5.3 Design for Shear (Part II)

This section covers the following topics.

- Design of Transverse Reinforcement
- Detailing Requirements
- Design Steps

5.3.1 Design of Transverse Reinforcement

When the shear demand (V_u) exceeds the shear capacity of concrete (V_c), transverse reinforcements in the form of stirrups are required. The stirrups resist the propagation of diagonal cracks, thus checking diagonal tension failure and shear tension failure.

The stirrups resist a failure due to shear by several ways. The functions of stirrups are listed below.

1) Stirrups resist part of the applied shear.
2) They restrict the growth of diagonal cracks.
3) The stirrups counteract widening of the diagonal cracks, thus maintaining aggregate interlock to a certain extent.
4) The splitting of concrete cover is restrained by the stirrups, by reducing dowel forces in the longitudinal bars.

After cracking, the beam is viewed as a plane truss. The top chord and the diagonals are made of concrete struts. The bottom chord and the verticals are made of steel reinforcement ties. Based on this truss analogy, for the ultimate limit state, the total area of the legs of the stirrups (A_{sv}) is given as follows.

$$\frac{A_{sv}}{s_v} = \frac{V_u - V_c}{0.87 f_y d_t} \qquad \textbf{(5-3.1)}$$

The notations in the above equation are explained.

s_v = spacing of the stirrups

d_t = greater of d_p or d_s

d_p = depth of CGS from the extreme compression fiber

d_s = depth of centroid of non-prestressed steel

f_y = yield stress of the stirrups

The grade of steel for stirrups should be restricted to Fe 415 or lower.

Design of Stirrups for Flanges

For flanged sections, although the web carries the vertical shear stress, there is shear stress in the flanges due to the effect of shear lag. Horizontal reinforcement in the form of single leg or closed stirrups is provided in the flanges. The following figure shows the shear stress in the flange at the face of the web.

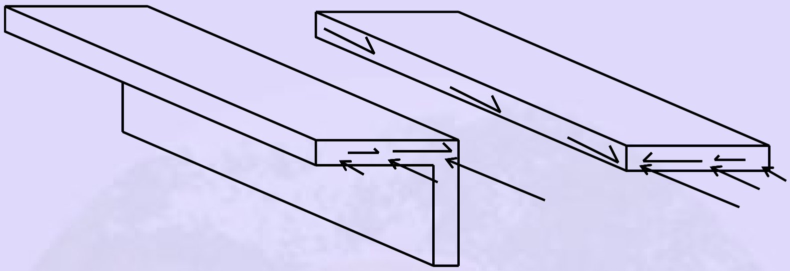


Figure 5-3.1 Shear stress in flange due to shear lag effect

The horizontal reinforcement is calculated based on the shear force in the flange. The relevant quantities for the calculation based on an elastic analysis are as follows.

1) Shear flow (shear stress × width)
2) Variation of shear stress in a flange (τ_f)
3) Shear forces in flanges (V_f).
4) Ultimate vertical shear force (V_u)

The following sketch shows the above quantities for an I-section (with flanges of constant widths).

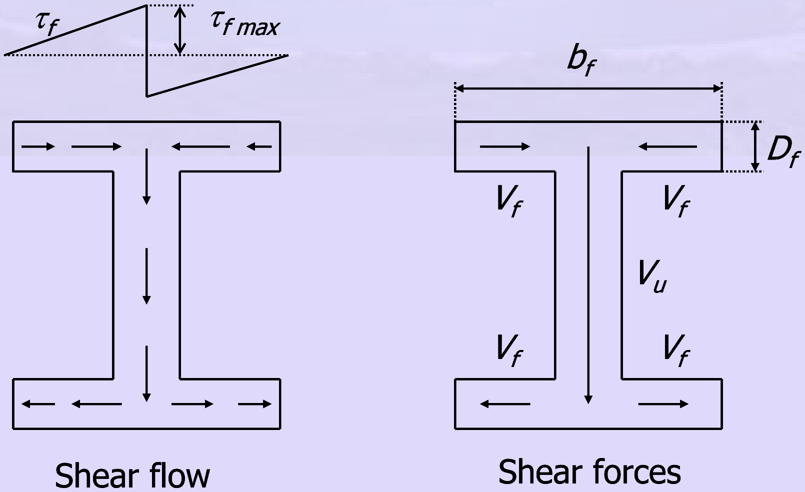


Figure 5-3.2 Shear flow and shear forces in an I-section

The design shear force in a flange is given as follows.

$$V_f = \frac{\tau_{f\,max}}{2}\frac{b_f}{2}D_f \qquad \textbf{(5-3.2)}$$

Here,

b_f = breadth of the flange

D_f = depth of the flange

$\tau_{f,max}$ = maximum shear stress in the flange.

The maximum shear stress in the flange is given by an expression similar to that for the shear stress in web.

$$\tau_{f\,max} = \frac{V_u A_1 \bar{y}}{I\,D_f} \qquad \textbf{(5-3.3)}$$

Here,

V_u = ultimate vertical shear force

I = moment of inertia of the section.

A_1 = area of half of the flange

= distance of centroid of half of the flange from the neutral axis at CGC.

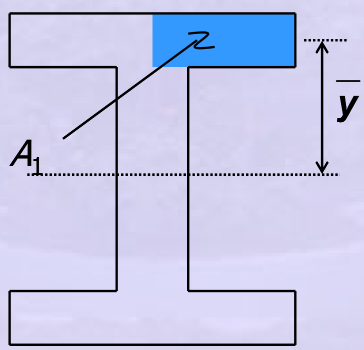


Figure 5-3.3 Cross-section of a beam showing the variables for calculating shear stress in the flange

The amount of horizontal reinforcement in the flange (A_{svf}) is calculated from V_f.

$$A_{svf} = \frac{V_f}{0.87 f_y} \qquad \textbf{(5-3.4)}$$

The yield stress of the reinforcement is denoted as f_y .

5.3.2 Detailing Requirements

The detailing requirements for the stirrups in **IS:1343 - 1980** are briefly mentioned.

Maximum Spacing of Stirrups

The spacing of stirrups (s_v) is restricted so that a diagonal crack is intercepted by at least one stirrup. This is explained by the following sketch.

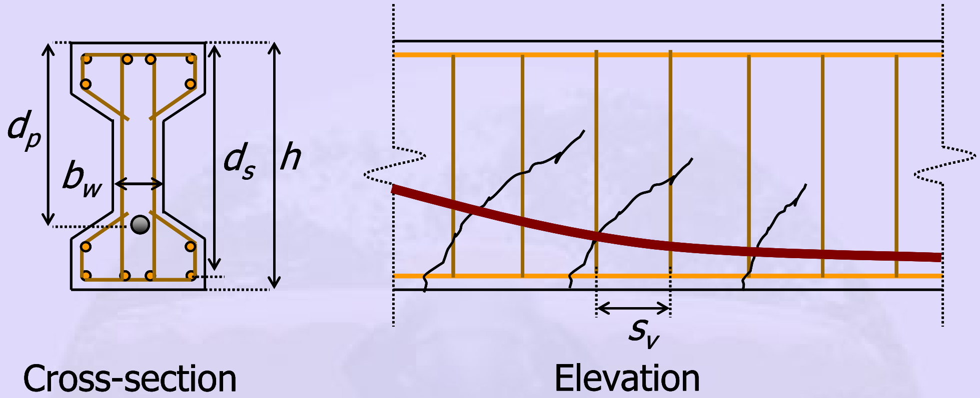


Figure 5-3.4 Cross-section and elevation of a beam showing stirrups

As per **Clause 22.4.3.2**, the maximum spacing is $0.75d_t$ or $4b_w$, whichever is smaller. When V_u is larger than $1.8V_c$, the maximum spacing is $0.5d_t$.

The variables are as follows.

b_w = breadth of web

d_t = greater of d_p or d_s

d_p = depth of CGS from the extreme compression fiber

d_s = depth of centroid of non-prestressed steel

V_u = shear force at a section due to ultimate loads

V_c = shear capacity of concrete.

Minimum Amount of Stirrups

A minimum amount of stirrups is necessary to restrict the growth of diagonal cracks and subsequent shear failure. For $V_u < V_c$, minimum amount of transverse reinforcement is provided based on the following equation.

$$\frac{A_{sv}}{bs_v} = \frac{0.4}{0.87f_y} \qquad \textbf{(5-3.5)}$$

b = breadth of the section

= b_w, breadth of the web for flanged sections.

If $V_u < 0.5V_c$ and the member is of minor importance, stirrups may not be provided.

Another provision for minimum amount of stirrups ($A_{sv,min}$) is given by **Clause 18.6.3.2** for beams with thin webs. The minimum amount of stirrups is given in terms of A_wh, the horizontal sectional area of the web in plan. The area is shown in the following sketch.

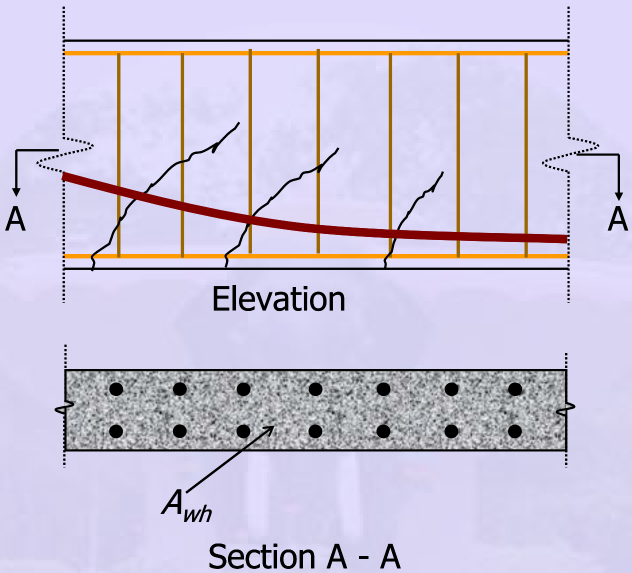


Figure 5-3.5 Elevation and horizontal section of a beam showing stirrups

In presence of dynamic load,

$A_{sv,min}$ = 0.3% A_{wh}

= 0.2% A_{wh}, when $h \leq 4b_w$

With high strength bars,

$A_{sv,min}$ = 0.2% A_{wh}

= 0.15% A_{wh}, when $h \leq 4b_w$

In absence of dynamic load, when $h > 4b_w$

$A_{sv,min}$ = 0.1% A_{wh}

There is no specification for $A_{sv,min}$ when $h \leq 4b_w$.

Anchorage of Stirrups

The stirrups should be anchored to develop the yield stress in the vertical legs.

1) The stirrups should be bent close to the compression and tension surfaces, satisfying the minimum cover.
2) Each bend of the stirrups should be around a longitudinal bar. The diameter of the longitudinal bar should not be less than the diameter of stirrups.
3) The ends of the stirrups should be anchored by standard hooks.
4) There should not be any bend in a re-entrant corner. In a re-entrant corner, the stirrup under tension has the possibility to straighten, thus breaking the cover concrete.

The following sketches explain the requirement of avoiding the bend of a stirrup at a re-entrant corner.

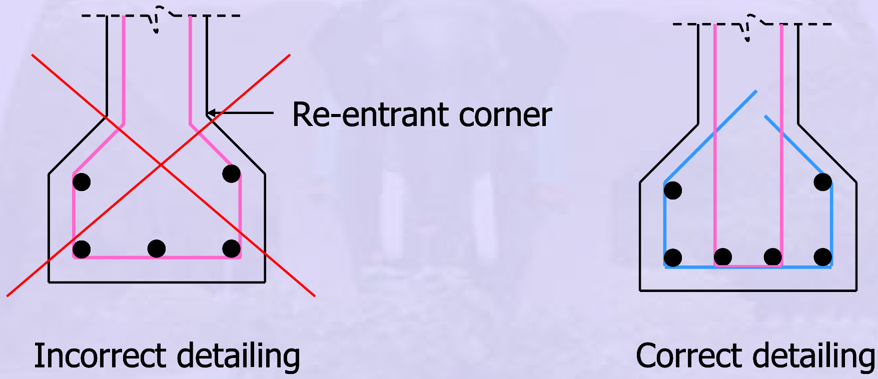


Figure 5-3.6 Cross-section of the bottom flange of a beam showing stirrups

Minimum Thickness (Breadth) of Web

To check web crushing failure, The **Indian Roads Congress Code IRC:18 - 2000** specifies a minimum thickness of the web for T-sections (**Clause 9.3.1.1**). The minimum thickness is 200 mm plus diameter of the duct hole.

5.3.3 Design Steps

The following quantities are known.

V_u = factored shear at ultimate loads. For gravity loads, this is calculated from V_{DL} and V_{LL}.

V_{DL} = shear due to dead load

V_{LL} = shear due to live load.

After a member is designed for flexure, the self-weight is known. It is included as dead load.

The grade of concrete is known from flexure design. The grade of steel for stirrups is selected before the design for shear. As per **IS:1343 - 1980**, the grade of steel is limited to Fe 415.

The following quantities are unknown.

V_c = shear carried by concrete

A_{sv} = total area of the legs of stirrups within a distance s_v

s_v = spacing of stirrups.

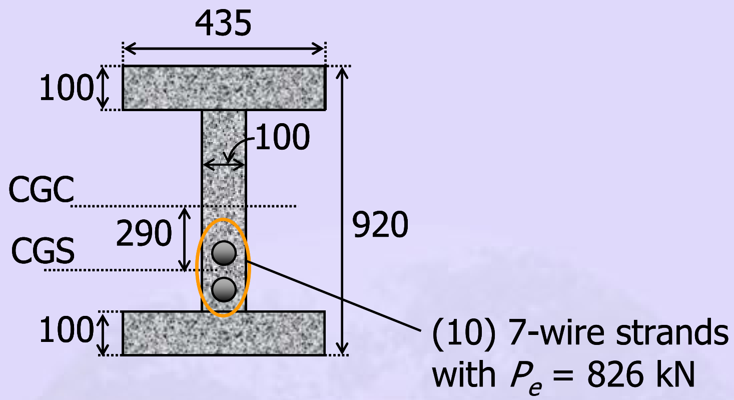
The steps for designing stirrups along the length of a beam are given below.

1) Calculate the shear demand V_u at the critical location.
2) Check $(V_u / bd_t) < \tau_{c,max}$. If it is not satisfied, increase the depth or breadth of the section. Here, b is the breadth of the web (b_w) and d_t is larger of d_p and d_s.
3) Calculate the shear capacity of concrete V_c from the lower of V_{co} and V_{cr}. In presence of inclined tendons or vertical prestress, the vertical component of the prestressing force (V_p) can be added to V_{c0}.
4) Calculate the requirement of shear reinforcement through A_{sv} / s_v . Compare the value with the minimum requirement.
5) Calculate the maximum spacing and round it off to a multiple of 5 mm.
6) Calculate the size and number of legs of the stirrups based on the amount required, type of section and space to accommodate.

Repeat the calculations for other locations of the beam, if the spacing of stirrups needs to be varied.

Example 5-3.1

Design the stirrups for the Type 1 prestressed beam with the following section (location of tendons shown at mid span).



Longitudinal reinforcement of 12 mm diameter is provided to hold the stirrups.

The properties of the sections are as follows.

$$A = 159{,}000 \text{ mm}^2,$$
$$I = 1.7808 \times 1010 \text{ mm}^4$$
$$A_p = 960 \text{ mm}^2$$

The grade of concrete is M 35 and the characteristic strength of the prestressing steel (f_{pk}) is 1470 N/mm^2. The effective prestress (f_{pe}) is 860 N/mm^2.

The uniformly distributed load including self weight, is w_T = 30.2 kN/m.

The span of the beam (L) is 10.7 m. The width of the bearings is 400 mm. The clear cover to longitudinal reinforcement is 30 mm.

Solution

1) Calculate V_u at the face of the support (neglecting the effect of compression in concrete).

$$V_u = 1.5 \times w_T \left(\frac{L}{2} - x\right)$$
$$= 1.5 \times 30.2 \times \left(\frac{10.7}{2} - 0.2\right)$$
$$= 233.3 \text{ kN}$$

Here, x denotes half of the width of bearing. x = 200 mm.

2) Check $(V_u / bd_t) < \tau_{c,max}$.

Effective depth d_t = total depth – cover – diameter of stirrups – ½ diameter of longitudinal bar.

Assume the diameter of stirrups to be 8 mm.

$$d_t = \left(920 - 30 - 8 - \frac{1}{2}\times 12\right)$$
$$= 876 \text{ mm}$$

$$\frac{V_u}{b_w d_t} = \frac{233.3\times 10^3}{100\times 876}$$
$$= 2.7 \text{ N/mm}^2$$

$\tau_{c,max}$ for M 35 is 3.7 N/mm^2. Hence, $(V_u / bd_t) < \tau_{c,max}$.

3) Calculate V_c from the lower of V_{c0} and V_{cr} .

$$V_{co} = 0.67\text{bD}\sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

Here,

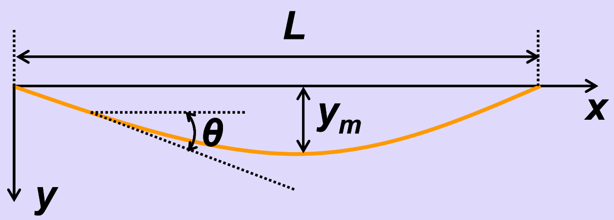
$$f_t = 0.24\sqrt{35}$$
$$= 1.42 \text{ N/mm}^2$$

$$f_{cp} = \frac{P_e}{A}$$
$$= \frac{826\times 10^3}{159{,}000}$$
$$= 5.19 \text{ N/mm}^2$$

$$V_{co} = 0.67 bD\sqrt{f_t^2 + 0.8 f_{cp} f_t}$$
$$= 0.67\times 100\times 920\sqrt{1.42^2 + 0.8\times 5.19\times 1.42}$$
$$= 173.4 \text{ kN}$$

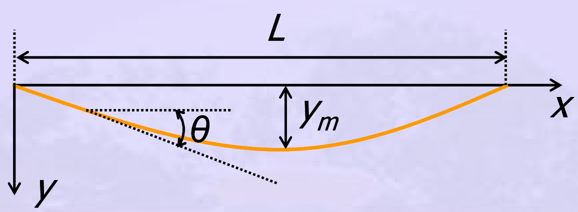
The vertical component of the prestressing force can be found out from the equation of the parabolic tendon.

$$y = \frac{4 y_m}{L^2} x (L - x)$$



The following is the expression of the slope of the parabolic tendon.

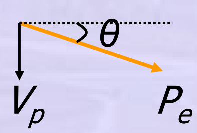
$$\tan\theta = \frac{dy}{dx} = \frac{4y_m}{L^2}(L - 2x)$$



At x = 0.2 m, y = 20 mm, dy/dx = 0.105 and θ = 6.0°.

$$\begin{aligned} V_p &= P_e \sin\theta \\ &= 826 \times 0.104 \\ &= 86.0\,\text{kN} \end{aligned}$$

$$\begin{aligned} V_{co} + V_p &= 173.4 + 86.0 \\ &= 259.4\ \text{kN} \end{aligned}$$



$$V_{cr} = \left(1 - 0.55\frac{f_{pe}}{f_{pk}}\right)\tau_c bd + M_0 \frac{V_u}{M_u}$$

Here,

$$\frac{f_{pe}}{f_{pk}} = \frac{860}{1470} = 0.58$$

$$\begin{aligned} \frac{100A_p}{bd} &= \frac{100 \times 960}{100 \times 480} \\ &= 2.0 \end{aligned}$$

$$\begin{aligned} d &= 460 + y \\ &= 460 + 20 \\ &= 480\ \text{mm} \end{aligned}$$

From Table 6, for M 35 concrete, τ_c = 0.86 N/mm^2.

$$M_0 = 0.8 f_{pt}\frac{I}{y}$$

Here,

$$\begin{aligned} f_{pt} &= -\frac{P_e}{A} - \frac{P_e y}{I} y \\ &= -\frac{826\times10^3}{159{,}000} - \frac{826\times10^3\times20}{1.7808\times10^{10}}\times20 \\ &= -5.19 - 0.02 \\ &= -5.21\,\text{N/mm}^2 \end{aligned}$$

$$\begin{aligned} M_0 &= 0.8\times5.21\times\frac{1.7808\times10^{10}}{20} \\ &= 3711.2\times10^6\,\text{Nmm} \\ &= 3711.2\,\text{kNm} \end{aligned}$$

At the critical section,

$$\begin{aligned} M_u &= 1.5 w_T \frac{x}{2}(L - x) \\ &= 1.5\times30.2\times\frac{0.2}{2}(10.7 - 0.2) \\ &= 47.6\,\text{kNm} \end{aligned}$$

Therefore,

$$V_{cr} = \left(1 - 0.55\frac{f_{pe}}{f_{pk}}\right)\tau_c bd + M_0\frac{V_u}{M_u}$$

$$\begin{aligned} V_{cr} &= (1 - 0.55\times0.58)\times\frac{0.86}{10^3}\times100\times480 \quad + \quad 3711.2\times\frac{233.3}{47.6} \\ &= 28.1 + 18204.8 \\ &= 18232.9\,\text{kN} \end{aligned}$$

The governing value of V_c is 259.4 kN.

$$\Rightarrow V_u < V_c\,.$$

4) Calculate A_{sv} / s_v .

Provide minimum steel.

$$\frac{A_{sv}}{b_w s_v} = \frac{0.4}{0.87 f_y}$$

5) Calculate maximum spacing

$s_v = 0.75\ d_t = 0.75 \times 876 = 656$ mm

$s_v = 4b_w = 4 \times 100 = 400$ mm

Select $s_v = 400$ mm.

6) Calculate the size and number of legs of the stirrups

Select $f_y = 250$ N/mm^2.

$$A_{sv} = b_w s_v \frac{0.4}{0.87 f_y}$$
$$= 100 \times 400 \times \frac{0.4}{0.87 \times 250}$$
$$= 73.6\ \text{mm}^2$$

Provide 2 legged stirrups of diameter 8 mm.

$$A_{sv,provided} = 2 \times 50.3$$
$$= 100.6\ \text{mm}^2$$

Check minimum amount of stirrups.

$$A_{sv,min} = 0.1\%\ A_{wh}$$
$$= \frac{0.1}{100} \times 100 \times 400$$
$$= 40\ \text{mm}^2$$

Provided amount of stirrups is larger. OK.

Provide same spacing of stirrups throughout the span.

Design of stirrups for flange

$$A_1 = \frac{1}{2} \times b_f \times D_f$$
$$= \frac{1}{2} \times 435 \times 100$$
$$= 21750\ \text{mm}^2$$

$$\bar{y} = 410\ \text{mm}$$

$$\tau_{f\,max} = \frac{V_u A_1 \bar{y}}{I\,D_f}$$
$$= \frac{233.3\times10^3 \times 21750\times 410}{1.7808\times10^{10} \times 100}$$
$$= 1.17\ \text{N/mm}^2$$

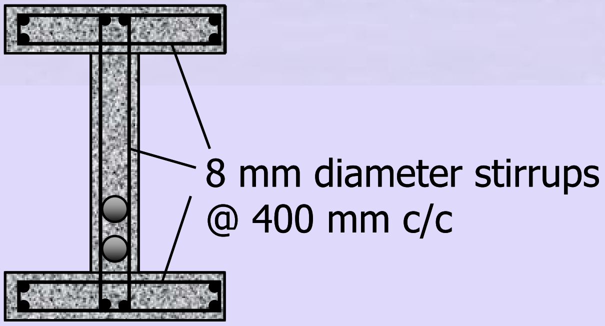
$$V_f = \frac{\tau_{f\,max}}{2}\frac{b_f}{2}D_f$$
$$= \frac{1.17}{2}\times\frac{435}{2}\times 100$$
$$= 12724\ \text{N}$$

$$A_{svf} = \frac{V_f}{0.87 f_y}$$
$$= \frac{12724}{0.87\times 250}$$
$$= 59.0\ \text{mm}^2$$

For minimum steel $$A_{svf} = D_f s_v \frac{0.4}{0.87 f_y}$$
$$= 100\times 400\times \frac{0.4}{0.87\times 250}$$
$$= 73.6\ \text{mm}^2$$

Provide 2 legged stirrups of diameter 8 mm.

Designed section



5.6 Design for Torsion (Part II)

This section covers the following topics.

- Design of Transverse Reinforcement
- Detailing Requirements
- Design Steps

5.6.1 Design of Transverse Reinforcement

For the design of the transverse reinforcement, the capacities of concrete to resist the torsion and shear need to be determined. To consider the simultaneous occurrence of flexural and torsional shears, a linear interaction between the two is considered.

The capacity of concrete to resist torsion is reduced from T_c, the capacity under pure torsion. Similarly, the capacity of concrete to resist shear is reduced from V_c, the capacity in absence of torsion.

Capacity of Concrete under Pure Torsion

The capacity of concrete is determined based on the plastic theory for torsion. The capacity is equal to the torque generating the first torsional crack (T_{cr}). For a reinforced concrete beam, T_{cr} is estimated by equating the maximum torsional shear stress (τ_{max}) caused by T_{cr} to the tensile strength of concrete ($0.2\sqrt{f_{ck}}$). The estimated tensile strength is less than that under direct tension because the full section does not plastify as assumed in the plastic theory.

The estimate of the cracking torque (T_{cr}) for a rectangular section is given below.

$$T_{cr} \approx 0.2\sqrt{f_{ck}}\frac{b^2D}{2}\left(1-\frac{b}{3D}\right)$$

$$T_{cr} = 0.1b^2D\left(1-\frac{b}{3D}\right)\sqrt{f_{ck}} \quad \textbf{(5-6.1)}$$

For flanged sections, the section is treated as a compound section. A compound section is a summation of rectangular sections.

The cracking torque is estimated as a summation of the capacities of the individual rectangular sections. Since the interaction between the rectangular sections is neglected in the summation, the estimate of the cracking torque is a lower bound estimate.

The following flanged section is shown as a compound section of five rectangles. For an individual rectangle, the short side is *b* and the long side is *D*.

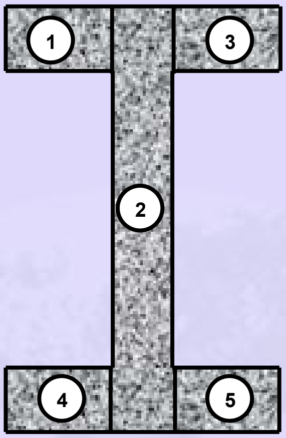


Figure 5-6.1 Flanged section as a compound section

The estimate of the cracking torque (T_{cr}) for a compound section is as follows.

$$T_{cr} = \sum 0.1 b^2 D\left(1 - \frac{b}{3D}\right)\sqrt{f_{ck}} \quad \textbf{(5-6.2)}$$

Here, the summation is for the individual rectangles.

For a prestressed concrete beam, the strength of concrete is multiplied by the factor λ_p, which is a function of the average effective prestress (f_{cp}).

$$\lambda_p = \sqrt{1 + \frac{12 f_{cp}}{f_{ck}}} \quad \textbf{(5-6.3)}$$

The value of f_{cp} is taken as positive (numeric value). It can be observed that the strength increases with prestress. The cracking torque (T_{cr}) and the capacity of concrete to resist torsion (T_c) for a prestressed concrete beam are thus estimated as follows.

$$T_c = T_{cr}$$

$$T_c = \sum 0.15 b^2 D\left(1 - \frac{b}{3D}\right)\lambda_p\sqrt{f_{ck}} \quad \textbf{(5-6.4)}$$

In the previous expression,

b = breadth of the individual rectangle

D = depth of the individual rectangle.

Interaction of Shear and Torsion

In presence of flexural shear, the torsional capacity of concrete reduces. Similarly, in presence of torsion, the flexural shear capacity of concrete reduces. This is referred to as interaction of shear and torsion. The capacity of concrete under shear is explained in Section 5.2, Design for Shear (Part I). A linear interaction of the shear and torsion capacities of concrete is considered as shown in the following figure. In the horizontal axis, the shear demand is normalised with respect to the capacity of concrete under flexural shear. In the vertical axis, the torsional demand is normalised with respect to the capacity of concrete under pure torsion.

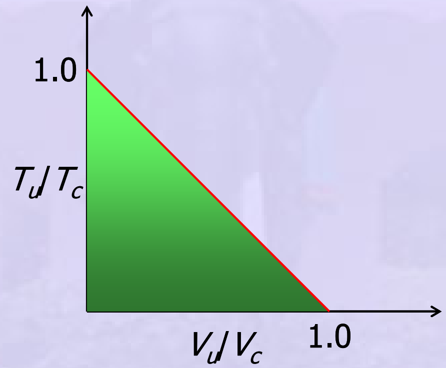


Figure 5-6.2 Interaction diagram for shear and torsion

The interaction equation is given as follows.

$$\frac{T_u}{T_c} + \frac{V_u}{V_c} = 1 \quad \textbf{(5-6.5)}$$

This is a linear interaction equation.

In the previous expression,

T_u = applied torsion at ultimate

V_u = applied shear at ultimate

T_c = capacity of concrete under pure torsion.

V_c = capacity of concrete under flexural shear.

Based on the interaction equation, the reduced capacity of concrete to resist torsion (T_{c1}) is given below.

$$T_{c1} = T_c\left(\frac{e}{e+e_c}\right) \le T_u/2 \quad \textbf{(5-6.6)}$$

T_{c1} is limited to $T_u/2$ to restrict concrete reaching its capacity.

The parameter e is the ratio of torsion and shear demands at ultimate. The parameter e_c is the ratio of the corresponding concrete capacities.

$$e = T_u/V_u \quad \textbf{(5-6.7)}$$

$$e_c = T_c/V_c \quad \textbf{(5-6.8)}$$

The reduced capacity of concrete to resist shear is given below.

$$V_{c1} = V_c\frac{e_c}{e+e_c} \quad \textbf{(5-6.9)}$$

Calculation of Transverse Reinforcement

The transverse reinforcement is provided in the form of **closed** stirrups enclosing the corner longitudinal bars. The amount (A_{sv}) is equal to the higher value determined from two expressions.

The first expression is based on the **Skew Bending Theory**.

$$A_{sv} = \frac{M_t s_v}{1.5 b_1 d_1 f_y} \quad \textbf{(5-6.10)}$$

The notations are as follows.

b_1 = distance between the corner longitudinal bars along the short side

d_1 = distance between the corner longitudinal bars along the long side.

M_t = additional bending moment from torsion.

s_v = spacing of the stirrups

f_y = characteristic yield stress of the stirrups.

The dimensions b_1 and d_1 are shown in the following sketch.

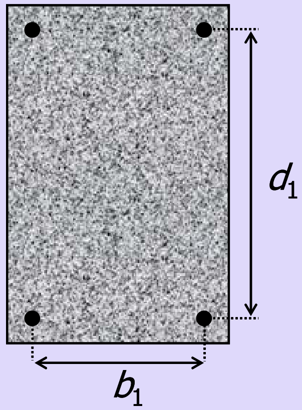


Figure 5-6.3 Dimensions between the corner bars

The second expression of A_{sv} is based on the concept of total shear.

$$A_{sv} = A_v + 2A_T \qquad \textbf{(5-6.11)}$$

The first component A_v corresponds to the flexural shear to be carried by the stirrups. The second component A_t corresponds to the torsional shear to be carried by the stirrups. The factor 2 considers that the torsional shear is additive to flexural shear in both the legs.

The following sketch shows the addition of flexural and torsional shears for a hollow section.

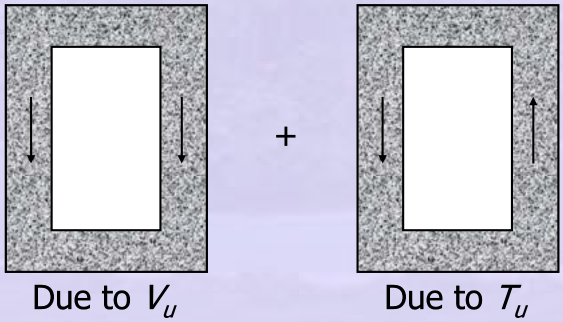


Figure 5-6.4 Distribution of flexural and torsional shears for a hollow section

The two shears are additive in the left web, whereas they are subtractive in the right web. Since, the stirrups have equal areas in the two legs, the torsional shear is considered additive to flexural shear in both the legs.

In solid sections, the two shears are not additive throughout the web. The flexural shear is distributed, whereas the torsional shear is restricted in the shear flow zone. Thus for solid sections, the expression of A_{sv} is conservative. The following sketch shows the addition of flexural and torsional shears for a solid section.

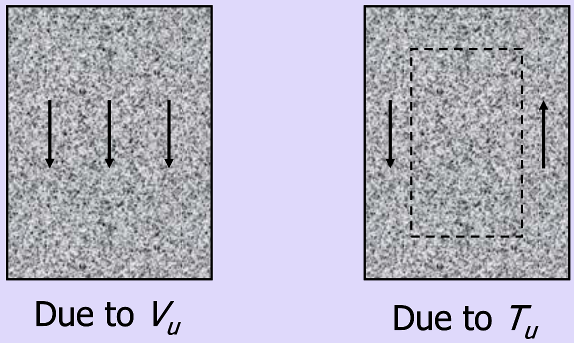


Figure 5-6.5 Distribution of flexural and torsional shears for a solid section

If the breadth of the web is large, the two shears can be designed separately. The stirrups for flexural shear can be distributed throughout the interior of the web. For torsional shear, closed stirrups can be provided in the peripheral shear flow zone.

The expressions of A_v and A_t are derived from the truss analogy for the ultimate limit state.

$$A_v = \frac{(V_u - V_{c1})s_v}{0.87 f_y d_1} \quad \textbf{(5-6.12)}$$

$$A_T = \frac{(T_u - T_{c1})s_v}{0.87 f_y b_1 d_1} \quad \textbf{(5-6.13)}$$

The minimum amount of transverse reinforcement is same as that for shear in absence of torsion.

$$\frac{A_{sv}}{bs_v} = \frac{0.4}{0.87 f_y} \quad \textbf{(5-6.14)}$$

5.6.2 Detailing Requirements

The detailing requirements for torsional reinforcement in **Clause 22.5.5**, **IS: 1343 - 1980** are briefly mentioned.

1) There should be at least one longitudinal bar in each corner. The minimum diameter of the longitudinal bars is 12 mm.

When any side is larger than 450 mm, provide side face reinforcement ($A_{s,\,sf}$), as per the following.

Minimum amount $A_{s,sf,min}$ = 0.1% bD

Maximum spacing s_{max} = 300 mm or b, whichever is less.

This amount is sufficient to check thermal and shrinkage cracks.

2) The closed stirrups should be bent close to the tension and compression surfaces satisfying the minimum cover. The stirrups should be perpendicular to the axis of the beam. Closed stirrups should not be made of pairs of U-stirrups lapping one another. This is clarified in the following sketch.

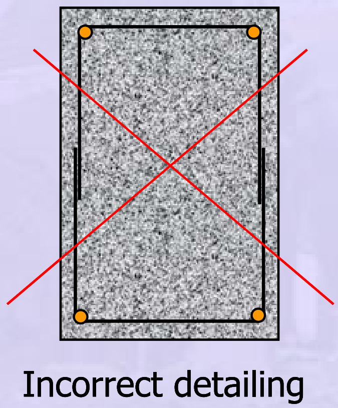


Figure 5-6.6 Detailing of closed stirrups

3) The maximum spacing is $(x_1 + y_1)/4$ or 200 mm, whichever is smaller. Here x_1 and y_1 are the short and long dimensions of the stirrups respectively.

4) Proper anchorage of stirrups as mentioned under detailing requirements of shear reinforcement. It is recommended to bend the ends of a stirrup by 135º and have 10 times the diameter of the bar (d_b) as extension beyond the bend. The following sketch clarifies the detailing of end hooks.

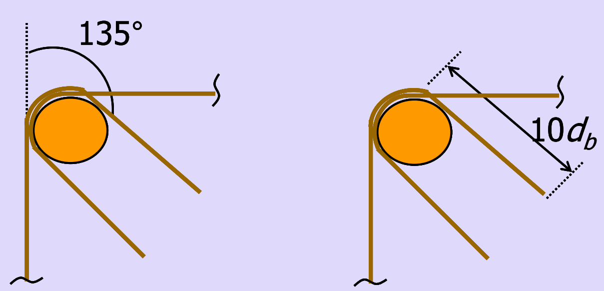


Figure 5-6.7 Detailing of end hooks for stirrups

5) The stirrups should be continued till a distance $h + b_w$ beyond the point at which it is no longer required. Here, h is the overall depth and b_w is the breadth of the web.

5.6.3 Design Steps

The following quantities are known at the selected section.

M_u = factored flexural moment

V_u = factored shear

T_u = factored torsional moment.

For gravity loads, these are calculated from the dead load and live load. The grades of concrete and steel are selected before design. As per **IS: 1343 - 1980**, the grade of steel for stirrups is limited to Fe 415.

For the design of longitudinal reinforcement, the following quantities are unknown.

The member cross-section.

M_{e1}, M_{e2}, M_{e3} = total equivalent flexural moment

A_p = amount of prestressing steel,

P_e = the effective prestress,

e = the eccentricity

A_s = area of longitudinal reinforcement

A_s' = area of longitudinal reinforcement in opposite face.

Prestressing steel A_p' may be provided in the opposite face.

For the design of stirrups, the following quantities are unknown.

V_{c1} = shear carried by concrete

T_{c1} = torsion carried by concrete

A_{sv} = total area of the legs of stirrups within a distance sv

s_v = spacing of stirrups.

The steps for designing longitudinal and transverse reinforcements for beams subjected to torsion are given.

1) Calculate M_u, V_u and T_u at a selected location. Select a suitable cross-section. For high value of T_u, as in bridges, a box section is preferred.

For longitudinal reinforcement

2a) Calculate M_{e1}.

2b) Design A_p and A_s. The design procedure involves preliminary design and final design, which are explained in the Section 4.2, Design of Sections for Flexure (Part I) and Section 4.3, Design of Sections for Flexure (Part II)

3a) Calculate M_{e2} if $M_u < M_t$.

3b) Design A_s'. The design procedure is similar for a reinforced concrete section. If A_p' is provided, the design is similar to a prestressed concrete section.

4a) Calculate M_{e3} if $M_u < M_t$.

4b) Check the adequacy of transverse bending based on the corner bars. If inadequate, design side face reinforcement ($A_{s,sf}$). $A_{s,sf}$ includes the corner bars. The design is similar to that for a reinforced concrete section.

For transverse reinforcement

5a) Calculate T_c, Eqn. **(5-6.4)**.

5b) Calculate V_c from the lower of V_{c0} and V_{cr}.

5c) Calculate e (if not calculated earlier) and e_c.

5d) Calculate T_{c1} and V_{c1}. Limit T_{c1} to $T_u/2$.

6) Calculate A_{sv} / s_v from the greater of the values given by Eqns. **(5-6.10)**, **(5-6.11)**, **(5-6.12)**, and **(5-6.13)**.

Compare the value with the minimum requirement Eqn. **(5-6.14)**.

7) Calculate maximum spacing and round it off to a multiple of 5 mm.

8) Calculate the size of the stirrups based on the amount required.

Repeat the calculations for other locations of the beam if the spacing of stirrups needs to be varied.

Example 5-6.1

Design a rectangular section to carry the following ultimate loads.

T_u = 44.5 kNm

M_u = 222.5 kNm (including an estimate of self-weight)

V_u = 89.0 kN.

The material properties are as follows.

f_{ck} = 35 N/mm^2

f_y = 250 N/mm^2

f_{pk} = 1720 N/mm^2

The prestressing is f_{pe} = 1035 N/mm^2.

Solution

1) Calculate M_{e1}.

Let D/b = 2

$$M_t = T_u\sqrt{1+\frac{2D}{b}}$$
$$= 44.5\sqrt{1+2\times 2}$$
$$= 99.5 \text{ kNm}$$

$$M_{e1} = M_u + M_T$$
$$= 222.5 + 99.5$$
$$= 322.0 \text{ kNm}$$

2) Select section. Design A_p and A_s.

Select

b = 250 mm

D = 500 mm

d = 450 mm.

Provide (2) 16 mm diameter corner bars. The flexural design results are as follows.

$$A_s = 2 \times 201$$
$$= 402 \text{ mm}^2.$$

Required amount of prestressing steel with $d_p = d = 450$ mm is $A_p = 484$ mm^2.

Provide 11 mm diameter strands with area = 70 mm^2.

Required number of strands = 484 / 70 = 6.8 → 7

Provided amount of prestressing steel

$$A_{p,prov} = 7 \times 70$$
$$= 490 \text{ mm}^2$$

3) Calculate M_{e2} .

Since $M_u > M_t$, design for M_{e2} is not required.

4) Calculate M_{e3} .

Since $M_u > M_t$, design for M_{e3} is not required.

5a) Calculate T_c .

$$\begin{aligned} f_{cp} &= \frac{P_e}{A} \\ &= \frac{f_{pe} \times A_p}{bD} \\ &= \frac{1035 \times 490}{250 \times 500} \\ &= 4.06 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \lambda_p &= \sqrt{1 + \frac{12 f_{cp}}{f_{ck}}} \\ &= \sqrt{1 + \frac{12 \times 4.06}{35}} \\ &= 1.55 \end{aligned}$$

$f_{cp} < 0.3\ f_{ck}$. OK

$$T_c = 0.15b^2D\left(1 - \frac{b}{3D}\right)\lambda_p\sqrt{f_{ck}}$$
$$= 0.15 \times 250^2 \times 500 \times \left(1 - \frac{1}{3 \times 2}\right) \times 1.55\sqrt{35}\ \text{Nmm}$$
$$= 35.8\ \text{kNm}$$

5b) Calculate V_c from the lower of V_{co} and V_{cr} .

$$\frac{100A_p}{bd} = \frac{100 \times 490}{250 \times 450}$$
$$= 0.43$$

From Table 6, for M 35 concrete, τ_c = 0.46 N/mm^2.

$$f_{pt} = -\frac{P_e}{A} - \frac{P_e e^2}{I}$$
$$= -\frac{507{,}150}{125{,}000} - \frac{507{,}150 \times 200^2}{2.604 \times 10^9}$$
$$= -11.85\ \text{N/mm}^2$$

Here,

e = 450 – ½ 500

= 200 mm

I = 250 × 5003 / 12

= 2.604 × 109 mm^4.

$$M_0 = 0.8 f_{pt}\frac{I}{y}$$
$$= 0.8 \times 11.85 \times \frac{2.604 \times 10^9}{200}$$
$$= 123.43\ \text{kNm}$$

$$V_{cr} = \left(1 - 0.55\frac{f_{pe}}{f_{pk}}\right)\tau_c bd + M_0\frac{V_u}{M_u}$$
$$= (1 - 0.55 \times 0.6) \times \frac{0.46 \times 250 \times 450}{10^3} + 123.43 \times \frac{89}{222.5}$$
$$= 84.0\ \text{kN}$$

Here,

f_{pe}/f_{pk} = 1035 / 1720

= 0.6.

$$
\begin{aligned}
V_{co} &= 0.67bD\sqrt{f_t^2 + 0.8f_{cp}f_t} \\
&= 0.67\times 250\times 500\sqrt{1.42^2 + 0.8\times 4.06\times 1.42} \\
&= 215.6 \text{ kN}
\end{aligned}
$$

Here,

$$
\begin{aligned}
f_t &= 0.24\sqrt{35} \\
&= 1.42 \text{ N/mm}^2
\end{aligned}
$$

$\therefore\ V_c = V_{cr} = 84.0$ kN

5c) Calculate e and e_c .

$$
\begin{aligned}
e &= \frac{T_u}{V_u} \\
&= \frac{44.5}{89.0} \\
&= 0.50 \text{ m}
\end{aligned}
\qquad\qquad
\begin{aligned}
e_c &= \frac{T_c}{V_c} \\
&= \frac{35.8}{84.0} \\
&= 0.43 \text{ m}
\end{aligned}
$$

5d) Calculate T_{c1} and V_{c1}.

$$
\begin{aligned}
T_{c1} &= T_c\frac{e}{e+e_c} \\
&= 35.82\times\frac{0.50}{0.50+0.43} \\
&= 19.26 \text{ kNm}
\end{aligned}
$$

$$
\begin{aligned}
V_{c1} &= V_c\frac{e_c}{e+e_c} \\
&= 84.0\times\frac{0.43}{0.50+0.43} \\
&= 38.84 \text{ kN}
\end{aligned}
$$

$$T_{c1} < \frac{T_u}{2} \text{ OK.}$$

6) Calculate A_{sv} / s_v

$$
\begin{aligned}
\frac{A_{sv}}{s_v} &= \frac{M_t}{1.5b_1d_1f_y} \\
&= \frac{99.5\times 10^6}{1.5\times 200\times 400\times 250} \\
&= 3.3 \text{ mm}^2\text{/mm}
\end{aligned}
$$

Estimated values

b_1 = 250 – 50

= 200 mm

d_1 = 500 – 100

= 400 mm.

$$\frac{A_v}{s_v} = \frac{V_u - V_{c1}}{0.87 f_y d_1}$$

$$= \frac{(89.0 - 38.8) \times 10^3}{0.87 \times 250 \times 400}$$

$$= 0.58 \text{ mm}^2/\text{mm}$$

$$\frac{A_T}{s_v} = \frac{T_u - T_{c1}}{0.87 f_y b_1 d_1}$$

$$= \frac{(44.5 - 19.26) \times 10^6}{0.87 \times 250 \times 200 \times 400}$$

$$= 1.45 \text{ mm}^2/\text{mm}$$

$$\frac{A_{sv}}{s_v} = \frac{A_V}{s_v} + 2 \times \frac{A_T}{s_v}$$

$$= 0.58 + 2 \times 1.45$$

$$= 3.48 \text{ mm}^2/\text{mm}$$

Minimum amount of stirrups

$$\frac{A_{sv}}{bs_v} = \frac{0.4}{0.87 f_y}$$

$$\frac{A_{sv}}{s_v} = \frac{0.4 \times 250}{0.87 \times 250}$$

$$= 0.46 \text{ mm}^2/\text{mm}$$

Select

A_{sv} / s_v = 3.48 mm^2/mm.

7) Calculate maximum spacing

$$s_v \leq \frac{x_1 + y_1}{4}$$
$$\leq \frac{204 + 422}{4}$$
$$\leq 156\ \text{mm}$$

Estimated values

x_1 = 250 – 46

= 204 mm

y_1 = 500 – 78

= 422 mm.

The other values of s_v do not govern.

8) Calculate the size of the stirrups

Select 2 legs of 12 mm diameter stirrups.

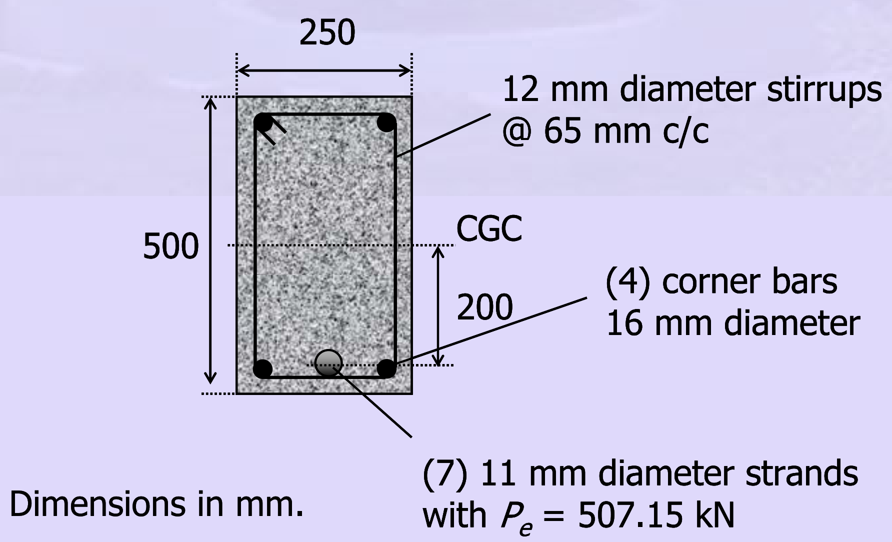
A_{sv} = 2 × 113

= 226 mm^2

$$s_v = \frac{226}{3.48}$$
$$= 65\ \text{mm}$$

The spacing can be increased by bundling the stirrup bars.

Designed section



As D > 450 mm, side face reinforcement is required.