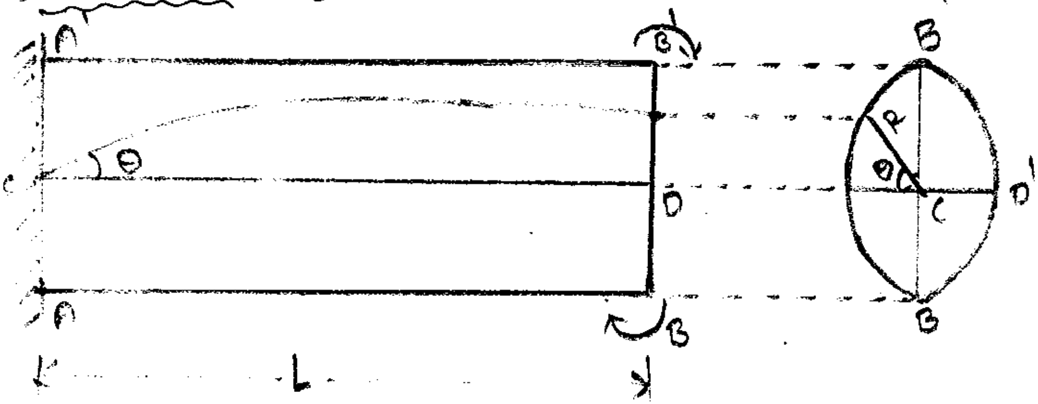
STRENGTH OF MATERIALS - II

UNIT- I

(1.) Make a neat sketch of a circular shaft subjected to a twisting moment. Show clearly the variation of shear angle, angles of twist and shear stress in shaft. Derive the torsion formula. What assumptions are taken while deriving formula for a circular shaft?

Ans: DERIVATION OF TORSION EQUATION:



Consider a shaft fixed at one end AA and is subjected to torque (T) at free end BB. Now distorsion at the outer the end BB.

Surface $\tau = \tan\phi = \phi = \frac{DD'}{CD} = \frac{DD'}{L}$ —— (1)

and shear strain ∴ $DD' = R\theta$ —— (2)

From 1 & 2

$$\phi = \frac{R\theta}{2} \quad \text{——} \quad (3)$$

Now Shear modulus or modulus of rigidity/ rigidity of modulus

$$C = \frac{\tau}{\phi} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$= \frac{\tau}{\frac{R\theta}{L}}$$

$$\Rightarrow \frac{\tau}{R} = \frac{C\theta}{L} \quad \text{——} \quad (4)$$

If θ is Constant where C and L are Constant

then

$$\Rightarrow \tau/R = \text{Constant}$$

Let q at r $\quad \tau/R = q/r$

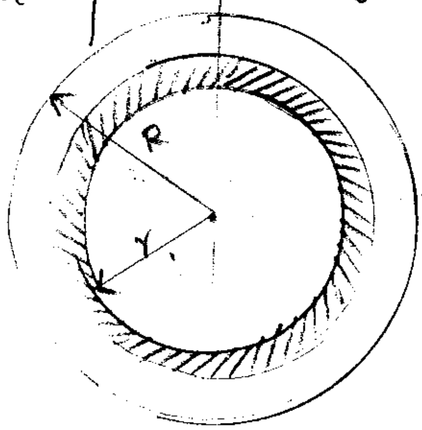
$$q/r = \frac{C\theta}{L} = \frac{\tau}{R} \quad \text{——} \quad (5)$$

from the Equation no: 5

— The shear stress in maximum at outer surface and is zero at the center of the shaft (axis of the shaft)

Consider an elemental circle ring of thickness (dr) at a distance 'r' from the center

∴ Area of the ring $= dA = (2\pi r)\,dr$



∴ Turning force on the ring

$$\Rightarrow dF = q \times dA = \left(\frac{T}{R} \times r\right) \times dr$$

∴ Turning moment on the ring

$= \text{Torque} = dT = dF \times r$

$$\Rightarrow dT = \left(T/R \times r^2\right) dA$$

∴ Total turning momentum or Total torque

$$\Rightarrow T = \int_0^R dT = \int_0^R \tau/R \cdot r^2 \cdot dA$$

$$\Rightarrow T = \tau/R \int_0^R r^2 \, dA$$

$$\Rightarrow T = \tau/R \, (J) \quad \text{——} \quad (6)$$

from eqn (5) & (6)

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$

where T = Torque / Turning moment / Twisting moment / Torsional Resistance of moment. (N-m)

J = polar moment of interia

$J = \int r^2 dA \; (m^4)$

τ = Shear stress at the outer surface (N/m^2)

R = Radius of the shaft

C = Shear modulus / modulus of Rigidity / Rigidity modulus (N/m^2)

θ = Angle of twist / twisting angle (Degree)

L = Length of the shaft.

Problem: Two shafts of the same material of same lengths are subjected to the same torque, if the first shaft is of a solid circular section and the second shaft is of hollow circular section, whose internal diameter is 2/3 of the diameter and the maximum shear stress developed in each shaft is the same, compare the weights of the shafts.

Sol: Given:

Let T = Torque transmitted by each shaft

τ = Max. shear stress developed in each shaft

D = Outer diameter of the solid shaft

D_o = Outer diameter of the hollow shaft

D_i = Inner diameter of the hollow shaft $= 2/3\, D_o$

W_s = Weight of the solid shaft

W_h = Weight of the hollow shaft

L = Length of each shaft

w = Weight density of the material of each shaft

Torque transmitted by the hollow shaft is given by equation

$$T = \frac{\pi}{16} \tau D^3 \qquad \text{—— (1)}$$

Torque transmitted by the hollow shaft is given by equation

$$T = \frac{\pi}{16}\tau\left[\frac{D_0^4 - D_i^4}{D_0}\right]$$

$$= \frac{\pi}{16}\tau\left[\frac{D_0^4 - (2/3 D_0)^4}{D_0}\right]$$

$$= \frac{\pi}{16}\tau\left[\frac{D_0^4 - \frac{16}{81}D_0^4}{D_0}\right]$$

$$= \frac{\pi}{16}\tau \times \frac{65 D_0^4}{81 \times D_0}$$

$$= \frac{\pi}{16}\tau \times \frac{65 D_0^3}{81} \quad \text{———} \quad (2)$$

As Torque transmitted by solid and hollow shafts are equal, hence equating equations (1) & (2)

$$\frac{\pi}{16}\tau D^3 = \frac{\pi}{16}\tau \times \frac{65}{81} \cdot D_0^3$$

Cancelling $\frac{\pi}{16}\tau$ from both sides we get,

$$D^3 = \frac{65}{81} \cdot D_0^3 .$$

$$D = \left[\frac{65}{81} \cdot D_0^3\right]^{1/3} = \left[\frac{65}{81}\right]^{1/3} D_0 = 0.929\, D_0 \quad \text{—— (3)}$$

Now weight of solid shaft,

W_s = weight density x Volume of solid shaft

= w x Area of cross-section x Length

$$= w \times \frac{\pi}{4} D^2 \times L \quad \text{—— (4)}$$

weight of hollow shaft,

W_h = w x Area of cross-section of hollow shaft x Length

$$= w \times \frac{\pi}{4}\left[D_0^2 - D_i^2\right] \times L$$

$$= w \times \frac{\pi}{4}\left[D_0^2 - (2/3\, D_0)^2\right] \times L$$

$$= w \times \frac{\pi}{4}\left[D_0^2 - \frac{4}{9} D_0^2\right] \times L$$

$$= w \times \frac{\pi}{4} \times \frac{5}{9} D_0^2 \times L \quad \text{—— (5)}$$

Dividing equation (4) by equation (5)

$$\frac{W_s}{W_h} = \frac{w \times \frac{\pi}{4} D^2 \times L}{w \times \frac{\pi}{4} \times \frac{5}{9} D_0^2 \times L} = \frac{9D^2}{5D_0^2}$$

$$= \frac{9}{5} \times \frac{(0.929\, D_0)^2}{D_0^2}$$

$$= \frac{9}{5} \times 0.929^2 \times \frac{D_0^2}{D_0^2}$$

$$= \frac{1.55}{1}$$

$$\therefore \frac{\text{Weight of Solid shaft}}{\text{weight of hollow shaft}} = \frac{1.55}{1}$$

③ problem: A hollow shaft of External diameter 120 mm transmits 300 kw power at 200 r.p.m. Determine the maximum internal diameter if the maximum stress in the shaft is not to Exceed 60 N/mm²

Sol: Given: External dia $D_0 = 120\,mm$

power $P = 300\,kW = 300{,}000\,W$

Speed $N = 200\ r.p.m$

Max. shear stress $\tau = 60\,N/mm^2$

Let D_i = Internal dia. of shaft

Using equation $P = \frac{2\pi N T}{60}$ (or) $300{,}000 = \frac{2\pi \times 200 \times T}{60}$

$$T = \frac{300{,}000 \times 60}{2\pi \times 200}$$

$= 14323.9 \text{ N-m}$

$= 14323.9 \times 1000 \text{ N/mm.}$

$= 14323900 \text{ N-mm}$

now using equation $T = \frac{\pi}{16} \times \tau \times \frac{(D_o^4 - D_i^4)}{D_o}$

$$\Rightarrow \frac{14323900 \times 16 \times 120}{\pi \times 60} = 120^4 - D_i^4$$

$$\Rightarrow 145902000 = 207360000 - D_i^4$$

$$D_i^4 = 207360000 - 145902000$$

$$= 61458000$$

$$D_i = (61458000)^{1/4}$$

$\therefore \quad D_i = 88.5 \text{ mm}$

(4) problem: A Solid shaft of diameter 80 mm is Subjected to a twisting moment of 8 MN-mm and bending moment of 5 MN-mm at a point. Determine:

(i) principal stresses and,

(ii) position of the plane on which they act.

Sol: Given: Diameter of shaft $D = 80$ mm

Twisting moment $T = 8$ MN-mm

$= 8 \times 10^6$ N-mm

Bending moment $M = 5$ MN-mm

$= 5 \times 10^6$ N-mm

The major principal stress is given by equation

We know that from theory of failure:

$$= \frac{16}{\pi D^3}\left(M \times \sqrt{M^2+T^2}\right)$$

$$= \frac{16}{\pi \times 80^3}\left(5 \times 10^6 + \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2}\right)$$

$$= \frac{16 \times 10^6}{\pi \times 80^3}\left(5 + \sqrt{25+64}\right)$$

$= 143.57$ N/mm^2.

Minor principal stress is given by equation

$\therefore$ Minor principle stress

$$= \frac{16}{\pi D^3}\left(M - \sqrt{M^2+T^2}\right)$$

$$= \frac{16}{\pi \times 80^3}\left(5\times10^6 - \sqrt{(5\times10^6)^2+(8\times10^6)^2}\right)$$

$$= \frac{16\times10^6}{\pi\times80}\left(5-\sqrt{25+64}\right)$$

$$= -44.1 \text{ N/mm}^2$$

$$= 44.1 \text{ N/mm}^2 \text{ (tensile)}.$$

Postition of plane is given by equation

$$\tan 2\theta = \frac{T}{M} = \frac{8\times10^6}{5\times10^6} = 1.6$$

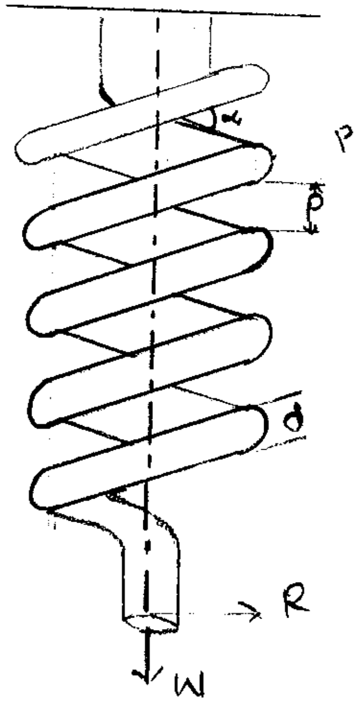
$$2\theta = \tan^{-1} 1.6 = 57^\circ 59.68'$$

$$\theta = 28^\circ 59.84'$$

STRENGTH OF MATERIALS - II

UNIT - I

(5) Q. Derive the formula to find the deflection of a closely helical spring subjected to an axial load?



In which the angle of helix α is so small so that if the axis of the spring is vertical.

Length of the spring $= l = 2\pi R n$

Twisting / Bending moment $= T = W \cdot R$

Shear stress due to torsion $=$ $\tau_1 = \frac{16T}{\pi d^3}$

Shear stress due to direct load $=$ $\tau_2 = W/A = \frac{4W}{\pi d^2}$

Total shear stress $=$ $\tau = \tau_1 + \tau_2 = \frac{8WD}{\pi d^3}$

$[V = \pi/4 \; d^2 \times L]$

Angle of Twist/ Rotation = $\theta_t = \frac{TL}{CJ}$

$$\theta_t = \frac{64 W R^2 n}{C d^4} \quad \left[J = 2I = \frac{\pi d^4}{32} \right]$$

Axial Deflection moment = $\delta = R\theta_t$

$$\delta = \frac{64 W R^3 n}{C d^4}.$$

Energy stored U = $\frac{1}{2} W \delta = \frac{1}{2} T \cdot \theta_t$

$$= \frac{\tau^2}{4C} \times V$$

Stiffness of the spring = $K = W/\delta$.

Where

d = Diameter of spring wire

P = pitch of the helical spring

n = number of coils

R = mean radius of spring coil

W = Axial load on spring

C = Modulus of rigidity

T = Max. shear stress induced in the wire

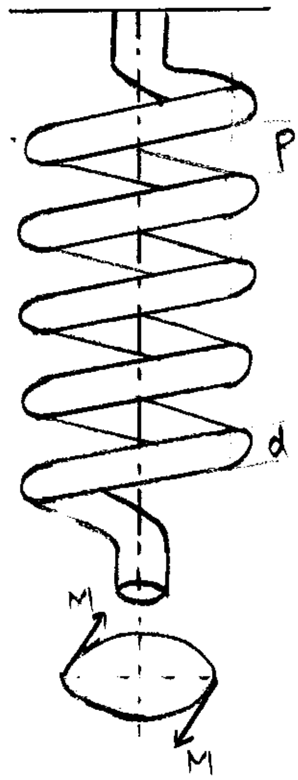
θ = Angle of twist in spring wire and

δ = deflection of spring due to axial load

l = length of the wire.

(5) Q. Derive the formula to find the deflection of closed helical spring - Axial couple / Torque 'M' & δ

closed Helical spring



Where

L = Length of the wire

M = Twisting or bending moment

n = number of coil

R = Mean radius of the spring coil

W = Axial load on spring

δ = deflection of the spring due to couple

θ = Angle of the twist in the spring wire

d = Diameter of the spring wire

f_b = bending stresses

Let Length of the Spring = $L = 2\pi R_2 n_2 = 2\pi R_1 n_1$

Axial torque M tends to wind up or unwind the Spring by producing a pure bending moment at all cross sections.

i.e The Radius of the coil changes from R_1 to R_2

The twisting / Bending moment = $M = EI\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$

$\therefore$ The Bending Stresses = $f_b = \frac{32M}{\pi d^3}$

So the Angle of the twist / the Rotation in the Spring wire = $\theta_b = \frac{ML}{EI} = 2\pi n_2 - 2\pi n_1$

bending moment $= \frac{M}{EI} = \frac{1}{R_2} - \frac{1}{R_1}$

Angle of twist $= \theta_b = \frac{128 M R_1 n_1}{E d^4}$

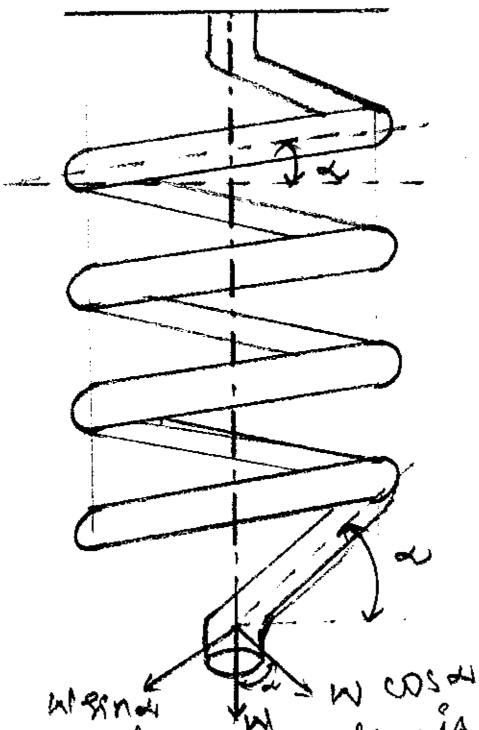
$\therefore$ Axial deflection movement $= \delta = R\theta_b$

Therefore the Energy stored $U = \frac{1}{2} W\delta = \frac{1}{2} M\theta_b$

$$= \frac{f_b^2}{8E} \times V$$

7) Q. Derive the deflection formula for an open coiled helical Spring subjected to an axial load?

open coiled helical spring



Where the bending couple is also considered in comparision with torsional couple.

Where:

L = Length of the spring wire

R = Mean Radius of the spring coil

τ = Max. shear stress induced in the wire

d = diameter of the spring wire

W = Axial load on spring.

A = Area of the cross section of the spring.

f_b = bending stresses

θ = Angle of the twist in the spring wire

δ = deflection of the spring due to axial load.

Let the length of the spring = $l = 2\pi R n \sec\alpha$.

So the twisting / Bending moment = $T = W \cdot R \cos\alpha$

$M = W \cdot R \cdot \sin\alpha$

Shear Stresses of the wire due to the torsion $= \tau_1 = \dfrac{16T}{\pi d^3}$

shear stresses to the direct load. $= \tau_2 = W/A = \dfrac{4W\sin\alpha}{\pi d^2}$

Total Shear Stress $= \tau = \tau_1 + \tau_2$

Bending stresses $= f_b = \dfrac{32M}{\pi d^3}$

The Angle of twist / Rotation $= \theta_t = \dfrac{Tl}{CJ}$, $\theta_b = \dfrac{ML}{EI}$

Total winding up monement $(\psi) = WRL \sin\alpha \cos\alpha \, [1/CJ - 1/EI]$