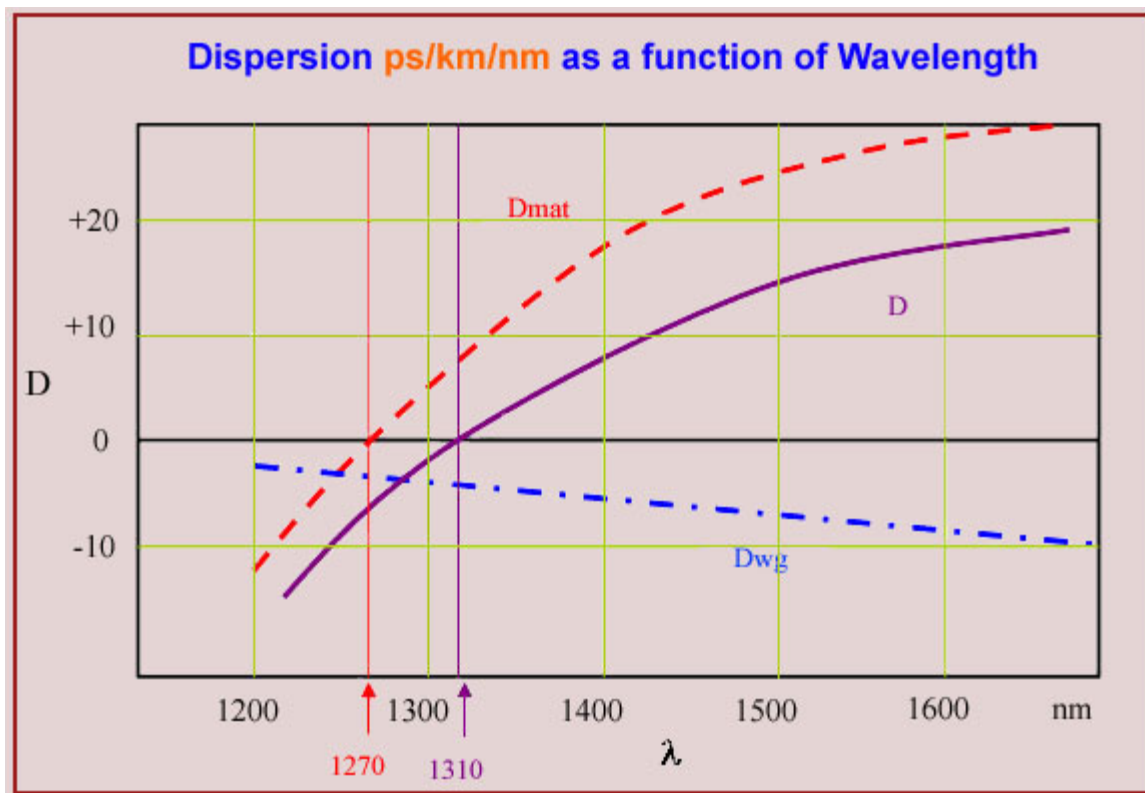
- **Note:** The material dispersion is zero at 1270nm. However, in a single mode fiber, we always get dispersion which is a combination of material and waveguide dispersion. Therefore

Total dispersion in a single mode fiber is zero at 1310nm.

- The dispersion is positive for wavelengths above 1310nm and negative below 1310nm.
- Note : The negative dispersion does not mean pulse compression. Due to dispersion the pulse always broadens and the broadening is

$$\tau = |D| \sigma_\lambda L$$

Where L is the length of the fiber and σ_λ is the spectral width of the source.



Inter-Modal Dispersion

- The inter-modal dispersion is due to difference in velocity of different modes inside a fiber.
- Obviously, the inter-modal dispersion takes place in a multi-mode fiber.

It has been shown earlier that the optical rays launched at different angles give different modal fields. The inter-modal dispersion can therefore be approximately calculated using the ray model.

- The inter-modal dispersion is approximately given as

$$D_{\text{inter}} \approx \frac{n_1 (NA)}{n_2 c}$$

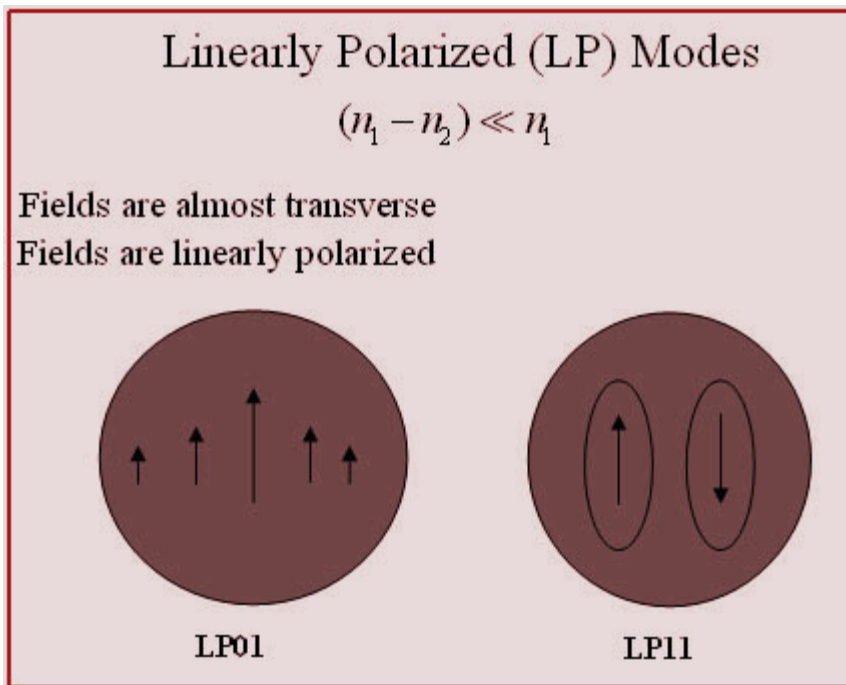
The unit of inter modal-dispersion is ps/Km.

- The magnitude of inter-modal dispersion is much larger compared to the chromatic dispersion. Therefore the chromatic dispersion is neglected while calculating the dispersion in a multi-mode fiber.

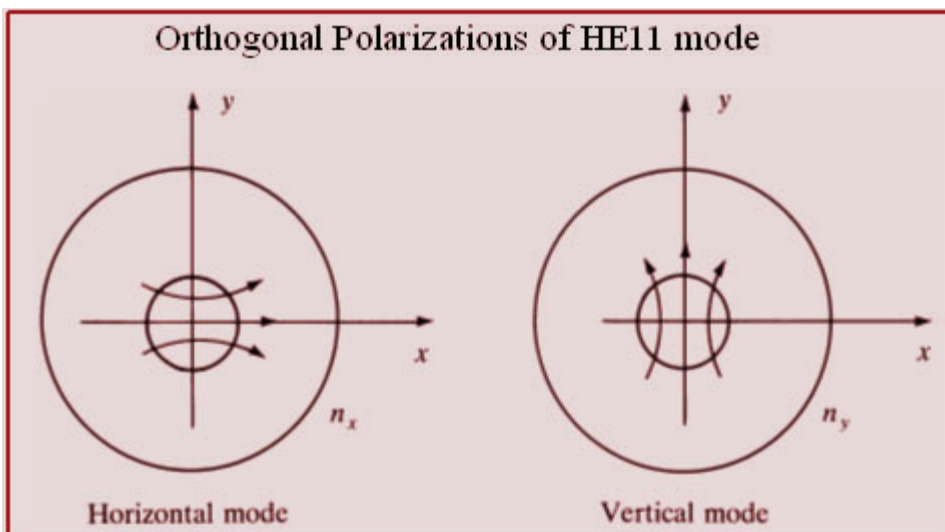
Polarization Mode Dispersion (PMD)

As the name suggests, the polarization mode dispersion is due unequal velocities of two

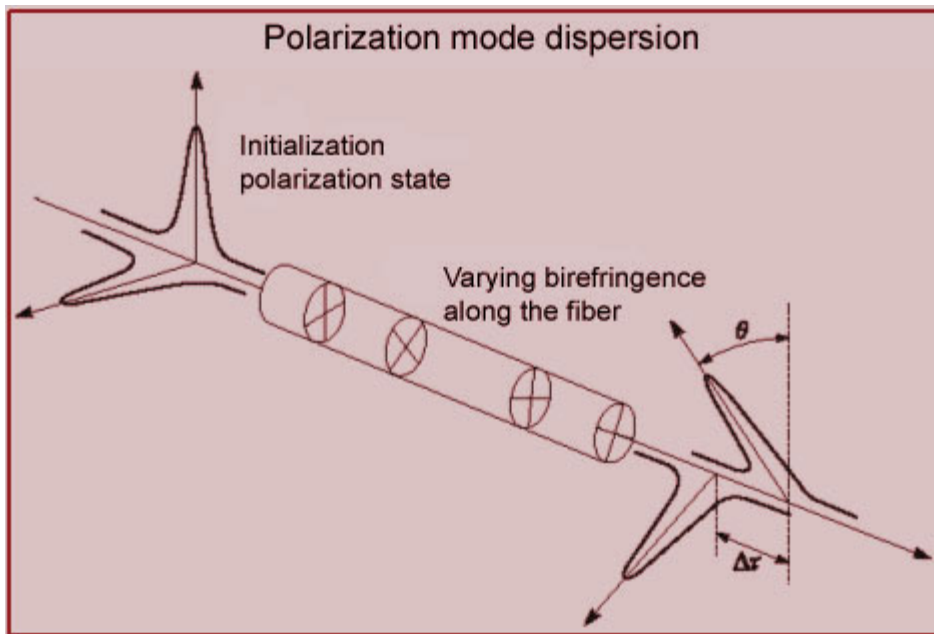
- orthogonal states of polarization.
- The PMD puts the ultimate restriction on the data rate on the long haul single mode optical fiber.
- In the weakly guiding approximation i.e. when $(n_1 - n_2) \ll n_1$, The lowest order HE_{11} mode becomes a linearly polarized, LP_{01} mode. That is over the cross-section of the fiber core the field has same direction. See Fig.



- A linearly polarized field can be resolved into two orthogonally polarized fields. The pulse energy gets divided into two polarization states as shown in Fig.



- Due to non-uniformity of the core radius, the effective modal index is different for the two polarizations. The difference in modal indices for two polarizations is called the **birefringence** of the fiber.
- The polarizations consequently travel with different velocities, splitting an optical pulse into two. This phenomenon is called **Polarization Mode Dispersion**.



- No one polarization systematically sees the same modal index. The optical pulse therefore has statistical fluctuation of the polarization.
- The pulse slowly broadens due to the statistical fluctuation of the velocities of the two orthogonal polarizations.
- The pulse broadening due to PMD is given as

$$\tau = D_{PMD} \sqrt{L}$$

Where D_{PMD} is the polarization mode dispersion of the SM fiber, and L is the length of the fiber. The unit of PMD is ps/\sqrt{km} .

Recap

In this lecture you have learnt the following

- **Material Dispersion**
- **Waveguide Dispersion**
- b-V Diagram
- Inter-Modal Dispersion
- Polarization Mode Dispersion (PMD)

Congratulations, you have finished Module 5. To view the next lecture select it from the left hand side menu of the page