**UNIT VI**

# **THE LAPLACE TRANSFORM**

**Learning Objectives:**

* To introduce the concept of various Laplace transform
* To introduce the concept of convergence of Laplace transform.
* To introduce the concept of representation of a Laplace transform in s domain

**Learning Outcomes:**

Students will be able to

* Apply properties of Laplace transform
* Solving the integro differential equations using Laplace transform
* Obtain the Laplace transform of standard signals

**Syllabus:**

Laplace transform of signals, Convergences of Laplace transform, Properties of region of convergence (ROC),Unilateral Laplace transform, Properties Unilateral Laplace transform, Initial value theorem, Final value theorem, Inversion of Unilateral and Bilateral Laplace transform, Relation between Laplace and Fourier Transforms

**Advantages of Laplace transform:**

1Signals which are not convergent in Fourier transform are convergent in Laplace transform.

2convolutions in time domain can be obtained by simple multiplication in S domain

3 Integro differential equations are converted into simple algebraic equations

**Laplace transform of signals:**

The Laplace transform of a [function](https://en.wikipedia.org/wiki/Function_(mathematics))*f* (*t*), defined for all [real numbers](https://en.wikipedia.org/wiki/Real_number) *t* ≥ 0, is the function *F*(*s*), which is a unilateral transform defined by



### Bilateral Laplace Transform

The *Bilateral Laplace Transform* is defined as follows:



**Inverse Laplace Transform:**

The inverse Laplace transform is defined as

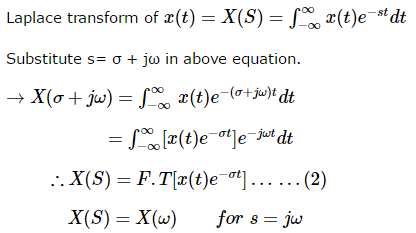


## Conditions for Existence of Laplace Transform:

Dirichlet's conditions are used to define the existence of Laplace transform. i.e.

* The function f(t) has finite number of maxima and minima.
* There must be finite number of discontinuities in the signal f(t),in the given interval of time.
* It must be absolutely integrable in the given interval of time. i.e.|f(t)|dt<∞

**Relationship between fourier transorm and laplace transform:**

****

**i.e., Fouriertransform is a special case of Laplace transform when Re[s] or  =0**

**Region of convergence:**

The range variation of σ for which the Laplace transform converges is called region of convergence.

## Properties of ROC of Laplace Transform:

* ROC contains strip lines parallel to jω axis in s-plane. 
* If x(t) is absolutely integral and it is of finite duration, then ROC is entire s-plane.
* If x(t) is a right sided sequence then ROC : Re{s} >σo.
* If x(t) is a left sided sequence then ROC : Re{s} <σo.
* If x(t) is a two sided sequence then ROC is the combination of two regions.

**PROPERTIES OF LAPLACETRANSFORM:**

**INVERSE LAPLACE TRANSFORM**:

The transfer function may of the form



Consider the proper partial fractions for the above expression \* Summary of Partial–Fraction Expansion

1. Expansion Structure:

Simple Roots (including complex conjugate)

=> could be complex.

Repeated Roots: m multiplicity

=>

1. Avoid complex number

For complex conjugates: 



**LAPLACE TRANSFORM OF STANDARD FUNCTIONS:**

|  |  |  |
| --- | --- | --- |
| **Entry** | **Laplace Domain** | **Time Domain**([note](javascript:void(0);)) |
| unit impulse | http://lpsa.swarthmore.edu/LaplaceZTable/images/img38.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table1.gif   unit impulse |
| unit step | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table13.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/img43.gif  ([note](javascript:void(0);)) |
| ramp | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table15.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table14.gif |
| parabola | http://lpsa.swarthmore.edu/LaplaceZTable/images/img3.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/img5.gif |
| tn (n is integer) | http://lpsa.swarthmore.edu/LaplaceZTable/images/img7.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/img9.gif |
| exponential | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table17.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table16.gif |
| time  multiplied exponential | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table19.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table18.gif |
| Asymptotic exponential | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table21.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table20.gif |
| double exponential | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table23.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table22.gif |
| asymptotic double exponential | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table2.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table3.gif |
| asymptotic critically damped | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table24.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table25.gif |
| sine | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table29.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table28.gif |
| cosine | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table31.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table30.gif |
| decaying sine | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table33.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table32.gif |
| decaying cosine | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table35.gif | http://lpsa.swarthmore.edu/LaplaceZTable/images/Table34.gif |

**Part- A**

1. **Objective Questions**
2. LT is the operator that transforms the signal in time domain into a signal
3. In a complex frequency domain called as S domain
4. In a real frequency domain called as S domain
5. In an imaginary frequency domain called as S domain
6. In a frequency domain called as S domain
7. LT is defined as
8. 𝑋 (𝑠) = b) 𝑋 (𝑠) =
9. 𝑋 (𝑠) = d) 𝑋 (𝑠) =
10. Inverse LT requires contour integration defined by
11. X(t) = ds
12. X(t) = ds
13. X(t) = ds
14. X(t) = ds
15. Laplace transform converts convolution of time –signals to
16. Addition
17. Subtraction
18. Multiplication
19. Division
20. LT can be converted to FT
21. By substituting s = jω
22. By substituting s = σ
23. By substituting s = σ +jω
24. By substituting s = -jω
25. The region of convergence (ROC) is defined as
26. the range of values of jω for which LT converges
27. the range of values of σ for which LT converges
28. the range of values of σ + jω for which LT converges
29. the range of values of s for which LT converges
30. Laplace transformation is a
31. Linear operation
32. Non-linear operation
33. Partially Linear operation
34. Partially Non-linear operation
35. Differentiation in s domain results in multiplication of the signal by
36. t b) -t c) –t/2 d) t/2
37. Time scaling of a signal by a factor of “ a” introduces
38. Scaling by a factor of a in s domain
39. Scaling by a factor of a2 in s domain
40. Scaling by a factor of 1/a in s domain
41. Scaling by a factor of 1/a2 in s domain
42. Region of convergence of a causal LTI system
43. Is the entire s-plane
44. Is the right –half of s-plane
45. right –half of s-plane
46. does not exist
47. **Descriptive Questions**
48. Define Laplace transform and write its advantages and applications.
49. Derive the relation between Laplace transform and Fourier transform.
50. State and prove any four properties of Laplace transform.
51. Define region of convergence(ROC) and write the properties of ROC of Laplace transform.
52. State and prove the initial and final value theorems.

**PART B**

1. **Multiple Choice Questions**
2. Laplace transform of a delta function is equal to
3. One b) three c) two d) zero
4. Laplace transform of the unit step function is equal to
5. b) c) d) s
6. Laplace transform of a signal x(t)= t u(t) is equal to

b) c) d) s

1. Laplace transform of =
2. b) c) d)
3. Laplace transform of is
4. b) c) d)
5. Laplace transform of ramp function is

a) b) c) d) none of the above

7. Laplace transform of tn is

a) nיִ /sn b) n /sn c) n/sn+1  d)n יִ/sn+1

8. If the Laplace transform of x(t) is , then the value of is

a) Cannot be determined b) zero c) unity d) infinity

9. The Laplace transform of δ(t)\*u(t) is

a)  b)

c) d)

10. The Laplace transform of x (t)= e -4tu(t) is

a) b)

c) d)

**II) Problems**

1. Find the laplace transform and ROC of x(t)= u(t)+u(t).

Ans:

1. Prove that the signals (a) x (t) = e –at u (t) and (b) x (t) = -e –at u (-t) have the same X(s) and differ only in ROC. Also plot their ROCs.

Ans: a) = , ROC: σ > -a b)= , ROC: σ < -a

1. Find the inverse Laplace transforms for the following signals.
2. b)

Ans: a) t e –t u (t) b) e –t sin t u(t)

1. Find the inverse laplace transform of X(s) = ; ROC : -4< Re(s) < -1

Ans: - e –t u (-t) - e –4t u (t)

1. Find the initial and final values of the following.

X(s) =

Ans: 1, 0

1. A signal has Laplace transform X(s) = ,Find Laplace transform Y(s) of the following signals: a) Y1(t) = x (2t) b) Y2(t) = t x (t)

Ans: Y1(s) = b) Y2(s) =