

UNIT - VI - Tuned Amplifiers

3.1 Introduction

To amplify the selective range of frequencies, the resistive load, R_C is replaced by a tuned circuit. The tuned circuit is capable of amplifying a signal over a narrow band of frequencies centered at f_r . The amplifiers with such a tuned circuit as a load are known as tuned amplifier.

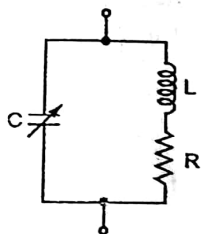


Fig. 3.1 Tuned circuit

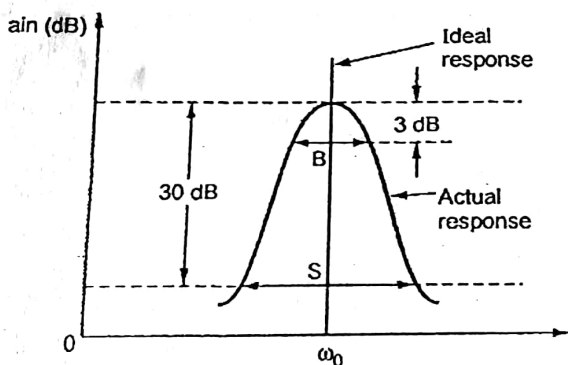


Fig. 3.2 Frequency response of a tuned amplifier

The Fig. 3.1 shows the tuned parallel LC circuit which resonates at a particular frequency. The resonance frequency and impedance of tuned circuit is given as,

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \dots (1)$$

$$\text{and } Z_r = \frac{L}{CR} \quad \dots (2)$$

The response of tuned amplifiers is maximum at resonant frequency and it falls sharply for frequencies below and above the resonant frequency, as shown in the Fig. 3.2.

As shown in the Fig. 3.2, 3 dB bandwidth is denoted as B and 30 dB bandwidth is denoted as S. The ratio of the 30 dB bandwidth (S) to the 3 dB bandwidth (B) is known as skirt selectivity.

At resonance, inductive and capacitive effects of tuned circuit cancel each other. As a result, circuit is like resistive and $\cos \phi = 1$ i.e. voltage and current are in phase. For frequencies above resonance circuit is like capacitive and for frequencies below resonance it is like inductive. Since tuned circuit is purely resistive at resonance it can be used as a load for amplifier.

3.1.1 Coil Losses

As shown in Fig. 3.1, the tuned circuit consists of a coil. Practically, coil is not purely inductive. It consists of few losses and they are represented in the form of leakage resistance in series with the inductor. The total loss of the coil is comprised of copper loss, eddy current loss and hysteresis loss. The copper loss at low frequencies is equivalent to the d.c. resistance of the coil. Copper loss is inversely proportional to frequency. Therefore, as frequency increases, the copper loss decreases. Eddy current loss in iron and copper coil are due to currents flowing within the copper or core caused by induction. The result of eddy currents is a loss due to heating within the inductors copper or core. Eddy current losses are directly proportional to frequency. Hysteresis loss is proportional to the area enclosed by the hysteresis loop and to the rate at which this loop is transversed. It is a function of signal level and increases with frequency. Hysteresis loss is however independent of frequency.

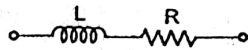


Fig. 3.3 Inductor with leakage resistance

As mentioned earlier, the total losses in the coil or inductor is represented by inductance in series with leakage resistance of the coil. It is as shown in Fig. 3.3.

3.1.2 Q Factor

Quality factor (Q) is important characteristics of an inductor. The Q is the ratio of reactance to resistance and therefore it is unitless. It is the measure of how 'pure' or 'real' an inductor is (i.e. the inductor contains only reactance). The higher the Q of an inductor the fewer losses there are in the inductor. The Q factor also can be defined as the measure of efficiency with which inductor can store the energy. The dissipation factor (D) that can be referred to as the total loss within a component is defined as 1/Q. The Fig. 3.4 shows the quality factor equations for series and parallel circuits and its relation with dissipation factor.

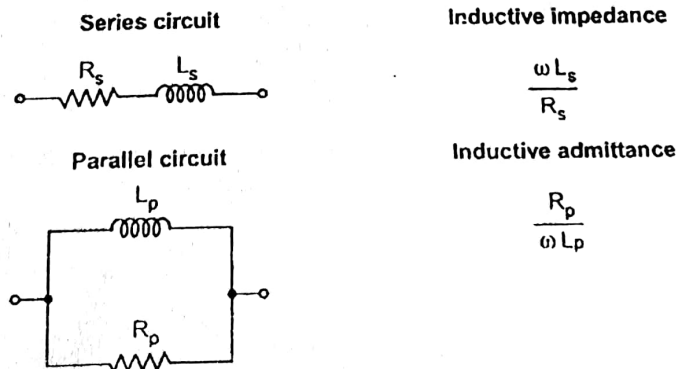


Fig. 3.4 Quality factor equations

$$\text{Quality factor equation } Q = \frac{1}{D} = \frac{\omega L_s}{R_s} = \frac{R_p}{\omega L_p}$$

3.1.3 Unloaded and Loaded Q

Unloaded Q is the ratio of stored energy to dissipated energy in a reactor or resonator. The unloaded Q or Q_U , of an inductor or capacitor is X/R_s , where X represents the reactance and R_s represents the series resistance. The loaded Q or Q_L , of a resonator is determined by how tightly the resonator is coupled to its terminations.

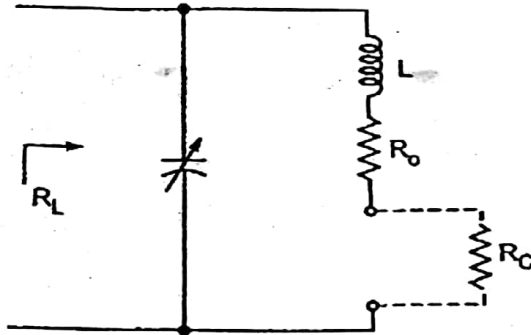


Fig. 3.5 Tuned load circuit

Let us consider the tuned load circuit as shown in the Fig. 3.5. Here, L and C represents tank circuit. The internal circuit losses of inductor are represented by R_o and R_C represents the coupled in load. For this circuit, we can write

$$R_o = \frac{\omega_o L}{Q_U} \text{ and } R_C = \frac{\omega_o L}{Q_L}$$

where Q_U is unloaded Q and Q_L is loaded Q.

The circuit efficiency for the above tank circuit is given as,

$$\eta = \frac{I^2 R_C}{I^2 (R_C + R_o)} = \frac{Q_U}{Q_U + Q_L} \times 100 \%$$

From above equation it can be easily realized that for high overall power efficiency, the coupled-in load R_C should be large in comparison to the internal circuit losses represented by R_o of the inductor.

The quality factor Q_L determines the 3 dB bandwidth for the resonant circuit. The 3 dB bandwidth for resonant circuit is given as,

$$BW = \frac{f_r}{Q_L}$$

where f_r represents the centre frequency of a resonator and BW represents the bandwidth.

If Q is large, bandwidth is small and circuit will be highly selective. For small Q values bandwidth is high and selectivity of the circuit is lost, as shown in the Fig. 3.6.

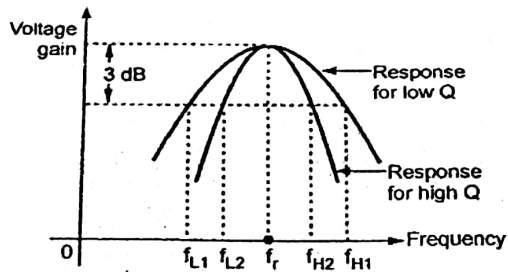


Fig. 3.6 Variation of 3dB bandwidth with variation in quality factor

Thus in tuned amplifier Q is kept as high as possible to get the better selectivity. Such tuned amplifiers are used in communication or broadcast receivers where it is necessary to amplify only selected band of frequencies.

3.1.4 Requirements of Tuned Amplifier

The basic requirements of tuned amplifiers are :

- The amplifier should provide selectivity of resonant frequency over a very narrow band.
- The signal should be amplified equally well at all frequencies in the selected narrow band.
- The tuned circuit should be so mounted that it can be easily tuned. If there are more than one circuit to be tuned, there should be an arrangement to tune all circuit simultaneously.
- The amplifier must provide the simplicity in tuning of the amplifier components to the desired frequency over a considerable range or band of frequencies.

3.1.5 Classification of Tuned Amplifier

We know that, multistage amplifiers are used to obtain large overall gain. The cascaded stages of multistage tuned amplifiers can be categorized as given below :

- Single tuned amplifiers
- Double tuned amplifiers
- Stagger tuned amplifiers.

These amplifiers are further classified according to coupling used to cascade the stages of multistage amplifier.

- Capacitive coupled
- Inductive coupled
- Transformer coupled.

3.2 Small Signal Tuned Amplifier

A common emitter amplifier can be converted into a single tuned amplifier by including a parallel tuned circuit as shown in Fig. 3.7. The biasing components are not shown for simplicity.

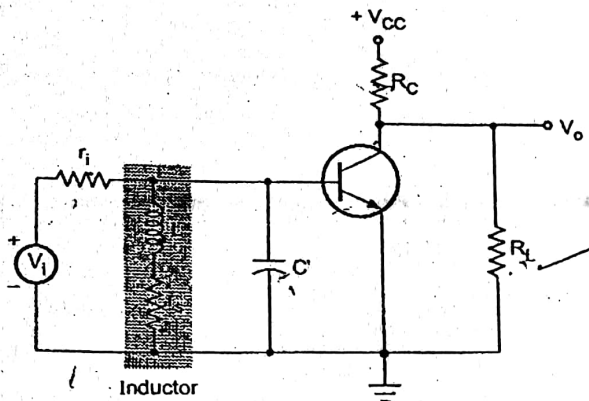


Fig. 3.7 Single tuned transistor amplifier

Before going to study the analysis of this amplifier we see the several practical assumptions to simplify the analysis.

Assumptions :

1. $R_L \ll R_C$
2. $r_{bb'} = 0$

With these assumptions, the simplified equivalent circuit for a single tuned amplifier is shown in Fig. 3.8.

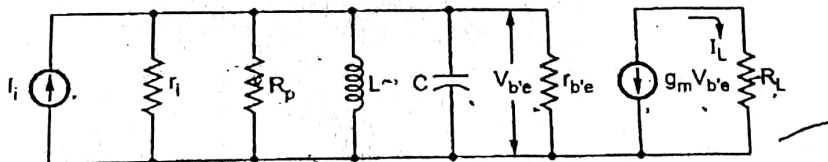


Fig. 3.8 Equivalent circuit of single tuned amplifier

where

$$C_{eq} = C' + C_{b'e} + (1 + g_m R_L) C_{b'c}$$

C' : External capacitance used to tune the circuit

$(1 + g_m R_L) C_{b'c}$: The Miller capacitance

r_s : Represents the losses in coil

The series RL circuit in Fig. 3.7 is replaced by the equivalent RL circuit in Fig. 3.8 assuming coil losses are low over the frequency band of interest, i.e., the coil Q high.

$$Q_c = \frac{\omega L}{r_c} \gg 1$$

... (1)

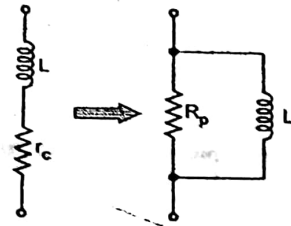


Fig. 3.9 Equivalent circuits

$$Y_1 = \frac{r_c}{\omega^2 L^2 + j\omega L}$$

$$Y_2 = \frac{1}{R_p} + \frac{1}{j\omega L}$$

∴ Therefore, equating Y_1 and Y_2 we get,

$$\frac{r_c}{\omega^2 L^2 + j\omega L} = \frac{1}{R_p} + \frac{1}{j\omega L}$$

$$\begin{aligned} \frac{1}{R_p} &= \frac{r_c}{\omega^2 L^2} \\ &= \frac{r_c^2}{r_c \omega^2 L^2} = \frac{1}{r_c Q_c^2} \end{aligned}$$

$$R_p = r_c Q_c^2 = \omega L Q_c$$

∴ $\omega L = Q_c r_c$ from equation (1) ... (2)

Looking at Fig. 3.8 we have,

$$R = r_i \parallel R_p \parallel r_{b'e}$$

... (3)

The current gain of the amplifier is then

$$A_i = \frac{-g_m R}{1 + j(\omega RC - R/\omega L)} = \frac{-g_m R}{1 + j\omega_0 RC(\omega/\omega_0 - \omega_0/\omega)} \quad \dots (4)$$

where $\omega_0^2 = \frac{1}{LC}$

The conditions for equivalence are most easily established by equating the admittances of the two circuits shown in Fig. 3.9.

$$\begin{aligned} Y_1 &= \frac{1}{r_c + j\omega L} = \frac{r_c - j\omega L}{r_c^2 + \omega^2 L^2} \\ &= \frac{r_c}{r_c^2 + \omega^2 L^2} - \frac{j\omega L}{r_c^2 + \omega^2 L^2} \end{aligned}$$

∴ $\omega L \gg r_c$ from equation (1)

$$\frac{1}{Q_c} = \frac{r_c}{\omega L} \Rightarrow Q_c = \frac{\omega L}{r_c}$$

We define the Q of the tuned circuit at the resonant frequency ω_o to be

$$Q_i = \frac{R}{\omega_o L} = \omega_o RC \quad \dots(5)$$

$$\therefore A_i = \frac{-g_m R}{1 + jQ_i(\omega/\omega_o - \omega_o/\omega)}$$

At $\omega = \omega_o$, gain is maximum and it is given as,

$$\therefore A_{i(\max)} = -g_m R \quad \dots(6)$$

The Fig. 3.10 shows the gain versus frequency plot for single tuned amplifier. It shows the variation of the magnitude of the gain as a function of frequency.

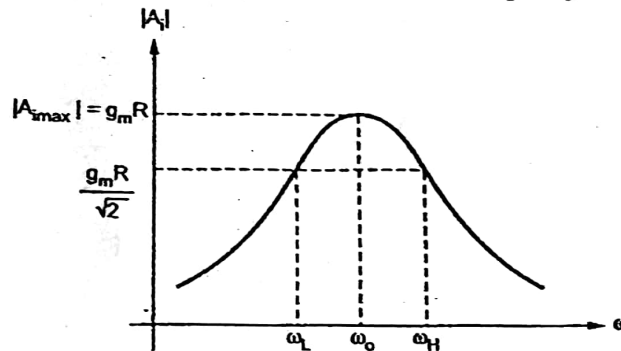


Fig. 3.10 Gain versus frequency for single tuned amplifier

At 3 dB frequency,

$$|A_i| = \frac{g_m R}{\sqrt{2}} \quad \dots(7)$$

\therefore At 3 dB frequency

$$1 + jQ_i [(\omega/\omega_o) - (\omega_o/\omega)] = \sqrt{2}$$

$$\therefore 1 + Q_i^2 \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right)^2 = 2 \quad \dots(8)$$

This equation is quadratic in ω^2 and has two positive solutions, ω_H and ω_L . After solving equation (8) we get 3 dB bandwidth as given below.

$$BW = f_H - f_L = \frac{\omega_o}{2\pi Q_i} = \frac{1}{2\pi RC} \quad \dots(9)$$

2π RC

$$\therefore BW = \frac{1}{2\pi RC}$$

► Example 3.1 : Design a single tuned amplifier for following specifications :

1. Centre frequency = 500 kHz

2. Bandwidth = 10 kHz

Assume transistor parameters : $g_m = 0.04$ S, $h_{fe} = 100$, $C_{b'e} = 1000$ pF and $C_{b'c} = 100$ pF. The bias network and the input resistance are adjusted so that $r_i = 4$ k Ω and $R_L = 510$ Ω .

Solution : From equation (9) we have,

$$BW = \frac{1}{2\pi RC}$$

$$\begin{aligned} \therefore RC &= \frac{1}{2\pi BW} = \frac{1}{2\pi \times 10 \times 10^3} \\ &= 15.912 \times 10^{-6} \end{aligned}$$

From equation (3) we have,

$$R = r_i \parallel R_p \parallel r_{b'e}$$

where

$$r_i = 4 \text{ k}\Omega$$

$$r_{b'e} = \frac{h_{fe}}{g_m} = \frac{100}{0.04} = 2500 \Omega$$

$$R_p = Q_c \omega_o L = \frac{Q_c}{\omega_o C}$$

$$\therefore R = 4 \times 10^3 \parallel 2500 \parallel \frac{Q_c}{\omega_o C}$$

$$C = \frac{1}{2\pi \times 10 \times 10^3 \times R}$$

$$\therefore C = \frac{1}{2\pi \times 10 \times 10^3 \times \left[4 \times 10^3 \parallel 2500 \parallel \frac{Q_c}{2\pi \times 500 \times 10^3 \times C} \right]}$$

The typical range for Q_c is 10 to 150. However, we have to assume Q such that value of C_p should be positive. Let us assume $Q = 100$.

$$\begin{aligned} \therefore C &= \frac{1}{2\pi \times 10 \times 10^3 \left[1538.5 \parallel \frac{1}{2\pi \times 5000 \times C} \right]} \\ &= \frac{1}{2\pi \times 10 \times 10^3 \left[\frac{1}{\frac{1}{1538.5} + 2\pi \times 5000 \times C} \right]} \end{aligned}$$

Solving for C we get,

$$C = 0.02 \mu\text{F}$$

We have,

$$C = C' + C_{v'e} + (1 + g_m R_L) C_{v'c}$$

\therefore

$$C' = C - [C_{v'e} + (1 + g_m R_L) C_{v'c}]$$

$$= 0.02 \times 10^{-6} - [1000 \times 10^{-12} + (1 + 0.04 \times 510) \times 100 \times 10^{-12}]$$

\therefore

$$C' = 0.01686 \mu\text{F}$$

We have,

$$\omega_0^2 = \frac{1}{LC}$$

\therefore

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi \times 500 \times 10^3)^2 \times 0.02 \times 10^{-6}}$$

$$= 5 \mu\text{H}$$

From equation (2) we have,

$$R_p = \omega L Q_c = 2\pi \times 500 \times 10^3 \times 5 \times 10^{-6} \times 100$$

$$= 1570 \Omega$$

\therefore

$$R = r_i \parallel R_p \parallel r_{v'e}$$

$$= 4 \times 10^3 \parallel 1570 \parallel 2500$$

$$= 777 \Omega$$

We have mid frequency gain as,

$$A_{i \max} = -g_m R = (-0.04)(777) = -31$$

3.3 Single Tuned FET Amplifier

The Fig. 3.11 shows the single tuned FET amplifier.

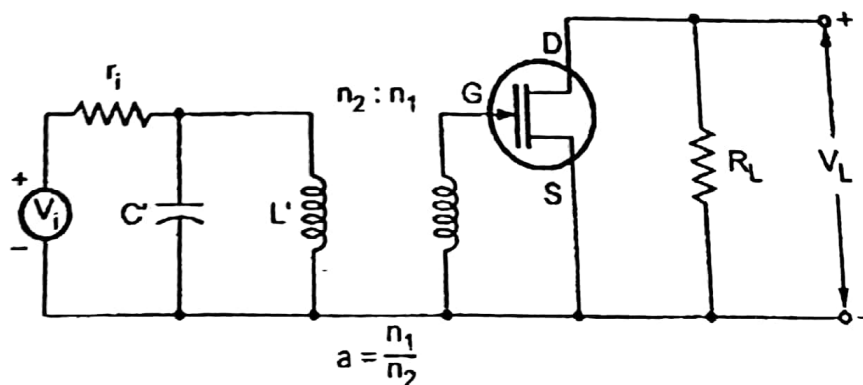


Fig. 3.11 Single tuned FET amplifier

The equivalent circuit for the given amplifier is as shown in the Fig. 3.12.

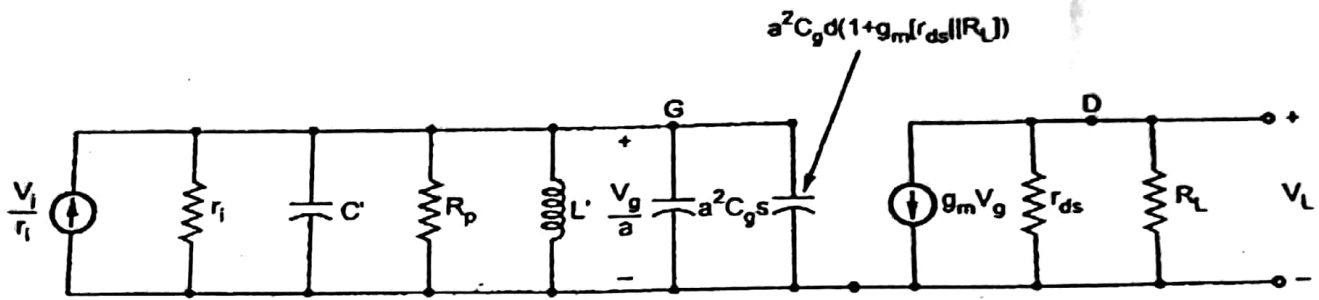


Fig. 3.12 Equivalent circuit of single tuned FET amplifier

The voltage gain is given by,

$$A_v = -a g_m (r_{ds} \parallel R_L) [(r_i \parallel R_p) / r_i] \quad \dots (1)$$

where

$$C_i = a^2 [C_{gs} + C_{gd} [1 + g_m (r_{ds} \parallel R_L)]] \quad \dots (2)$$

$$Q_i = \omega_o (r_i \parallel R_p) (C' + C_i) \quad \dots (3)$$

$$\omega_o^2 = \frac{1}{L(C' + C_i)} \quad \dots (4)$$

At centre frequency, i.e., at $\omega = \omega_o$ gain is

$$A_{v \max} = -a g_m (r_{ds} \parallel R_L) \frac{R_p}{r_i + R_p} \quad \dots (5)$$

The 3 dB bandwidth is given by,

$$BW = \frac{1}{2\pi(r_i \parallel R_p)(C' + C_i)} \quad \dots (6)$$

3.4 Single Tuned Capacitive Coupled Amplifier

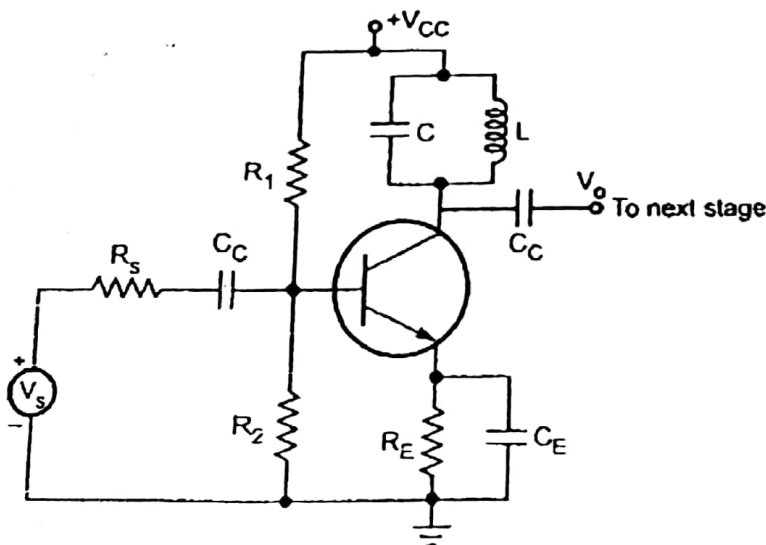


Fig. 3.13 Single tuned capacitive coupled transistor amplifier

Single tuned multistage amplifier circuit uses one parallel tuned circuit as a load in each stage with tuned circuits in all stages tuned to the same frequency. Fig. 3.13 shows a typical single tuned amplifier in CE configuration.

As shown in Fig. 3.13 tuned circuit formed by L and C acts as collector load and resonates at frequency of operation. Resistors R_1 , R_2 and R_E along with capacitor C_E provides self bias for the circuit.

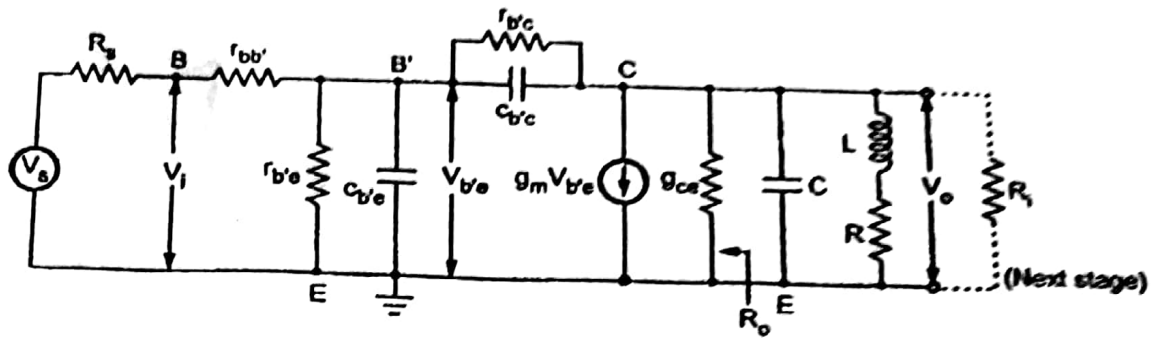


Fig. 3.14 Equivalent circuit of single tuned amplifier

The Fig. 3.14 shows the equivalent circuit for single tuned amplifier using hybrid α parameters.

As shown in the Fig. 3.14, R_i is the input resistance of the next stage and R_o is the output resistance of the current generator $g_m V_{b'e}$. The reactances of the bypass capacitor C_E and the coupling capacitors C_C are negligibly small at the operating frequency and hence these elements are neglected in the equivalent circuit shown in the Fig. 3.14.

The equivalent circuit shown in Fig. 3.14 can be simplified by applying Miller's theorem. Fig. 3.15 shows the simplified equivalent circuit for single tuned amplifier.

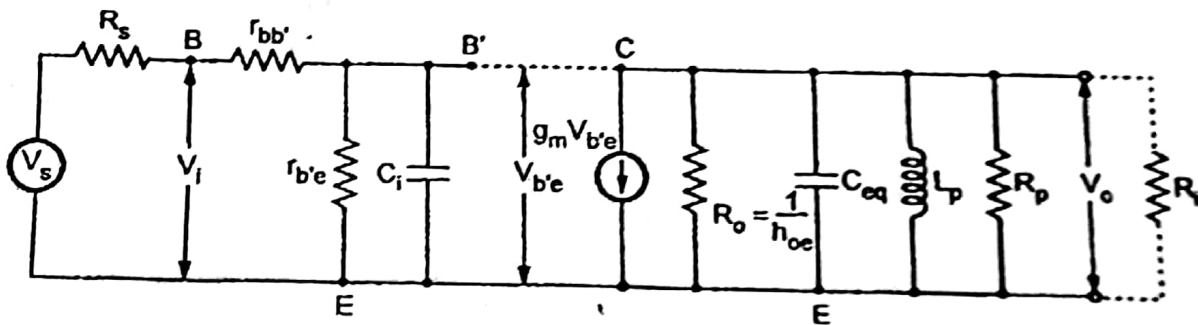


Fig. 3.15 Simplified equivalent circuit for single tuned amplifier

Here C_i and C_{eq} represent input and output circuit capacitances, respectively. They can be given as,

$$C_i = C_{b'e} + C_{b'c} (1 - A) \quad \text{where } A \text{ is the voltage gain of the amplifier.} \quad \dots(1)$$

$$C_{eq} = C_{b'c} \left(\frac{A-1}{A} \right) + C \quad \text{where } C \text{ is the tuned circuit capacitance.} \quad \dots (2)$$

The g_{ce} is represented as the output resistance of current generator $g_m V_{b'e}$.

$$g_{ce} = \frac{1}{r_{ce}} = h_{oe} - g_m h_{re} = h_{oe} = \frac{1}{R_o} \quad \dots (3)$$

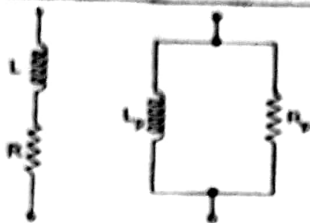


Fig. 3.16

The series RL circuit is represented by its equivalent parallel circuit. The conditions for equivalence are most easily established by equating the admittances of the two circuits shown in Fig. 3.16.

Admittance of the series combination of RL is given as,

$$Y = \frac{1}{R + j\omega L}$$

Multiplying numerator and denominator by $R - j\omega L$ we get,

$$\begin{aligned} Y &= \frac{R - j\omega L}{R^2 + \omega^2 L^2} = \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2} \\ &= \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega^2 L}{\omega(R^2 + \omega^2 L^2)} \\ &= \frac{1}{R_p} + \frac{1}{j\omega L_p} \end{aligned}$$

$$\text{where } R_p = \frac{R^2 + \omega^2 L^2}{R} \quad \dots (4)$$

$$\text{and } L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L} \quad \dots (5)$$

Centre frequency

The centre frequency or resonant frequency is given as,

$$f_r = \frac{1}{2\pi \sqrt{L_p C_{eq}}} \quad \dots (6)$$

$$\text{where } L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L}$$

$$\begin{aligned} \text{and } C_{eq} &= C_{tr} \left(\frac{A-1}{A} \right) + C \\ &= C_o + C \end{aligned} \quad \dots (7)$$

Therefore, C_{eq} is the summation of transistor output capacitance and the tuned circuit capacitance.

Quality factor Q

The quality factor Q of the coil at resonance is given by,

$$Q = \frac{\omega_r L}{R} \quad (8)$$

where ω_r is the centre frequency or resonant frequency.

This quality factor is also called unloaded Q. But in practice, transistor output resistance and input resistance of next stage act as a load for the tuned circuit. The quality factor including load is called as loaded Q and it can be given as follows :

The Q of the coil is usually large so that $\omega L \gg R$ in the frequency range of operation.

From equation (4) we have,

$$R_p = \frac{R^2 + \omega^2 L^2}{R} = R + \frac{\omega^2 L^2}{R}$$

$$\text{As } \frac{\omega^2 L^2}{R} \gg 1, R_p = \frac{\omega^2 L^2}{R} \quad \dots(9)$$

From equation (5) we have,

$$L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L} = \frac{R^2}{\omega^2 L} + L$$

$$\approx L \quad \because \omega L \gg R \quad \dots (10)$$

From equation (9), we can express R_p at resonance as,

$$R_p = \frac{\omega_r^2 L^2}{R}$$

$$= \omega_r Q_r L \quad \because Q_r = \frac{\omega_r L}{R} \quad \dots (11)$$

Therefore, Q_r can be expressed in terms of R_p as,

$$Q_r = \frac{R_p}{\omega_r L} \quad \dots(12)$$

The effective quality factor including load can be calculated looking at the simplified equivalent output circuit for single tuned amplifier.

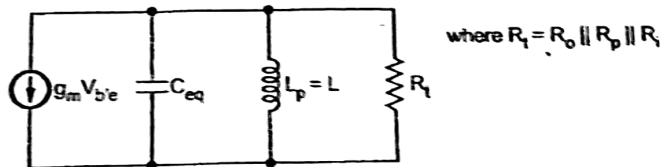


Fig. 3.17 Simplified output circuit for single tuned amplifier

$$\text{Effective quality factor } Q_{\text{eff}} = \frac{\text{Susceptance of inductance } L \text{ or capacitance } C}{\text{Conductance of shunt resistance } R_t}$$

$$= \frac{R_t}{\omega_r L} \text{ or } \omega_r C_{\text{eq}} R_t \quad \dots (13)$$

Voltage gain (A_v)

The voltage gain for single tuned amplifier is given by,

$$A_v = -g_m \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \times \frac{R_t}{1 + 2jQ_{eff}\delta}$$

where

$$R_t = R_o \parallel R_p \parallel R_i$$

δ = Fraction variation in the resonant frequency

$$A_v \text{ (at resonance)} = -g_m \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \times R_t$$

$$\therefore \left| \frac{A_v}{A_v \text{ (at resonance)}} \right| = \frac{1}{\sqrt{1 + (2Q_{eff}\delta)^2}} \quad \dots (14)$$

3 dB bandwidth

The 3 dB bandwidth of a single tuned amplifier is given by,

$$\begin{aligned} \Delta f &= \frac{1}{2\pi R_t C_{eq}} \\ &= \frac{\omega_r}{2\pi Q_{eff}} \quad \because Q_{eff} = \omega_r R_t C_{eq} \quad \dots (15) \end{aligned}$$

$$= \frac{f_r}{Q_{eff}} \quad \because \omega_r = 2\pi f_r \quad \dots (16)$$

➔ **Example 3.2 :** A single tuned RF amplifier uses a transistor with an output resistance of 50 K, output capacitance of 15 pF and input resistance of next stage is 20 kΩ. The tuned circuit consists of 47 pF capacitance in parallel with series combination of 1 μH inductance and 2 Ω resistance. Calculate

- i) Resonant frequency
- ii) Effective quality factor
- iii) Bandwidth of the circuit

Solution : i) Resonant frequency f_r is given as,

$$\begin{aligned} f_r &= \frac{1}{2\pi \sqrt{LC_{eq}}} \\ &= \frac{1}{2\pi \sqrt{1 \mu\text{H} \times (15 \text{ pF} + 47 \text{ pF})}} \\ &= 20.2 \text{ MHz} \end{aligned}$$

ii) Effective quality factor is given as,

$$\begin{aligned} Q_{eff} &= \omega_r C_{eq} R_t \\ &= 2\pi f_r C_{eq} \times (R_o \parallel R_p \parallel R_i) \end{aligned}$$

$$\text{where } R_p = \frac{\omega_r^2 L^2}{R} = \frac{(2\pi \times 20.2 \times 10^6)^2 (1 \times 10^{-6})^2}{2}$$

$$= 8054 \Omega$$

$$\therefore Q_{\text{eff}} = 2\pi \times 20.2 \times 10^6 \times (15 \text{ pF} + 47 \text{ pF}) \times (50 \text{ K} \parallel 8.054 \text{ K} \parallel 20 \text{ K})$$

$$= 40.52$$

iii) Bandwidth of the circuit is given as,

$$\text{BW} = \frac{f_r}{Q_{\text{eff}}} = \frac{20.2 \times 10^6}{40.52}$$

$$= 498.5 \text{ kHz}$$

➔ **Example 3.3 :** A single tuned transistor amplifier is used to amplify modulated RF carrier of 600 kHz and bandwidth of 15 kHz. The circuit has a total output resistance, $R_t = 20 \text{ k}\Omega$ and output capacitance $C_o = 50 \text{ pF}$. Calculate values of inductance and capacitance of the tuned circuit.

Solution : Given : $f_r = 600 \text{ kHz}$

$$\text{BW} = 15 \text{ kHz}$$

$$R_t = 20 \text{ k}\Omega$$

$$C_o = 50 \text{ pF}$$

$$\therefore C_{\text{eq}} = (50 \text{ pF} + C)$$

$$Q_{\text{eff}} = \frac{f_r}{\text{BW}} = \frac{600 \text{ kHz}}{15 \text{ kHz}}$$

$$= 40$$

i) We know that,

$$Q_{\text{eff}} = \omega_r C_{\text{eq}} R_t$$

$$\therefore C_{\text{eq}} = \frac{Q_{\text{eff}}}{\omega_r R_t} = \frac{40}{2\pi \times 600 \times 10^3 \times 20 \times 10^3}$$

$$= 530.5 \text{ pF}$$

$$C_{\text{eq}} = (50 \text{ pF} + C)$$

$$\therefore C = 530.5 \text{ pF} - 50 \text{ pF}$$

$$= 480.5 \text{ pF}$$

ii) We know that,

$$f_r = \frac{1}{2\pi \sqrt{LC_{\text{eq}}}}$$

$$\therefore L = \frac{1}{(2\pi f_r)^2 C_{\text{eq}}} = \frac{1}{(2\pi \times 600 \times 10^3)^2 \times 530.5 \times 10^{-12}}$$

$$= 132.6 \mu\text{H}$$

3.5 Double Tuned Amplifier

Fig. 3.18 shows double tuned RF amplifier in CE configuration. Here, voltage developed across tuned circuit is coupled inductively to another tuned circuit. Both tuned circuits are tuned to the same frequency.

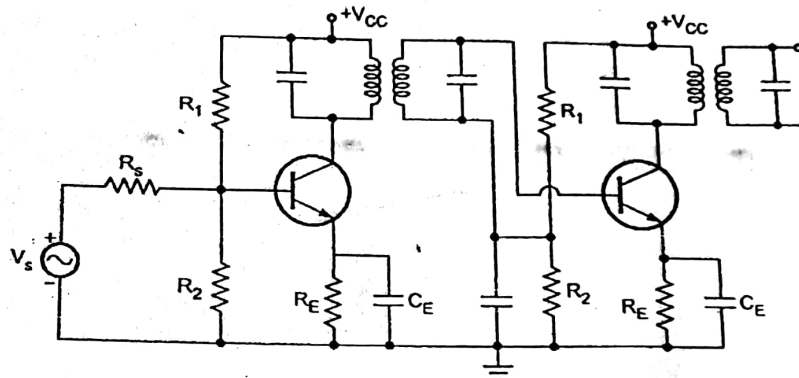


Fig. 3.18 Double tuned amplifier

The double tuned circuit can provide a bandwidth of several percent of the resonant frequency and gives steep sides to the response curve. Let us analyze the double tuned circuit.

Analysis

The Fig. 3.19 (a) shows the coupling section of a transformer coupled double tuned amplifier. The Fig. 3.19 (b) shows the equivalent circuit for it. In which transistor is replaced by the current source with its output resistance (R_o). The C_1 and L_1 are the tank circuit components of the primary side. The resistance R_1 is the series resistance of the inductance L_1 . Similarly on the secondary side L_2 and C_2 represents tank circuit components of the secondary side and R_2 represents resistance of the inductance L_2 . The resistance R_i represents the input resistance of the next stage.

The Fig. 3.19 (c) shows the simplified equivalent circuit for the Fig. 3.19 (b). In simplified equivalent circuit the series and parallel resistances are combined into series elements. Referring equation (9) we have,

$$R_p = \frac{\omega^2 L^2}{R} \text{ i.e. } R = \frac{\omega^2 L^2}{R_p}$$

where R represents series resistance and R_p represents parallel resistance.

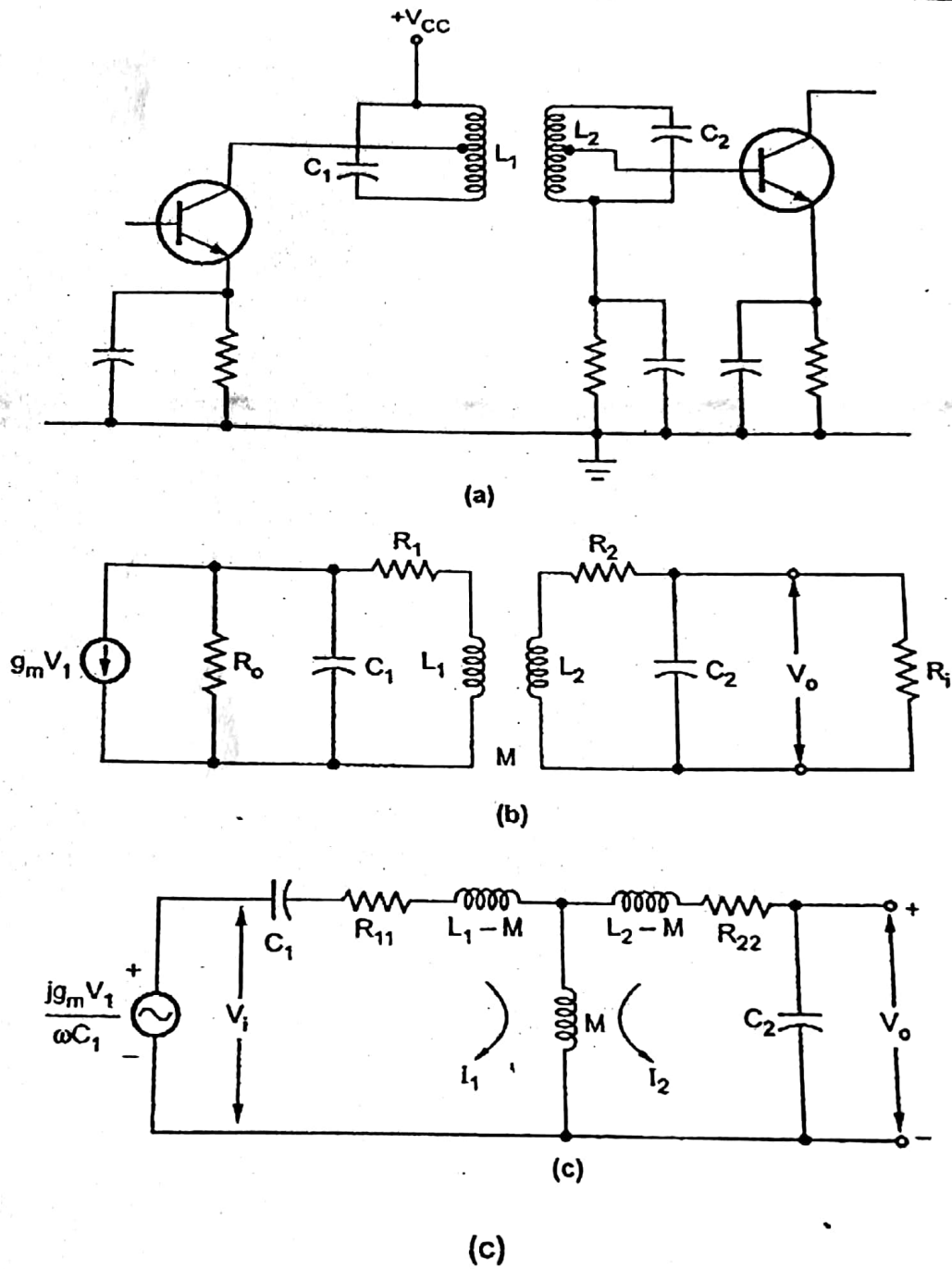


Fig. 3.19 Equivalent circuits for double tuned amplifier

Therefore we can write,

$$R_{11} = \frac{\omega_o^2 L_1^2}{R_o} + R_1$$

$$R_{12} = \frac{\omega_o^2 L_2^2}{R_i} + R_2$$

In the simplified circuit the current source is replaced by voltage source, which is now in series with C_1 . It also shows the effect of mutual inductance on primary and secondary sides.

$$\text{We know that, } Q = \frac{\omega_r L}{R}$$

Therefore, the Q factors of the individual tank circuits are

$$Q_1 = \frac{\omega_r L_1}{R_{11}} \text{ and } Q_2 = \frac{\omega_r L_2}{R_{22}} \quad \dots(1)$$

Usually, the Q factors for both circuits are kept same. Therefore, $Q_1 = Q_2 = Q$ and the resonant frequency $\omega_r^2 = 1/L_1 C_1 = 1/L_2 C_2$.

Looking at Fig. 3.19 (c), the output voltage can be given as,

$$V_o = -\frac{j}{\omega_r C_2} I_2 \quad \dots (2)$$

To calculate V_o/V_1 it is necessary to represent I_2 in terms of V_1 . For this we have to find the transfer admittance Y_T . Let us consider the circuit shown in Fig. 3.20. For this circuit, the transfer admittance can be given as,

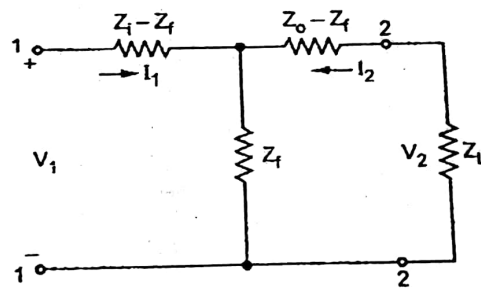


Fig. 3.20

$$Y_T = \frac{I_2}{V_1} = \frac{I_2}{I_1 Z_{11}} = \frac{A_i}{Z_{11}}$$

$$= \frac{Z_f}{Z_f^2 - Z_i (Z_o + Z_L)}$$

where

$$Z_{11} = \frac{V_1}{I_1} = Z_i - \frac{Z_f^2}{Z_o + Z_L} \text{ and}$$

$$A_i = \frac{I_2}{I_1} = \frac{-Z_f}{Z_o + Z_L}$$

The simplified equivalent circuit for double tuned amplifier is similar to the circuit shown in Fig. 3.20 with

$$Z_1 = j \omega_r M$$

$$Z_1 = R_{11} + j \left(\omega L_1 - \frac{1}{\omega C_1} \right)$$

$$Z_o + Z_L = R_{22} + j \left(\omega L_2 - \frac{1}{\omega C_2} \right)$$

The equations for Z_1 , Z_1 and $Z_o + Z_L$ can be further simplified as shown below.

$$Z_1 = j \omega_r M = j \omega_r k \sqrt{L_1 L_2}$$

where, k is the coefficient of coupling.

Multiplying numerator and denominator by $\omega_r L_1$ for Z_1 we get,

$$\begin{aligned} Z_1 &= \frac{R_{11} \omega_r L_1}{\omega_r L_1} + j \omega_r L_1 \left(\frac{\omega L_1}{\omega_r L_1} - \frac{1}{\omega C_1 \omega_r L_1} \right) \\ &= \frac{\omega_r L_1}{Q} + j \omega_r L_1 \left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega} \right) && \because Q = \frac{\omega_r L}{R_{11}} \text{ and } \frac{1}{\omega_r L} = \omega_r C \\ &= \frac{\omega_r L_1}{Q} + j \omega_r L_1 (2\delta) && \because \frac{\omega}{\omega_r} - \frac{\omega_r}{\omega} = 1 + \delta - (1 - \delta) = 2\delta \\ &= \frac{\omega_r L_1}{Q} + (1 + j2Q\delta) \end{aligned}$$

$$Z_o + Z_L = R_{22} + j \left(\omega L_2 - \frac{1}{\omega C_2} \right)$$

By doing similar analysis as for Z_1 we can write,

$$Z_o + Z_L = \frac{\omega_r L_2}{Q} + (1 + j2Q\delta)$$

Then

$$Y_T = \frac{Z_i}{Z_i^2 - Z_i(Z_o + Z_L)} = \frac{1}{Z_i - Z_i(Z_o + Z_L)/Z_i}$$

$$Y_T = \frac{1}{j\omega_r k\sqrt{L_1 L_2} - \left[\frac{\omega_r L_1}{Q} (1 + j2Q\delta) \left\{ \frac{\omega_r L_2}{Q} (1 + j2Q\delta) \right\} \right]}$$

$$Y_T = \frac{kQ^2}{\omega_r \sqrt{L_1 L_2} [4Q\delta - j(1 + k^2Q^2 - 4Q^2\delta^2)]} \quad \dots (3)$$

Substituting value of L_2 , i.e. $V_i \times Y_T$ we get,

$$V_o = \frac{-j}{\omega_r C_2} \frac{jg_m V_i}{\omega_r C_1} \left[\frac{kQ^2}{\omega_r \sqrt{L_1 L_2} [4Q\delta - j(1 + k^2Q^2 - 4Q^2\delta^2)]} \right]$$

$$\therefore V_i = \frac{jg_m V_1}{\omega C_1}$$

$$\therefore A_v = \frac{V_o}{V_i} = g_m \omega_r^2 L_1 L_2 \left[\frac{kQ^2}{\omega_r \sqrt{L_1 L_2} [4Q\delta - j(1 + k^2Q^2 - 4Q^2\delta^2)]} \right]$$

$$\therefore \frac{1}{\omega_r C} = \omega_r L$$

$$= \left[\frac{g_m \omega_r \sqrt{L_1 L_2} kQ^2}{4Q\delta - j(1 + k^2Q^2 - 4Q^2\delta^2)} \right] \quad \dots (4)$$

Taking the magnitude of equation (4) we have,

$$|A_v| = g_m \omega_r \sqrt{L_1 L_2} Q \frac{kQ}{\sqrt{1 + k^2Q^2 - 4Q^2\delta^2 + 16Q^2\delta^2}} \quad \dots (5)$$

The Fig. 3.21 shows the universal response curve for double tuned amplifier plotted with kQ as a parameter.

The frequency deviation δ at which the gain peaks occur can be found by maximizing equation (4), i.e.

$$4Q\delta - j(1 + k^2Q^2 - 4Q^2\delta^2) = 0 \quad \dots (6)$$

We know that, the 3 dB bandwidth for single tuned amplifier is $2 f_r/Q$. Therefore, the 3 dB bandwidth provided by double tuned amplifier ($3.1 f_r/Q$) is substantially greater than the 3 dB bandwidth of single tuned amplifier.

Compared with a single tuned amplifier, the double tuned amplifier

1. Possesses a flatter response having steeper sides.
2. Provides larger 3 dB bandwidth.
3. Provides large gain-bandwidth product.

3.6 Effect of Cascading Single Tuned Amplifier on Bandwidth

In order to obtain a high overall gain, several identical stages of tuned amplifiers can be used in cascade. The overall gain is the product of the voltage gains of the individual stages. Let us see the effect of cascading of stages on bandwidth.

Consider n stages of single tuned direct coupled amplifiers connected in cascade. We know that the relative gain of a single tuned amplifier with respect to the gain at resonant frequency f_r is given from equation (14) of section 3.4.

$$\left| \frac{A_v}{A_v(\text{at resonance})} \right| = \frac{1}{\sqrt{1+(2\delta Q_{\text{eff}})^2}}$$

Therefore, the relative gain of n stage cascaded amplifier becomes

$$\left| \frac{A_v}{A_v(\text{at resonance})} \right|^n = \left[\frac{1}{\sqrt{1+(2\delta Q_{\text{eff}})^2}} \right]^n = \frac{1}{[1+(2\delta Q_{\text{eff}})^2]^{\frac{n}{2}}}$$

The 3 dB frequencies for the n stage cascaded amplifier can be found by equating

$$\left| \frac{A_v}{A_v(\text{at resonance})} \right|^n = \frac{1}{\sqrt{2}}$$

$$\therefore \left| \frac{A_v}{A_v(\text{at resonance})} \right|^n = \frac{1}{[1+(2\delta Q_{\text{eff}})^2]^{\frac{n}{2}}} = \frac{1}{\sqrt{2}}$$

$$\therefore [1+(2\delta Q_{\text{eff}})^2]^{\frac{n}{2}} = 2^{\frac{1}{2}}$$

$$\therefore [1+(2\delta Q_{\text{eff}})^2]^n = 2$$

$$\therefore 1+(2\delta Q_{\text{eff}})^2 = 2^{\frac{1}{n}}$$

$$\therefore 2\delta Q_{\text{eff}} = \pm \sqrt{2^{\frac{1}{n}} - 1}$$

Substituting for δ , the fractional frequency variation, i.e.

$$\delta = \frac{\omega - \omega_r}{\omega_r} = \frac{f - f_r}{f_r}$$

$$\therefore 2 \left(\frac{f - f_r}{f_r} \right) Q_{\text{eff}} = \pm \sqrt{2^{\frac{1}{n}} - 1}$$

$$\therefore 2 (f - f_r) Q_{\text{eff}} = \pm f_r \sqrt{2^{\frac{1}{n}} - 1}$$

$$\therefore f - f_r = \pm \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^{\frac{1}{n}} - 1}$$

Let us assume f_1 and f_2 are the lower 3 dB and upper 3 dB frequencies, respectively. Then we have

$$f_2 - f_r = + \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^{\frac{1}{n}} - 1} \text{ and similarly,}$$

$$f_r - f_1 = + \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^{\frac{1}{n}} - 1}$$

The bandwidth of n stage identical amplifier is given as,

$$\begin{aligned} BW_n &= f_2 - f_1 = (f_2 - f_r) + (f_r - f_1) \\ &= \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^{\frac{1}{n}} - 1} + \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^{\frac{1}{n}} - 1} \\ &= \frac{f_r}{Q_{\text{eff}}} \sqrt{2^{\frac{1}{n}} - 1} \\ &= BW_1 \sqrt{2^{\frac{1}{n}} - 1} \quad \dots (1) \end{aligned}$$

where BW_1 is the bandwidth of single stage and BW_n is the bandwidth of n stages.

►►► **Example 3.4 :** The bandwidth for single tuned amplifier is 20 kHz. Calculate the bandwidth if such three stages are cascaded. Also calculate the bandwidth for four stages.

Solution : i) We know that,

$$\begin{aligned} BW_n &= BW_1 \sqrt{2^{\frac{1}{n}} - 1} = 20 \times 10^3 \times \sqrt{2^{\frac{1}{3}} - 1} \\ &= 10.196 \text{ kHz} \end{aligned}$$

$$\text{ii) } BW_n = 20 \times 10^3 \times \sqrt{2^{\frac{1}{4}} - 1} = 8.7 \text{ kHz}$$

The above example shows that bandwidth decreases as number of stages increase.

3.7 Effect of Cascading Double Tuned Amplifiers on Bandwidth

When a number of identical double tuned amplifier stages are connected in cascade, the overall bandwidth of the system is thereby narrowed and the steepness of the sides of the response is increased, just as when single tuned stages are cascaded. The quantitative relation between the 3 dB bandwidth of n identical double tuned critically coupled stages compared with the bandwidth Δ_2 of such a system can be shown to be 3 dB bandwidth for

$$n \text{ identical stages double tuned amplifiers} = \Delta_2 \times \left(2^{1/n} - 1\right)^{\frac{1}{4}} \quad \dots (1)$$

where $\Delta_2 = 3 \text{ dB bandwidth of single stage double tuned amplifier}$

Key Point: The equation (1) assumes that the bandwidth Δ_2 is small compared with the resonant frequency.

➔ **Example 3.5 :** The bandwidth for double tuned amplifier is 20 kHz. Calculate the bandwidth if such three stages are cascaded.

Solution : We know that for double tuned cascaded stages,

$$\begin{aligned} BW_n &= BW_1 \times \left(2^{1/n} - 1\right)^{\frac{1}{4}} \\ &= 20 \text{ K} \times \left(2^{1/3} - 1\right)^{\frac{1}{4}} \\ &= 14.28 \text{ kHz} \end{aligned}$$

➔ **Example 3.6 :** A three stage double tuned amplifier system is to have a half power BW of 20 kHz centred on a centre frequency of 450 kHz. Assuming that all stages are identical, determine the half power bandwidth of single stage. Assume that each stage couple to get maximum flatness.

Solution : We get maximum flat response when each stage is critically coupled. When stages are critically coupled we have

$$\begin{aligned} BW_n &= BW_1 \times (2^{1/n} - 1)^{1/4} \\ BW_n &= \frac{BW_n}{(2^{1/n} - 1)^{1/4}} \end{aligned}$$

For $n = 3$

$$\begin{aligned} BW_n &= \frac{BW_n}{(2^{1/3} - 1)^{1/4}} \\ &= \frac{20 \times 10^3}{(2^{1/3} - 1)^{1/4}} \\ &= 28.01 \text{ kHz} \end{aligned}$$

don i st - 0.707
 3dB
 26

3.8 Staggered Tuned Amplifier

We have seen that double tuned amplifier gives greater 3 dB bandwidth having steeper sides and flat top. But alignment of double tuned amplifier is difficult. To overcome this problem two single tuned cascaded amplifiers having certain bandwidth are taken and their resonant frequencies are so adjusted that they are separated by an amount equal to the bandwidth of each stage. Since the resonant frequencies are displaced or staggered, they are known as staggered tuned amplifiers. The advantage of staggered tuned amplifier is to have a better flat, wideband characteristics in contrast with a very sharp, rejective, narrow band characteristic of synchronously tuned circuits (tuned to same resonant frequencies). Fig. 3.23 shows the relation of amplification characteristics of individual stages in a staggered pair to the overall amplification of the two stages.

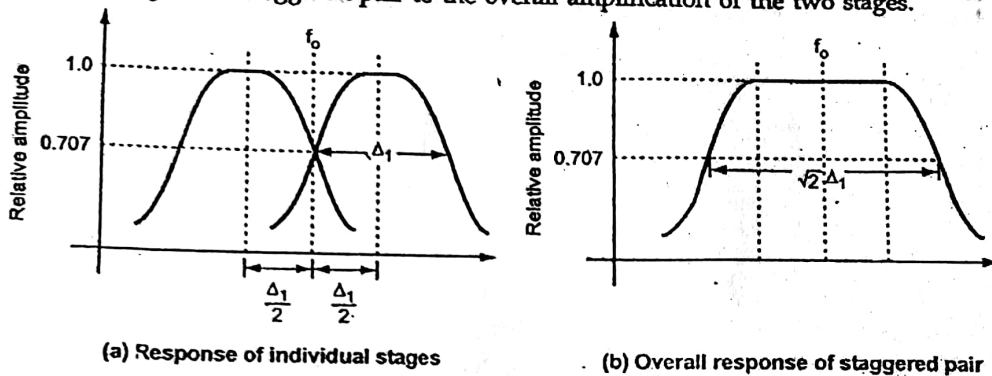


Fig. 3.23

The overall response of the two stage stagger tuned pair is compared in Fig. 3.24 with the corresponding individual single tuned stages having same resonant circuits. Looking at Fig. 3.24, it can be seen that staggering reduces the total amplification of the centre frequency to 0.5 of the peak amplification of the individual stage and at the centre frequency each stage has an amplification that is 0.707 of the peak amplification of the individual stage. Thus the equivalent voltage amplification per stage of the staggered pair is 0.707 times as great as when the same two stages are used without staggering. However,

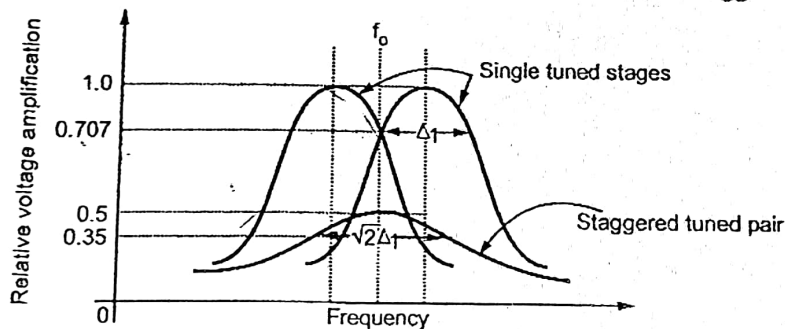
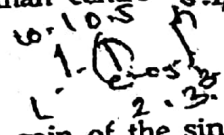


Fig. 3.24 Response of individually tuned and staggered tuned pair

the half power (3 dB) bandwidth of the staggered pair is $\sqrt{2}$ times as great as the half power (3 dB) bandwidth of an individual single tuned stage. Hence the equivalent gain bandwidth product per stage of a stagger tuned pair is $0.707 \times \sqrt{2} = 1.00$ times that of the individual single tuned stages.

The stagger tuned idea can easily be extended to more stages. In case of three stage staggering, the first tuning circuit is tuned to a frequency lower than centre frequency while the third circuit is tuned to higher frequency than centre frequency. The middle tuned circuit is tuned at exact centre frequency.



Analysis

From equation (14) of section 3.4 we can write the gain of the single tuned amplifier as,

$$\frac{A_v}{A_v \text{ (at resonance)}} = \frac{1}{1 + 2jQ_{eff} \delta}$$

$$= \frac{1}{1 + jX} \text{ where } X = 2Q_{eff} \delta$$

Since in stagger tuned amplifiers the two single tuned cascaded amplifiers with separate resonant frequencies are used, we can assume that the one stage is tuned to the frequency $f_r + \delta$ and other stage is tuned to the frequency $f_r - \delta$. Therefore we have,

$$f_{r1} = f_r + \delta$$

and

$$f_{r2} = f_r - \delta$$

According to these tuned frequencies the selectivity functions can be given as,

$$\frac{A_v}{A_v \text{ (at resonance)}_1} = \frac{1}{1 + j(X+1)} \text{ and}$$

$$\frac{A_v}{A_v \text{ (at resonance)}_2} = \frac{1}{1 + j(X-1)}$$

The overall gain of these two stages is the product of individual gains of the two stages.

$$\therefore \frac{A_v}{A_v \text{ (at resonance)}_{cascaded}} = \frac{A_v}{A_v \text{ (at resonance)}_1} \times \frac{A_v}{A_v \text{ (at resonance)}_2}$$

$$= \frac{1}{1 + j(X+1)} \times \frac{1}{1 + j(X-1)}$$

$$= \frac{1}{2 + 2jX - X^2} = \frac{1}{(2 - X^2) + (2jX)}$$

$$\therefore \left| \frac{A_v}{A_v \text{ (at resonance)}_{cascaded}} \right| = \frac{1}{\sqrt{(2 - X^2)^2 + (2X)^2}}$$

$$= \frac{1}{\sqrt{4 - 4X^2 + X^4 + 4X^2}} = \frac{1}{\sqrt{4 + X^4}}$$

Substituting the value of X we get,

$$\left| \frac{A_v}{A_v \text{ (at resonance)}} \right|_{\text{cascaded}} = \frac{1}{\sqrt{4 + (2Q_{\text{eff}}\delta)^4}} = \frac{1}{\sqrt{4 + 16Q_{\text{eff}}^4 \delta^4}}$$

$$= \frac{1}{2\sqrt{1 + 4Q_{\text{eff}}^4 \delta^4}}$$

3.9 Large Signal Tuned Amplifiers

The output efficiency of an amplifier increases as the operation shifts from class A to class C through class AB and class B. As the output power of a radio transmitter is high and the efficiency is of prime concern, class B and class C amplifiers are used at the output stages in transmitters.

The operation of class B and class C amplifiers are non-linear since the amplifying elements remain cut-off during a part of the input signal cycle. The non-linearity generates harmonics of the signal frequency at the output of the amplifier. In the push-pull arrangement where the bandwidth requirement is no limited, these harmonics can be eliminated or reduced. When a narrow bandwidth is desired, a resonant circuit is employed in class B and class C tuned RF power amplifiers to eliminate the harmonics.

3.9.1 Class B Tuned Amplifier

The Fig. 3.25 shows the class B tuned amplifier. It works with a single transistor by sending half sinusoidal current pulses to the load. The transistor is biased at the edge of the conduction. Eventhough the input is half sinusoidal, the load voltage is sinusoidal because a high Q RLC tank shunts harmonics to ground. The negative half is delivered by

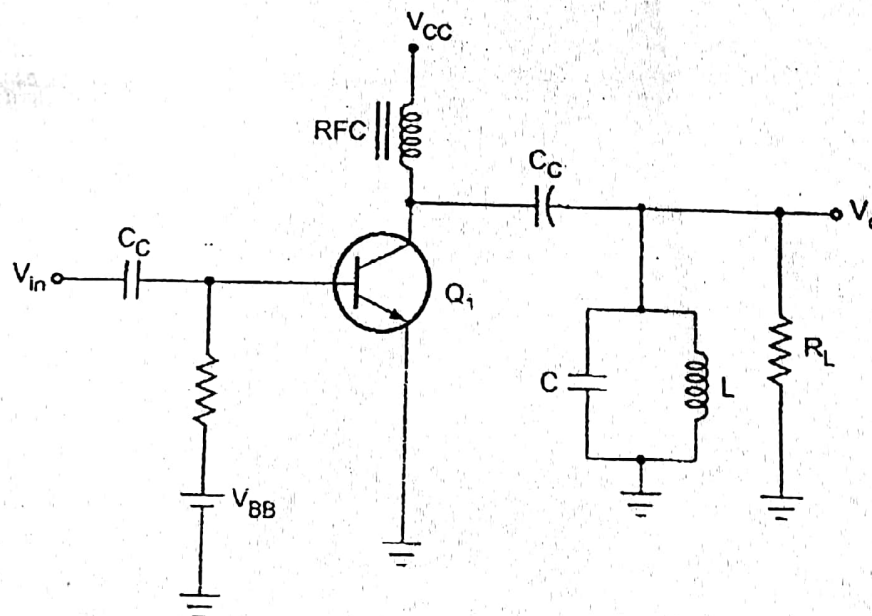


Fig. 3.25 Class B tuned amplifier

the RLC tank. The Q factor of the tank needs to be large enough to do this. This is analogous to pushing someone on a swing. We only need to push in one direction, and the reactive energy stored will swing the person back in the reverse direction.

3.9.2 Class C Tuned Amplifier

The amplifier is said to be class C amplifier, if the Q point and the input signal are selected such that the output signal is obtained for less than a half cycle, for a full input cycle.

Due to such a selection of the Q point, transistor remains active, for less than a half cycle. Hence only that much part is reproduced at the output. For remaining cycle of the input cycle, the transistor remains cut-off and no signal is produced at the output.

The current and voltage waveforms for a class C amplifier operation are shown in the Fig. 3.26.

Looking at Fig. 3.26, it is apparent that the total angle during which current flows is less than 180° . This angle is called the conduction angle, θ_c .

