UNIT – III

Objective:

Syllabus: Sampling: Sampling theorem for band limited signals- Explanation, reconstruction of signal from samples, Aliasing, Sampling Techniques- Impulse, Natural and Flat top sampling.

Outcomes:
Students will be able to

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- Understand the concept of sampling.
- Know different types of sampling techniques.

Learning Material

Sampling Theorem:

Statement: A band limited signal f(t) with F(w)=0 for $|w| \geq w_m$ can be represented into and uniquely determined from its samples f(nT) if the sampling frequency $f_s \geq 2 f_m$, where f_m is the highest frequency component present in it. That is for signal recovery, the sampling frequency must be at least twice the highest frequency present in the signal.

- Let f(t) is a continuous time band limited signal to be sampled which has no spectral components above f_m.
- Let $\delta_T(t)$ is an impulse train which samples at a rate of fs Hz
- Then the sampled signal

$$\bar{f}(t) = f(t)\delta_T(t) = \sum_n f(nT)\delta(t - nT)$$

- The Fourier transform of sampled signal is

$$\bar{F}(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(\omega - n\omega_s)$$

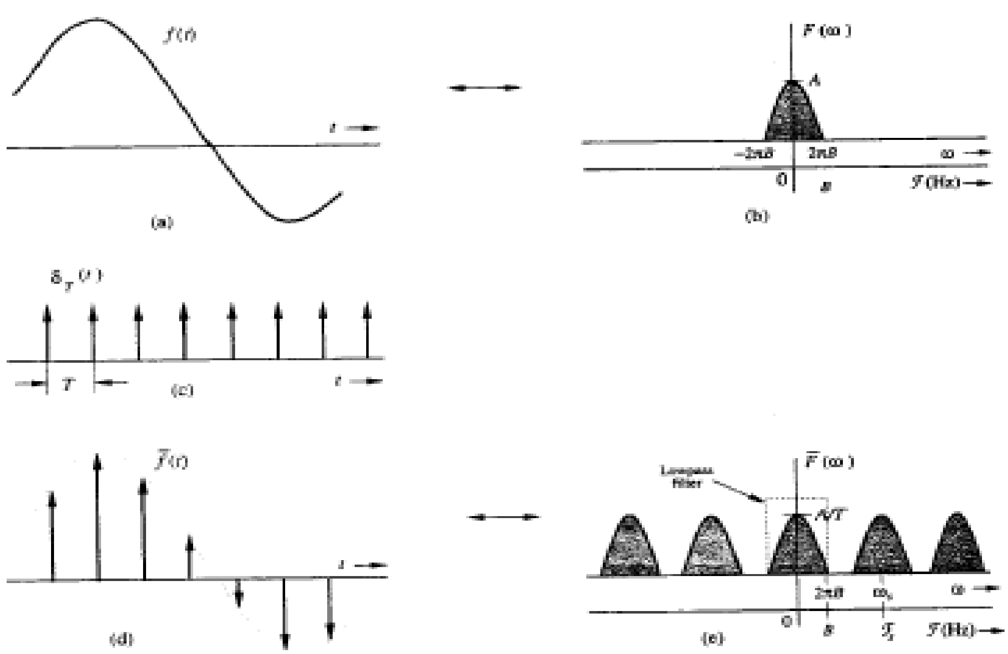


Fig. sampled signal and its Fourier spectrum.

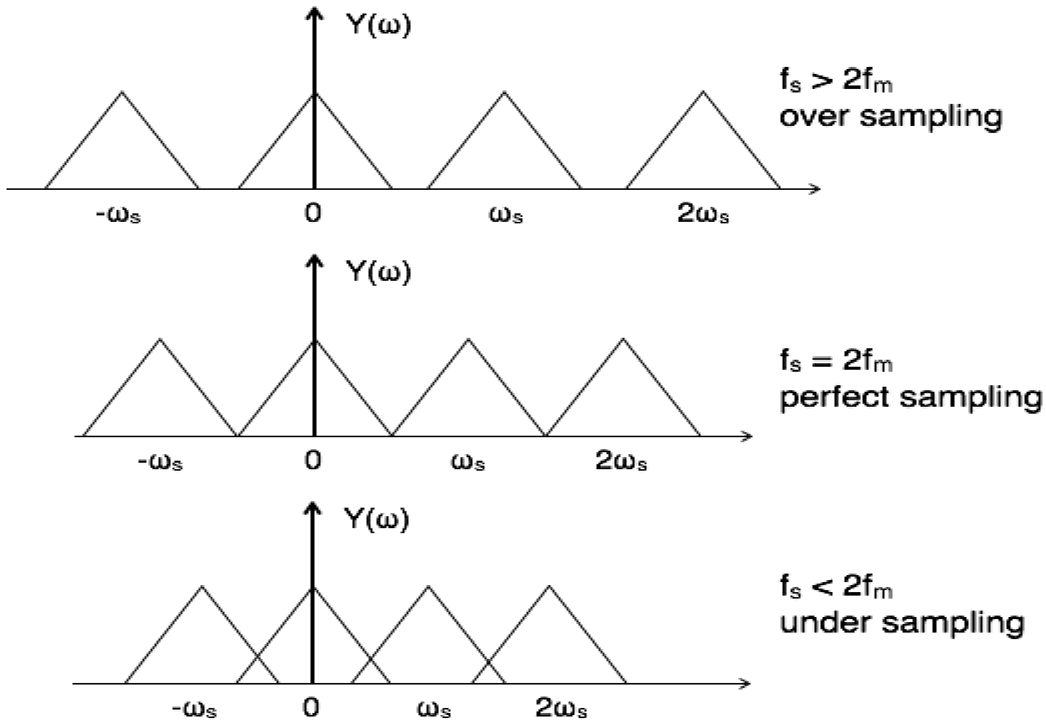


Fig. Effect of under sampling and over sampling.

- Nyquist rate of sampling is the theoretical minimum sampling rate at which a signal can be sampled and still be reconstructed from its samples without any distortion.
- A signal sampled at greater than Nyquist rate is said to be over sampled and signal sampled is less than its Nyquist rate is said to be under sampled.

<u>Reconstruction of signal from it's samples:</u>

- The original function can be recovered by passing the sampled function through a low pass filter with a cutoff frequency f_m
- This can be obtained by multiplying the sampled signal by a gate function

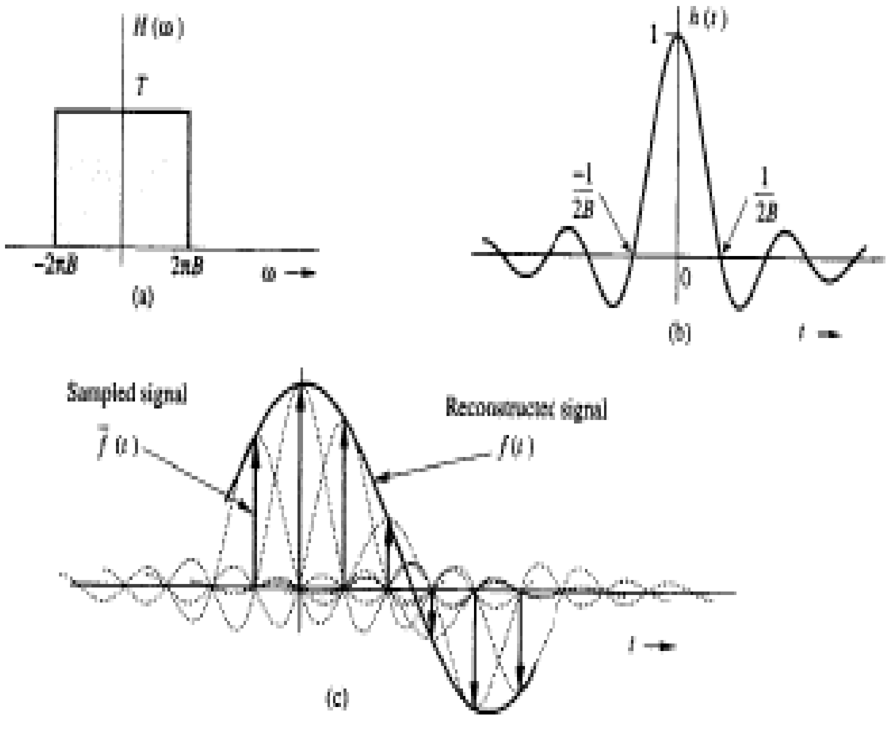
$$h(t) = \text{rect}\left(\frac{t}{T}\right) = \text{rect}(2Bt)$$

and

$$H(\omega) = T\,\text{sinc}\left(\frac{\omega T}{2}\right) = \frac{1}{2B}\,\text{sinc}\left(\frac{\omega}{4B}\right)$$

The resultant output of the filter is

$$
\begin{aligned}
f(t) &= \sum_k f(kT)h(t-kT) \\
&= \sum_k f(kT)\ \mathrm{sinc}\,[2\pi B(t-kT)] \\
&= \sum_k f(kT)\ \mathrm{sinc}\,(2\pi Bt - k\pi)
\end{aligned}
$$

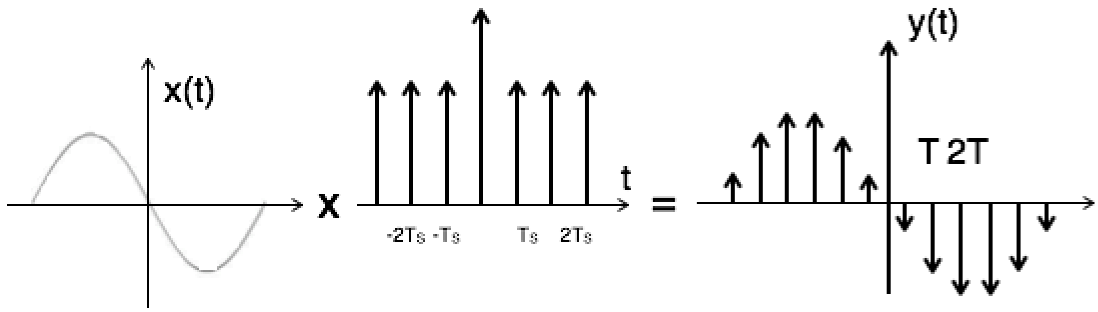


<u>**Sampling techniques:**</u>

There are three types of sampling techniques

<u>Impulse sampling:</u>

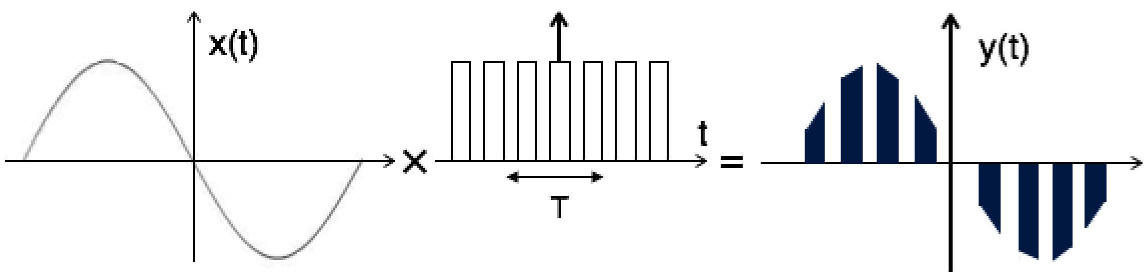
- Impulse sampling can be performed by multiplying input signal x(t) with impulse train $\sum_{n=-\infty}^{\infty}\delta(t-nT)$ of period 'T'. Here, the amplitude of impulse changes with respect to amplitude of input signal x(t). The output of sampler is given by



$$y(t) = x(t) \times \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

Natural Sampling:

- Natural sampling is similar to impulse sampling, except the impulse train is replaced by pulse train of period T. i.e. you multiply input signal x(t) to pulse train $\sum_{n=-\infty}^{\infty} p(t-nT)$ as shown below



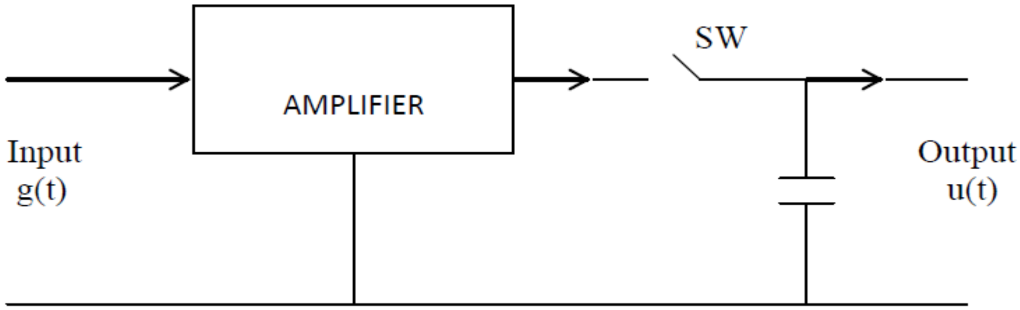
$$y(t) = x(t) \times \sum_{n=-\infty}^{\infty} p(t-nT)$$

Flat top sampling:

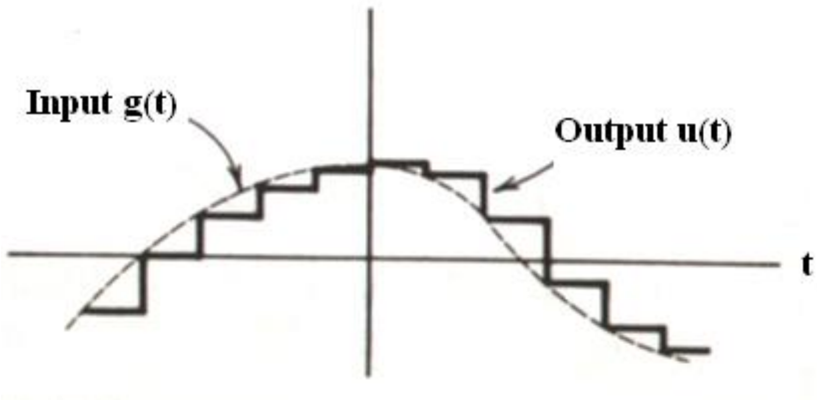
- The Sample-and-Hold circuit consists of an amplifier of unity gain and low output impedance, a switch and a capacitor; it is assumed that the load impedance is large.

- The switch is timed to close only for the small duration of each sampling pulse, during which time the capacitor charges up to a voltage level equal to that of the input sample.

- When the switch is open , the capacitor retains the voltage level until the next closure of the switch. Thus the sample-and-hold circuit produces an output waveform that represents a staircase interpolation of the original analog signal.



a) Sample and Hold Circuit



b) Sampled signal

Assignment-Cum-Tutorial Questions

A. Questions testing the remembering / understanding level of students

I) Objective Questions

1. When does aliasing occur? How can it be avoided? Give the condition to avoid aliasing

2. What is Nyquist rate?

3. What is guard band?

4. According to sampling theorem, the signal should be sampled at

   a) $f_s \geq 2f_m$ b) $f_s < 2f_m$ c) $f_s = \frac{f_m}{2}$ d) None of the above

5. What is Over sampling?

6. What are the conditions for existence of Fourier Transform?

7. What is zero order hold?

8. According to sampling theorem, aliasing problem occures at

   a) $f_s \geq 2f_m$ b) $f_s < 2f_m$ c) $f_s = \frac{f_m}{2}$ d) None of the above

II) Descriptive Questions

1. State and prove the sampling theorem.

2. Explain various sampling techniques.

3. Explain the signal recovery from its sampled signals.

B. Question testing the ability of students in applying the concepts.

I) Multiple Choice Questions:

1. The Nyquist rate for the signal $x(t) = 5\sin 200\pi t + 8\cos 500\pi t$

   (a) 200Hz (b)500Hz (c) 700Hz (d) 300Hz