DATASTRUCTURES
UNIT–IV

TREES

A tree is a non-linear data structure that is used to represents hierarchical relationships between individual data items.

"A tree is a finite set of one or more nodes such that, there is a specially designated node called root. The remaining nodes are partitioned into n>=0 disjoint sets T1,T2,..Tn, where each of these set is a tree T1,...Tn are called the subtrees of the root."

REPRESENTATION OF TREES

Root: An Unique node in the tree to which subtrees are attached.

Branch: Branch is the link between the parent and its child.

Leaf: A node with no children is called a leaf.

Subtree: A Subtree is a subset of a tree that is itself a tree.

Degree: The total number of sub-trees of a node is called the degree of the node. The maximum degree in the tree is called degree of a tree.

Parent: The node having the sub-branches.

Children: The nodes branching from a particular node X are called children of X.

Siblings: Children of the same parent are said to be siblings.

Ancestors: Ancestors of a node are all the nodes along the path from root to that node. Hence root is ancestor of all the nodes in the tree.

Level:
Level of a node is defined by letting root at level one. If a node is at level L, then its children are at level L + 1.

Height or depth: The height or depth of a tree is defined to be the maximum level of any node in the tree.

Climbing: The process of traversing the tree from the leaf to the root is called climbing the tree.

Descending: The process of traversing the tree from the root to the leaf is

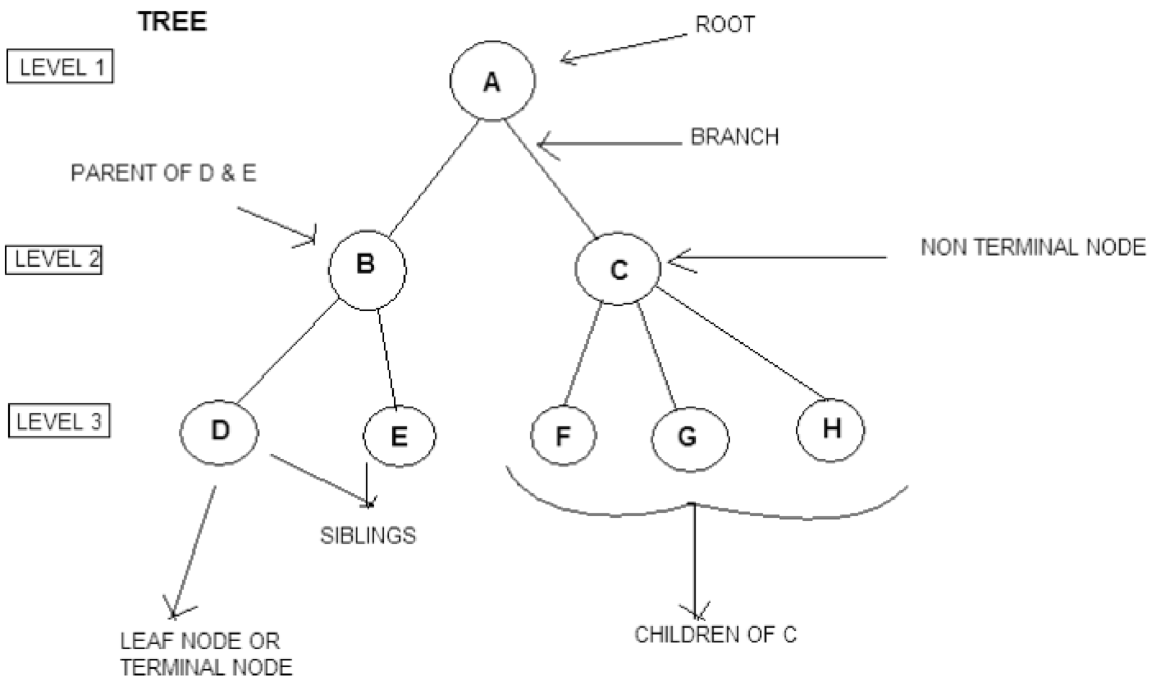
Called descending the tree.

Forest: It is a collection of disjoint trees. It is obtained by removing root.

Non–Terminals: The nodes other than root node and leaf nodes.

Predecessor: Consider the node X, then the node previous to node X is called predecessor node.

Successor: Consider the node X, then the node that comes next to node X is called successor node.

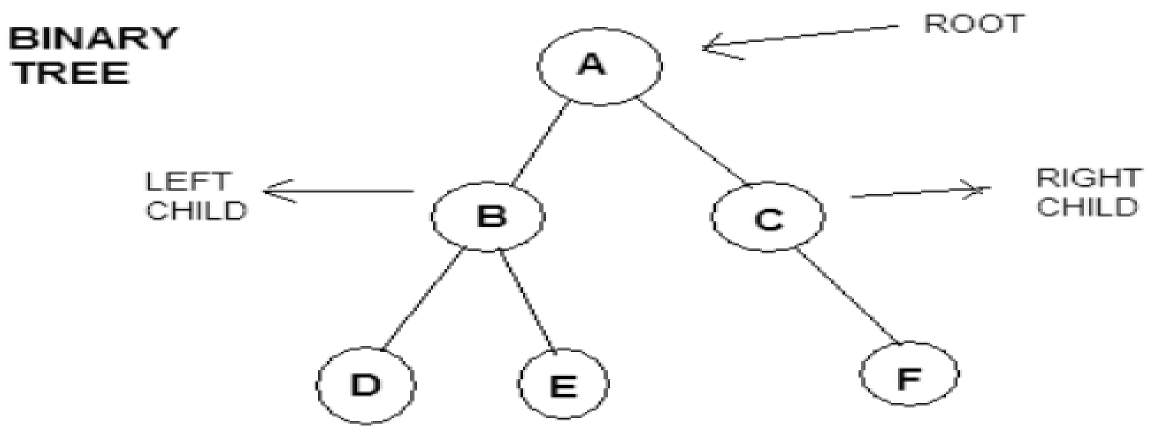


BINARYTREES

A binary tree is a tree either empty or consists two disjoint binary trees called the left subtree and right subtree.

Left child: The node present to the left of the parent node is called the left child.

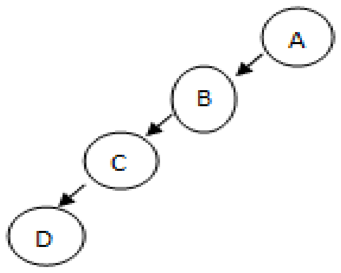
Right child: The node present to the right to the parent node is called the right child.



TYPES OF BINARY TREES

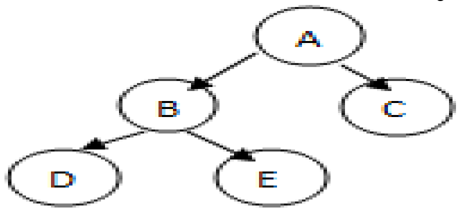
Skewed Binary tree

If the new nodes in the tree are added only to one side of the binary tree then it is a skewed binary tree.



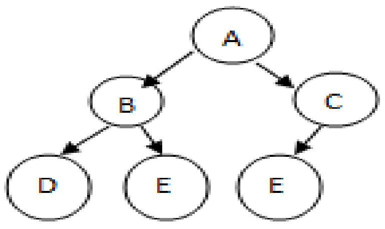
Strictly binary tree

If the binary tree has each node consisting of either two nodes or no nodes at all, then it is called a strictly binary tree.



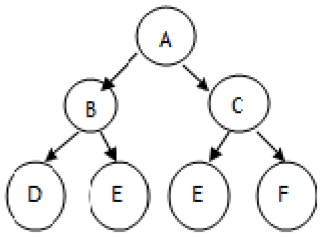
Complete binary tree

A complete binary tree is a binary tree in which every level except possibly the last is completely filled, and all nodes are as far left.



Fully binary tree

A fully binary tree is a binary tree in which every node other than the leaves has two children.



THE ABSTRACT DATA TYPE OF BINARY TREES

```
ADT Binary _tree
{
   instances:
        A finite set of nodes either empty or consisting of a root node,left
        Binary tree, right Binary tree
  operations:
        for all bt,bt1,bt2 ,Bin tree, item Єelement

        Bin tree create()          - creates an empty binary tree

        Boolean Is                 - if(bt==empty) return true else return false
        empty(bt)

        Bin tree Make bt((bt1,item,bt2)-return binary tree whose left sub tree is
                                      bt1 and whose right sub tree is bt2 and
                                      whose root node contains data item.
        Bin tree Lchild(bt)           -if(Is empty(bt) return error else return
                                      the left sub tree of bt.
        Bin tree Rchild(bt)           -if(Is empty(bt) return error else return
                                      the right sub tree of bt.
        Bin tree Data(bt)             -if(Is empty(bt) return error else return
                                      the data in the root node of bt.
}
```

PROPERTIES OF BINARY TREES

Some of the important properties of a binary tree are as follows:

1. The maximum number of nodes on level i of a binary tree is 2^{i-1},i>=1
2. The maximum number of nodes in a binary tree of depth k is 2^{k}-1, k>=1.
3. The total numberofedges in a full binary tree with n node is n- 1.

BINARY TREE REPRESENTATION

There are two ways in which a binary tree can be represented. They are:

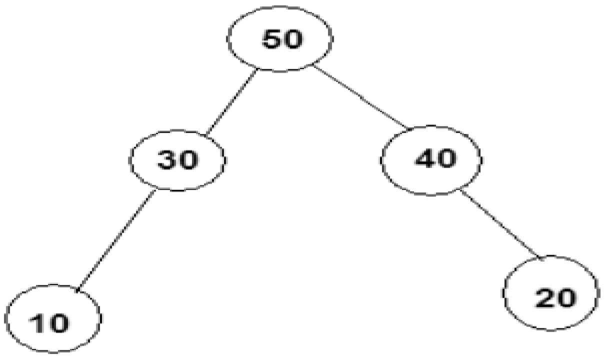
1.Array representation of binary trees.

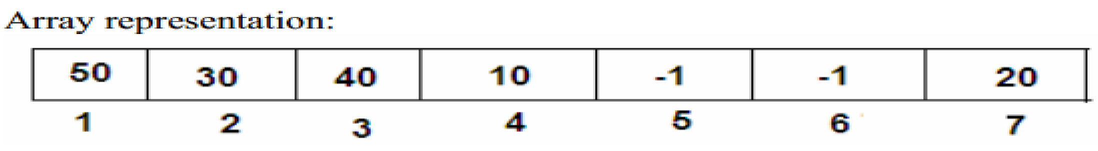
2.Linked representation of binary trees.

1. ARRAY REPRESENTATION OF BINARYTREES

When arrays are used to represent the binary trees, then an array of size 2^k is declared, where k is the depth of the tree.For example if the depth of the binary tree is 3, then maximum 2^3-1=7 elements will be present in the node and hence the array size will be 8.This is because the elements are stored from position one leaving the position 0 vacant.

But an array of bigger size is declared so that later new nodes can be added to the existing tree.The following binary tree can be represented using arrays as shown.




Array representation:

50	30	40	10	-1	-1	20
1	2	3	4	5	6	7

The root element is always stored in position1.The left child of node i is stored in position 2i and right child of node is stored in position 2i+1. The formulas for identifying the parent, left child and right child of a particular node.

Parent (i)=i/2,if I≠1.If i=1then I is the root node and root does not has parent.
Leftchild (i)=2i,if 2i≤2n,where n is the maximum number of elements in the tree. If 2i > n,then i has no left child.
Rightchild (i)=2i+1,if 2i+1≤2n.If 2i+1>n,then i has no right child.

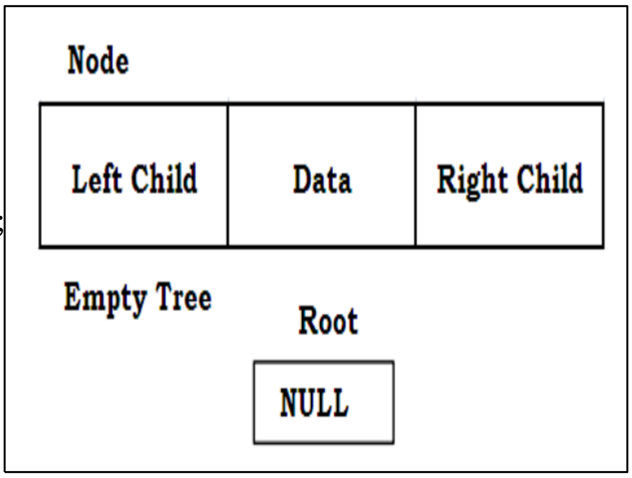
The empty positions in the tree where no node is connected are represented in the array using -1,indicating absence of a node.Using the formula,we can see that for a node 3,the parent is 3/2 is1.Referring to the array locations, we find that 50 is the parent of 40.The left child of node 3 is

2*3 is 6.But the position 6 consists of -1 indicating that the left child does note x is t for the node 3.Hence 50 does not have a left child.The right child of node 3 is 2*3+1 is 7.The position 7 in the array consists of 20.Hence, 20 is the right child of 40.

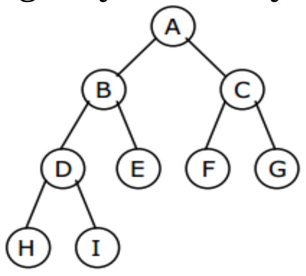
2. LINKED REPRESENTATION:

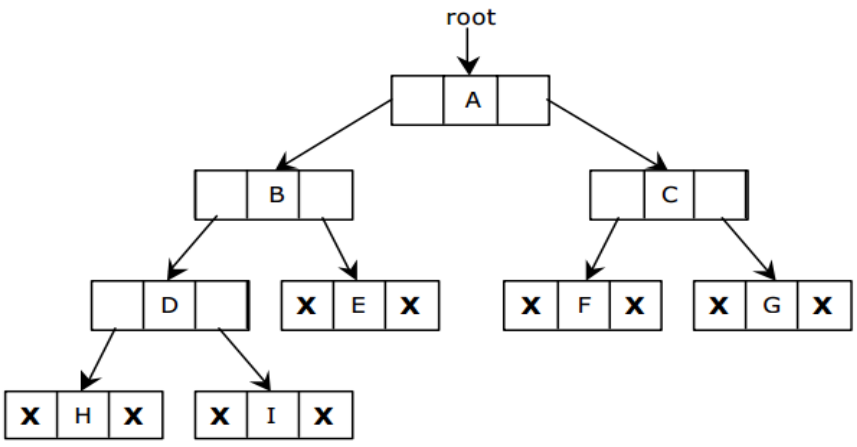
In linked representation of binary trees, instead of arrays, pointers are used to connect the various nodes of the tree.Hence each node of the binary tree consists of three parts namely, the data, left and right.The data part stores the data, left part stores the address of the left child and the right part stores the address of the right child.

```
Struct binary tree
{
        Struct binary tree *Left Child;
        int data;
        Struct binary tree *Right Child;
};
Struct binary tree node;
node *root = NULL;
```



Logically the binary tree in linked form can be represented as shown.





The pointers storing NULL value indicates that there is no node attached it. Traversing through this type of representation is very easy.

The left child of a particular node can be accessed by following the left link of that node and the right child of a particular node can be accessed by following the right link of that node.

BINARY TREE TRAVERSALS

A tree traversal is a method of visiting every node in the tree.By visit, we mean that some type of operation is performed.For example,we may want to print the contents of the nodes.There are three standard ways of traversing a binary tree T with root R.They are:
(i) Pre order Traversal

(ii) In order Traversal

(iii) Post order Traversal

Pre order Traversal

(1) Process the root R.

(2) Traverse the left sub tree of R in preorder.
(3) Traverse the right sub tree of R in preorder.

In order Traversal

(1) Traverse the left sub tree of R in inorder.
(2) Process the root R.
(3) Traverse the right sub tree of R in inorder.

Post order Traversal

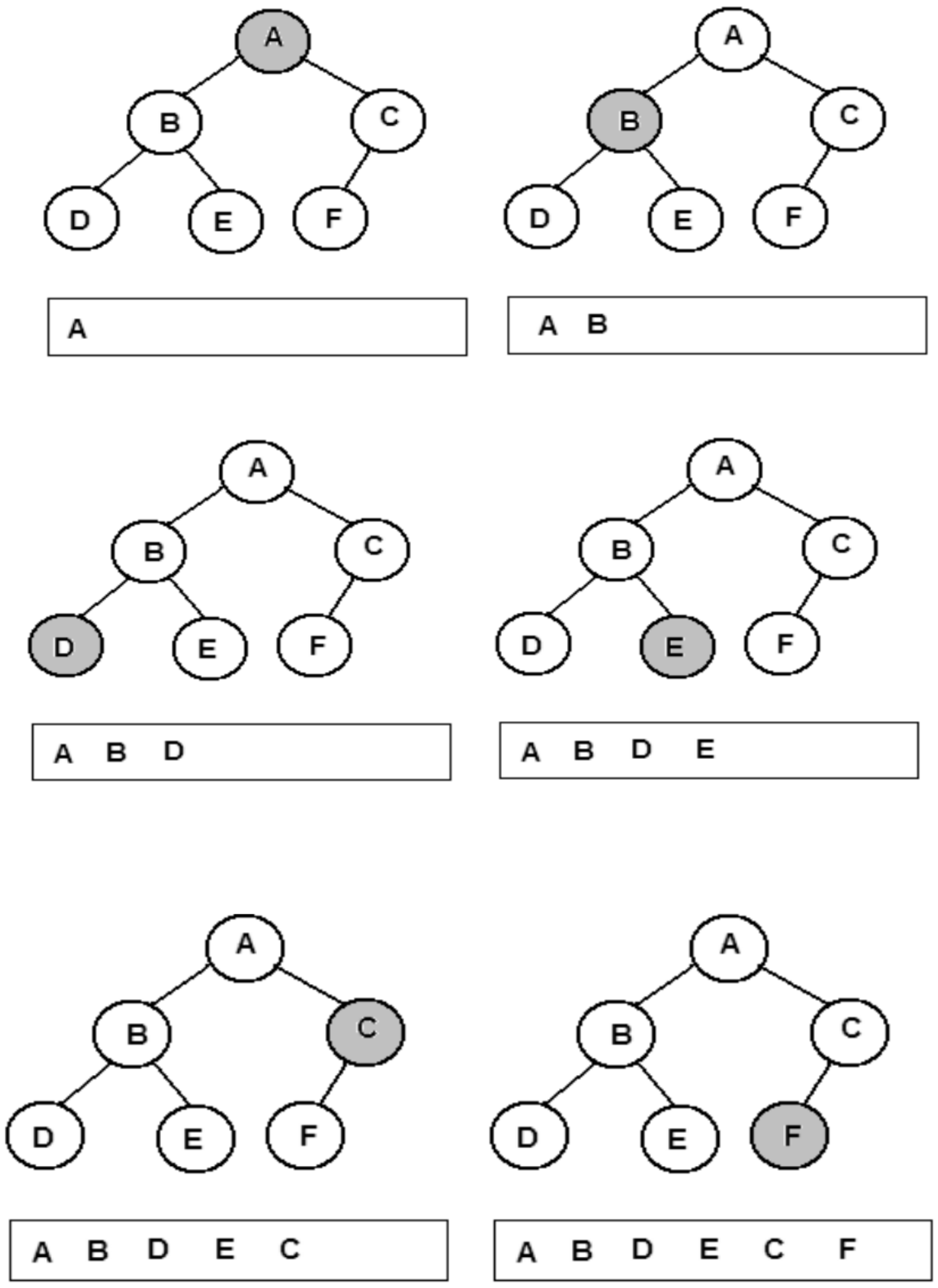
(1) Traverse the left sub tree of R in post order.
(2) Traverse the right subtree of R in post order.
(3) Process the root R.

Observe that each algorithm m contains the same three steps,and that the left subtree of Risal ways traversed before the right subtree.The difference between the algorithms is the time at which the root R is processed.The three algorithms are sometimes called the node-left-right (NLR)traversal,the left-node-right(LNR)traversal and the left-right-node (LRN) traversal. Traversal algorithms using recursive approach.

Pre order Traversal

In the pre order traversal, the node element is visited first and then the left sub tree of the node and then the right sub tree of the node is visited. Consider we have 6 nodes in the tree A,B,C,D,E,F.The traversal always starts from the root of the tree.The node A is the root and hence it is visited first. The value at this node is processed.

Now we check if there existsany leftchild for this node if so apply the pre order procedure on the left subtree.Now check if there is any right sub tree for the node A,the pre order procedure is applied on the right sub tree.Consider the left sub tree for node A,B is now considered as the root of the left subtree of A and pre order procedure is applied.Hence we find that B is processed next and then it is checked if B has a left subtree. This recursive method is continued until all the nodes are visited.



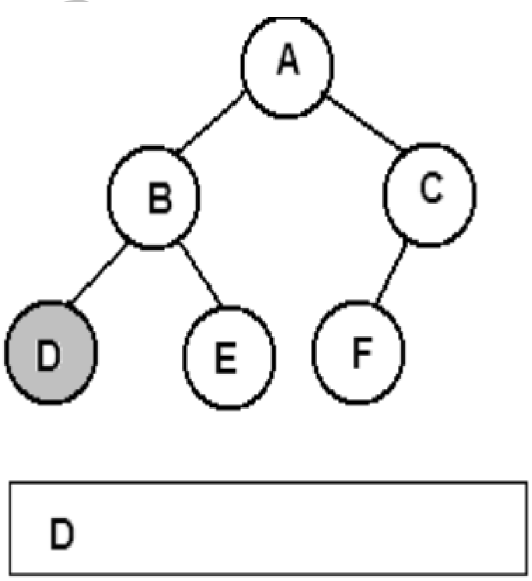
Algorithm for Preorder

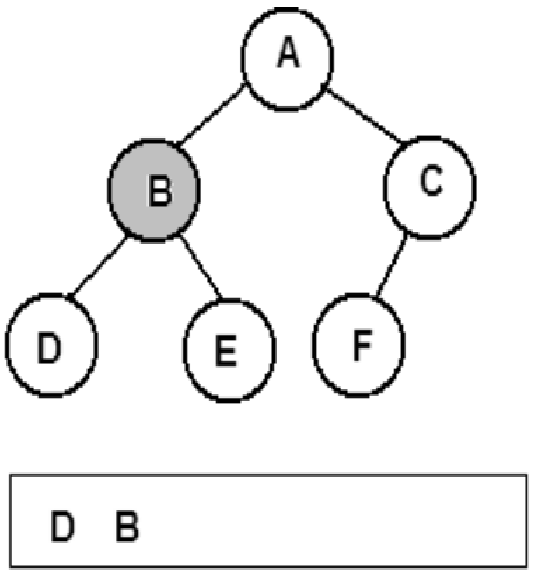
```
PREORDER( ROOT)
Temp = ROOT
If temp = NULL
        return
displaytemp -> data
Iftemp - > left ≠NULL
                PREORDER ( temp - > left )
If temp -> right≠ NULL
        PREORDER ( temp - > right )
```

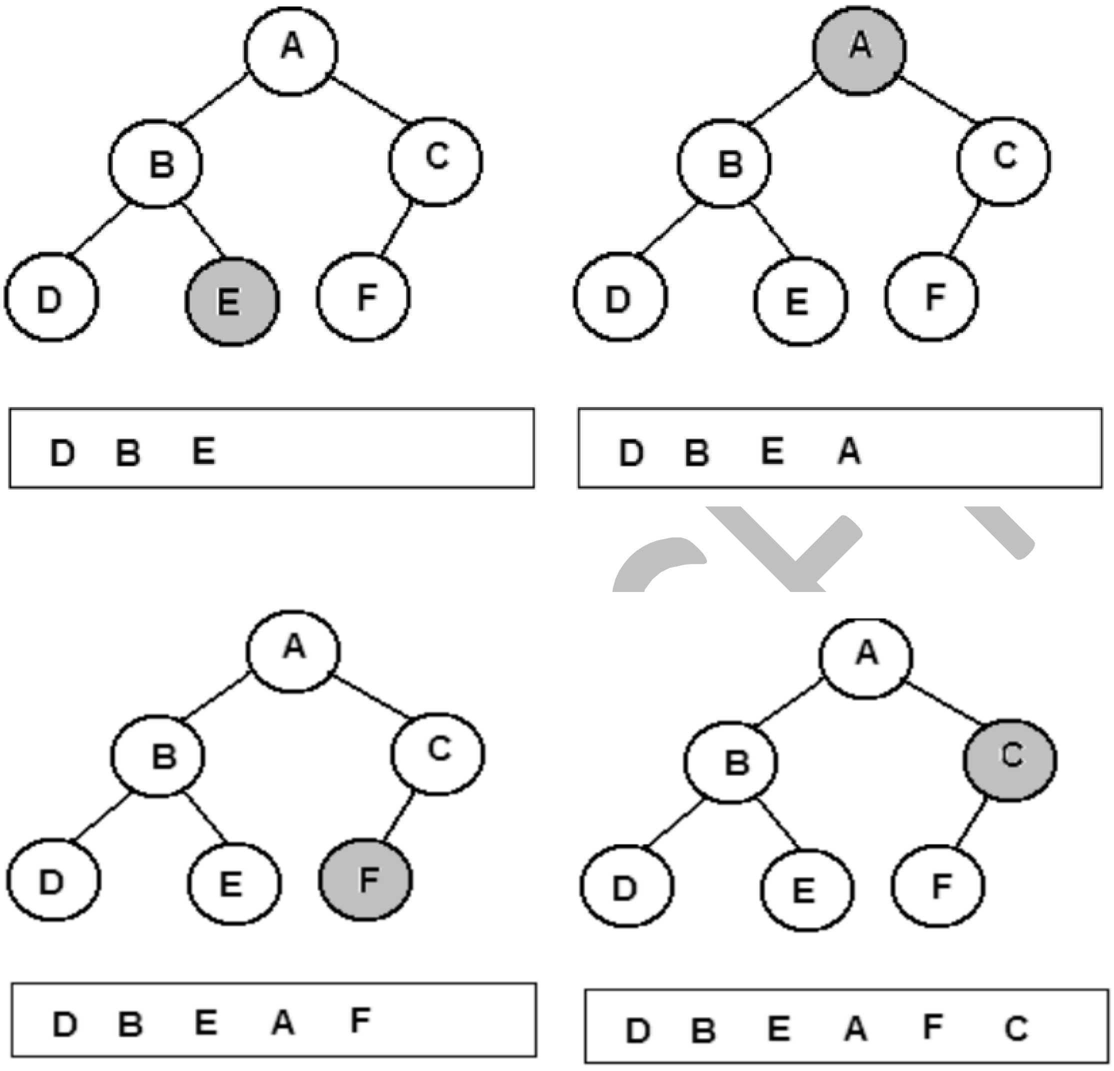
In order Traversal

In the In order traversal method,the left subtree of the node element is visited first and then the node element is processed and at last the right sub tree of the node element is visited.Forexample,the traversal starts with the root of the binary tree.The node A is the root and it is checked if it has the left sub tree.Then the in order traversal procedure is applied on the left subtree of the node A.

Now we find that node D does not have left subtree.Hence the node D is processed and then it is checked if here is a right sub tree for node D. Since there is no right sub tree,the control returns back to the previous function which was applied on B.Since left of B is already visited,now B is processed.It is checked if B has the right subtree.If so apply the in roder traversal method on the right sub tree of the node B.This recursive procedure is followed till all the nodes are visited.





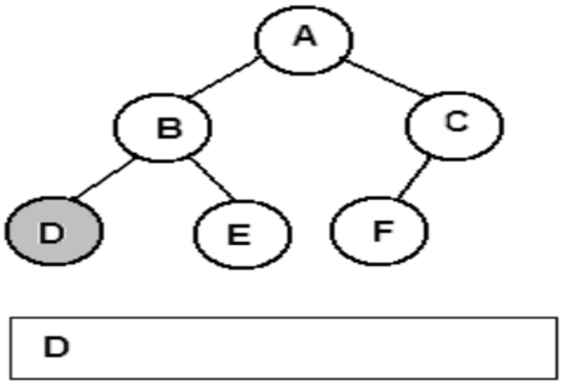


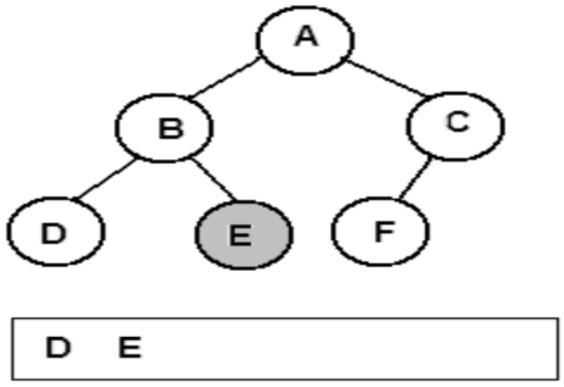
Algorithm for In orderINORDER(ROOT) Temp = ROOT

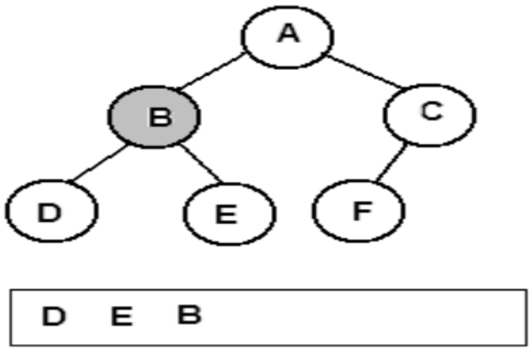
```
If temp = NULL
        return
Iftemp - > left≠ NULL
                INORDER ( temp- >left )
displaytemp -> data
If temp -> right≠NULL
        INORDER ( temp- >right )
```

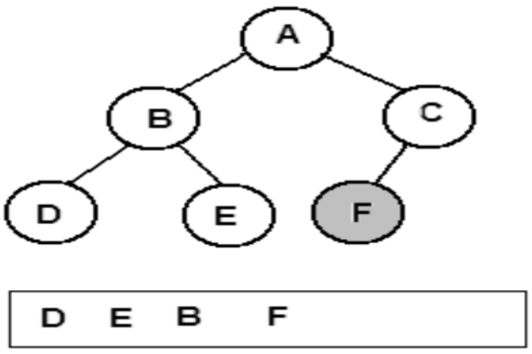
Post order Traversal

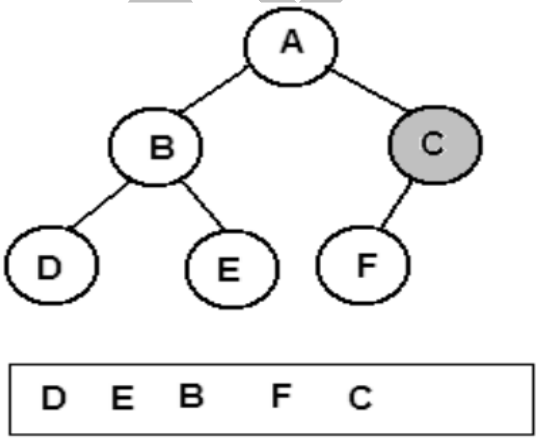
In the post order traversal method the left subtree is visited first,then the right sub tree and at last the node element is processed.Forexample,A is the root node.Since A has the left sub tree the post order traversal method is applied recursively on the left sub tree of A.Then when left sub tree of A is completely is processed, the post order traversal method is recursively applied on the right sub tree of the node A.If right sub tree is completely processed, then the node element A is processed.

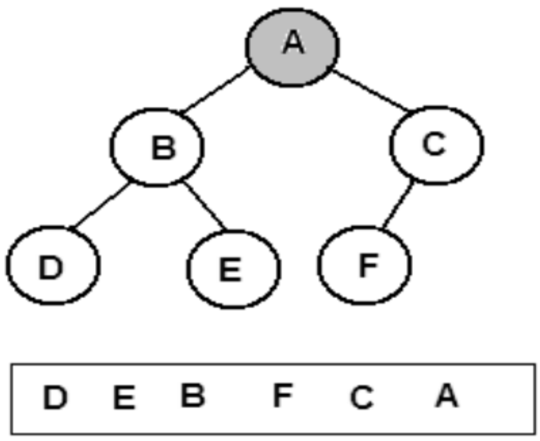












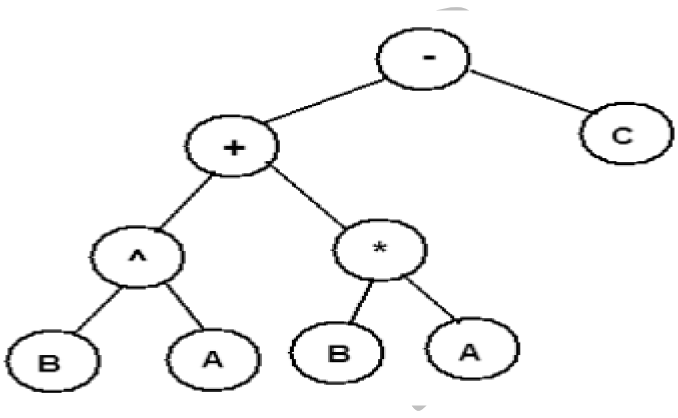
Algorithm for Postorder

```
POSTORDER( ROOT )
Temp = ROOT
If temp = NULL
    return
```

```
Iftemp - > left≠ NULL
                POSTORDER ( temp-> left )
If temp -> right≠NULL
        POSTORDER ( temp- > right )
displaytemp -> data
```

EXPRESSION TREES

The trees are many times used to represent an expression and if done so,those types of trees are called expression trees.The following expression is represented using the binary tree,where the leaves represent the operands and the internal nodes representthe operators.
B ^ A + B * A – C



If the expression tree is traversed using pre order,in order and post order traversal methods, then we get the expressions in prefix, infix and postfix forms as shown.

- + ^ B A * B A - C
B ^ A + B * A - CB
A ^ B A * C –

THREADED BINARY TREES

In binary tree, the leaf nodes have no children.Therefore the left and right fields of the leaf nodes are made NULL.But, NULL wastes memory space so to avoid NULL in the node we will set threads.

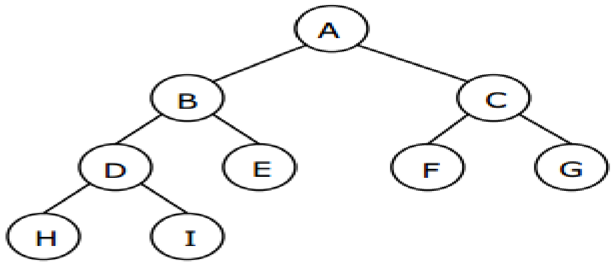
THREADS

Threads are links that point to its predecessor node and successor node. To construct threads we use the following rules.

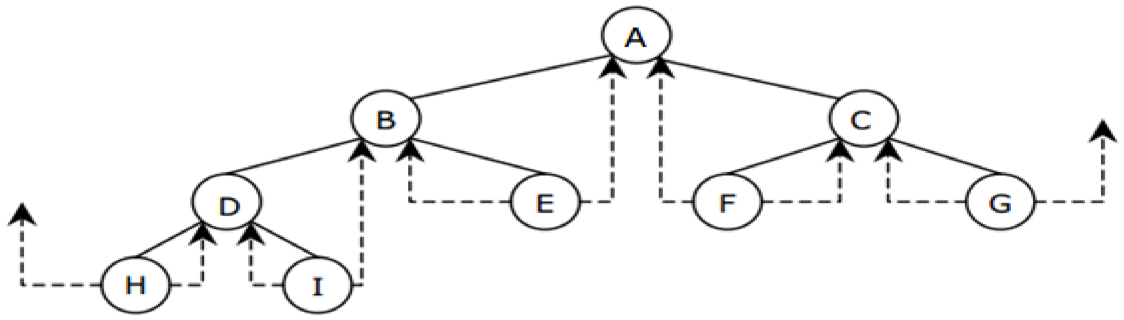
- ✓ If ptr->left child is NULL, replace ptr->left child with a pointer to its

inorder predecessor of ptr
- ✓ If ptr -> right child is NULL, replace ptr -> right child with a pointer to its in order successor of ptr

Let us consider the binary tree as follows



The corresponding threaded binary tree is as follows



Since INORDER Traversal for above binary tree : HDIBEAFCG

The structure of a threaded binary tree is as follows

```
struct threadedbtree
{
        int leftthread, rightthread;
        int data;
        struct threadedbtree *leftchild;
        struct threadedbtree *rightchild;
};
```

INORDER TRAVERSAL OF A THREADED BINARY TREE

The basic idea in inorder threaded binary tree is that the leftthread should point to the predecessor and the right thread points to inorder successor. The head node is the starting node and the root node of the trees is attached to the left of the head node.

There are two additional fields in each node named as left thread and rightthread set initially to 0.To explain about inorder thread traversing of a

binary tree let us consider the values for creating a threaded binary tree
10, 8, 6, 12, 9, 11, 14

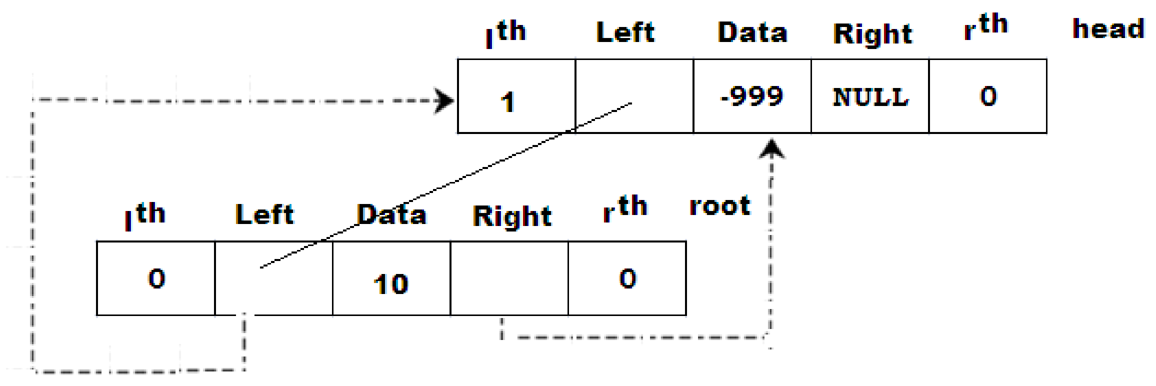
Initially, create a head node of the tree

lth	Left	Data	Right	rth
0	NULL	-999	NULL	0

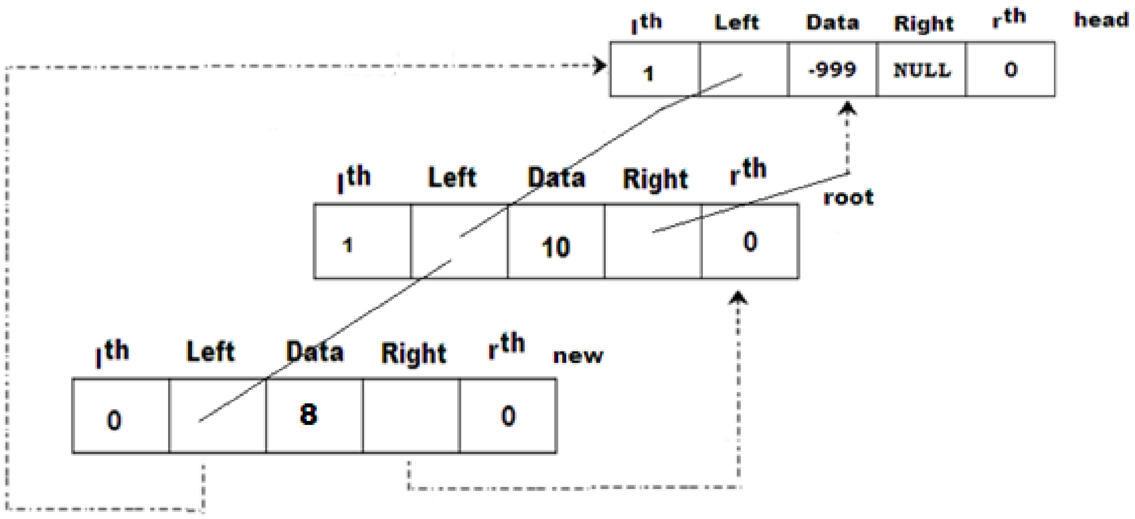
Now let us take the first value 10,this will be the root node and attached to the left of head node as follows

lth	Left	Data	Right	rth
0	NULL	10	NULL	0

The NULL links of the roots left and right will be pointedto the head node as follows



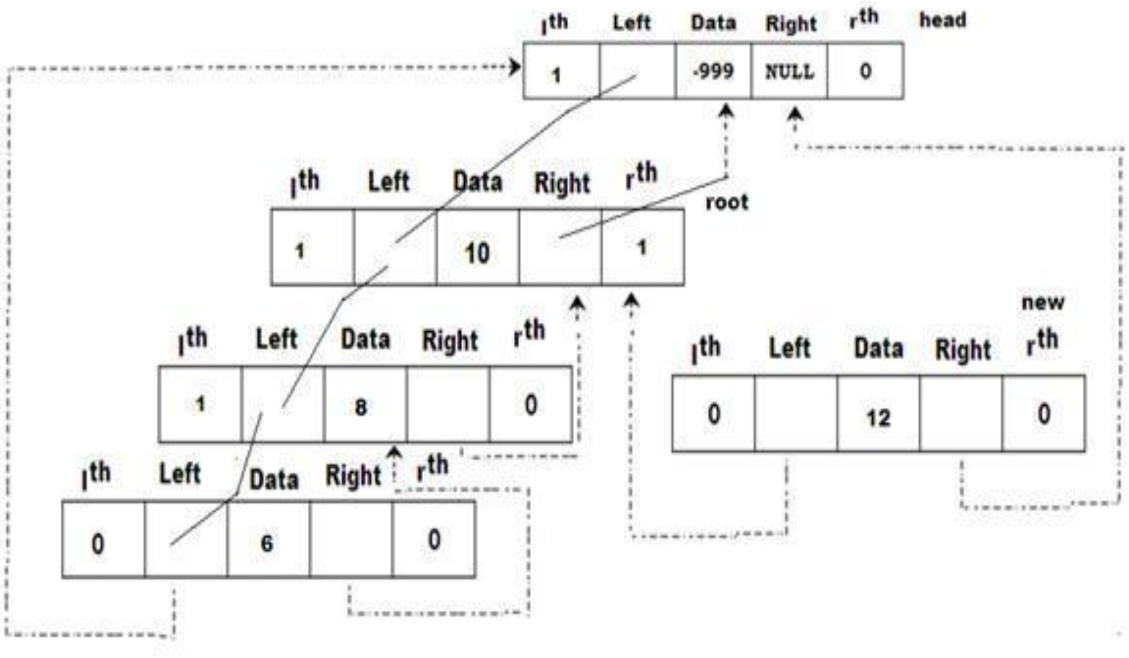
Next comes 8 now 8 is compared with root as it is lessthan attach 8 as the left child of the root 10.



new - > left = root- > left
new - > right = root
root - > left = new
root - > lth = 1

The left link of node 8 points to its inorder predecessor and right link of the node 8 points to its inorder successor.

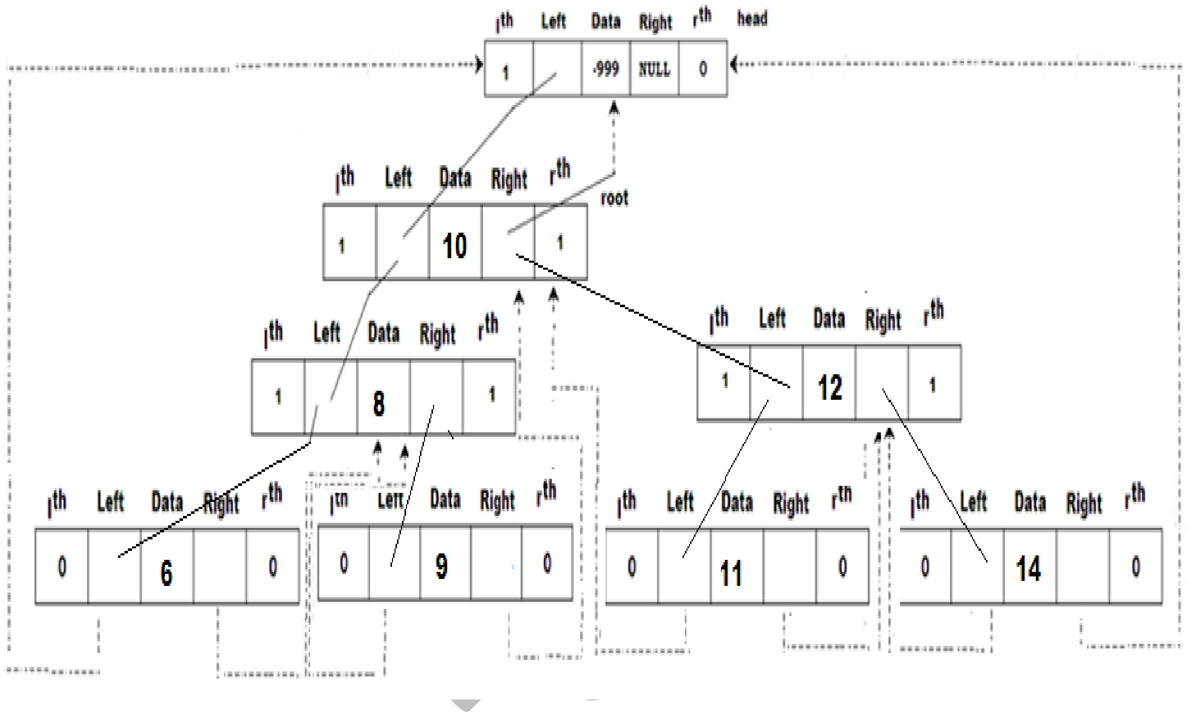
Similarly,the next node 6 is attached to the left of the node 8.The next node is 12 when compared with the root node 10 it is greater so we attach the node 12 to the right of the root node 10 which is as follows.



new - > right = root- > right
new - > left = root
root - >right = 1

root - > right = new

Similarly we construct the remaining the nodes to the threaded binary tree by comparing with root node for the nodes 9, 11, 14 as follows

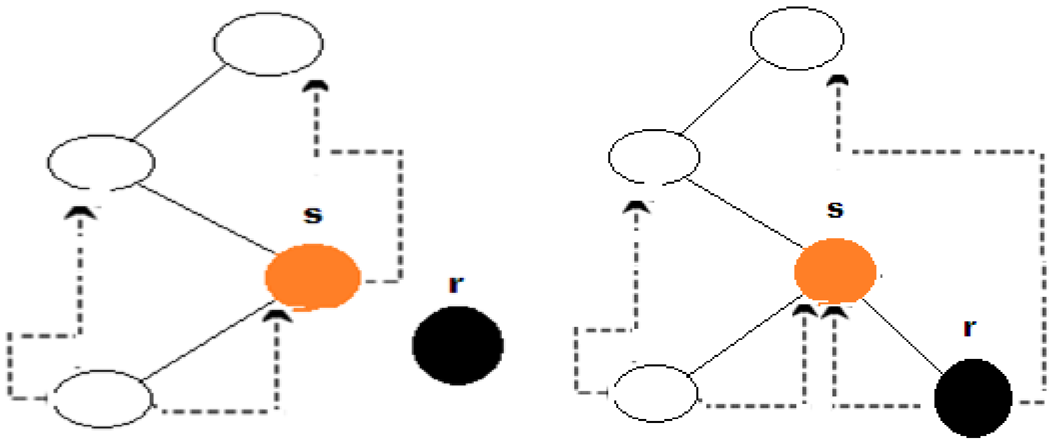


INSERTING A NODE INTO A THREADED BINARY TREE

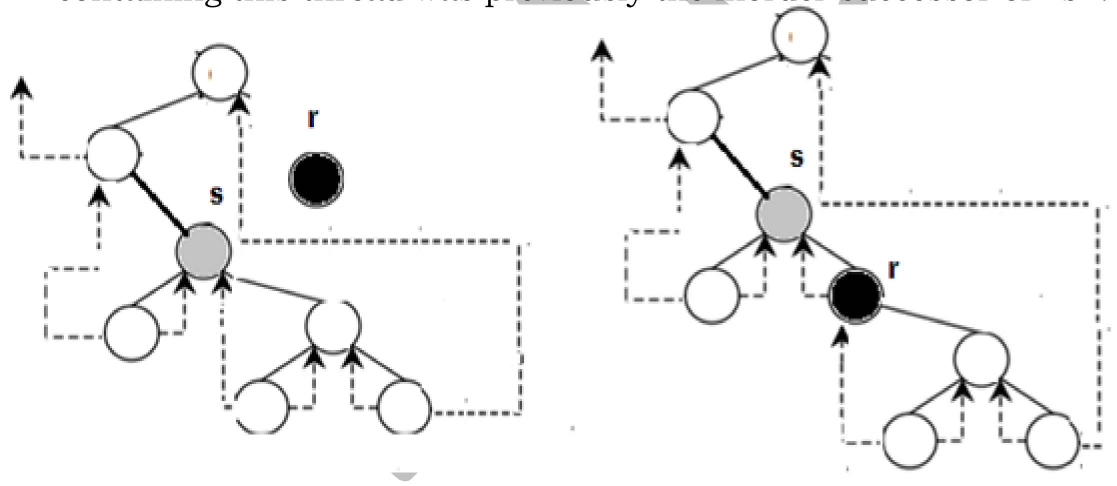
Let us consider now how to insert the node into the threaded binary tree. The case we consider here is inserting the node **“r”** as the right child of a node **“s”.**The cases for insertion are

- ✓ If **“**s**”** has an empty right sub tree, then the insertion is simple as

  shown below



- ✓ If the right subtree of **"**s**"** is not empty,then this right subtree is made the right subtree of **"r"** after insertion.Then **"r"** becomes inorder predecessor of a node that has leftthread == true and consequently there is a thread which has to be updated to point to **"r"**.the node containing this thread was previously the inorder successor of **"s"**.



HEAPS

PRIORITY QUEUES

Heaps are used to implement priority queues. In this type of queues the element to be deleted is one with highest(lowest)priority. We can insert the element at arbitrary priority can be inserted into the queue. The ADT of max priority is as follows.

```
Abstract Datatype MaxPriorityQueue
{
        instances:
                A collection of n>0 elements, each element has a key
```

operations:

for all q €MaxPriorityQueue, item €Element, n €integers

MaxPriorityQueuecreate()	- creates an empty dictionay
Boolean Isempty(q,n)	- if(n>0) return true else return false
Elementtop(q,n)	-if(!isempty(q,n))return an instance of the largest element in q else return error.
Elementpop(q,n)	-if(!isempty(q,n))return an instance of the largest element in q and remove it from the heap else return error.
MaxPriorityQueuepush(q,item,n)	-insert item in to p q and return the resulting priority queue.

}

EXAMPLE OF PRIORITY QUEUES

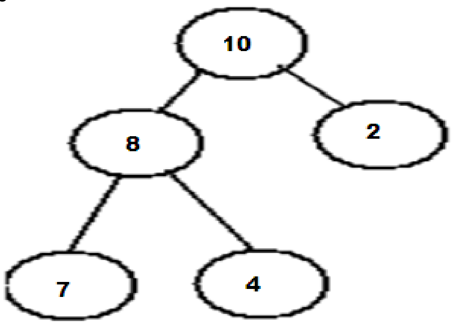
Consider that we are selling the services of a machine. Each user pays a fixed amount per their use.

But the time needed by the each user is different. Now we want to maximize the returns from the machine under the assumption that the machine is not idle. This can be maintained by using a priority queue of all persons waiting to use the machine. Whenever the machine becomes idle, the user with the smallest time requirement is selected. Hence a min priority queue is required.

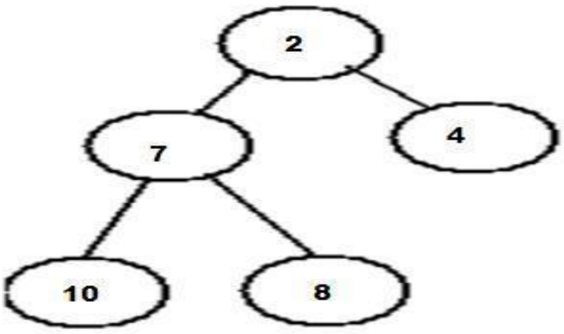
If each user needs the same amount of time on the machine but they are ready to pay different amounts for the service, then a priority queue based on the amount of payment can be maintained. Whenever the machine is idle then the user paying more amounts will be selected. This requires a max priority queue.

DEFINITION OF A MAXHEAP

A maxheap is a complete binary tree that is also a maxtree.A max tree is a tree in which the key value in each node is larger than the key values of its children if any.

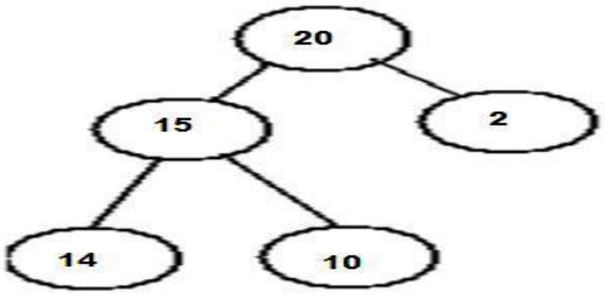


A minheap is a complete binary tree that is also a mintree.A min tree is a tree in which the key value in each node is smaller than the key val ues of its children if any.

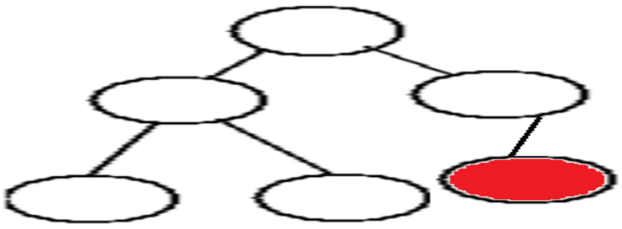


INSERTION INTO A MAXHEAP

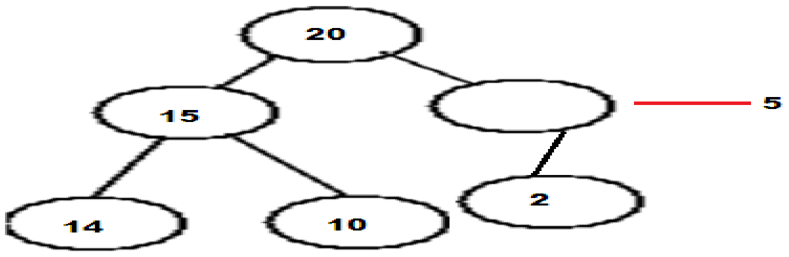
Let us consider a max heap of five elements.



When an element is added to this heap, the resulting is six element heap and it is a complete binary tree. To determine the correct place for the element to be inserted we use bubbling up process that begin at newnode and move to the root. The node we want to insert bubbles up to ensure a max heap.

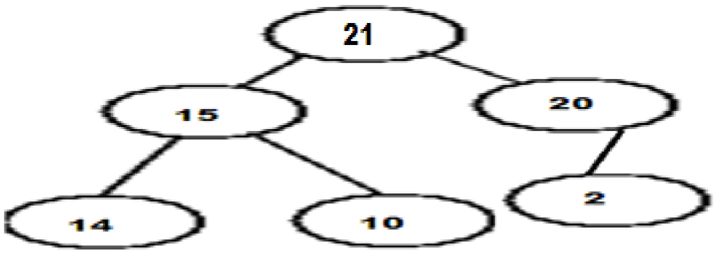


If the element we want to insert is with key value1,it may be inserted as the left child of 2.But if the key value we want to insert is 5then we cannot insert as left child of 2 because heap property fails. So 2 is moved down as left child and the place for 5 is the oldplace of 2.

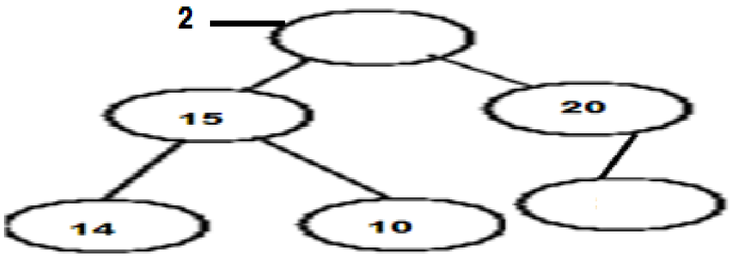


DELETION FROM A MAXHEAP

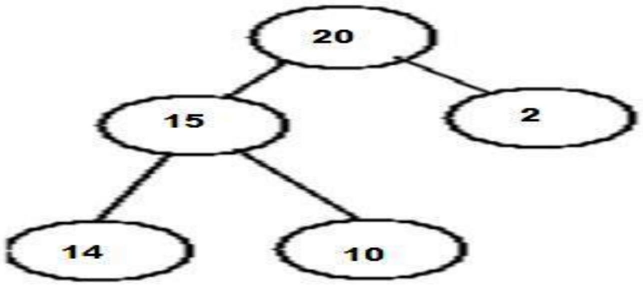
When an element is to be deleted from the maxheap it is taken from the root of the heap.For example, a deletion from the heap results in removal of element 21 then the heap will have only five elements.



To do this were move the element in position 6. Now we have right structure. But the root is vacant and the element 2 is not in the heap. If 2 is inserted into the root then the result binary tree is notmax heap.



The element at the root should be largest in the tree a part from left and right child. This element is 20.It is moves to the root and creates vacancy at position 3. Since it has no children we insert 2 at this place.



BINARY SEARCH TREES

A dictionary is a collection of pairs, each pair has a key and an associated item. We assume no two pairs have the same key. The ADT of a dictionary is shown below

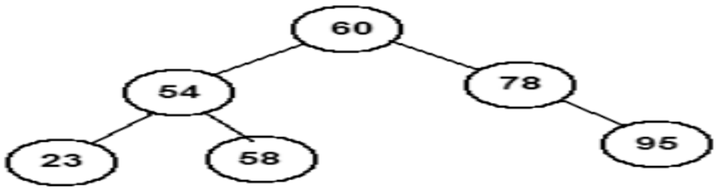
```
Abstract Datatype dictionary
{
        instances:
                a collection of pairs where n>0 each pair has a key and an
                associated item

        operations:
                for all d Єdictionary, item Є Item, k Єkey, n Єintegers
        dictionary create()       - creates an empty dictionary
        Boolean Isempty(bt)       - if(n>0) return true else return false
        Elementsearch(d,k)        -return item with key k otherwise return NULL
                                       if no such element.
        Element delete(d,k)       - delete and return item with key k.
        Void insert(d,item,k)     - insert item with key k into d.
}
```

A Binary Search Tree (BST) is a binary tree. It may be empty or it may if not empty than it satisfies the following properties.

- ✓ Each node has exactly one key and the keys in the tree are distinct
- ✓ The keys if any in the left sub tree are smaller than the key in the root
- ✓ The keys if any in the right sub tree are larger than the key in the root
- ✓ The left and right sub trees are also binary search trees.

The reason why we go for a Binary Search tree is To improve the searching efficiency. The average case time complexity of the search operation in a binary search tree is O(log n).

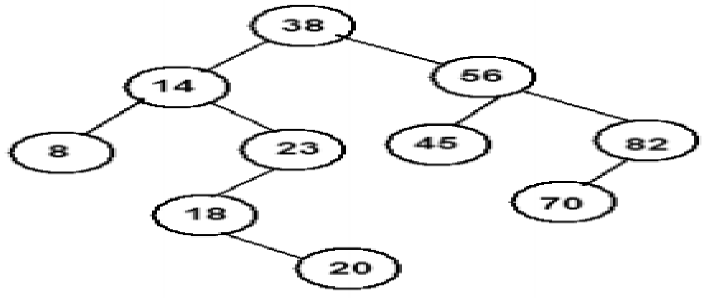


Consider the following list of numbers. A binary search tree can be constructed using this list of numbers, as shown.

38, 14, 8, 23, 18, 20, 56, 45, 82,70

Initially 38 is taken and placed as the root node. The next number 14 is taken and compared with 38. As 14 is lesser than 38, it is placed as the left child of 38. Now the third number 8 is taken and compared starting from the root node 38. Since is 8 is less than 38 move towards left of 38. Now 8 is compared with 14, and as it is less than 14 and also14 does not have any child, 8 is attached as the left child of 14.

This process is repeated until all the numbers are inserted in to the tree. Remember that if a number to be inserted is greater than a particular node element, then we move towards the right of the node and start comparing again.

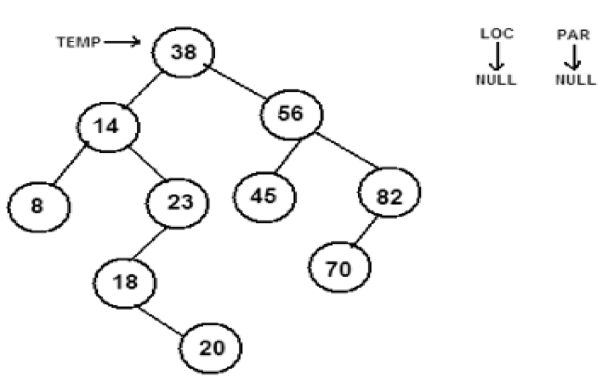


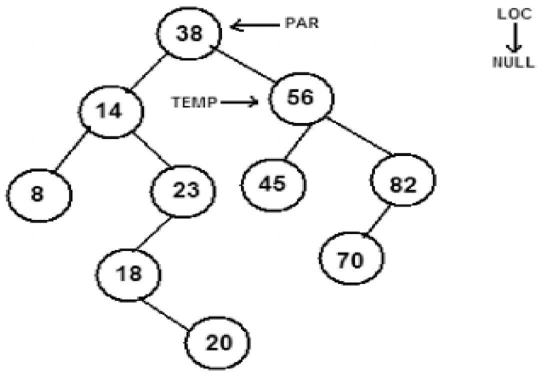
SEARCH OPERATION IN A BINARY SEARCH TREE

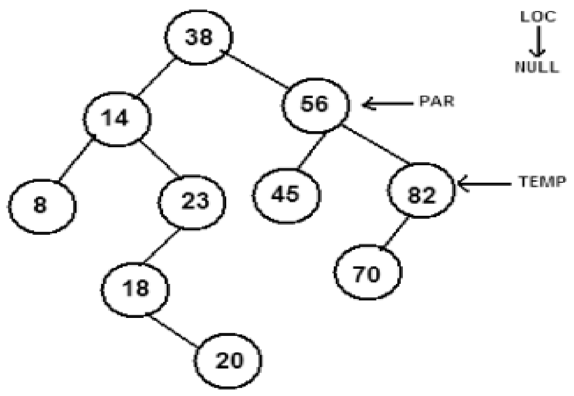
The search operation on a BST returns the address of the node where the element is found. The pointer LOC is used to store the address of the node where the element is found. The pointer PAR is used to point to the parent of LOC. Initially the pointer TEMP is made to point to the root node.

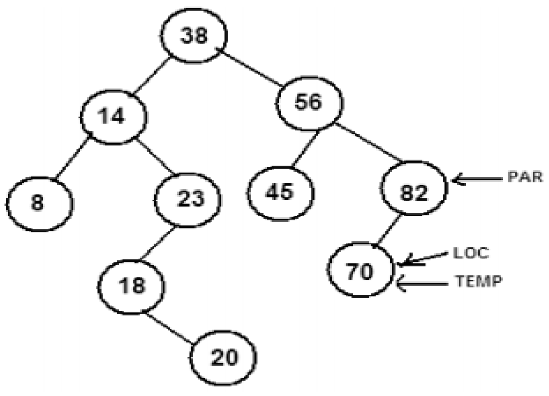
Let us search for a value 70 in the following BST. Let k=70.The k value is compared with 38.As k is greater that 38, move to the right child of 38, i.e., 56. K is greater than 56 and hence we move to the right child of 56, which is 82.Now since k is lesser than 82, temp is moved to the left child of 82. The k value matches here and hence the address of this node is stored in the pointer LOC.

Every time the temp pointer is moved to the next node, the current node is made pointed by PAR. Hence we get the address of that node where the k value is found, and also the address of its parent node though PAR.









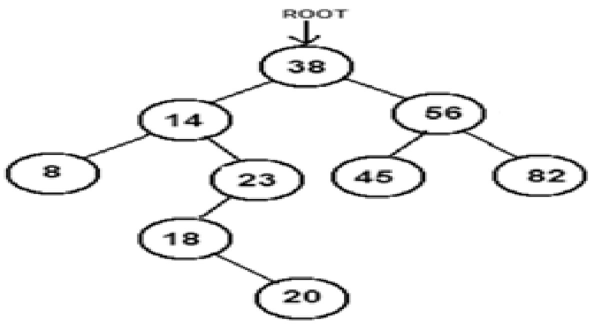
AlgorithmSEARCH(ROOT,k)

```
temp=ROOT,
par= NULL,
loc = NULL
while temp≠ NULL
        If  k =temp - >data
                loc=temp
                break
        If   k <temp - >data
                par=temp
        temp=temp- >left
         else
               par=temp
               temp=temp->right
```

INSERT INTO A BINARYSEARCH TREE

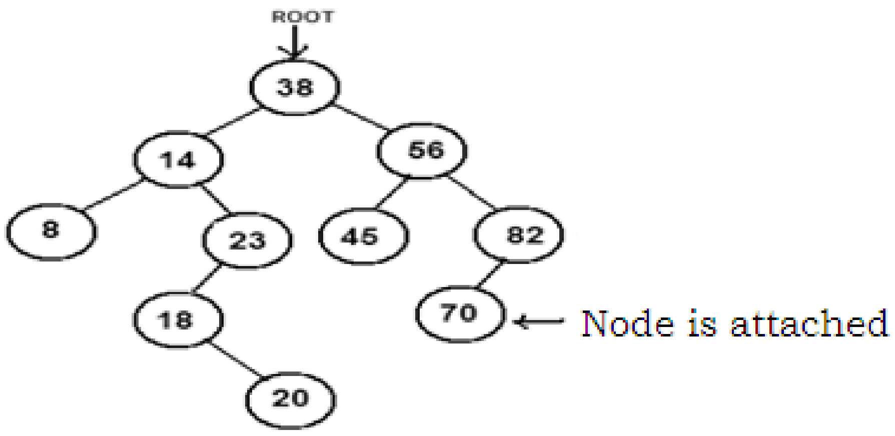
The BST itself is constructed using the insert operation described below. Consider the following list of numbers. A binary tree can be constructed using this list of numbers.

38, 14, 8, 23, 18, 20, 56, 45, 82.



For example we want to insert the element is 70.While inserting a node in to the binary search tree first we have find the appropriate position in the binary search tree. We start comparing the node value 70 with the root if it is greater than the root then it is inserted on the right branch of the root else on the left branch of the root.

Now compare the node 70 with root node 38. As node 70 is greater than the root 38 we will move to the right subtree. Now compare node 70 with the node 56 as it greater then move to right and compare node 70 with node 82 as it lessthan the node 82 we attach 70 as left child of node82. The diagram is shown below.



<u>Algorithm INSERT(ROOT,k)</u>

1.Read the value for the node which is to be created and store it in a node called new.
2. Initially if(root!=NULL) then root = new
3. Again read the next value of node created in new
4.If(new->data<root->data)then attach the newnode as a left child of root otherwise attach the new node as a right child of root
5.Repeat step3 and 4 for constructing required binary search tree completely.

DELETION FROM A BINARYSEARCHTREE

The deletion of a node from a binary search tree occurs with three possibilities

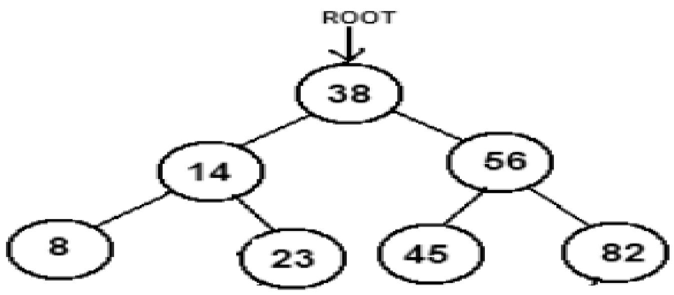
1.Deletion of a leaf node.

2.Deletion of a node having one child.

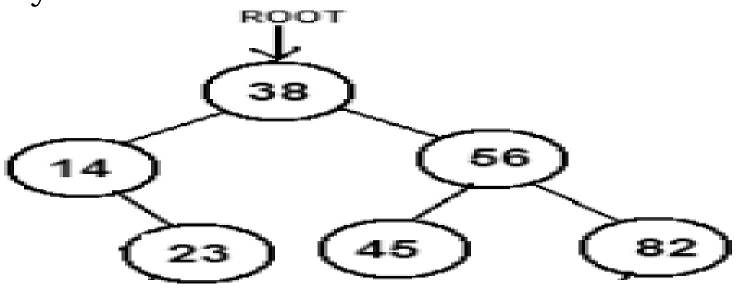
3.Deletion of a node having two children.

1.Deletion of a leaf node

This is the simplest deletion in which we can simple remove it from the tree. For example consider the binary search tree as follows.



From the above tree diagram the node we want to delete is the node 8, then we will set the left pointer of its parent (node14) to NULL. Then after deletion the binary search tree is as follows.

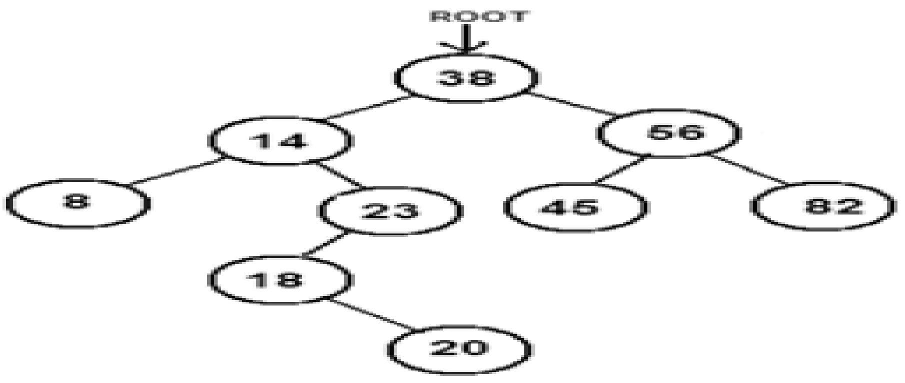


<u>Algorithm</u>

```
if(temp - > left == NULL && temp- > right == NULL)
        if(parent - > left == temp)
                parent - > left = NULL
        else
               parent - > right = NULL
```

2.DeletionOf a node having one child

The node if we wantto delete is having onlyone child(i.e. either left or right child), delete it and replace it with its child.From the diagram the node we want to delete is having the value
23 then we simple copy node 18 at the place of 23 and set thenode free.



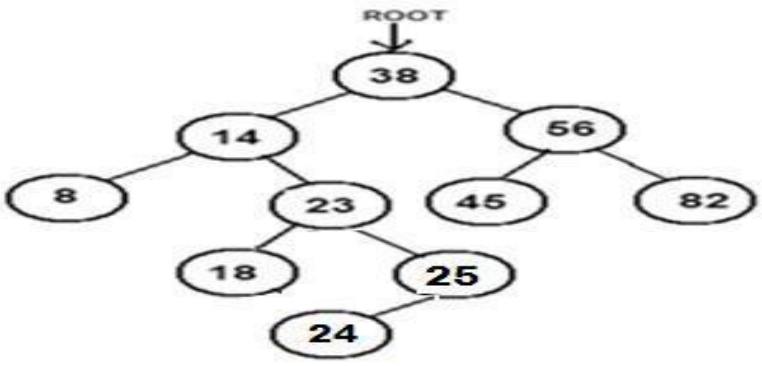
Algorithm

```
if(temp - > left !=NULL && temp - > right == NULL)
if(parent - > left ==temp)
parent - > left = temp - > left
else
parent - > right = temp - > left
temp == NULL
delete temp
```

3.Deletion of a node having two children

Suppose the node to be deleted is called N.Were place the value of N with either its in-order successor (the left-most child of the right subtree) or the in-order precedessor (the right-most child of the the left subtree).

The node if we want to delete is having two children .From the diagram the node we want to delete is having the value 23 then we find the inorder successor of the node 23 and it is copied at the place of 23 and set the node 25 left pointer to NULL.



Algorithm

```
if(temp - > left != NULL && temp -> right != NULL)
        parent = temp
        temp_succ = temp - > right
        while(temp_succ - > left != NULL)
                parent = temp_succ
                temp_succ = temp_succ- > left
                temp - > data = temp_succ- > data
                parent- > right = NULL
```

HEIGHT OF A BINARYSEARCH TREE

The height of a binary search tree with “n” elements can become as large as “n”. For instance, when the values like1,2,--------n are inserted in to the empty binary search tree. If insertions and deletions are made at random then the height of the binary search tree is O(log n) on average.

Search trees with worst case height of O(logn) are called balanced search trees. These trees permit insertions, deletions and searches to be performed at time O(h).for example, AVLtrees, Red/Black Trees, B-Trees, 2 – 3 Trees etc.

****