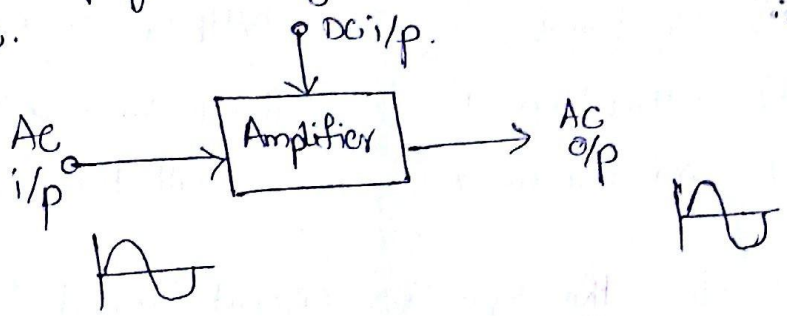


Unit-VI

Oscillators

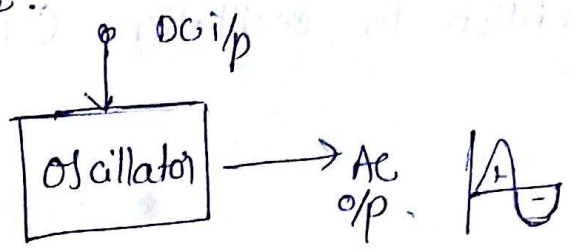
⇒ An Amplifier to amplify the signal it requires AC i/p and DC i/p as shown in figure.



⇒ Oscillator:- which circuit is used to provide or generate a periodic voltage with out an ac input signal is called oscillator.

→ To generate the periodic voltage, the circuit is supplied with energy from a dc source.

→ It is also called DC to AC Converter.



* Classification of oscillators:-

Oscillator can be classified in different ways.

1. According to the waveform generated:

- a) Sinusoidal oscillator (or) Harmonic oscillator
- b) Non-Sinusoidal oscillator (or) Relaxation oscillators.

2. According to the fundamental mechanism used.

- a) feedback oscillators
- b) -ve resistance oscillators. [Tunnel diode, VJT oscillator]

3. According to frequency generated.

- i) Audio frequency oscillator (AFO) \rightarrow ω_0 20 kHz
- ii) Radio " " RFO \rightarrow 20k - 30MHz
- iii) Very high " " (VHF)_{cs} \rightarrow 30M - 300MHz
- iv) Ultra high " " (UHF)_{on} \rightarrow 300M - 3GHz
- v) microwave " " oscillator \rightarrow above 3GHz

4. According to the type of circuit used.

- i) LC oscillator
- ii) RC oscillator.

-o-

* Condition for oscillation (Barkhausen Condition)

\Rightarrow oscillator produce continuous waveform with out any ac signal. Even when no external signal is applied, any conductor or active device produce - ever present noise will cause some small signal at the output of the amplifier.

\Rightarrow A small feedback network (β) is used to give that small noise signal on the i/p side which is nearest frequency of tank circuit. Then this feedback signal is amplified by amplifier.

\Rightarrow If the amplifier gain has is more than $1/\beta$, then the o/p increases and there by the feedback signal becomes larger.

\Rightarrow This process continues and the o/p goes on increasing. But as the signal level increases, the gain of the amplifier decreases and equal to $1/\beta$. Then o/p voltage remains constant at frequency ω_0 , called frequency of oscillation.

\Rightarrow The essential conditions for maintaining oscillation are:

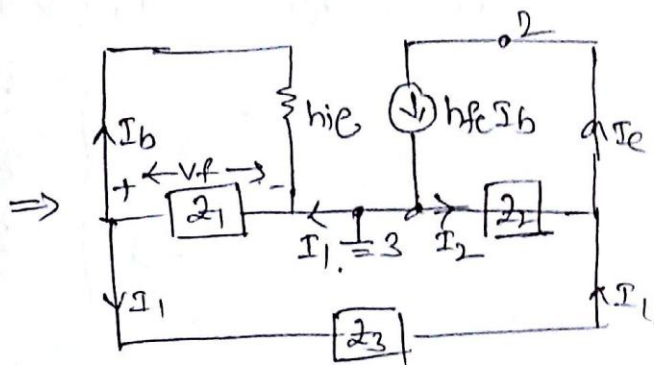
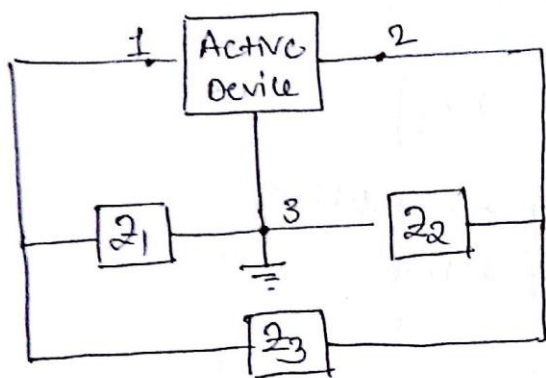
1) $|A\beta| = 1$ i.e. the magnitude of loop gain must be unity.

2) The total phase shift around the closed loop is zero or 360°

-o-

* General form of an LC Oscillator:-

(2)



$$\Rightarrow Z' = Z_1 \parallel h_{ie} \Rightarrow Z' = \frac{Z_1 h_{ie}}{Z_1 + h_{ie}}$$

\Rightarrow Total load impedance Z_L b/w 2 & 3 is $Z_2 \parallel (Z' + Z_3)$

$$\begin{aligned} \therefore Z_L &= \frac{Z_2 (Z' + Z_3)}{Z_2 + Z' + Z_3} \\ &= \frac{Z_2 \left(\frac{Z_1 h_{ie}}{Z_1 + h_{ie}} + Z_3 \right)}{Z_2 + \frac{Z_1 h_{ie}}{Z_1 + h_{ie}} + Z_3} \\ &= \frac{Z_1 Z_2 h_{ie} + Z_1 Z_2 Z_3 + Z_3 Z_2 h_{ie}}{Z_1 + h_{ie}} \\ &= \frac{Z_1 Z_2 + Z_2 h_{ie} + Z_1 h_{ie} + Z_1 Z_3 + Z_3 h_{ie}}{Z_1 + h_{ie}} \\ &= \frac{Z_2 [Z_1 h_{ie} + Z_1 Z_3 + Z_3 h_{ie}]}{h_{ie}(Z_1 + Z_2 + Z_3) + Z_1 Z_2 + Z_1 Z_3} \end{aligned}$$

$$\begin{aligned} A &= \frac{-h_{fe} Z_L}{h_{ie}} \\ &= \frac{-h_{fe} \cdot \frac{Z_2 (Z_1 h_{ie} + Z_1 Z_3 + Z_3 h_{ie})}{h_{ie}(Z_1 + Z_2 + Z_3) + Z_1 Z_2 + Z_1 Z_3}}{h_{ie}} \end{aligned}$$

$$\beta = \frac{V_f}{V_o}$$

$$\text{where } V_o = -I_1 (z_1 + z_3)$$

$$= -I_1 \left[\frac{z_{hie}}{z_{thie}} + z_3 \right]$$

$$= -I_1 \left[\frac{z_{hie} + z_1 z_3 + z_3 hie}{z_{thie}} \right]$$

$$V_f = -I_1 z_1$$

$$= -I_1 \left[\frac{z_{hie}}{z_{thie}} \right]$$

$$\beta = \frac{-I_1 \left[\frac{z_{hie}}{z_{thie}} \right]}{-I_1 \left[\frac{z_{hie} + z_1 z_3 + z_3 hie}{z_{thie}} \right]}$$

$$= \frac{z_{hie}}{z_{hie} + z_1 z_3 + z_3 hie}$$

$$\therefore A\beta = 1$$

$$\frac{-hfe z_2 [hie(z_1 z_3 + z_3 hie)]}{hie [hie(z_1 + z_2 + z_3) + z_1 z_2 + z_1 z_3]} \times \frac{hie z_1}{(z_{hie} + z_1 z_3 + z_3 hie)} = 1$$

$$\Rightarrow \frac{-hfe z_2 z_1}{hie(z_1 + z_2 + z_3) + z_1 z_2 + z_1 z_3} = 1$$

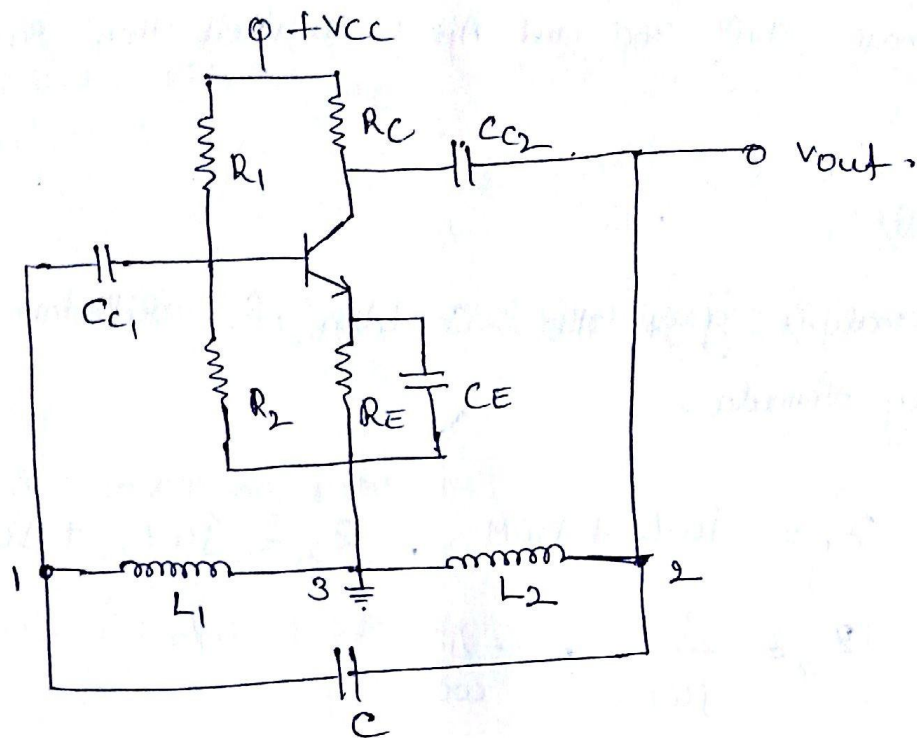
$$hie(z_1 + z_2 + z_3) + z_1 z_2 + z_1 z_3 = -hfe z_1 z_2$$

$$\therefore hie(z_1 + z_2 + z_3) + z_1 z_2 + z_1 z_3 + hfe z_1 z_2 = 0$$

$$hie(z_1 + z_2 + z_3) + z_1 z_2 (1 + hfe) + z_1 z_3 = 0$$

This is the general expression for LC oscillator.

* Hartley oscillator



- The Hartley oscillator consists of a CE amplifier circuit and feedback tank circuit consisting of L_1 , L_2 and C , determine frequency.
- The resistors R_1 , R_2 and R_E provide the necessary DC bias to the transistor. C_E is a bypass capacitor and C_{C1} , C_{C2} are coupling capacitors.

* operation -

- when the supply voltage $+V_{CC}$ is ON, the ^{transient} current is produced in the tank circuit and damped oscillations are set up in the circuit.
- As terminal '3' is earthed, it is at zero potential. If terminal 1 is at +ve and terminal 2 is negative. Thus the phase difference b/w the terminals 1 and 2 is always 180° . In the CE mode, the transistor provides the phase difference 180° b/w input and o/p. The total phase shift is 360° .

→ If the feedback is adjusted so that the loop gain $A\beta=1$ the circuit acts as an oscillator.

→ If the phase shift 360° and $A\beta=1$ satisfied then sustained oscillations produced.

→ Analysis:-

This analysis gives the condition for oscillation and oscillating frequency formula.

$$\text{Let } Z_1 = j\omega L_1 + j\omega M, \quad Z_2 = j\omega L_2 + j\omega M$$

$$Z_3 = \frac{1}{j\omega C} \quad \text{or} \quad \frac{-j}{\omega C}$$

→ The condition for LC oscillator for oscillations

$$h_{ie}(Z_1 + Z_2 + Z_3) + (1 + h_{fe})Z_1 Z_2 + Z_1 Z_3 = 0 \quad \text{--- (1)}$$

$$\rightarrow Z_1 + Z_2 + Z_3 = j\omega(L_1 + L_2 + 2M) - \frac{j}{\omega C}$$

$$= j\omega\left(L_1 + L_2 + 2M - \frac{1}{\omega^2 C}\right)$$

$$\rightarrow Z_1 Z_3 = (j\omega L_1 + j\omega M) \times \frac{1}{j\omega C} = \frac{j\omega(L_1 + M)}{j\omega C}$$

$$= \frac{L_1 + M}{C}$$

$$Z_1 Z_2 = (j\omega L_1 + j\omega M)(j\omega L_2 + j\omega M)$$

$$= j\omega(L_1 + M)j\omega(L_2 + M)$$

$$= j^2 \omega^2 (L_1 + M)(L_2 + M)$$

$$= -\omega^2 (L_1 + M)(L_2 + M)$$

then eq (1) becomes

$$\rightarrow \text{Subic } (L_1 + L_2 + 2M - \frac{1}{\omega^2 C}) - \omega^2 (L_1 + M) (L_2 + M) C + h_p C + \frac{L_1 + M}{C} = 0.$$

$$\text{Subic } (L_1 + L_2 + 2M - \frac{1}{\omega^2 C}) - \omega^2 (L_1 + M) \left[(L_2 + M) C + h_p C - \frac{1}{\omega^2 C} \right] = 0 \quad \text{--- eq (2)}$$

\Rightarrow making imaginary part = 0.

$$\text{Subic } [L_1 + L_2 + 2M - \frac{1}{\omega^2 C}] = 0$$

$$L_1 + L_2 + 2M - \frac{1}{\omega^2 C} = 0$$

$$L_1 + L_2 + 2M = \frac{1}{\omega^2 C} \quad \text{--- eq (3)}$$

$$\omega^2 C = \frac{1}{L_1 + L_2 + 2M}$$

$$\omega^2 = \frac{1}{(L_1 + L_2 + 2M) C}$$

$$\omega = \frac{1}{\sqrt{(L_1 + L_2 + 2M) C}}$$

$$2\pi f = \frac{1}{\sqrt{(L_1 + L_2 + 2M) C}}$$

$$f = \frac{1}{2\pi \sqrt{(L_1 + L_2 + 2M) C}}$$

$$\therefore f = \frac{1}{2\pi \sqrt{L_{eq} C}} \quad \text{where } L_{eq} = L_1 + L_2 + 2M.$$

→ Condition for oscillation.

Make real part = 0.

$$\omega^2 (L_1 + M) \left[(L_2 + M) (1 + hf_c) - \frac{1}{\omega^2 C} \right] = 0$$

$$\Rightarrow (L_2 + M) (1 + hf_c) - \frac{1}{\omega^2 C} = 0$$

$$(L_2 + M) (1 + hf_c) = \frac{1}{\omega^2 C} \quad \text{from eq (2)}$$

$$\frac{1}{\omega^2 C} = L_1 + L_2 + 2M$$

$$(L_2 + M) (1 + hf_c) = L_1 + L_2 + 2M$$

$$1 + hf_c = \frac{L_1 + L_2 + 2M}{L_2 + M} = \frac{(L_1 + M) + (L_2 + M)}{(L_2 + M)}$$

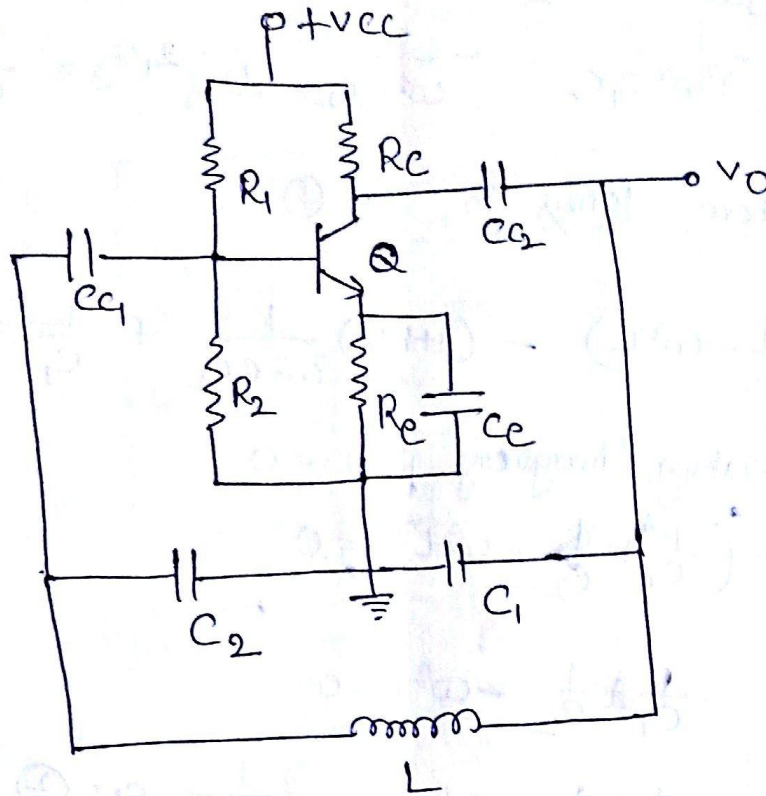
$$1 + hf_c = 1 + \left(\frac{L_1 + M}{L_2 + M} \right)$$

$$hf_c = \frac{L_1 + M}{L_2 + M}$$

$$\therefore \beta = hf_c = \frac{L_1 + M}{L_2 + M}$$

Colpitts oscillator:

(5)



- The Colpitts oscillator is also same as Hartley oscillator. But the tank circuit has two capacitance and one inductance L .
- In this oscillator also 180° phase shift provided by CE Amplifier and remaining phase shift 180° provided by feedback n/w.
- when the circuit is turned ON C_1 and C_2 are charged and after some time capacitors discharged through the inductor L setting up the frequency of oscillation.

Analysis:-

The frequency of oscillation is in general form

$$h_{ie}(z_1 + z_2 + z_3) + (1 + h_{fe})z_1 z_2 + z_1 z_3 = 0 \quad \text{--- eq (1)}$$

where, in Colpitts oscillator

$$z_1 = \frac{1}{j\omega C_1}, \quad z_2 = \frac{1}{j\omega C_2}, \quad z_3 = j\omega L.$$

$$\rightarrow z_1 + z_2 + z_3 = \frac{1}{j\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} - \omega^2 L \right)$$

$$\rightarrow z_1 z_2 = \frac{1}{j^2 \omega^2 C_1 C_2} = -\frac{1}{\omega^2 C_1 C_2}, \quad z_1 z_3 = \frac{L}{C_1}$$

put the above terms in eq ①.

$$\frac{hfc}{j\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} - \omega^2 L \right) - (1+hfc) \frac{1}{\omega^2 C_1 C_2} + \frac{L}{C_1} = 0.$$

making imaginary part = 0.

$$-\frac{hfc}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} - \omega^2 L \right) = 0$$

$$\frac{1}{C_1} + \frac{1}{C_2} - \omega^2 L = 0$$

$$\frac{1}{C_1} + \frac{1}{C_2} = \omega^2 L \quad \text{--- eq ②}$$

$$\omega^2 = \frac{1}{L} \cdot \frac{C_1 + C_2}{C_1 C_2}$$

$$\omega^2 = \frac{1}{L \frac{C_1 C_2}{C_1 + C_2}}$$

$$\omega = \frac{1}{\sqrt{L \frac{C_1 C_2}{C_1 + C_2}}}$$

$$2\pi f = \frac{1}{\sqrt{L \frac{C_1 C_2}{C_1 + C_2}}}$$

$$f = \frac{1}{2\pi \sqrt{L C_{eq}}}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

→ making real part = 0.

$$\frac{L}{C_1} - \frac{(1+hfc)}{\omega^2 C_1 C_2} = 0$$

$$\frac{L}{C_1} = \frac{(1+hfc)}{\omega^2 C_1 C_2} \Rightarrow$$

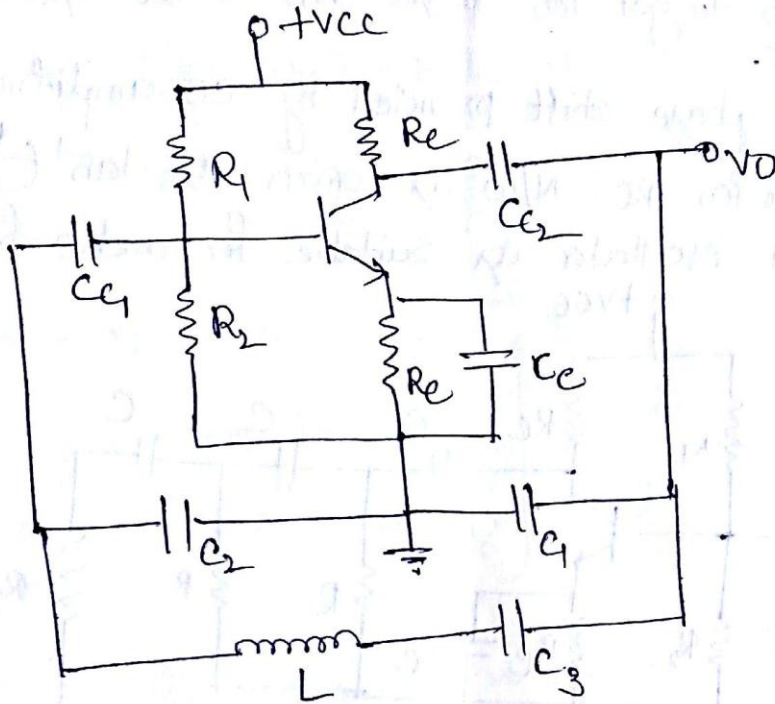
$$L \omega^2 C_2 = 1+hfc$$

$$\omega^2 L = \frac{C_1 + C_2}{C_1 C_2}$$

$$C_2 \left(\frac{C_1 + C_2}{C_1 C_2} \right) = 1+hfc$$

$$1+hfc = 1 + \frac{C_2}{C_1} \Rightarrow \boxed{hfc = \frac{C_2}{C_1}}$$

* Clapp oscillator:-



- In the Clapp oscillator C_1, C_2 and L series with C_3 are considered as feedback tank circuit.
- The additional C_3 improves the frequency stability of the oscillator.
- The frequency of oscillation is

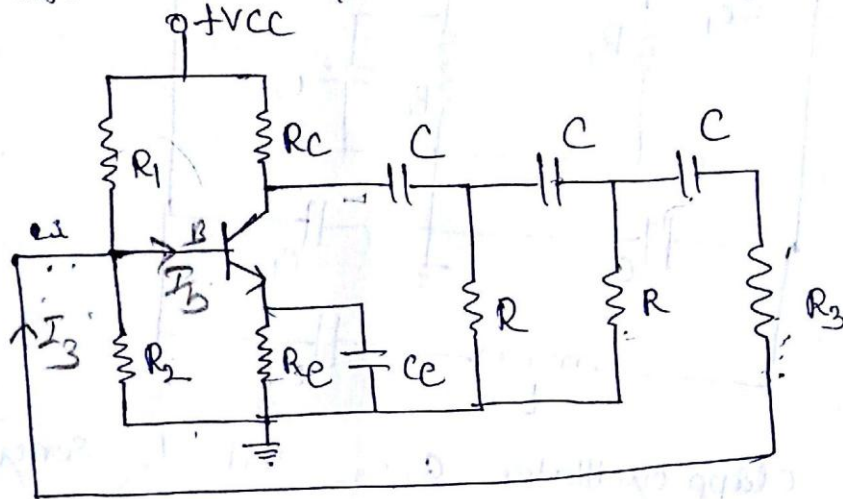
$$f_0 = \frac{1}{2\pi \sqrt{L C_{eq}}}$$

where $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

or $C_{eq} = \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_1 C_3}$

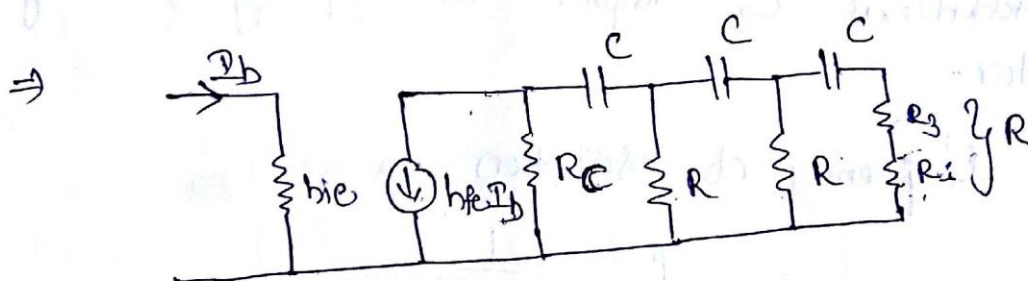
* RC phase shift oscillator :-

- In the RC phase shift oscillator each RC-network provides phase shift of 60° . so to get 180° phase shift 3-RC-N/w are combined in this oscillator.
- Additional 180° phase shift provided by CE amplifier.
- The phase angle for RC-N/w is given $\theta = \tan^{-1} \left(\frac{XC}{R} \right) \Rightarrow \theta = 60^\circ$
- RC-phase shift oscillator is suitable for audio frequencies.

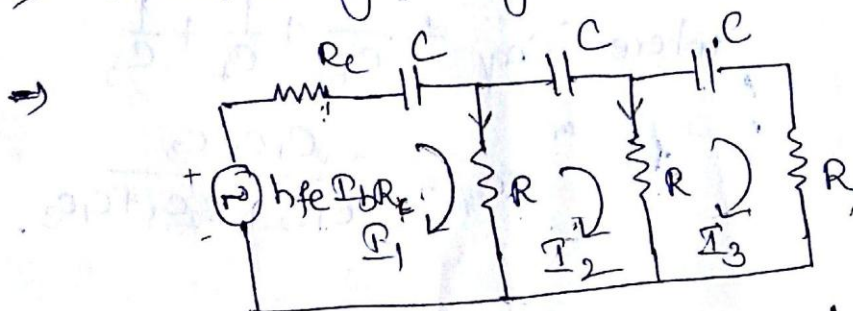


$$\begin{aligned} \rightarrow R_3 + R_i &= R \\ \rightarrow R_3 &= R - R_i \end{aligned}$$

The approximate hybrid-equivalent circuit is given by.



→ Remodified by using theorem's model.



where $A = \frac{-hfe I_B}{I_B}$

$$\beta = \frac{I_3}{-hfe I_B}$$

$$A\beta = \frac{-hfe \times I_3}{-hfe I_B}$$

$$A\beta = \frac{I_3}{I_B} = 1$$

$$\text{Let } X_C = \frac{1}{\omega C} \Rightarrow Z_C = -j X_C (\omega) = \frac{-j}{\omega C}$$

→ The mesh equations are. to find out I_3 value. (7)

$$I_1(R_C + R - jX_C) - I_2 R = -h_f e I_B R_C \quad \text{--- (1)}$$

$$-I_1 R + I_2(2R - jX_C) - R I_3 = 0 \quad \text{--- (2)}$$

$$-I_2 R + I_3(2R - jX_C) = 0 \quad \text{--- (3)}$$

→

$$R \begin{bmatrix} R+R_C - jX_C & -R & 0 \\ -R & 2R - jX_C & -R \\ 0 & -R & 2R - jX_C \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -h_f e I_B R_C \\ 0 \\ 0 \end{bmatrix}$$

⇒ find $I_3 = \frac{\Delta_3}{\Delta}$ (Cramer's rule)

$$\Delta_3 = \begin{bmatrix} R+R_C - jX_C & -R & -h_f e I_B R_C \\ -R & 2R - jX_C & 0 \\ 0 & -R & 0 \end{bmatrix} \quad \begin{matrix} R+R_C - jX_C \\ -h_f e I_B R_C \cdot R^2 \end{matrix}$$

$$\Delta = R^3 \begin{bmatrix} 1 + \frac{R_C}{R} - \frac{jX_C}{R} & -1 & 0 \\ -1 & 2 - \frac{jX_C}{R} & -1 \\ 0 & -1 & 2 - \frac{jX_C}{R} \end{bmatrix}$$

$$\text{let } k = \frac{R_C}{R}, \quad \alpha = \frac{X_C}{R}$$

$$= R^3 \begin{bmatrix} 1+k-j\alpha & -1 & 0 \\ -1 & 2-j\alpha & -1 \\ 0 & -1 & 2-j\alpha \end{bmatrix}$$

$$= R^3 \left[(1+k-j\alpha) \left[(2-j\alpha)^2 - 1 \right] + 1 \left[j\alpha - 2 \right] \right]$$

$$= R^3 \left[(1+k-j\alpha) (4 - \alpha^2 + 4j\alpha - 1) + j\alpha - 2 \right]$$

$$= R^3 \left[3 - \alpha^2 + 4j\alpha + 3k - \alpha^2 k + 4jk\alpha - 3j\alpha + j\alpha^3 + 4\alpha^2 + j\alpha - 2 \right]$$

$$\Delta = R^3 [(1+3k) - (5+k)\alpha^2] - j [(6+4k)\alpha - \alpha^3]$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-hfe I_b R_c R^2}{R^3 [(1+3k) - (5+k)\alpha^2] - j [(6+4k)\alpha - \alpha^3]}$$

$$I_3 = \frac{-hfe I_b \frac{R_c}{R}}{[(1+3k) - (5+k)\alpha^2] - j [(6+4k)\alpha - \alpha^3]}$$

$$\frac{I_3}{I_b} = \frac{-hfe k}{[(1+3k) - (5+k)\alpha^2] - j [(6+4k)\alpha - \alpha^3]} = 1$$

$$\Rightarrow [(1+3k) - (5+k)\alpha^2] - j [(6+4k)\alpha - \alpha^3] = -hfe k$$

make imaginary part = 0

$$(6+4k)\alpha - \alpha^3 = 0$$

$$(6+4k)\alpha = \alpha^3$$

$$\alpha^2 = 6+4k$$

$$\left(\frac{X_c}{R}\right)^2 = 6+4k \Rightarrow \left(\frac{1}{\omega CR}\right)^2 = 6+4k \Rightarrow \frac{1}{\omega CR} = \sqrt{6+4k}$$

$$\omega = \frac{1}{RC\sqrt{6+4k}}$$

Frequency of oscillation.

$$f = \frac{1}{2\pi RC\sqrt{6+4k}}$$

put real imaginary part = 0 : $\alpha^2 = 6+4k$ in eq 0

$$1+3k - \alpha^2(5+k) = -hfe k$$

$$1+3k - (6+4k)(5+k) = -hfe k$$

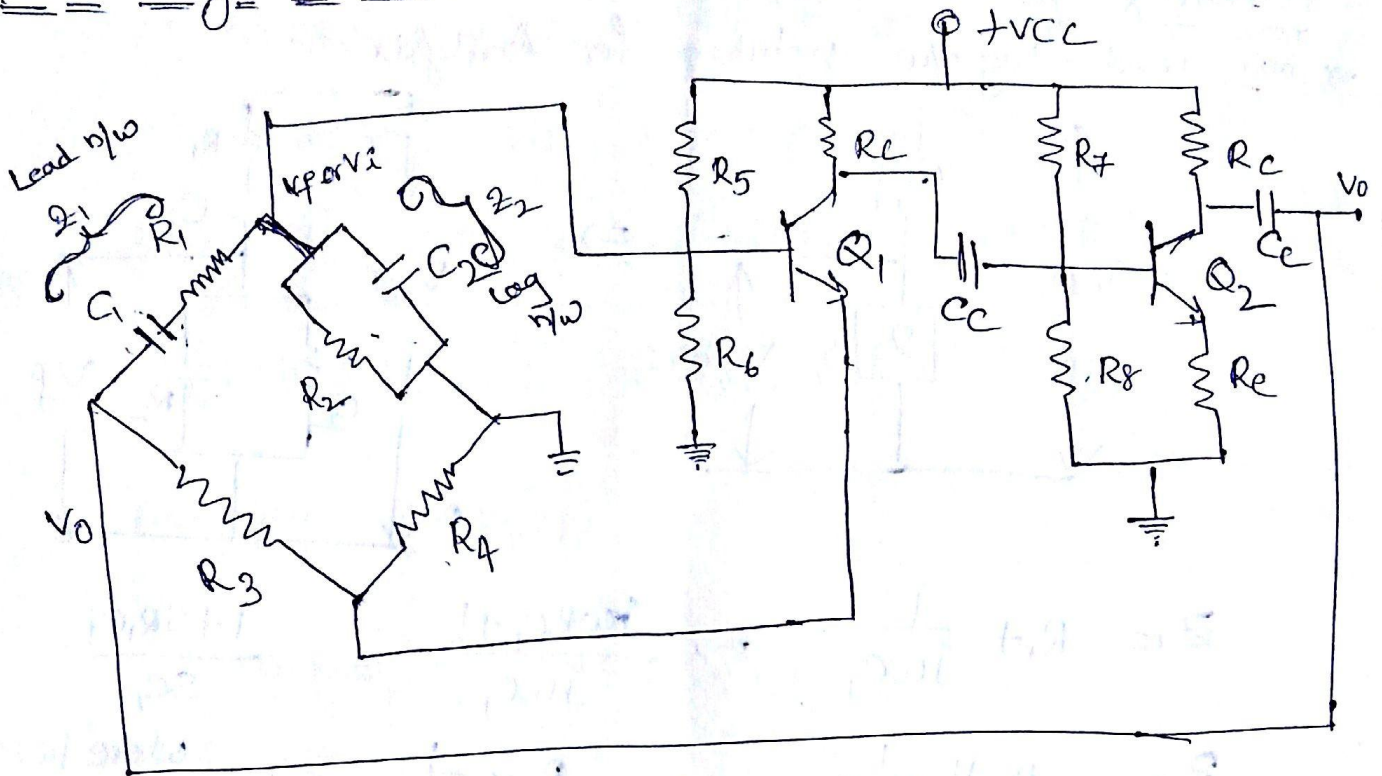
$$+hfe = \frac{4k^2 + 23k + 29}{k}$$

$$\therefore hfe = \frac{29}{k} + 23 + 4k$$

condition for oscillation.

* Wien Bridge Oscillator :-

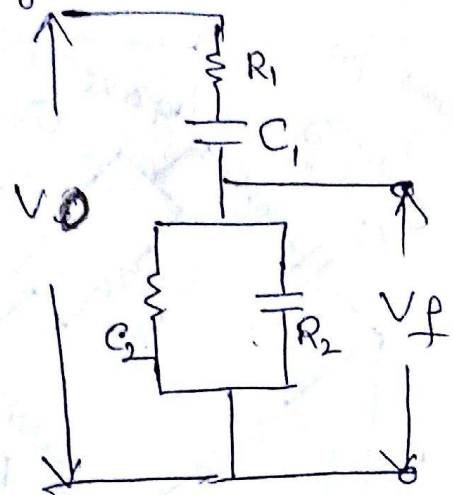
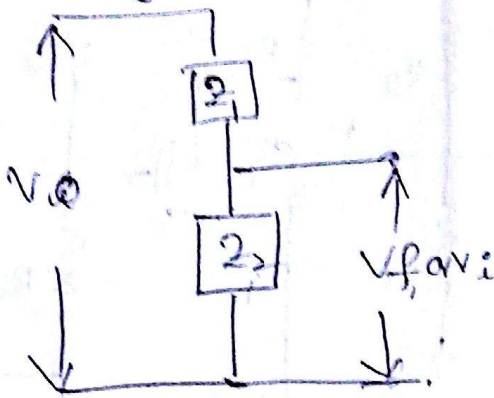
(P)



- The wien bridge oscillator consists of a two stage RC-coupled amplifier and a balanced bridge. where two stage Amplifier produce phase shift of 360° . Then there is no need to provide extra phase shift by feedback network.
- The feedback bridge consists of Lead-lag circuit which provides positive feedback and connected to V_p of the 1st stage Amplifier.
- In this lead-lag network series combination of $R_1 + C_1$ is lead network and parallel combination of $R_2 + C_2$ is lag network.
- The R_3 and R_4 in bridge is voltage divider which gives -ve feedback and connected to emitter of the amplifier.
- These oscillators are used for commercial audio signal generators.

Conditions for oscillations -

→ only lead-lag n/w is taken for Analysis



$$Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{j\omega R_1 C_1 + 1}{j\omega C_1} = \frac{1 + sR_1 C_1}{sC_1}$$

$$Z_2 = R_2 \parallel \frac{1}{j\omega C_2} \Rightarrow \frac{R_2 \times \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{j\omega R_2 C_2 + 1} = \frac{R_2}{1 + sR_2 C_2}$$

where $j\omega = s$

$$\Rightarrow \beta = \frac{V_f}{V_0} ?$$

from the above ckt.

$$\Rightarrow V_f = \frac{Z_2}{Z_1 + Z_2} \cdot V_0$$

$$\therefore \beta = \frac{V_f}{V_0} = \frac{Z_2}{Z_1 + Z_2}$$

$$\Rightarrow \beta = \frac{\frac{R_2}{1 + sR_2 C_2}}{\frac{1 + sR_1 C_1}{sC_1} + \frac{R_2}{1 + sR_2 C_2}} = \frac{R_2 (sC_1)}{1 + sR_1 C_1 + sR_1 C_1 + sR_2 C_2 + 1 + R_2 sC_1}$$

I have found this circuit in a book
 I have found this circuit in a book
 I have found this circuit in a book

$$\beta = \frac{j\omega R_2 C_1}{(1 - \omega^2 R_1 R_2 C_1 C_2) + j\omega (R_1 C_1 + R_2 C_2 + C_1 R_2)}$$

Rationalising the above expression

$$\beta = \frac{j\omega R_2 C_1 [(1 - \omega^2 R_1 R_2 C_1 C_2) - j\omega (R_1 C_1 + R_2 C_2 + C_1 R_2)]}{(1 - \omega^2 R_1 R_2 C_1 C_2)^2 + \omega^2 (R_1 C_1 + R_2 C_2 + C_1 R_2)^2}$$

Imaginary part = 0

A = 1
Aβ = 1.

$$\omega R_2 C_1 (1 - \omega^2 R_1 R_2 C_1 C_2) = 0$$

$$\omega^2 R_1 R_2 C_1 C_2 = 1$$

$$\Rightarrow \omega^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

$$(2\pi f)^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

$$\therefore f = \frac{1}{2\pi \sqrt{R_1 R_2 C_1 C_2}}$$

If $R_1 = R_2 = R, C_1 = C_2 = C$

$$f = \frac{1}{2\pi RC} \Rightarrow \omega = \frac{1}{RC}$$

$$\Rightarrow \beta = \frac{\frac{1}{(RC)^2} \cdot 3(RC)^2 + 0}{0 + \frac{1}{(RC)^2} [3(RC)^2]^2} = \frac{3}{9} = \frac{1}{3}$$

Aβ = 1 ⇒ A = 3
Answer: A = 3

* Crystal oscillators :-

- Crystal oscillator is basically a tuned oscillator. It uses a piezo-electric crystal as a resonant tank circuit.
- ⇒ The crystal provides a high degree of frequency stability.
- So, where we require great stability, the crystal oscillators are preferred. Ex:- communication transmitters, digital clocks etc.

↳ A quartz crystal exhibits a very important property known as piezoelectric effect.

When an A.C voltage is applied across the crystal it vibrates at the frequency of the applied voltage.

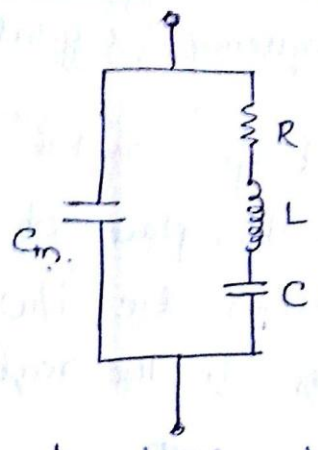
(or) when a crystal is subjected to mechanical stress it produces an A.C voltage. This phenomenon is called piezo electric effect.

→ The other substances that exhibit piezoelectric effect besides Quartz are Rochelle salt and tourmaline.
Rochelle salt → greatest piezo electric activity - but mechanically weak. they break easily. → microphones, headset, loudspeakers

Tourmaline → least piezo electric activity - most expensive.
~~Quartz~~ → → Rarely used for high frequencies

↳ Quartz is a compromise b/w Rochelle salt and tourmaline.
↳ It is inexpensive and readily available in nature.
↳ It is used mainly for RF oscillators.

⇒ For use in electronic oscillators, the crystal material is suitably cut and then mounted b/w two metal plates as shown in figure. The electrical equivalent circuit of the crystal also shown in figure.



- The frequency of crystal is depends on the cut and how the crystal is mounted.
- The outstanding feature of crystals is compared with discrete LC tank circuits is high Quality factor Q. due to this high frequency stability achieved.
- The crystal has two different frequencies
 - 1) The Inductance L resonates with series capacitance C' and produces series resonant frequency f_s.

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC}$$

$$\therefore f_s = \frac{1}{2\pi\sqrt{LC}}$$

↳ Above frequency f_s, the series branch LCR has inductive reactance only. This inductance resonates with capacitance C_m. called parallel resonant frequency f_p.

$$X_{C_m} = X_L - X_C$$

$$\frac{1}{\omega C_m} = \omega L - \frac{1}{\omega C} \Rightarrow \frac{1}{\omega C_m} + \frac{1}{\omega C} = \omega L$$

$$= \omega L \left(\frac{C + C_m}{C C_m} \right)$$

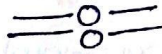
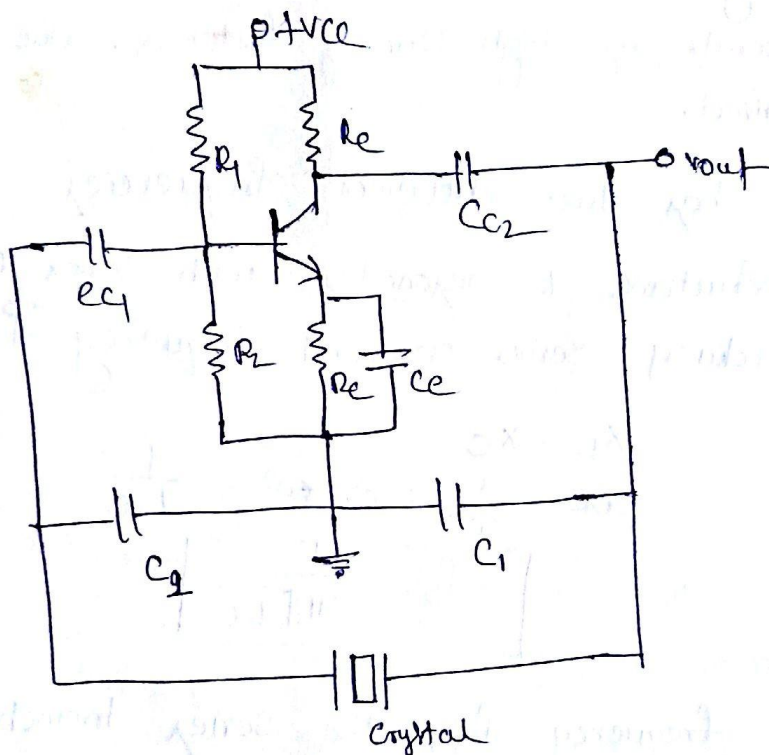
$$\frac{C + C_m}{\omega C C_m} = \omega L$$

$$\Rightarrow \left[f_p = \frac{1}{2\pi\sqrt{L C_{eq}}} \right] \quad C_{eq} = \frac{C C_m}{C + C_m} \Rightarrow \omega^2 = \frac{1}{L \frac{C C_m}{C + C_m}}$$

→ Above this f_p frequency crystal has capacitive reactance only.

→ only b/w f_s and f_p crystal acts as inductor. If the crystal is used in the place of inductor in an oscillator ckt, the frequency of oscillation lie b/w f_s and f_p - differ by very small amount → this is the reason to give great frequency stability.

→ The following circuit represents the Colpitts crystal oscillator.



* Q. In the Hartley oscillator $L_2 = 0.4 \text{ mH}$ and $C = 0.004 \mu\text{F}$. If the frequency of the oscillator is 120 kHz . Find the value of L_1 . Neglect mutual inductance.

$$f_0 = \frac{1}{2\pi \sqrt{L_{eq} C}}$$

$$L_{eq} = L_1 + L_2$$

$$f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2) C}}$$

$$(2\pi f_0)^2 = \frac{1}{(L_1 + L_2) C}$$

$$L_1 = \frac{1}{4\pi^2 f_0^2 C} - L_2$$

$$= \frac{1}{4 \times \pi^2 (120 \times 10^3)^2 \times (0.004 \times 10^{-6})} - 0.4 \times 10^{-3}$$

$$\therefore \underline{0.04 \text{ mH}}$$

* Q. In a transistorized Hartley oscillator, the two inductances are 2 mH and $20 \mu\text{H}$ while the frequency is to be changed from 950 kHz to 2050 kHz . Calculate the range over which the capacitor is to be varied.

Sol:- If $L_1 = 2 \text{ mH}$, $L_2 = 20 \mu\text{H}$, $f_1 = 950 \text{ kHz}$, $f_2 = 2050 \text{ kHz}$.

$$f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2) C}} \Rightarrow C = \frac{1}{4\pi^2 (L_1 + L_2) f_0^2}$$

$$\text{If } f_0 = 950 \text{ kHz} \Rightarrow C = \underline{13.89 \text{ pF}}$$

$$\text{If } f_0 = 2050 \text{ kHz} \Rightarrow C = \underline{2.98 \text{ pF}}$$

range of capacitance is from 2.98 pF to $\underline{13.89 \text{ pF}}$

* In a Hartley oscillator, the value of the capacitor in the tuned circuit is 500 pF and the two sections of coil have inductance $38 \mu\text{H}$ and $12 \mu\text{H}$. Find the frequency of oscillation and the feedback factor β .

Sol:- $f_0 = 1 \text{ MHz} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)C}}$

$$\beta = \frac{L_1}{L_2} = 3.166$$