**ST.MARY’S GROUP OF INSTITUTIONS GUNTUR**

**DEPARTMENT OF ECE**

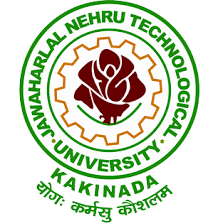
**NAME OF THE FACULTY:** CH.NAGA PHANEENDRA

**SUBJECT:** RVSP

**YEAR & SEM:** II - I

**STUDY MATERIAL**

**A.C.Y:2019-20**



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| **JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY KAKINADA** |
| **KAKINADA-533003, ANDHRAPRADESH** |
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#### II Year - I Semester L T P C

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**RANDOM VARIABLES & STOCHASTIC PROCESSES**

**OBJECTIVES:**

* To give students an introduction to elementary probability theory, in preparation for courses on statistical analysis, random variables and stochastic processes.
* To mathematically model the random phenomena with the help of probability theory concepts.
* To introduce the important concepts of random variables and stochastic processes.
* To analyze the LTI systems with stationary random process as input.
* To introduce the types of noise and modeling noise sources.

#### UNIT I

**THE RANDOM VARIABLE :** Introduction, Review of Probability Theory, Definition of a Random Variable, Conditions for a Function to be a Random Variable, Discrete, Continuous and Mixed Random Variables, Distribution and Density functions, Properties, Binomial, Poisson, Uniform, Gaussian, Exponential, Rayleigh, Conditional Distribution, Conditional Density, Properties.

#### UNIT II

**OPERATION ON ONE RANDOM VARIABLE – EXPECTATIONS** : Introduction, Expected Value of a Random Variable, Function of a Random Variable, Moments about the Origin, Central Moments, Variance and Skew, Chebychev’s Inequality, Characteristic Function, Moment Generating Function, Transformations of a Random Variable: Monotonic Transformations for a Continuous Random Variable, Nonmonotonic Transformations of Continuous Random Variable.

#### UNIT III

**MULTIPLE RANDOM VARIABLES** : Vector Random Variables, Joint Distribution Function, Properties of Joint Distribution, Marginal Distribution Functions, Conditional Distribution and Density, Statistical Independence, Sum of Two Random Variables, Sum of Several Random Variables, Central Limit Theorem: Unequal Distribution, Equal Distributions.

**OPERATIONS ON MULTIPLE RANDOM VARIABLES:** Joint Moments about the Origin, Joint Central Moments, Joint Characteristic Functions, Jointly Gaussian Random Variables: Two Random Variables case, N Random Variables case, Properties, Transformations of Multiple Random Variables, Linear Transformations of Gaussian Random Variables.

#### UNIT IV

**RANDOM PROCESSES – TEMPORAL CHARACTERISTICS:** The Random Process Concept, Classification of Processes, Deterministic and Nondeterministic Processes, Distribution and Density Functions, Concept of Stationarity and Statistical Independence. First-Order Stationary Processes, Second-order and Wide-Sense Stationarity, Nth-order and Strict-Sense Stationarity, Time Averages and Ergodicity, Autocorrelation Function and its Properties, Cross-Correlation Function and its Properties, Covariance Functions, Gaussian Random Processes, Poisson RandomProcess.

#### UNIT V

**RANDOM PROCESSES – SPECTRAL CHARACTERISTICS:** The Power Density Spectrum: Properties, Relationship between Power Density Spectrum and Autocorrelation Function, The Cross-Power Density Spectrum, Properties, Relationship between Cross-Power Density Spectrum and Cross-Correlation Function.

#### UNIT VI

**LINEAR SYSTEMS WITH RANDOM INPUTS :** Random Signal Response of Linear Systems: System Response – Convolution, Mean and Mean-squared Value of System Response, Autocorrelation Function of Response, Cross-Correlation Functions of Input and Output, Spectral Characteristics of System Response: Power Density Spectrum of Response, Cross-Power Density Spectra of Input and Output, Band pass, Band-Limited and Narrowband Processes, Properties, Modeling of Noise Sources: Resistive (Thermal) Noise Source, Arbitrary Noise Sources, Effective Noise Temperature, Average Noise Figure, Average Noise Figure of cascadednetworks.

#### TEXT BOOKS:

1. Probability, Random Variables & Random Signal Principles, Peyton Z. Peebles, TMH, 4th Edition,2001.
2. Probability, Random Variables and Stochastic Processes, Athanasios Papoulis and S.Unnikrisha, PHI, 4th Edition, 2002.

#### REFERENCE BOOKS:

1. Probability Theory and Stochastic Processes – B. Prabhakara Rao, BSPublications
2. Probability and Random Processes with Applications to Signal Processing, Henry Stark and John W. Woods, Pearson Education, 3rd Edition.
3. Schaum's Outline of Probability, Random Variables, and RandomProcesses.
4. An Introduction to Random Signals and Communication Theory, B.P. Lathi, International Textbook,1968.
5. Random Process – Ludeman , JohnWiley
6. Probability Theory and Random Processes, P. Ramesh Babu, McGrawHill,2015.

**UNIT-I**

**INTRODUCTION TO PROBABILITY THEORY & RANDOM VARIABLE**

**Objective:**

* To gain the knowledge of the basic probability concepts and standard distributions which can describe real life phenomena

**Syllabus:**

Definitions, scope and history; limitation of classical and relative-frequency-based definitions, Sets, fields, sample space and events; axiomatic definition of probability, Probability on finite sample spaces, Joint and conditional probabilities, independence, total probability; Bayes’ rule and applications.

**Learning Outcomes:**

At the end of the unit student will be able to:

1. Define sample space, events, the terms related to probability theory and set theory
2. Identify the limitations of classical and relative-frequency probabilities
3. Explain probability theory in axiomatic approach
4. Determine joint and conditional probabilities
5. Apply joint and conditional probabilities for the representation of total probability
6. Distinguish dependent and independent events
7. Demonstrate Baye’s theorem

**Probability theory** is the branch of [mathematics](https://en.wikipedia.org/wiki/Mathematics) concerned with [probability](https://en.wikipedia.org/wiki/Probability), the analysis of [random](https://en.wikipedia.org/wiki/Statistical_randomness) phenomena.

An **experiment** is a situation involving chance or probability that leads to results called outcomes.

An **outcome** is the result of a single trial of an experiment.

An **event** is one or more outcomes of an experiment. It is a subset of sample space.

**Probability** is the measure of how likely an event is.

**Mutually exclusive:** The random experiment results in the occurrence of only one of the n outcomes. E.g. if a coin is tossed, the result is a head or a tail, but not both. That is, the outcomes are defined so as to be mutually exclusive.

**Equally likely:** Each outcome of the random experiment has an equal chance of occurring.

**Random experiment**: A random experiment is a process leading to at least two possible outcomes with uncertainty as to which will occur.

**Sample space:** The collection of all possible outcomes of an experiment.

***Several concepts of probability have evolved over the time: They are***

The classical approach

The relative frequency approach

The axiomatic approach

***The classical approach***

The classical approach of probability applies to equally probable events, such as the outcomes of tossing a coin or throwing dice; such events were known as "equipossible".

Probability = number of favorable equipossible / total number of relevant equipossible.

, where ‘N’ is the number of all possible outcomes.

**Example:** If one tosses a coin there are two mutually exclusive outcomes: head or tail. Of these two outcomes, one is associated with the attribute heads; one is associated with the attribute tails. If the coin is fair each outcome is equally likely. In which case, Pr[head]= nA/n =1/2 , where n=2 and nA is the number of possible outcomes associated with a head (1).

***Disadvantages:***

* A Basic assumption in the definition of classical probability is that n is a finite number; that is, there are only a finite number of possible outcomes. If there are an infinite number of possible outcomes, the probability of an outcome is not defined in the classical sense.

**Examples:**

The roll of a die: There are 6 equally likely outcomes. The probability of each is 1/6.

Draw a card from a deck: There are 52 equally likely outcomes.

The roll of two die: There are 36 equally likely outcomes (6x6): 6 possibilities for the first die, and 6 for the second. The probability of each outcome is 1/36.

***Note:*** An important thing to note is that classical probabilities can be deduced from knowledge of the sample space and the assumptions. Nothing has to be observed in terms of outcomes to deduce the probabilities.

**The Relative Frequency Approach**

What if n is not finite?

What if the outcomes are not equally likely?

In both the above cases classical definition of probability is not applicable.

In such cases, how a probability might be defined for an outcome that has event (attribute) A.

**Definition:** One might take a random sample from the population of interest and identify the proportion of the sample with event( attribute) A. That is, calculate

No. of observations in the sample that posses event

Relative frequency of A in the sample = -----------------------------------------------------------------

No. of observations in the sample

Then assume ”Relative freq of A in the sample” is an estimate of Pr[A].

***Example:*** For example, one tosses a coin, which might or might not be fair, 100 times and observes heads on 52 of the tosses. One’s estimate of the probability of a head is 0.52. Frequency probability allows estimating probabilities when Classical probability provides no insight.

Experimental approach, the same experiment has to be repeated *n* times. If the *A* occurs *n*(*A*) times, then the relative frequency of the event *A* is: 

And the probability of the event *A* is 

*Limitation: Experiment has to be performed infinite number of times which may not be a feasible option.*

**Axiomatic Approach**

The axiomatic approach builds up probability theory from a number of assumptions (axioms).

To define Axioms, there should be some sample space, i.e., the collection of all possible outcomes of an experiment.

The axiomatic approach introduces a *probability space* as its main component

Example: if the experiment is tossing a coin Ι = {H, T}.

***Event:*** A subset of sample space.

For defining the value of P(A), there are nevertheless certain axioms which should always hold for internal consistency.

*Axioms of probability theory*

1. P(A) should be a number between 0 and 1 Or P(A)≥0

2. If A represents a certain event then P(A)=1 Or P(S)=1

3. If A1 and A2 are mutually exclusive events then P(A1 or A2) = P(A1)+P(A2) Or P() =

***Note:*** if the number of outcomes is finite and equally likely then one has the Classical world of probability. Also note that the Frequency definition assumes the existence of the probability function Pr[A]. The axiomatic approach subsumes the Classical and Frequency approaches.

**JOINT PROBABILITY:**

A joint probability is a statistical measure where the likelihood of two events occurring together and at the same point in time is calculated. Joint probability is the probability of event Y occurring at the same time event X occurs.

P(X∩Y) = P(X) + P(Y) – P(XÚY)

***Example:*** A joint probability cannot be calculated when tossing a coin on the same [flip](http://www.investopedia.com/terms/f/flip.asp). However, the joint probability can be calculated on the probability of rolling a 2 and a 5 using two different dice.

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| **Addition Rule 1:** | When two events, A and B, are mutually exclusive, the probability that A or B will occur is the sum of the probability of each event. |
|  | P(A ∪ B) = P(A) + P(B)  *Example:* A single 6-sided die is rolled. What is the probability of rolling a 2 or a 5? |

**Addition Rule 2:** When two events, A and B, are not mutually exclusive, the probability that A or B will occur is P(A ∪ B) = P(A) + P(B) – P(A ∩ B).

*Example:* In a math class of 30 students, 17 are boys and 13 are girls. In a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?

**INDEPENDENT EVENTS:**

Two events, A and B, are **independent** if the fact that A occurs does not affect the probability of B occurring.

*Example:* A dresser drawer contains one pair of socks with each of the following colors: blue, brown, red, white and black. Each pair is folded together in a matching set. You reach into the sock drawer and choose a pair of socks without looking. You replace this pair and then choose another pair of socks. What is the probability that you will choose the red pair of socks both times?

There are a couple of things to note about this experiment. Choosing a pairs of socks from the drawer, replacing it, and then choosing a pair again from the same drawer is a [compound event](javascript:x1096653463('compound_event')). Since the first pair was replaced, choosing a red pair on the first try has no effect on the probability of choosing a red pair on the second try. Therefore, these events are independent.

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| **Multiplication Rule 1:** | When two events, A and B, are independent, the probability of both occurring is: |
|  | P(A ∩ B) = P(A) **·** P(B) |

**DEPENDENT EVENTS:**

Two events are **dependent** if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed.

*Example:* A card is chosen at random from a standard deck of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

The outcome of choosing the first card has affected the outcome of choosing the second card, making these events dependent.

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| **Multiplication Rule 2:** | When two events, A and B, are dependent, the probability of both occurring is: |
|  | P(A ∩ B)  =  P(A) **·** P(B|A) |

*Example:* A teacher needs two students to help him with a science demonstration for his class of 18 girls and 12 boys. He randomly chooses one student who comes to the front of the room. He then chooses a second student from those still seated. What is the probability that both students chosen are girls?

**CONDITIONAL PROBABILITY:**

*Problem:* A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

*Analysis:* This problem describes a conditional probability since it asks us to find the probability that the second test was passed given that the first test was passed.

Let A and B are two events which are dependent, i.e., occurrence of B depends on occurrence of A, assuming A has already occurred, then the probability of both occurring is

P (B/A) = P(A∩B)/P(A) given P(A) ≠ 0

**Problem:** The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

P(Absent|Friday)  =  P(Friday and Absent)  =  0.03  =  0.15  =  15%

P(Friday) 0.2

**Exhaustive Events:**

When a sample space SS is partitioned into some mutually exclusive events such that their union is the sample space itself then the events are called exhaustive events or collectively events.

Suppose a die is tossed and the sample space is S={1,2,3,4,5,6}

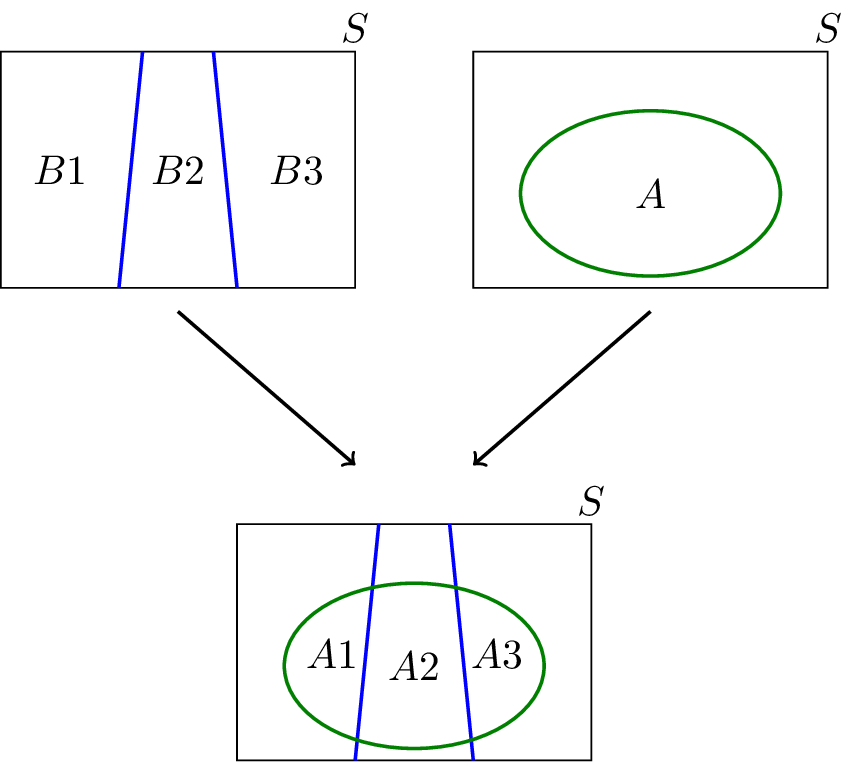
Let A={1,2}, B={3,4,5} and C={6} Hence the events A,B and C are **mutually exclusive** because A∩B∩C=ϕ and A∪B∪C=S

**TOTAL PROBABILITY:**

***Example:*** In a certain country there are three provinces, call them B1, B2, and B3 (i.e., the country is partitioned into three disjoint sets B1, B2, and B3). One may be interested in the total forest area in the country. Suppose that it is known that the forest area in B1, B2, and B3 are 100km2, 50km2, and 150km2, respectively. What is the total forest area in the country?

**Answer:** 100km2+50km2+150km2=300km2

That is, one can simply add forest areas in each province (partition) to obtain the forest area in the whole country. This is the idea behind the law of total probability, in which the *area of forest* is replaced by *probability of an event*A. In particular, if one wants to find P(A), one can look at a partition of S, and add the amount of probability of A that falls in each partition. One have already seen the special case where the partition is B and Bc: it is seen for any two events A and B,



P(A)=P(A∩B)+P(A∩Bc)

and using the definition of conditional probability,

P(A∩B)=P(A|B)P(B) we can write

P(A)=P(A|B)P(B)+P(A|Bc)P(Bc)

One can state a more general version of this formula which applies to a general partition of the sample space S.

Law of Total Probability:  
If B1,B2,B3,⋯ is a partition of the sample space S, then for any event A we have

P(A)=∑P(A∩Bi)=∑P(A|Bi)P(Bi)

***Example:***

There are three bags that each contains 100 marbles:

* Bag 1 has 75 red and 25 blue marbles;
* Bag 2 has 60 red and 40 blue marbles;
* Bag 3 has 45 red and 55 blue marbles.

One of the bags is chosen at random and then pick a marble from the chosen bag, also at random. What is the probability that the chosen marble is red?

***Solution:***

Let R be the event that the chosen marble is red. Let Bi be the event that the chosen Bag is ith one. It is already known that

P(R|B1)=0.75, P(R|B2)=0.60, P(R|B3)=0.45

One choose the partition as B1,B2,B3 Note that this is a valid partition because, firstly, the Bi's are disjoint (only one of them can happen), and secondly, because their union is the entire sample space as one the bags will be chosen for sure, i.e., P(B1∪B2∪B3) = 1. Using the law of total probability, one can write

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| P(R) | =P(R|B1)P(B1)+P(R|B2)P(B2)+P(R|B3)P(B3) |
|  | =(0.75)(1/3)+(0.60) (1/3)+(0.45) (1/3) |
|  | =0.60 |

**BAYES’ THEOREM:**

One of the most useful results in conditional probability is stated i.e., Bayes’ rule. Suppose P(A|B) is known, one is interested in the probability P(B|A). Using the definition of conditional probability, one have

P(A|B)P(B)=P(A∩B)=P(B|A)P(A)

Dividing by P(A), one obtains

**P(B|A) = [ P(A|B)P(B) ] / [ P(A) ]**

which is the Bayes’ rule. Often, in order to find P(A) in Bayes’ formula one need to use the law of total probability, so sometimes Bayes’ rule is stated as

P(Bj|A) = [ P(A|Bj)P(Bj) ] / [ ∑ P(A|Bi)P(Bi) ]

Where B1, B2 ……Bn form a partition of the sample space.

*Example:* In the above example, suppose it is observed that the chosen marble is red. What is the probability that Bag 1 was chosen?

Here P(R|Bi) is known but P(B1|R) is desired, so this is a scenario in which one can use Bayes' rule. One have

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| --- | --- |
| P(B1|R) | = [ P(R|B1)P(B1) ] / [ P(R) ] |
|  | = [ 0.75×0.5 ] / [ 0.6 ] |
|  | =5 / 12 |

**Assignment-Cum-Tutorial Questions**

**A. Questions testing the remembering / understanding level of students**

***I) Objective Questions***

1) If event *A* occurs *n*(*A*) times, then the relative frequency of *A* is…..……

And the probability of the event *A* is…………………

2) Probability space consists of …………...., …………… &…………..

3) If A is subset of B, B is subset of A, then A and B are called…………sets.

a) Disjoint b) Equal c) Complement d) Null

4) Any subset of the sample space *S* is called ……………….

A sample point of *S* is often referred to as ……………………

5) If A being an event, P (A) is non negative, and its range being …….………..

with P(S) =………..

6) The conditional probability of two sets A and B is

P (A/B)=……………………………….

a) P(A∩B)/P(A) b) P(A∩B)/P(B) c) P(AUB)/P(A) d) P(AUB)/P(B)

7) According to Total Probability theorem,

P(B/A1)P(A1)+P(B/A2)P(A2)+….P(B/An)P(An)=………..

a) P(An) b) P(B) c) P(An/B) d) P(B/An).

8) Which of the following statement is true for all Bm∩Bn=ϕ according to Baye’s Theorem?

i) P(Bn/A)= ÷

ii) P(A/Bn)= ÷

iii) P(Bm/Bn)= ÷

.iv) P(Am/Bn)= ÷

9) Two events, A and B, are mutually exclusive and each have a nonzero probability and if the event A is known to occur, the probability of the occurrence of event B is

a) One c) zero

b) Any positive value. d) Any value between 0 and 1

10) If A and B are any two sets which are mutually exclusive, then P(AUB)

=……………..

***II) Descriptive Questions***

1. Define probability using axiomatic and relative frequency approaches.

2. Explain the following terms with suitable examples

a) Sample Space and types b) Events and types

c) Trial and Outcome d) Set and types

3. Define the Joint and Conditional Probabilities and explain their properties

4. State and Explain with necessary expressions, the Total Probability theorem

5. State and Explain with necessary expressions, the Baye’s theorem

6. Explain the following:

a) Union and Intersection of sets using Venn diagrams

b) Mathematical Modeling of an experiment

c) Mutually Exclusive and Equally likely Events

d) Independent Events

**B. Question testing the ability of students in applying the concepts.**

***I) Multiple Choice Questions:***

1. Acommittee of 5 persons is to be selected randomly from a group of 5 men and 10 women. The probability that the committee consists of all women is……….

a) 0.043 b) 0.084 c) 0.054 d) 0.063

2. What is *P(A* / B) if A subset of B?

a) 1 b) 0 c) P(A)/P(B) d) P(B)/P(A)

3. Two cards are drawn at random from a deck. Determine the probability that both are aces.

a) 0.004 b) 0.04 c) 0.4 d) 1

4. Given that P(A)=0.9, P(b)= 0.89, P(AB)= 0.75, then P(AUB)=



a) 0.95 b) 0.59 c) 0.095 d) 0.059

5. Using De Morgans Theorem, = …………….

a) ) = b) )= U



c) ) = d) )= U



6. A lot of 100 semiconductor chips contains 20 that are defective. Two chips are selected at random, without replacement, from the lot. The probability that the first one selected is defective is………..

a) 0.2 b) 0.1 c) 0.02 d) 0.01

7. Two numbers are chosen at random from among the numbers 1 to 10 without replacement. Determine the probability that the second number chosen is 5.

a) 0.2 b) 0.1 c) 0.02 d) 0.01

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| 8. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card (Jack, Queen and King only)?  a)1/13 b)3/13 c)1/4 d)9/52 |
|  |

9. The six sides of a fair die are numbered from 1 to 6.The die is rolled four times.How many sequences of the four resulting numbers are possible?

a) 64 b) 63 c) 66 d) 62

10. A set of 4 elements has ………..subsets.

a) 24 b) 42 c) 44 d) 22

***II) Descriptive Questions:***

1. Anexperiment consists of tossing two dice.

Identify the following:

(a) The sample space S.

(b) The event ***A*** that the sum of the dots on the dice equals 7.

(c) The event B that the sum of the dots on the dice is greater than 10.

(d) The event C that the sum of the dots on the dice is greater than 12**.**

2. Consider the experiment of tossing a fair coin repeatedly and counting the number of tosses required until the first head appears.

*(a)*Find the probability that the first head appears on an even-numbered toss.

(b) Find the probability that the first head appears on an odd-numbered toss.

3. Consider a telegraph source generating two symbols, dots and dashes. We observed that the dots were twice as likely to occur as the dashes. Determine the probabilities of dot's occurring and the dash's occurring

4. A missile can be accidentally launched if two relays A and B both have failed. The probabilities of A and B failing are known to be 0.01 and 0.03 respectively. It is also known that B is more likely to fail (probability 0.06) if A has failed. Determine:

a) The probability of an accident missile launch

b) The probability that A will fail if B has failed

c) Are the events “A fails” and “B fails” statistically independent?

5.A number is selected at random from (1, 2, . . . , 100). Given that the number selected is divisible by *2,* determine the probability that it is divisible by 3 or 5.

6. Two manufacturing plants produce similar parts. Plant 1 produces 1,000 parts, 100 of which are defective. Plant 2 produces 2,000 parts, 150 of which are defective. A part is selected at random and found to be defective. What is the probability that it came from plant 1?

7. In three boxes there are capacitors as shown in table. An experiment consists of first randomly selecting a box, assuming each has the same likelihood of selection, and then selecting a capacitor from the chosen box.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Capacitors(µF)** | **Box1** | **Box2** | **Box3** | **Totals** |
| 0.01 | 20 | 95 | 25 | 140 |
| 0.1 | 55 | 35 | 75 | 165 |
| 1.0 | 70 | 80 | 145 | 295 |
| **Totals** | 145 | 210 | 245 | 600 |

1. Determine the probability of selecting a 0.01µF capacitor, given that box2 is selected?
2. If a 0.01µF capacitor is selected, determine the probability that it comes from box3?

8. Suppose that a laboratory test to detect a certain disease has the following statistics. Let *A*= event that the tested person has the disease *B* = event that the test result is positive .It is known that *P(B* / *A****)*** = 0.99 and *P(B* / *A)* = 0.005 and 0.1 percent of the population actually has the disease. What is the probability that a person has the disease given that the test result is positive?

**C. Questions testing the analyzing / evaluating ability of students**

1. A binary symmetry channel of the communication system transmits three symbols 0, 1 and 2. Define appropriate events Ai,Bi,i-1,2,3, to represent symbols after and before the channel, respectively.Assume channel transition probabilities are all equal at P(Ai/ Bj)=0.1, i≠j, and are P(Ai/ Bj)=0.8 for i=j=1,2,3, while symbol transmission probabilities are P(B1)=0.5, P(B2)=0.3, and P(B3)=0.2.

a) Evaluate the received symbol probabilities P(A1),P(A2) and P(A3)

b) Evaluate the a posteriori probabilities of the system

2. Consider the switching network shown in Fig. It is equally likely that a switch will or will not work. Evaluate the probability that a closed path will exist between terminals a and b.



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**UNIT-II**

**OPERATION ON ONE RANDOM VARIABLE**

**Objective:**

* To get familiarize with the one dimensional random variables and operations.
* To gain the knowledge of the standard density & distributions which can describe real life phenomena

**Syllabus:**

Definition of random variables, continuous and discrete random variables, cumulative distribution function (cdf) for discrete and continuous random variables; probability density functions (pdf) and properties. Expectation: mean, variance and moments of a random variable, Moment generating and characteristic functions and their properties. Binomial, Poisson, Uniform, Exponential, Gaussian and Rayleigh distributions. Transformations of random variable.

**Learning Outcomes:**

At the end of the unit student will be able to:

1. Define random variable
2. Distinguish continuous and discrete random variables
3. Representation of cumulative and density functions
4. Calculate Nth order moments of a random variable
5. Find moments through characteristic and moment generating functions
6. Understand different distribution functions
7. Explain the transformations of random variables

**Learning Material**

**Definition of random variable**

* A random variable X(s) is a single-valued real function that assigns a real number called the value of X(s) to each sample point of S.
* Often, r.v. or R.V is used to denote the random variable.
* Clearly a random variable is not a variable but it is a function.
* The sample space S is termed the domain of the r.v. X, and the collection of all numbers [values of X(s)] is termed as the range of the r.v. X. Thus the range of X is a certain subset of the set of all real numbers.
* Note that two or more different sample points might give the same value of X(s), but two different numbers in the range cannot be assigned to the same sample point.

**Conditions for a function to be a random variable:**

1. The set {X ≤ x} shall be an event for any real number x.

This set corresponds to those points s in the sample space for which the random variable X(s) does not exceed the number x.

The probability of this event, P{X ≤ x} is equal to the sum of probabilities of all elementary events corresponding to {X ≤ x}.

1. Probabilities of the events {X = ∞} and {X = -∞} be 0.

P{X = ∞} = 0 and P{X = -∞} = 0

**Classification of random variabes:**

1. Discrete random variable
2. Continuous random variable
3. Mixed random variable

**Discrete random variable:**

A Discrete random variable is one having only discrete values. Sample space for a discrete r.v can be discrete, continuous or even a mixture of dicrete and continuous points.

**Continuous random variable:**

A continuous random variable is one having only continuous range of values. Sample space for a continuous r.v can’t be a discrete or mixed.

X is a continuous r.v. only if its range contains an interval (either finite or infinite) of real numbers

**Mixed random variable**

A mixed random variable is one having both discrete and continuous values. This is usually the least important type of random variable.

**Cumulative distribution function (cdf):**

The distribution function [or cumulative distribution function (cdf)] of X is the function defined by

***FX(x) = P{X ≤ x}***

*Where x is any real value ranging from -∞ to ∞*

**Properties:**

1. FX(-∞) = 0
2. FX(∞) = 1
3. 0 ≤ FX(x) ≤ 1
4. FX(x1) ≤ FX(x2) if x1 < x2
5. P { x1 < X ≤ x2} = FX(x2) - FX(x1)
6. FX(x+) = FX(x)

**Cumulative distribution function (cdf) for discrete random variables:**

***FX(x) =***

Where u(x) is unit-step function; u(x) =

**Cumulative distribution function (cdf) for continuous random variables:**

***FX(x) =***

**Probability Density Functions (Pdf) And Properties:**

The function fX(x) is called the probability density function (pdf) and is derivative of cdf.

***fX(x) =***

**Properties:**

1. 0 ≤ fX(x); for all x
2. FX(x) =
3. P { x1 < X ≤ x2 } =

**Distribution and Density Functions:**

1. Discrete Random Variables
2. Binomial Distribution and Density Function
3. Poisson Distribution and Density Function
4. Continuous Random Variables
5. Uniform Distribution and Density Function
6. Exponential Distribution and Density Function
7. Rayleigh Distribution and Density Function
8. Gaussian Random Variable

Gaussian Distribution and Density Function

**DISCRETE DISTRIBUTION AND DENSITY FUNCTIONS**

**Binomial Random Variate:**

**Density Function**

Let 0 < p < 1 and N = 1,2,…., then

**fx(x) = pk (1-p)N-k  δ(x-k)** where



**Distribution Function**

Integration of binomial density function

**Fx(x) = pk (1-p)N-k  u(x-k)**



**Poisson Random Variate:**

**Density Function; fX(x) = δ(x-k)**

****

**Distribution Function; FX(x) = u(x-k)**

****

Where b > 0 is a real constant;

1. b = Np if N → ∞ and p →0
2. b = λT; λ is average rate and T is time interval of duration

**CONTINUOUS DISTRIBUTION AND DENSITY FUNCTIONS**

**Uniform Random Variate:**

**Density function**



**Distribution Function**

****

**Exponential Random Variate:**

**Density function**

***fX*(x)=**



**Distribution Function**

FX(x)=

Fx(x) =



**Rayleigh Random Variate:**

**Density function**

**fX(x) =** x ≥ a;

0 x < a

fx(x) =



**Distribution Function**

Fx(x) **= 1 -** x ≥ a

0 x < a



**Gaussian Random Variate:**

A random variable X is called Gaussian if its density function has the form of ***fX (x) =***

Where are real constants

Its Maximum value occurs at x = ax. Its spread about the point x=ax Is related to σx. Function decreases to 0.6.7 times of its maximum at

x = ax + σx and x = ax – σx

Distribution function FX (x) is integral of fX (x) and is given by

***Fx(x) =***



**Normalized Gaussian Function**

If ax = 0 and σx = 1then normalized function **F(x) =**

Which is a function of x only and x ≥ 0.

For negative value of x ,**F(-x) = 1 - F(x) is used.**

Let u = (ᶓ - ax) / σx and substitute in Fx(x); then

***FX(x) =***

And is clearly equivalent to ***FX(x) = FX***

* Function F(x) can be evaluated b approximation. F(x) = 1 – Q(x)

Where Q(x) = is known as Q-function

* Q-approximation; Q(x) ≈ ; x ≥ 0;

If a=0.039 and b=5.510 then **Q(x) ≈**

**Conditional Distribution Function:**

Fx(x|B) = P { X ≤ x|B} = where X(s) and s is subset of B

**Properties:**

1. FX(-∞|B) = 0
2. FX(∞|B) = 1
3. 0 ≤ FX(x|B) ≤ 1
4. FX(x1|B) ≤ FX(x2|B) if x1 < x2
5. P { x1 < X ≤ x2|B } = FX(x2|B) - FX(x1|B)
6. FX(x+|B) = FX(x|B)

**Conditional Density Function:**

*fX(x*|B*) =*

**Properties:**

1. 0 ≤ fX(x|B); for all x
2. FX(x|B) =
3. P { x1 < X ≤ x2|B } =

**Methods of defining conditioning event:**

Define event B in terms of X; so let B = {X ≤ b}; where b lies in -∞ to ∞; then

Fx(x|X ≤ B) = x < b

1 x ≥ b

fX(x|X ≤ B) = = x < b

1. x ≥ b

**Note:** Fx(x|X ≤ B) ≥ Fx(x)



**Operations on One Random Variable**

**Expected value:** Defined by “Mathematical expectation of X” or “statistical average of X” or “mean value of X” or “expected value of X” and denoted by E[X] or .

* For discrete random variable E[X] = =
* For continuous random variable E[X] = = fX(x)dx
* Conditional Expected value E[X|B] = fX(x|B)dx

Define event B in terms of X; so let B = {X ≤ b}; where b lies in -∞ to ∞; then E[X|B] = E[X| X ≤ b] =

**Moments:**

1. **Moments About Origin:**

**mn = E[Xn] = n fX(x) dx**

m0 = E[X0] = 0 fX(x) dx = 1 → area of the functionfX(x)

m1 = E[X1] = 1fX(x) dx = →expected value

1. **Central Moments:**

**µn = E[(X-)n] = n fX(x) dx**

µ0 = E[(X-)0] = 0 fX(x) dx = 1 → area of functionfX(x)

µ1 = E[(X-)1] = 1 fX(x) dx = 0

**Variance:** Second Central Moment

µ2 = σx2 = E[(X-)2] = 2 fX(x) dx = m2 – m12

Standard Deviation (σx): Positive Square root of variance; it is measure of spread of fX(x) about mean.

**Skew:** Third Central Moment

µ3 = E[(X-)3] = 3 fX(x) dx → It is a measure of asymmetry of fX(x) about x = m1=

**Coefficient of skewness:** Normalized third central moment µ3/σx3

**Functions that give moments:**

1. **Characteristic Function:**
2. **Φx(ω) = E[] = fX(x)dx; Φ**x(ω) isaFourier Transform with **ω** is reversed.
3. So fX(x) can be obtained from **Φ**x(ω) by applying inverse Fourier transform; **fx(x) = Φx(ω) dω;**
4. nth moment of X is given by **mn = (-j)n Φx(ω)|ω=0**
5. **Φ**x(ω) ≤**Φ**x(0) = 1
6. **Moment Generating Function:**
7. **Mx(v) = E[] = fX(x)dx;**
8. nth moment of X is given by **mn = Mx(v)|v=0**

**Transformations of a random variable:**

* Transformations are used to transform (change) one random variable X into another random variable Y.
* Y = T(X) 
* X can be discrete, continuous or mixed random variable.
* Transformation T can be linear, nonlinear, segmented, staircase etc.
* Depending on form of X and T; Only three cases are considered. They are:

1. X continuous and T continuous and either monotonically increasing or decreasing

* Monotonically increasing



* Monotonically decreasing

****

* fY(y) = fX[T-1(y)] => fY(y) = fX(x)

1. X continuous and T continuous but non-monotonic



* fY(y) =

1. X discrete and T continuous

fY(y) = Where yn = T(xn)

FY(y) = P(yn) = P(xn)

|  |  |  |
| --- | --- | --- |
| **Random Variable** | **Mean** | **Variance** |
| Bernoulli | p | (1-p) |
| Binomial | np | np(1-p) |
| Poisson | λ | λ |
| Uniform | (a+b)/2 | (b-a)2/12 |
| Rayleigh | 1/ λ | 1/ λ2 |
| Gaussian | µx | σx |

**Assignment-Cum-Tutorial Questions**

**A. Questions testing the remembering / understanding level of students**

***I) Objective Questions***

1. If X is a random variable, ‘x’ being real, x1 < x2 єx, then which of the following is true?

a)FX(x1)= FX(x2) b) FX(x1)≥ FX(x2) c) FX(x1)≤ FX(x2) d) FX(x1)≠ FX(x2)

2.=……………….=…………….

3…………….is monotonous increasing stair step whereas ………..is piecewise continuous function.

4. Area under the graph of pdf, is…………………………..

5. =……………………

a) b) c) d)

6. Mean of X is the……….order moment about………..

a) I, b) II, (0, 0) c) I, (0, 0) d) II,

7. Variance of a continuous random variable X is, Var(X) =……………

a)E{X-2E(X)}2 b)E{E(X)-X}2 c)E(X2) - { E(X)}2 d) E(2

8. Match the following:

1.  1) Normal

1.  2)Binomial
2.  3)Uniform
3.  4)Exponential

9. *fx*  =………………

10. X being a continuous random variable with fX*(x)* as its pdf, if a one-to-one transformation y=g(x), then the inverse transformation x=g-1(y) has a pdf

a)*fY*(y)= *fX(x)* b) *fY*(y)= *fX(x)*  c) *fX*(x)= *fY(y)*  d) *fX*(x)= *fY(y)*

11. Which of the following distribution is used to approximate the number of telephone calls in a call centre switching unit at various intervals of time?

a)Binomial b)Rayleigh c)Exponential d)Poisson

***II) Descriptive Questions***

1. Differentiate Continuous and discrete random variables with suitable examples.
2. Define and explain the Cumulative Distribution Function and its properties.

3. Define and explain the Probability Density Function and its properties.

4. Explain the following random variables with necessary expressions and neat

sketches:

a)Binomial b) Gaussian c)Uniform d)Exponential e)Poisson f)Rayleigh

5. Explain briefly the following:

a) Moments about origin b) Moments about mean

6. Define and explain Moment Generating function and its properties.

7. Define and explain Characteristic function and its properties.

8. Explain about a) Monotonic Transformation of a random variable

b) Non-monotonic Transformation of a random variable

**B. Question testing the ability of students in applying the concepts.**

***I) Multiple Choice Questions:***

1. Consider the experiment of tossing a coin three times. Let X be the r.v. giving the number of heads obtained. Suppose that the tosses are independent and the probability of a head is ***p.*** The range of X is………… and the probability, *P(X* = 2) =……

a){0,1,2,3},p(1-p)3 b) ){0,1,2}, p(1-p)3

c) ){0,1,2,3},3(1-p)p2 d) ){0,1,2},3(1-p)p2

2. Consider the function given by the following equation,

, If X is the r.v. whose cdf is given by F(x), then P(0<X≤1/4) is………

a) 1/2 b)1 c)1/4 d)1/3

1. Let X be a continuous r.v. X with pdf,



where *k* is a constant, the value of k is………and P(1/4 <X≤2)=……..

a)1, b)2 , c) 3, d)0.2 ,

1. A r.v. X is called a *Rayleigh* r.v. if its pdf is given by

, then cdf FX(x) is……..

a)1-exp(-x2/2σ2 ) b) exp(-x2/2σ2 ) c) 1+exp(-x2/2σ2 ) d) exp(-x2/2σ2 )-1

1. Consider a discrete r.v. X whose pmf is given by ,,

then mean and variance of X are……&……….

1. 0, 1 b)1,0.121 c) 0, 2/3 d)0.451, 0.25

6. **A** binary source generates digits 1 and **0** randomly with probabilities **0.6** and **0.4,** respectively. What is the probability that at least three 1s will occur in a five-digit sequence?

a) 0.23 b) 0.683 c) 0.231 d) 0.451

7. All manufactured devices and machines fail to work sooner or later. Suppose that the failure rate is constant and the time to failure (in hours) is an exponential r.v. X with parameter λ. Measurements show that the probability that the time to failure for computer memory chips in a given class exceeds 104 hours is e-t (approx.0.368). The value of the parameter λ is………

a) 104 b) 10-4 c) 103 d) 10-3

8. If X be a r.v. with cdf *FX(x)* and pdf *fX(x).* Let Y =. *ax* + b, where a and b are real constants and *a* ≠0, the pdf of Y in terms of *fX(x) is….……….*

a)fX() b)fX() c)fX() d)fX()

9. The pdf of Y if X is a uniform r.v. over (-π/2, π/2), for the transformation Y = tan ***X***

is……………………

a) b)

c)2) d)

10. The moment generating function of a discrete r.v. X with E(Xk)=0.8, k=1,2,... is

a) 0.2+0.8et b) 0.8+0.2et c) 0.2+0.8e2t d) 0.8+0.2e2t

***II) Descriptive Questions:***

1***.*** An information source generates symbols at random from. a four-letter alphabet (a, *b,* ***c,*** d} with probabilities P(a) = 1/2,P(b) = 1/4, and P(c) = P(d) =1/8**. A** coding scheme encodes these symbols into binary codes as follows:



Let X be the r.v. denoting the length of the code, that is, the number of binary symbols (bits).

(a) What is the range of X?

(b) Assuming that the generations of symbols are independent, find the probabilities P(X = 1),P(X = 2), P(X = **3),** and P(X > **3).**

**c)** Sketch the cdf ***FX(x)*** of X and specify the type of X.

Find (i) P(X **≤**1), (ii) P(l < X ≤2), (iii) P(X > 1), and (iv) P(l ≤ X **≤** 2).

2. The pdf .of a continuous r.v. X is given by

***,*** Find the corresponding cdf FX(x) and sketch fX(x) and FX(x).

3. **A** noisy transmission channel has a per-digit error probability ***p*** = 0.01.

(a) Calculate the probability of more than one error in 10 received digits.

(b) Repeat (a), using the Poisson approximation

4. The number of telephone calls arriving at a switchboard during any 10-minute period is known to be a Poisson r.v. X with *λ=* 2.

*(a)* Find the probability that more than three calls will arrive during any 10-minute period.

*(b)* Find the probability that no calls will arrive during any 10-minute period.

5. Assume that the length of a phone call in minutes is an exponential r.v. X with parameter λ=1/10***.*** If someone arrives at a phone booth just before you arrive, find the probability that you will have to wait *(a)* less than *5* minutes, and (b) between 5 and 10 minutes.

6. The radial miss distance [in meters (m)] of the landing point of a parachuting sky diver from the center of the target area is known to be a Rayleigh r.v. X with parameter **σ2** = 100.

***(a)*** Find the probability that the sky diver will land within a radius of 10 m from the center of the target area.

(b) Find the radius ***r*** such that the probability that X > ***r*** is ***e-t*** (approx. 0.368).

7. If X is a uniform **r.v.** over (- 1, 2).with pdf *fX(x).* Let Y = *X2****.*** Find and sketch the pdf of Y.

8. Let X be a uniform r.v. over (0, 1) and **Y** = ***ex.***

(a) Find **E(Y)** by using ***fY(y).***

*(b)* Find **E(Y)** by using ***fX(x).***

**C. Questions testing the analyzing / evaluating ability of students**

1. Suppose the depth of water, measured in meters, behind a dam is described by an exponential random variable with density fX(x) = (1/13.5) u(x) exp(-x/13.5).There is an emergency overflow at the top of dam that prevents the depth from exceeding 40.6m .there is a pipe placed 32 m below the overflow that feeds water to a hydro electric generator.

a) What is the probability that water is wasted through emergency overflow?

b) Given that water is not wasted in overflow, what is the probability that the generator will have the water to drive it.

c) What is the probability that water will be too low to produce power?

2. Show that the characteristic function for a Gaussian random variable X, having zero mean is ФX(ω) =exp(-σx2ω2/2). Find all the moments of X using ФX(ω) .

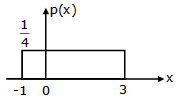
**D. GATE:**

1. Consider two identically distributed zero-mean random variables U and V. Let the cumulative distribution functions of U and 2V be F(x) and G(x) respectively. Then, for all values of x **[GATE-2013]**

(A) F (x) - G(x) ≤ 0 (B) F (x) - G(x) ≥ 0

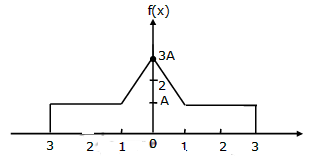
(C) (F (x) - G(x)).x ≤ 0 (D) (F (x) - G(x)).x ≥ 0

1. Let X be a random variable which is uniformly chosen from the set of positive odd numbers less than 100. The expectation E[X] is **\_\_\_\_**\_\_\_\_\_\_. **[GATE-2014]**
2. The second moment of Poisson-distributed random variables is 2. The mean of the random variable is \_\_\_\_\_. **[GATE-2016]**
3. A fair die with faces {1, 2, 3, 4, 5, and 6} is thrown repeatedly till „3‟ is observed for the first time. Let X denotes the number of times the die is thrown. The expected value of X is\_\_\_\_\_\_\_
4. For a random variable x following the probability density function, P(x) shown in figure the mean and the variance are, respectively, **[GATE-1992]**



(a) 1/2 and 2/3 (b) 1 and 4/3 (c) 1 and 2/3 (d) 2 and 4/3

1. The function shown in figure can represent a probability density function of A \_\_\_\_\_\_\_.

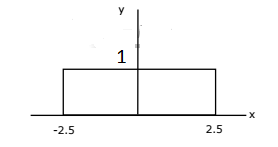
 **[GATE-1996]**

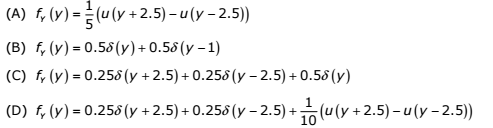
1. 20. A probability density function is given by p(x) = K -∞<x<∞, the value of K should be

(a) (b) (c) (d) **[GATE- 1997, 2006]**

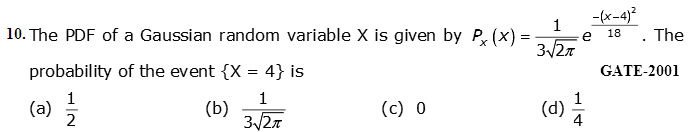
1. A uniformly distributed random variable X with probability density function Where u (.) is the unit step function is passed through a transformation given in the figure below. The probability density function of the transformed randomvariable Y would be

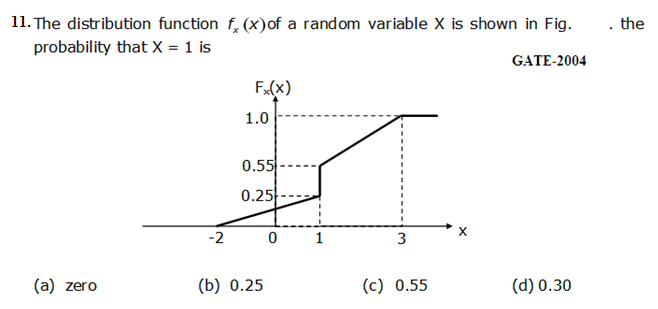
** [GATE-2006]**

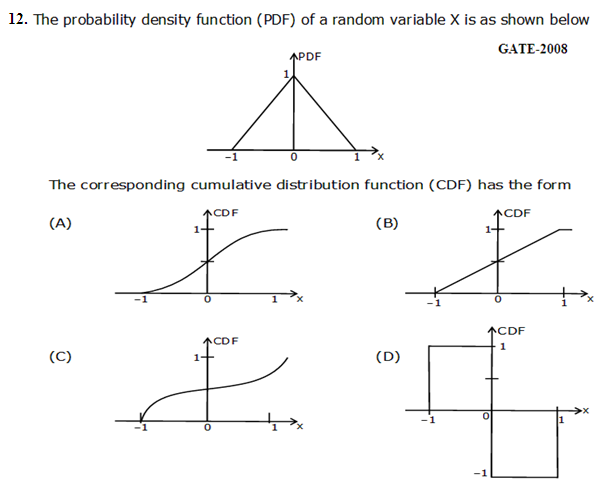
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** [GATE-2007]**

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**UNIT III**

**MULTIPLE RANDOM VARIABLES**

**Objective:**

* To familiarize with the concept of two dimensional random variables and operations.

**Syllabus:**

Jointly distributed random variables, conditional, joint density and distribution functions, Function of two random variables; Sum of two independent random variables, Central limit theorem (for IID random variables), Joint moments, covariance and correlation; independent, uncorrelated and orthogonal random variables.

**Learning Outcomes:**

At the end of the unit student will be able to:

1. Define multiple random variables
2. Explain the joint and conditional distribution and density functions
3. Represent the cumulative and density functions of multi random variables
4. Understand Central limit theorem
5. Calculate Joint moments, covariance and correlation of rvs
6. Distinguish between Correlation and Uncorrelation and dependence and Independence concepts of rvs

**Learning Material**

* **Jointly Distributed Random Variables (Bivariate Random Variables or 2-D Random Vector):**
* If X,Y are discrete r.v.s’ defined on Sample Space S, then the pair (X,Y) is called Jointly ***Distributed RV or a Bivariate RV or 2D Random Vector****.*
* If each of X and Y associates a real number with every element of S
* If X &Y discrete, then {X,Y} discrete
* If X & Y continuous, then {X,Y} discrete/continuous
* If one of X & Y is discrete, then {X,Y} is mixed

**JOINT DISTRIBUTION FUNCTIONS**

Let A= {X≤x} and B= {Y≤y} be two events defined on Sample Space and P(A)= *FX(x),P(B)* = *FY(y)*, then the *joint cumulative distribution function* (or joint cdf) of *X* and *Y ,* denoted by *FXY(x, y)* is the function defined by

*FXY(x, y)* = *P(X* ≤*x, Y*≤ *y)=P(A* n B)

*FXY(x, y)=*

**Properties of *FXY(x, y):***

The joint cdf of two r.v.'s has many properties like to those of the cdf of a single r.v.:

1. 0 ≤Fxy(x, y) ≤ **1**

1. a) FXY(-∞, -∞) =0 c) FXY(-∞, y)=0

b) FXY(x, -∞)=0

1. FXY(∞,∞ )=1
2. FXY(x, y) is a monotonic and non decreasing function of both x and y.
3. The probability of the joint event {x1<X≤ x2, y1<Y≤ y2} is given by

P{x1<X≤ x2, y1<Y≤ y2}= FXY(x2, y2)+ FXY(x1, y1)- FXY(x1, y2)- FXY(x2, y1)

1. The marginal distribution functions are given by FXY(x, ∞)= FX(x) and FXY(∞, y)= FY(y)

**Independent Random Variables:**

* If X and Y are independent r.v.'s, then 

**Continuous Random Variables-Joint Probability Density Functions:**

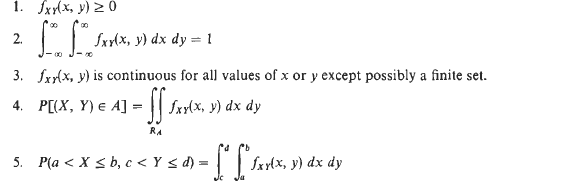
**Joint Probability Density Functions:**

Let (X, Y) be a continuous bivariate r.v. with cdf FXY(x, y) and let C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png

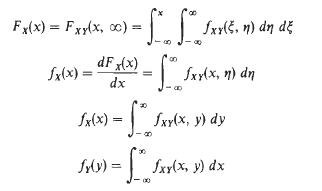
In the above equation, the function *fXY*(x, y) is called the joint probability density function (joint pdf) of (X, Y) and is given by

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**Properties of *fXY*(x, y):**



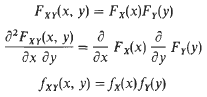
**Marginal Probability Density Functions:**



* The pdf's *f* X(x) and *f* Y(y) are referred to as the marginal pdf's of X and Y, respectively.

**Independent Random Variables(w.r.t cdf and pdf)** :

* If X and Y are independent r.v.'s,then



The above equation is the condition for the continuous r.v.'s X and Y are independent r.v.'s

**Conditional Distributions:**

***Point Conditioning:***

Two rvs’ X and y are considered.The distribution of an rv X when the distribution function of an rv Y is known at some value of y, is defined as the conditional distribution function of X. It can be expressed as:

FX(x/Y=y)= and the conditional density function is

expressed as:fX(x/Y=y) = C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png

***Interval conditioning:***

Assume that event b is defined in the interval y1<Y≤y2 for the rv Y i.e., B={ y1<Y≤y2 } and p(B) is non zero i.e., P(B)=P{ y1<Y≤y2 }≠0, then the conditional distribution function is given by

FX(x/( y1<Y≤y2))= and the conditional density function is given by

*f*X(y/(x1<X≤x2))=

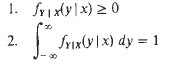
**Conditional Probability Density Functions:**

If (X, Y) is a continuous bivariate r.v. with joint pdf *f*XY(x, y), then the ***conditional*** pdf of ***Y,*** given that X = x, is defined by

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Similarly, C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png

**Properties of Conditional Probability Density Function:**



As in the discrete case, if X and Y are independent, then, C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png

**Sum of Two random variables:**

X and Y be two random variables defined on sample space such that {X≤x} and {Y≤y} are the respective events associated with them then,W=X+Y is called the sum of two rvs’ with {W≤w} be the resulting event.

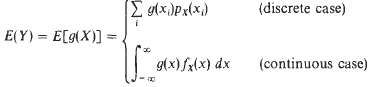
If FX(x) and FY(y) are the cdfs of X and y respectively, then FW(w)=P{W≤w}=P{X+Y≤w}

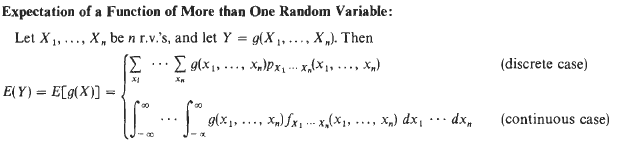
The pdf of W is written as *fW(w)=*

**EXPECTATION**

**Expectation of a Function of One Random Variable:**

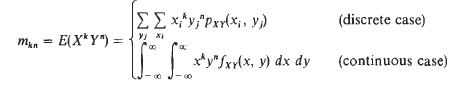
The expectation of Y = g(X) is given by



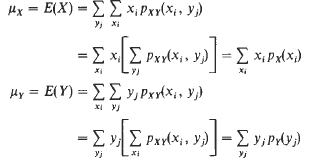


**COVARIANCE AND CORRELATION COEFFICIENT**

The (k, n)th moment of a bivariate r.v. (X, Y) is defined by

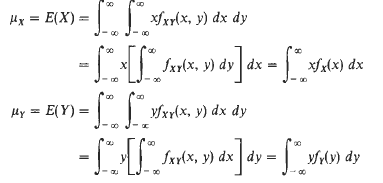


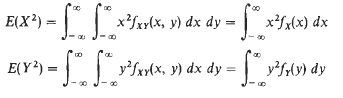
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Similarly,

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The (1, 1)th joint moment of (X, Y), C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png is called the correlation of X and Y. If E(XY) = 0, then

X and Y are orthogonal

The covariance of X and Y, denoted by Cov (X, Y) or σX,, is defined by

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If Cov(X, Y) = **0,** then X and Y are uncorrelated

X and Y are uncorrelated if C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png

The correlation coefficient,denoted by ρ(X, Y) or ρXY, is defined by

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**Correlation:**

* If X and y are two rvs then their second order joint moment (about origin) m11 is called the correlation of X and Y.

RXY=m11=E[XY]

If two rvs are statistically independent, then E[XY]=E[X] E[Y]

**Orthogonality:**

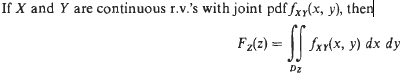
* If two rvs X and Y are said to be orthogonal if their joint occurrence is zero.

i.e., *fXY(x,y)=0 and* RXY=m11=E[XY]=0

**FUNCTIONS OF TWO RANDOM VARIABLES**

* Given two r.v.'s X and Y and a function g(x, y), the expression C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png defines a new r.v. Z. With ***z*** a given number,

**gXY(x,** y) ***≤*** z. Then and cdf is C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png



**The Central Limit Theorem:**

The central limit theorem is one of the most remarkable results in probability theory. There are many versions of this theorem. In its simple form, the central limit theorem is stated as follows: Let X1, X2,…….Xn be a sequence of independent, identically distributed random variables each with mean µ and variance σ2 .Let

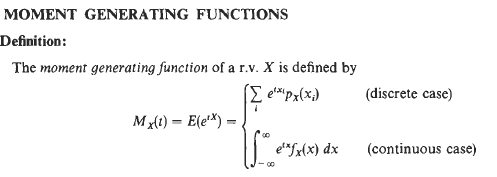
Zn = =

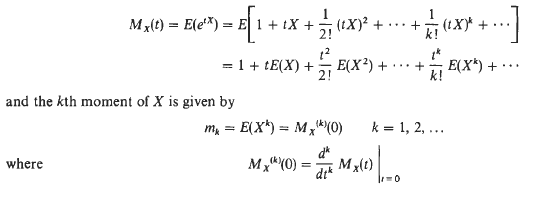
Where is defined by the above eq. then the distribution of Zn  tends to the standard normal as n →∞ ; that is =N(0; 1)

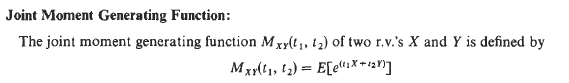
Or

=

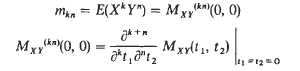
Where is the cdf of a standard normal random variable.. thus the central limit theorem tells us that for large n, the distribution of the sum Sn = is approximately normal regard less of the form of the distribution of the individual Xi ‘s .







The (k, n) joint moment of X and Y is given by



The joint moment generating function of n r.v.'s C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png

If X1,X2,X3,…..X~~n~~ be n independent rvs’ then the moment is given by

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**Assignment-Cum-Tutorial Questions**

**A. Questions testing the remembering / understanding level of students**

***I) Objective Questions***

1. Two dimensional product space is known as -----------------------------------------------------
2. The value of joint distribution function FX,Y(∞,∞) = -------------------------------------------
3. If X and Y are statistically independent, then joint distribution function FX,Y(X,Y) = -------------------------------------------
4. Central Limit theorem is mostly applicable to -----------------------------------------------
5. If Y = X1+X2+…… +XN, where X1,X2, …… ,XN are statistically independent RVs, then fY(y) =

a) fX1(x1)+fX2(x2)+…….+ fXN(xN) b) fX1(x1)-fX2(x2)+ …….+ fXN(xN)

c) fX1(x1)\*fX2(x2)\*…….\*fXN(xN) d) fX1(x1) fX2(x2)……. fXN(xN)

1. The distribution function of one random variable X conditioned by a second random variable Y with interval is known as

a) moment generation b) point conditioning

c) expectation d) interval conditioning

1. If the joint probability of X and Y is PX,Y (x,y) = k(x+y) for x=1,2 and y=1,2 then value of k is

a) 1/6 b) 1/12 c) 1/3 d) ¼

1. Let X be the means of N random variables. If Y=X1+X2+…+XN, then

E[Y] =

a) b) c) d)

1. The (n+k)th order joint central moment of the random variables X and Y is µnk= ------------------------------------
2. Which of the following is marginal distribution function

a) FX,Y (∞,Y) = FX (X), b) FX,Y (∞,Y) = FY (Y),

c) FX,Y (∞,Y) = fY (Y), d) FX,Y (∞,Y) = fX (X)

***II) Descriptive Questions***

1. Define the following:
2. Joint probability density function b) Joint probability distribution function
3. State and prove the properties of Joint probability density function?
4. State and prove the properties of Joint probability distribution function?
5. Explain the following for two rvs’

a) Mean b) mean square value C) skew d) skewness

e) Correlation f) covariance

1. Determine PDF of sum of two random variables.
2. State and prove central limit theorem for i.i.d. rvs.

**B. Question testing the ability of students in applying the concepts.**

***I) Multiple Choice Questions:***

1. Two random variables X and Y have means 1 and 2 respectively and variance 4 and 1 respectively. Their correlation coefficient is 0.4. New random variables W and V are V = -X + 2Y, W = X + 3Y. The correlation and correlation coefficient of V and W is ……

a) 22.2 and 0.08 b) 0.222 and 0.8 c) 0.08 and 2.22 d) 2.22 and 0.8

1. Which of the relation is correct?

a) b) c) d)

1. If X and Y are two independent random variables such that E[X] = λ1, variance of X is σ12, E[Y] = λ2, variance of Y is σ22, then the co-variance of [X,Y] = --------

a)σ12 σ22 + λ12 σ22 + σ12 λ22 b)σ12 σ22 + λ12 σ22 - σ12 λ22

c)σ12 σ22 - λ12 σ22 + σ12 λ22 d)σ12 σ21 + λ12 σ22 +σ21 λ22

4) For a given valid joint density function fXY(x,y) = , the value of constant b (in terms of a) is----

a) ln b) ln c) ln d) ln

5) Consider an experiment of tossing a fair coin twice. Let (X,Y) be a bivariate r.v., where X is the number of heads that occurs in two tosses and Y is the number of tails that occurs in two tosses. P(X=2, Y=0), P(X=2, Y=0) and P(X=2, Y=0) are

1. ¼,¼,½ b)1,1,½ c) ½, ½, ½ d) ¼, ¼, ¼

6) The Joint cdf of a bivariate r.v (X,Y) is given by FXY(x,y)=, then P(X>x, Y>y) is

1. + b) c) 1,1 d)

7) Consider an experiment of drawing randomly three balls from an urn containing two red, three white and four blue balls. Let (X,Y) be a bivariate r.v, where X and Y denote number of red and white balls chosen. Marginal pmf’s of X and Y are shown below, then A, B and C values are

**Table P(X,Y)(i,j)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| i | J | | | |
| 0 | 1 | 2 | 3 |
| 0 | 4/84 | A | 12/84 | 1/84 |
| 1 | 12/84 | 24/84 | B | 0 |
| 2 | C | 3/84 | 0 | 0 |

1. 18/84, 6/84, 4/84 c) 18/84, 4/84,6/84
2. 67/84, 72/84, 68/84 d) 4/84,6/84, 18/84

8) The Joint pdf of a bivariate r.v (X,Y) is given by fXY(x,y)=, then P(0 < Y < ½ | X= 1) is

a) 1 b) 5/32 c) 1/8 d) 0

9) Density function of two random variables (X,Y) is fXY(x,y)= 4u(x)u(y) then mean value of the function is

a) 9/4 b) 3/9 c) 4/9 d) ¾

10) The correlation coefficient between X and Y from the given data is

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | 1 | 2 | 3 | 4 |
| P(x) | 0.2 | 0.4 | 0.1 | 0.3 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Y | 1 | 2 | 3 | 4 |
| P(y) | 0.25 | 0.25 | 0.15 | 0.35 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (X,Y) | (1,1) | (2,2) | (3,3) | (4,4) |
| P(x,y) | ½ | 1/8 | ¼ | 1/8 |

a) 1.12 b) 1.2 c) -0.93 d) 0.93

***II) Descriptive Questions:***

1. Consider an experiment of tossing a fair coin twice. Let (X, Y) be a bivariate r.v., where X is the number of heads that occurs in the first toss and Y is the number of tails that occurs in the second toss.
2. Find and sketch the joint density function of (X, Y).
3. Find and sketch the joint distribution function.
4. The joint pdf of a bivariate r.v. (X, Y) is given by

fXY(x,y) = where k is a constant.

1. Find the value of k.
2. Find the marginal pdf's of X and Y.
3. Are X and Y independent?
4. Joint probability density function is fX,Y(x,y)=

a) Find and sketch FX,Y(x,y)

b) if a < b, find P{X+Y ≤ 3a/4}

4) Statistically independent random variables X and Y have moments m10=2, m20=14, m11=-6 and m02=12. Find the moment µ22.

5) Three statistically independent random variables X1, X2 and X3 having mean values as 3, 6 and -2 respectively. Find the mean values of the following functions

a) g(X1, X2, X3) = X1 + 3X2 + 4X3

b) g(X1, X2, X3) = X1 X2 X3

c) g(X1, X2, X3) = -2X1X2 - 3X1X3 + 4 X2X3

6) A joint density is given as fX,Y(x,y)=

a) Find all the joint moments mnk, n and k = 0,1,…..

b) Find all the joint central moments µnk, n and k = 0,1,…..

7) Probability density functions of two statistically independent random variables X and Y are fX(x) = ½ u(x-1)e–(x+1)/2 and fY(y) = ¼ u(y-3)e–(y-3)/4. Find the probability density of the sum W = X + Y

8) Consider the binary communication channel. Let X, Y be random variables, where X is the input to the channel and Y is the output of the channel. Let P(X=0) = 0.5, P(Y=1 | X=0) = 0.1 and P(Y=0 | X=1) = 0.2

a) Find the joint pmf’s of (X,Y)

b) Find the marginal pmf’s of (X,Y)

c) Are X and Y are independent?

**C. Questions testing the analyzing / evaluating ability of students:**

1. The time it takes to drive to work is a random variable Y. Because of traffic, driving time depends on the (random) time of departure, denoted X, which occurs in an interval of duration T0 that begins at 7:30 A.M, each day. There is a minimum driving time T1 required, regardless of the time of departure. The joint density of X and Y is known to be

fX,Y (x,y) = c(y- T1)3 u(y- T1) [u(x)- u(x- T0)] exp[-(y- T1)(x+1)]

where c=(1+ T0)3 / 2[(1+ T0)3-1]

1. Find the average driving time that results when it is given that departure occurs at 7:30 A.M. Evaluate your results for T0 = 1h.
2. Repeat part (a) given that departure time is 7:30 A.M. plus T0.
3. What is the average time of departure if T0 = 1h? (Hint: Note that point conditioning applies.)
4. Random variables R and Θ have joint density function

f R,Θ (r ,θ) =

a) Find P{0< R ≤ 1, 0< Θ ≤ π/2}.

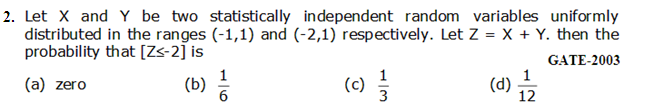
b) Find f R(r / Θ = π).

c) Find f R(r / Θ ≤ π) and compare to the result found in part (b), and explain the comparison.

**D. GATE:**

1. Two random variables X and Y are distributed according to **[GATE-2016]**

The probability P(X + Y ≤1) is \_\_**\_**\_\_\_\_

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**UNIT IV**

**CLASSIFICATION OF RANDOM PROCESSES**

**Objective:**

To familiarize with the concept of random process, classification and examples of random processes.

**Syllabus:**

Definition and examples – first order, second order, strictly stationary, wide-sense stationary and ergodic processes, Examples of random processes: white noise, Gaussian and Poisson processes.

**Learining Outcomes:**

At the end of the unit student will be able to:

1. Define a random process and difference between a random variable and random process.
2. Classify the random processes
3. Define stationary concept and types.
4. Verify the given random process is stationary or not.
5. Verify the ergodicity.

**LEARINING MATERIAL**

**INTRODUCTION:**

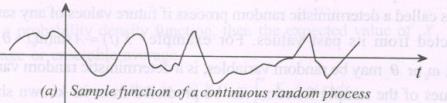
* In many cases the signals of interest are very complex due to the randomness of the world around them, which leaves them noisy and often corrupted.
* This often causes the information contained in the signal to be hidden and distorted. For this reason, it is important to understand these random signals and how to recover the necessary information.

The random processes are also called as stochastic processes which deal with randomly varying time wave forms such as any message signals and noise. They are described statistically since the complete knowledge about their origin is not known. So statistical measures are used. Probability distribution and probability density functions give the complete statistical characteristics of random signals. A random process is a function of both sample space and time variables. And can be represented as {X ≤x(s,t)}.

Deterministic and Non-deterministic processes: In general a random process may be deterministic or non deterministic. A process is called as deterministic random process if future values of any sample function can be predicted from its past values. For example, X(t) = A sin (ω0t+ϴ), where the parameters A, ω0 and ϴ may be random variables, is deterministic random process because the future values of the sample function can be detected from its known shape. If future values of a sample function cannot be detected from observed past values, the process is called non-deterministic process.

Classification of random process: Random processes are mainly classified into four types based on the time and random variable X as follows.

Continuous Random Process: A random process is said to be continuous if both the random variable X and time t are continuous. The below figure shows a continuous random process. The fluctuations of noise voltage in any network is a continuous random process.



Discrete Random Process: In discrete random process, the random variable X has only discrete values while time, t is continuous. The below figure shows a discrete random process. A digital encoded signal has only two discrete values a positive level and a negative level but time is continuous. So it is a discrete random process.



Continuous Random Sequence: A random process for which the random variable X is continuous but t has discrete values is called continuous random sequence. A continuous random signal is defined only at discrete (sample) time intervals. It is also called as a discrete time random process and can be represented as a set of random variables {X(t)} for samples tk, k=0, 1, 2,….



Discrete Random Sequence: In discrete random sequence both random variable X and time t are discrete. It can be obtained by sampling and quantizing a random signal. This is called the random process and is mostly used in digital signal processing applications. The amplitude of the sequence can be quantized into two levels or multi levels as shown in below figures (d) and (e)



**STATIONARY PROCESS:**

From the definition of a **random process**, we know that all random processes are composed of random variables, each at its own unique point in time. Because of this, random processes have all the properties of random variables, such as mean, correlation, variances, etc.

When dealing with groups of signals or sequences it will be important for us to be able to show whether or not these statistical properties hold true for the entire random process. To do this, the concept of ***stationary processes*** has been developed. The general definition of a stationary process is:

**Definition : *stationary process***

* A random process where all of its statistical properties do not vary with time
* Processes whose statistical properties do change are referred to as ***nonstationary***

### **First-Order Stationary Process:**

* A random process is classified as ***first-order stationary*** if its first-order probability density function remains equal regardless of any shift in time to its time origin.

fX(x1; t1) = fX(x1; t1+∆)

* The physical significance of this equation is that our density function, fX(x1), is completely independent of t1 and thus any time shift, t
* The most important result of this statement, and the identifying characteristic of any first-order stationary process, is the fact that the mean is a constant, independent of any time shift

E(X(t)] = constant

E[X(t1+∆)] = E[X(t1)]

### **Second-Order and Strict-Sense Stationary Process:**

* A random process is classified as ***second-order stationary*** if its second-order probability density function does not vary over any time shift applied to both values.
* In other words, for values x1 and x2 then we will have the following be equal for an arbitrary time shift τ.

fX(x1, x2; t1, t2) = fX(x1, x2; t1+∆, t2+∆)

τ = t2 – t1

* From this equation we see that the absolute time does not affect our functions, rather it only really depends on the time difference between the two variables.
* These random processes are often referred to as ***strict sense stationary (SSS)*** when **all** of the distribution functions of the process are unchanged regardless of the time shift applied to them.
* For a second-order stationary process, we need to look at the [**autocorrelation function**](http://cnx.org/content/m10676/latest/) to see its most important property. Since we have already stated that a second-order stationary process depends only on the time difference, then all of these types of processes have the following property:

|  |  |  |
| --- | --- | --- |
| *Rxx*(*t*,*t*+*τ*) | = | *E*[*X*(*t*+*τ*) ] |
|  | = | *Rxx*(*τ*) |

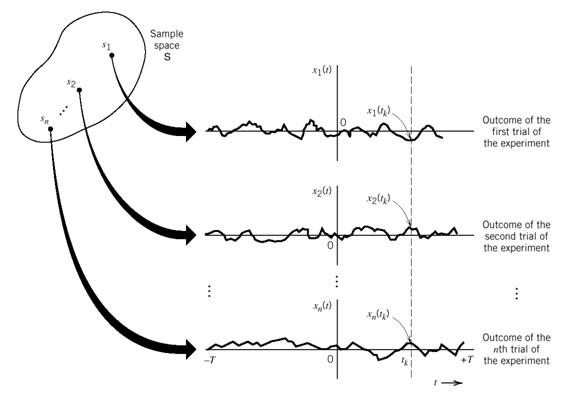
### **Wide-Sense Stationary Process:**

* Many practical problems require that we deal with the autocorrelation function and mean value of a random process.
* Problem solutions are greatly simplified if these quantities are not dependent on absolute time. Of course, **SSS** is sufficient to guarantee these characteristics. However, it is often more restrictive than necessary, and a more relaxed form of stationarity is desirable.
* The most useful form is the Wide Sense Stationary process, defined for which two conditions are true:

***X*=*E{x*[*n*] } =constant and *E*[*X*(*t*+*τ*) ] =*Rxx*(*τ*)**

**Time Averages and Ergodicity:**

* For a random process X(t) there can be two types of averages computed. They are (1) Ensemble averages (2) Time averages. Ensemble averages are computed over the ensemble of the random process by considering all the sample functions.



**Figure:** An ensemble of sample functions

* Consider the ensemble of a random process as shown in the above figure, at T= t1 take the corresponding Y-coordinates from the sample functions (let 20). Then, their average is sum of all the 20 Y-coordinates divided by 20. This is an example for ensemble average.
* Time averages are computed over the entire time.
* The time average of a quantity is defined as (x̄)



* If the time averages of the process are equal to the ensemble averages of X(t) [i.e.,E[X(t)]], then the process is said to be ergodic process.

E{A[x(t)] }= E[X(t)]

* Any time average will be independent of time for an ergodic process, since ensemble averages are same as time averages, it implies that the ensemble averages are also independent of time, which is the condition for stationarity.
* Thus, every ergodic process is a stationary process, but the converse in not true.

**Ergodicity implies the following:**

* Some stationary processes may be such that almost every member (sample function) of the ensemble posses the same statistical performance as possessed by the ensemble. Then, the statistical behavior of the ensemble (overall random process) can be determined from only one time function (sample function). Such a process is called ergodic process.

**Gaussian Random Process:**

* Let X(t) be a random process and let X(t1), X(t1), X(t1),………. X(t1) be the random variables obtained from X(t) at t=t1,t2,…..tn sec respectively.
* Let all these random variables be expressed in the form of a matrix
* Then, X(t) is referred to as normal or Gaussian process if all the elements of X are jointly Gaussian i.e., density of X is multivatiate Gaussian density.

**Poisson Random Process:**

* Poisson process describes the number of times that some event has occurred as a function of time, and the event is considered to occur at random times.
* **Example:** Arrival of a customer at a bank, telephone call coming in a telephone exchange.
* In the above examples, a single event occurs ata random time and counting the number of such occurrences with time is considered by the Poisson process. Hence, Poisson process is also referred to as Poisson counting process.

**Conditions required defining the Poisson process:**

1. We require that the event occur only once in any vanishingly small interval of time. In essence, we require that only one event can occur at a time.
2. We require that occurrence times be statistically independent so that the number that occurs in any given time interval is independent of the number in any other non-overlapping time interval.

* As a consequence of the two conditions in that the number of event occurrences in any finite interval of time is described by the Poisson distribution where the average rate of occurrences is denoted by λ.
* Thus replacing λ with λt, the probability of exactly k occurrences over a time interval (0,t) is

P[X(t)=k] = (λt)k e-λt/(k!) k=0,1,2,…..

**White Noise:**

* A very commonly use random process is white noise. White noise is often used to model the thermal noise in electronic systems.
* The random process X(t) is called white noise if X(t) is a stationary Gaussian random process with zero mean and flat power spectral density,

**SX(f) = N0/2, for all *f***

**Assignment cum tutorial questions**

1. **Questions testing the remembering and understanding level of students.**
2. **Objective questions:**
3. Time average of x(t) is A[x(t)]= \_\_\_\_\_\_\_\_\_\_\_\_\_
4. For an ergodic process,
5. Mean is necessarily zero (b) mean square value is infinity
6. Mean square value is independent of time (d) all time averages are zero
7. A R.P is a R.V that is a function of
8. Time (b) temperature (c) both (d) none
9. A stationary R.P X(t) will have its\_\_\_\_ properties not affected by a shift in time.
10. Mathematical (b) normal (c) statistical (d) none
11. All strict sense stationary processes are wide sense stationary. True or false.
12. If all the statistical properties of X(t) are not affected by time shift is referred as WSS. (TRUE/FALSE)
13. \_\_\_\_\_\_\_\_ averages are computed by considering all the sample functions.
14. Time (b) ensemble (c) both (d) none.
15. If the future value of the sample function can be predicted based on its past values, then the process is referred to as
16. Deterministic (b) Non-deterministic (c) Independent (d) Statistical

**(ii) Descriptive questions:**

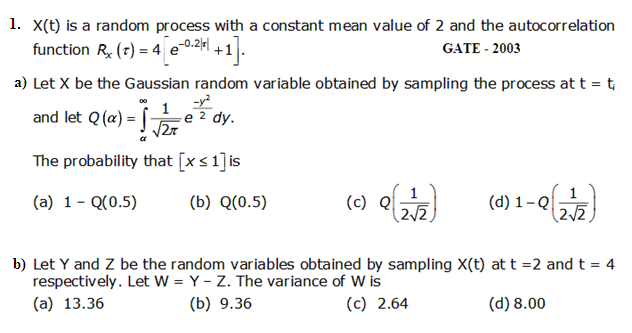
1. Define a random process and classify random processes with neat sketch
2. When is a random process said to be ergodic ?
3. Define and distinguish wide sense stationary & strict sense stationary random processes
4. Explain the following:

a) White Noise

b) Poisson R.P.

c) Guassian R.P.

1. **Questions testing the ability of students in applying the concepts.**
2. **Multiple Choice Questions:**
3. For the random process X(t)= Acosωt where ω is a constant and A is uniform over (0,1) the mean square value is
4. 1/3 (b) (1/3)cosωt (c) (1/3)cos2ωt (d) 1/9
5. X(t) is a R.P defined as X(t) = cosΩt, where Ω is a uniform R.V over (0, ω0). Then, mean of X(t) is zero at t=
6. 3π/2ω0 (b) π/ω0 (c) π/2ω0 (d) π/4ω0
7. A random process is defined as X(t) = Acos(ωt+θ), where ω and θ are constants and A is a random variable. Then, X(t) is stationary if
8. E[A]=2 (b) E[A]=0
9. A is Gaussian with non zero mean (d) A is Rayleigh with non zero mean.
10. The mean square value for the Poisson process X(t) with parameter λt is
11. λt (b)(λt)2 (c) λt + (λt)2 (d) λt - (λt)2
12. Difference of two independent Poisson processes is :
13. Poisson process (b) not a Poisson process
14. Process with mean=0 (d) Process with var = 0
15. For a Poisson random process with b=λt, the probability of exactly K occurrences over the time interval (0,t) is P[X(t)=K] is (K = 0,1,2,….)
16. (λte-λt)/K! (b) (e-λt)/K! (c) (tte-λt)/K! (d) [(λt)te-λt]/K!
17. **Problems :**
18. A random process Y(t) is given as Y(t) = X(t)cos(ωt+θ), where X(t) is a WSS R.P, ‘ω’ is a constant and ‘θ’ is a random phase independent of X(t), uniformly distributed on (π, -π). Find E[Y(t)]
19. If a random process X (t) =Acosωt + Bsinωt is given, where A and B are uncorrelated zero mean random variables having the variance σ2. Show that X(t) is wide sense stationary.
20. A random process is given as X(t) =At, where A is an uniformly distributed random variable on (0,2). Find whether X(t) is WSS or not.
21. If Y1(t) =X1cosωt + X2sinωt and Y2(t) = X1sinωt + X2cosωt where X1 and X2 are zero mean independent random variables with unity variance. Show that the random processes Y1(t) and Y2(t) are individually WSS.
22. A random process Y(t) = X(t)-X(t+τ) is defined in terms of a process. X(t) that is at least WSS. (a) show that mean value of Y(t) s zero even if X(t) has a non zero mean value. (b) If Y(t) =X(t)+X(t+τ). Find E(Y(t)] and σ2 of Y.
23. Telephone calls are initiated through an exchange of the average rate of 75 per minute and are described by a Poisson process. Find the probability that more than 3 calls are initiated in any 15 second period.
24. A random process is defined by X(t)=A, where A is a continuous random variable uniformly distributed on (0,1). (a) Classify the process. (b) Is it deterministic?
25. The two-level semi random binary process is defined by X(t) = A or –A ; (n-1)T< t<nT where the levels A and –A occur with equal probability, T is a positive constant, and n= 0, ±1, ±2, …. (a) Sketch a typical sample function. (b) Classify the process. (c) is the process deterministic?
26. **Questions testing the analyzing and evaluating ability of students:**
27. Define a random process X(t) as follows: (1) X(t) assumes only one of two possible levels 1 or -1 at any time , (2) X(t) switches back and forth between its two levels randomly with time, (3) the number of level transitions in any time interval τ is a Poisson random variable, that is, the probability of exactly k transitions, when the average rate of transitions is λ, is given by [(λτ)k/k!] exp(-λτ), (4) transitions occurring in any time interval are statistically independent of transitions in any other interval, and (5) the levels at the start of any interval are equally probable. X(t) is usually called the random telegraph process. It is an example of a discrete random process.
28. Find the autocorrelation function of the process.
29. Find probabilities P[X(t)=1] and P[X(t)=-1] for any t.
30. What is E[X(t)]?
31. Discuss the stationarity of X(t).
32. **GATE:**

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**UNIT V**

**RANDOM PROCESSES – SPECTRAL CHARACTERISTICS**

**Objective:**

* To understand the temporal and spectral characteristics of random processes

**Syllabus:**

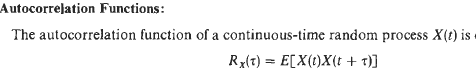
Auto Correlation, Cross Correlation-Properties, Power Spectral Density, Cross Spectral Density-Properties, Wiener Khintchine Relation, Relationship between Cross Power Spectrum and Cross Correlation Function

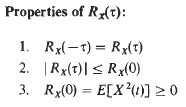
**Learning Outcomes:**

At the end of the unit student will be able to:

1. Define Auto Correlation Function and its properties
2. Explain the Cross Correlation and its properties
3. Understand Wiener Khintchine Relation and calculate Power Spectral Density, Cross Spectral Density and understand the properties.
4. Distinguish between Relationship between Cross Power Spectrum and Cross Correlation Function.

**Learning Material**



 *This is o*ften called the *average power* of *X(t)*

In the case of a discrete-time random process *X(n),* the autocorrelation function of *X(n)* is defined by

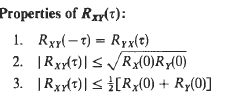
*Rx(k)* = *E[X(n)X(n* **+** *k)]*

**Cross-Correlation Functions**

The cross-correlation function of two continuous-time jointly **WSS** random processes *X(t)* and

*Y(t)* is defined by

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* Two processes *X(t)* and *Y(t)* are called *(mutually) orthogonal* if

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* Similarly, the cross-correlation function of two discrete-time jointly **WSS** random processes *X(n)* and

*Y(n)* is defined by

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**Power Spectral Density:**

The *power spectral density* (or *power spectrum) Sx(w)* of a continuous-time random process *X(t)* is defined as the Fourier transform of C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png

C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png**,** Thus, taking the inverse Fourier transform of *Sx(w),one gets*

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The above two relations are known as the ***Wiener-Khinchine relations****.*

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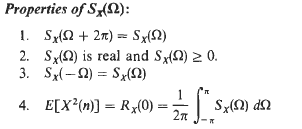
Similarly, the power spectral density Sx(Ω) of a discrete-time random process X(n) is defined as the

Fourier transform of Rx(k):

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Thus, taking the inverse Fourier transform of Sx(Ω)

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**Cross Power Spectral Densities:**

The cross power spectral density (or cross power spectrum) Sxy(w) of two continuous-time random

processes X(t) and Y(t) is defined as the Fourier transform of RXY(C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png):

C:\Users\JASWANTH\AppData\Local\Temp\msohtmlclip1\01\clip_image001.png Thus, taking the inverse Fourier transform of Sx(w),one gets,

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***Properties of SX(w)*** :

Unlike Sx(w), which is a real-valued function of w, Sxy(w), in general, is a complex-valued function.

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Similarly, the cross power spectral density SXY(w) of two discrete-time random processes X(n) and

Y(n) is defined as the Fourier transform of Rxy(τ):

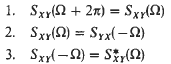
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Thus, taking the inverse Fourier transform of Sxy(Ω),

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***Properties of SXY (Ω):***

Unlike ***SX (Ω),*** which is a real-valued function of ***w,*** Sxy(*Ω*), in general, is a complex-valued function



**Weiner khintzine Relation:**

This states the relationship between the Power spectral density and Auto Correlation function.

The inverse Fourier Transform of Power Spectral density is equal to Time average of Auto Correlation Function. The proof of it is given below:

**Relationship between Power Spectrum and autocorrelation function:**

The inverse Fourier Transform of Power Spectral Density is equal to Time Average of Auto Correlation Function and it is expressed as:

(ω)dω = A[(t,t+T)]

Consider L.H.S.,and in that consider the power spectral density, the formula for power spectral density is,

Sxx(ω)=)

=

E[x(t1)X(t2)]=R*xx*(t1,t2) -T<(t1 and t2)<T

S*xx*(ω)=

Substitute the above equation in actual L.H.S.,

=

=

=> =

Integration of Impulse within the interval assumes the value of unity.

= -T < t+< T

A[R*xx*(t,t+] = dt

S*xx*(=

A[R*xx*(t,t+] <--> S*xx*((Hence Proved)

S*xx*(

R*xx (*τ*)*

They form Fourier Transform pairs.

R*xx*(

**Relationship Between Cross-Power Spectrum And Cross-correlation Function:**

The relation between the cross spectral density and cross correlation function is given by:

S*xy(*

Proof: Consider two random processes X(t) and Y(t) respectively. Their Fourier Transform being:

X*t*(

Yt(

X\*t(

Considering the Cross Power Spectral Density,

S*xy*(

=

=

=>

=

=

=

(Hence Proved)

This is the relation between Cross Power Spectrum and Cross Correlation Function.

**Assignment cum Tutorial Questions**

**A) Questions testing the understanding and remembering level of students.**

**(i) Multiple choice questions.**

1. X(t1) = X1 and X(t2)=X2. The correlation between X1 and X2 is R(t1,t2)=

a) b)

c) d)

2. The auto covariance COVX(t1,t2) of a random process X(t) is

a) RXX(t1,t2)-E[X(t1)] b) RXX(t1,t2)-E[X(t2)]

c) RXX(t1,t2)-E[X(t1)]E[X(t2)] d) RXX(t1,t2)+E[X(t1)]+E[X(t2)]

3. The mean of a R.P X(t) is the expected value of the random variable X at time t, i.e., the mean

m(t)=

a) b) c) d)

4. If RXY=0, then X and Y are

a) Independent b) Orthogonal c) Independent & Orthogonal d) Statistically Independent

5. A random process is defined as X(t)=cos(ω0t+θ), where θ is a uniform random variable over

(-π, π). The second moment of the process is

a) 0 b) ½ c) ¼ d) 1

6. Let X(t) and Y(t) be two random processes with respective auto correlation functions Rxx(τ) and Ryy(τ). Then |Rxy(τ)| is

a) = b) ≥ c) ≤ d) >

7. A stationary continuous process X(t) with autocorrelation function Rxx(τ) is called

autocorrelation ergodic for all τ, if and only if

a) b)

c) d)

8. The power density spectrum of XT(t) =X(t) for –T<t<T; XT(t) =0 , elsewhere; is SXX(ω) =

a) b) c) d)

9. The average power of the random process having PSD Sxx(ω) is Pxx =

a) zero b) c) d)

10. The time average of the autocorrelation function and the power spectral density form a pair

of

a) Fourier transform b) Laplace transform c) Z-transform d) convolution

11. The power spectral density of WSS is always

a) negative b) non-negative c) finite d) can be negative or positive

12. For a WSS process, PSD at zero frequency gives

a) the area under the curve of a power spectral density

b) area under the curve of its autocorrelation

c) mean of the process

d) variance of the process

13. The mean square value of a WSS process equals

a) the area under the PSD b) the area under the Rxx(τ)

c) zero d) mean of the process

14. The cross spectral density Syx(ω) =

a) Sxy(ω) b) Sxy(-ω) c) Syx(-ω) d) –Syx(ω)

15. The cross power Pxy is given by

a) b) c) d)

**(ii) Descriptive Questions**

1. A random process X[n] is stationary. If it is known that E[X(10)] = 10 and var(X[10]) = 1,

Then determine E[X[100]] and var(X[100]).

2. State and prove the properties of auto correlation & cross correlation.

3. State and prove the properties of power spectral density & cross power spectral density.

4. State and prove Weiner Khintchine relation

**B. Questions testing the ability of the students in applying the concepts.**

**(i) Multiple Choice Questions:**

1. A random process is defined as X(t) =Acost(ω1t+θ), where X(t) is a uniform random variable over (0,2π). Then Rxx(τ) is

a) A2cosωcτ b) cosωcτ c) cosωcτ d) A2cosωcτ

2. The autocorrelation function of a stationary random process X(t) is Rxx(τ) = 25+ . The

mean and variance are

a) 4, 25 b) 25, 4 c) 21, 2 d) 5, 4

3. The mean square value of a random process whose autocorrelation function is

a) b) c) d)

4. A random process has the PSD Sxx(ω) = . The average power in the process is

a) 1.06 W b) 2.06 W c) 0.06W d) 0W

5. If Sxx(ω) = , Syy(ω) = and X(t) and Y(t) are of zero mean where U(t) = X(t) + Y(t), then Sxu(ω) is

a) b) c) d)

6. A WSS process, X(t), has an autocorrelation function Rxx(τ)=. Then PSD is

a) b) c) d)

7. The autocorrelation function of a WSS process is R(τ) = K. Then its spectral density is

S(ω) =

a) b) c) d) [

8. If Sxx(ω) = η/2 for -2πB≤ω≤2πB, then Rxx(τ) is

a) ηBsinc(2πB b) Bsinc(2πB c) sinc(2πB d) sinc(2πB

**(ii) Descriptive Questions:**

1. A random process is the sum of WGN and a deterministic sinusoid and is given as X[n] =

U[n] + sin(2πf0n) for all n, where U[n] is WGN with variance . Determine the mean and

covariance sequences.

3. A random process is defined as X[n] = AU[n] for all n, where A ~ N(0, ) and U[n] is WGN

with variance , and A is independent of U[n] for all n. Find the mean and covariance

sequences. What type of random process is X[n]?

4. Let X(t) = cos(ωt + θ) and Y(t) = sin(ωt + θ) where θ is a random variable uniformly

distributed in [π, -π]. Find the cross-covariance of X(t) and Y(t).

5. Suppose process Y(t) consists of a desired signal X(t) plus noise N(t): Y(t) = X(t) + N(t). Find

the cross-correlation between the observed signal and the desired signal assuming that X(t)

and N(t) are independent random processes.

6. Consider a WSS random process X(t) with RX(τ)=e−a|τ|, where a is a positive real number. Find

the PSD of X(t).

7. The cross spectral density of two random process X(t) and Y(t) is Sxy(ω) = 1+ for –k<w<k,

and zero elsewhere, where k>0. Find the cross correlation function between the processes.

8. Prove that cross correlation function and cross spectral density are Fourier transform pair.

9. Consider a random process x(t) = cost(ωt+θ) where ω is a real constant and θ is a uniform

random variable in (0, π/2). Find the average power in the process.

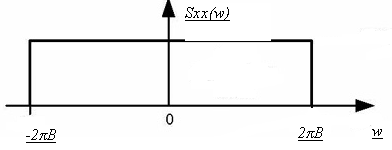
10. Find the autocorrelation function of the following power spectral densities.

a) = b) =

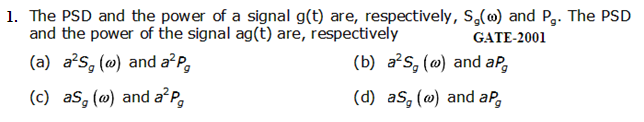
**C.Questions testing the analyzing and evaluating ability of students:**

1. Determine the auto correlation function Rxx(τ) and the power of the low pass random process

with power spectral density Sxx(ω) = η/2, which is shown as



**D. GATE:**

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1. The power spectral density of a real stationary random process X(t) is given by

Sx(f) = 1/w; if |f|≤ w and = 0; if |f|>w

The value of the expectation E[πX(t)(t-w)] is\_\_\_\_\_\_\_\_\_\_. **[GATE-2014]**

1. Consider random process X(t) = 3V(t) −8, where V (t) is a zero mean stationary random

process with autocorrelation Rv(τ) = 4e−5|τ|. The power is X(t) is\_\_\_\_\_\_ **[GATE-2016]**

1. The power spectral density of a deterministic signal is given by |sin(f)/f2| where f is frequency. The autocorrelation function of the signal in the time domain is

(a) a rectangular pulse (b) a delta function **[GATE-1997]**

(c) a sine pulse (d) a triangular pulse

5. The spectral density of a real valued random process has **[GATE-1998]**

(a) an even symmetry (b) an odd symmetry

(c) a conjugate symmetry (d) no symmetry

**UNIT VI**

**LINEAR SYSTEMS WITH RANDOM INPUTS**

**Objective:**

To understand and describe the response of a linear system when the random processes and white-noise are applied.

**Syllabus:**

Linear time invariant system, System transfer function, linear systems with random inputs, Auto correlation and cross correlation functions of input and output. Examples with white-noise as input

**Learning Outcomes:**

At the end of the unit student will be able to:

1. Define the response of a Linear Time Invariant(LTI) system and its System Transfer Function
2. Express Mean and Mean-Squared Value of System Response
3. Express Auto correlation and Cross correlation functions of input and output
4. Express Power Density Spectrum and Cross Power Density Spectrum of System Response
5. Calculate System Response of the functions(Mean, Mean-Squared Value, Auto correlation, Cross correlation, Power Density Spectrum and Cross Power Density Spectrum) when white –noise is an input

**LEARNING MATERIAL**

**Linear System:**

In general, the linear system will cause the response y(t) operating on x(t) and is represented as **y(t) = L[x(t)]**; where L is an operator representing the action of system on x(t).

A system is said to be linear if its response to a sum of inputs xn(t), n=1,2,….,N is equal to the sum of responses taken individually.

Thus if xn(t) causes a response yn(t), n=1,2,….,N, then for a linear system; yn(t) = L[xn(t)]

= L[αnxn(t)]

= L[αnxn(t)]

**yn(t) = αnyn(t)]** must valid;

where αn are arbitrary constants and N may be infinite.

From the definition and by the property of impulse function; x(t) is represented as **x(t) = .**

By substituting x(t)in y(t)= L[x(t)];

y(t) = L[**]**

and let the operator operates on the time function; then

y(t) **=**

let **=** andis defined as the impulse response of linear system.

**y(t) =**

The above expression is the output response of a linear system and is completely determined by its impulse response.

**Linear Time-Invariant System:**

A general linear system is said to be linear time-invariant if the impulse response does not depend on the time that the impulse is applied.

Thus if an impulse (t), occurring at t=0, causes the response h(t), then an impulse 𝛅(t-), occurring at t= must causes the response h(t-) if the system is time-invariant; i.e.  **= .**

Hence output response of a linear time-invariant system is given by

**y(t) =**

and is also known as convolution integral of and is given by

**y(t) = =**

by a suitable change of variables, alternative form defined below is also valid

**y(t) = =**

**Time-Invariant System Transfer Function [H(ω)]:**

H(ω), Y(ω) and X(ω) are the respective Fourier transforms of h(t), y(t) and x(t). **Y(ω) = F[y(t)] = y(t) e-jwt dt**

= e-jwt dt

= e-jw(𝛏) e-jw(t-𝛏)dt

= e-jw(𝛏) e-jw(t-𝛏)dt

**Y(ω) = X(ω) H(ω)**

Where the function **H(ω)**  is called the transfer function of the system. Hence Fourier transform for the response of any linear time-invariant system is the product of transfer function of input signal and transfer function for the network impulse response.

**Causal System:**

A linear time-invariant system is said to be causal if it does not respond to the application of input signal. Mathematically this implies that y(t) = 0 for t < t0 if x(t) = 0 for t < t0 and requires that h(t) = 0 for t < 0; where t0 is any real constant

**Stable System:**

A linear time-invariant system is said to be stable if its response to any bounded input is bounded; i.e. if |x(t)| < M then |y(t)| < MI for a stable system; where M is a constant and I is a constant independent of input and given by I = ; having the impulse response h(t) will be stable.

**Linear Systems With Random Inputs:**

**System Response-Convolution:**

* When x(t) is a random signal and h(t) is the networks impulse response then the networks response y(t) is given the convolution integral. **y(t) = .**
* If a random process X(t) is applied to a linear time-invariant system whose impulse response is h(t) then it produces a new random process Y(t); given by **Y(t)= =**

**Mean Value of System Response:**

Consider X(t) is a WSS; thenMean Value of System Response is obtained by;E[Y(t)] = E[

=

=

**=**

Thus mean value of system response Y(t) equals the mean value of X(t)

times the area under the impulse response if X(t) is a WSS.

**Mean-Squared Value of System Response:**

Mean-Squared Value of System Response is obtained by;

E[Y2(t)]= E[2] = E[[

=

**=** E[]

**=**

Consider input is WSS; then **E[ = Rxx(𝛏1-𝛏2);** hence

**E[Y2(t)] =**

**Auto Correlation Function Of Input And Output:**

Let X(t) be WSS; then autocorrelation function of Y(t) is obtained by

RYY (t, t+τ) = E[Y(t) Y(t+τ)]

=E[ =

As X(t) is assumed WSS, then the above expression reduces to

RYY (τ) = RXX

1. Y(t) is WSS if X(t) is WSS because RYY (τ) does not depend on t and E[Y(t)] is a constant.
2. RYY (τ) is the two-fold convolution of the input autocorrelation function with the networks impulse response given by

**RYY (τ) = RXX (τ) \* h(-τ) \* h(τ)**

**Cross Correlation Function Of Input And Output:**

The cross-correlation function of X(t) and Y(t) is

RXY (t, t+τ) = E[X(t) Y(t+τ)]

= E[X(t) ]

=

If X(t) is WSS, the above expression reduces to

RXY (τ) = RXX(τ-𝛏)

Which is the convolution RXX (τ) with h(τ)

**RXY (τ) = RXX (τ) \* h(τ)**

Similarly we will obtain

**RYX (τ) =RXX(τ-𝛏) = RXX (τ) \* h(-τ)**

**Relation between autocorrelation and cross correlation:**

From the expressions of autocorrelation function it is clear that

**RYY (τ) = RXY (τ) \* h(-τ)**

**RYY (τ) = RYX (τ) \* h(τ)**

**Power Density Spectrum of Response:**

Power density spectrum φYY(ω)of the response of a linear time-invariant system having a transfer function H(ω) is given by

**φYY(ω) = φXX(ω)|H(ω)|2**

where φXX(ω) is the power spectrum of input process X(t) and |H(ω)|2 is thepower transfer function of the system.

**Cross-Power Density Spectrum of Input and Output Response:**

Cross-Power density spectrum φXY(ω) and φYX(ω) of the response of a linear time-invariant system having a transfer function H(ω) are given by; **φXY(ω) = φXX(ω) H(ω)**

**φYX(ω) = φXX(ω)H(-ω)** respectively

**ASSIGNMENT CUM TUTORIAL QUESTIONS**

**A) Questions testing the understanding and remembering level of students.**

**(i) Multiple choice questions.**

1. A system is said to be linear system if it satisfies
2. Principle of superposition c) Principle of homogeneity
3. a and c d) Reciprocity principle
4. For an LTI system, the response y(t) for any input x(t), with impulse response is determined by using the following integral
5. Convolution b) Fourier c) Laplace d) b and c
6. The cross correlation between X(t) and y(t) is RXY(τ) =
7. h(τ)\* RXX(τ) b) h(-τ)\* RXX(τ) c) h(-τ)\* RXY(τ) d) h(τ)\* RYX(τ)
8. A random process X(t) of mean 3 is applied to a delay element. The mean of output process is
9. 2 b) 3 c) 1.5 d) 9
10. If PXX(ω) is the power spectrum of the input response X(t) and |H(ω)2| is the transfer function of the system, then the average power PYY is =
11. The output power density of Y(t) can be obtained by, PYY(ω) =
12. The cross power density of X(t) and Y(t) can be obtained by PYX(ω) =
13. H(ω)PXX(ω) b) H\*(ω)PXX(ω) c) H(ω)/PXX(ω) d) H\*(ω)/PXX(ω)

**(ii) Descriptive Questions**

1) Define the following systems

i) Linear System ii) Causal System iii) Stable System

1. Derive the relation for Time-Invariant System Transfer Function.
2. Derive the expression for the following
3. System Response
4. Mean Value of System Response
5. Mean Square Value of System Response & with White Noise as input
6. Auto-Correlation of System Response and with White Noise as input
7. Cross-Correlation of System Response and with White Noise as input
8. Derive the relation between psds of input and output random process of an LTI System.

**B) Questions testing the ability of the students in applying the concepts.**

**(i) Multiple Choice Questions:**

1. A stationary random process X(t) has auto-correlation function RXX(τ) = 10 + 5cos(2τ) + 10e-2|τ|; then dc average power of X(t) is
2. 10W b) 15W c) 25W d) 20W
3. A stationary random process X(t) has auto-correlation function RXX(τ) = 10 + 5cos(2τ) + 10e-2|τ|; then average power of X(t) is
4. 10W b) 15W c) 25W d) 20W
5. Mean Square Value of the output response for a system having h(t)=e-tu(t) and input of white noise with psd No/2 is
6. No/2 b) No/4 c) No/8 d) No/16
7. The psd of a random process is given by φXX(ω)=. Is it a valid psd. State True or False.
8. A WSS RP X(t) with psd ΦXX(ω) is applied at the input of a delay system as shown below; then psd of Y(t) is

X(t) Y(t)

***Σ***

***Delay 2T***

1. 2cos2(ωT) ΦXX(ω) c) 4cos2(ωT) ΦXX(ω)
2. 2cos2(ωT) ΦXX(ω) d) 4cos2(ωT) ΦXX(-ω)
3. Transfer function for the following circuit is

R

L

Input Output

C

1. (1+ω2RC)/( 1-ω2LC+jωRC) c) (1-ω2RC)/( 1+ω2LC+jωRC)
2. (1-ω2RC)/( 1-ω2LC+jωRC) d) (1+ω2RC)/( 1+ω2LC+jωRC)
3. A stationary random process X(t) and Y(t) has a psd given by φXX(ω)= and φYY(ω)=. Let another stationary random process be U(t)=X(t)+Y(t). assume that X(t) and Y(t) are uncorrelated with zero mean; then φUU(ω)=
4. ΦXX(ω)+ ΦXY(-ω)+ ΦYX(-ω)+ ΦYY(ω) c) ΦXX(ω)+ ΦXY(ω)+ ΦYX(ω)+ ΦYY(ω)
5. ΦXX(-ω)+ ΦXY(-ω)+ ΦYX(-ω)+ ΦYY(-ω) d) ΦXX(-ω)+ ΦXY(-ω)+ ΦYX(-ω)+ ΦYY(-ω)

**(ii) Descriptive Questions:**

1. A random process X(t) is applied to a network with impulse response h(t)=e-btu(t), where b>0 is constant. The cross correlation X(t) with the output Y(t) is known to have the form RXY(τ)=τe-bτu(τ). Find autocorrelation of Y(t).
2. A random process X(t) whose mean value is 2 and autocorrelation function is RXX(τ)=4e-2|τ| is applied to a system whose transfer function is . Find the mean value, autocorrelation, power density spectrum and average power of the output signal Y(t).
3. Two systems have transfer functions H1(ω) and H2(ω).
4. Show that transfer function H(ω) of the cascade of the two i.e. output of first feeds the input of the second system is H(ω)= H1(ω) H2(ω)
5. For a cascade of N systems with transfer functions Hn(ω); n=1,2,……N.

Show that H(ω)=

1. The autocorrelation of a WSS random process X(t) is given by RXX(τ)=Acos(ω0τ); where A and ω0 are constants. Find PSD.
2. Let jointly WSS processes X1(t) and X2(t) cause responses Y1(t) and Y2(t) respectively; from a linear time-invariant system with impulse responses h(t). if the sum X(t)=X1(t)+X2(t) is applied the response is Y(t). Find a) E[Y(t)]; b) RYY(t, t+τ) in terms of h(t) and characteristics of X1(t) and X2(t)
3. Let X1(t) and X2(t) are WSS processes. Show that
4. φYY(ω)=|H(ω)2|[φx1x1(ω) + φx1x2(ω) + φx2x1(ω) + φx2x2(ω)]
5. if X1(t) and X2(t) are statistically independent; then

φYY(ω)=|H(ω)2|[φx1x1(ω) + φx2x2(ω) + 4π ]

1. Consider a WSS random process X(t) with PSD as φXX(ω). Another random process is given by Y(t) = X(t+T)+ X(t-T); where T is a constant. Find the PSD of Y(t) ie. φYY(ω)

**C) Questions testing the analyzing and evaluating ability of students:**

1. A stationary random signal X(t) has an autocorrelation function RXX(τ)=10e-|τ|. It is added to a white noise for which No/2 = 10-3 and the sum is applied to a filter having a transfer function H(ω)=2/(1+jω)3. Find i) power spectrum and ii) average power in the output signal.

**D. GATE:**

1. (a) A Gaussian random variable with zero mean and variance σ is input to a limiter with

input output characteristic given by **[GATE-1991]**

eout=ein for |ein|<σ

eout=σ for|ein|≥σ

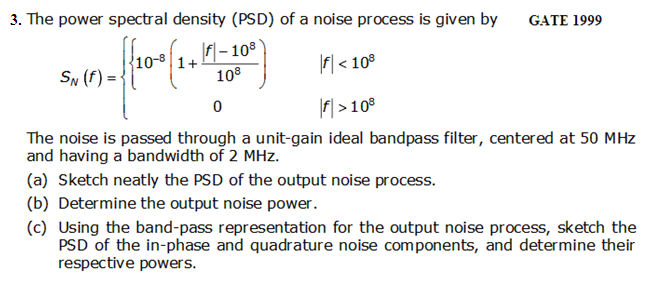
eout= -σ for |ein|≤σ

Determine the probability density function of the output random variable.

1. A random process X(t) is wide sense stationary. If Y(t)=X(t)-X(t-a). Determine the autocorrelation function RY(i) and power spectral density Sy(w) of Y(t) in terms of those of Y(t).
2. The probability density function of the envelope of narrow band Gaussian noise is

(a) Poisson (b) Gaussian **[GATE-1998]**

(c) Rayleigh (d) Rician

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