2.1 Losses in Prestress (Part I)

This section covers the following topics.

- Introduction
- Elastic Shortening

The relevant notations are explained first.

Notations

Geometric Properties

The commonly used geometric properties of a prestressed member are defined as follows.

A_c = Area of concrete section
= Net cross-sectional area of concrete excluding the area of prestressing steel.

A_p = Area of prestressing steel
= Total cross-sectional area of the tendons.

A = Area of prestressed member
= Gross cross-sectional area of prestressed member.
$= A_c + A_p$

A_t = Transformed area of prestressed member
= Area of the member when steel is substituted by an equivalent area of concrete.
$= A_c + mA_p$
$= A + (m - 1)A_p$

Here,

m = the modular ratio = E_p/E_c
E_c = short-term elastic modulus of concrete
E_p = elastic modulus of steel.

The following figure shows the commonly used areas of the prestressed members.

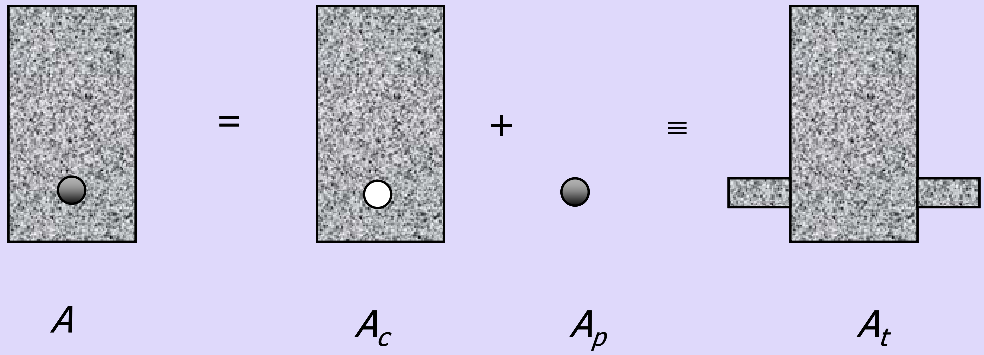


Figure 2-1.1 Areas for prestressed members

CGC = Centroid of concrete
= Centroid of the gross section. The CGC may lie outside the concrete (Figure 2-1.2).

CGS = Centroid of prestressing steel
= Centroid of the tendons. The CGS may lie outside the tendons or the concrete (Figure 2-1.2).

I = Moment of inertia of prestressed member
= Second moment of area of the gross section about the CGC.

I_t = Moment of inertia of transformed section
= Second moment of area of the transformed section about the centroid of the transformed section.

e = Eccentricity of CGS with respect to CGC
= Vertical distance between CGC and CGS. If CGS lies below CGC, e will be considered positive and vice versa (Figure 2-1.2).

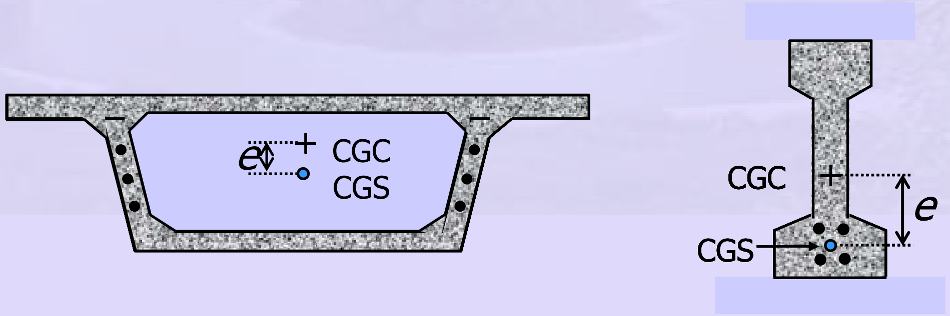


Figure 2-1.2 CGC, CGS and eccentricity of typical prestressed members

Load Variables

P_i = Initial prestressing force
= The force which is applied to the tendons by the jack.

P_0 = Prestressing force after immediate losses

= The reduced value of prestressing force after elastic shortening, anchorage slip and loss due to friction.

P_e = Effective prestressing force after time-dependent losses

= The final value of prestressing force after the occurrence of creep, shrinkage and relaxation.

2.1.1 Introduction

In prestressed concrete applications, the most important variable is the prestressing force. In the early days, it was observed that the prestressing force does not stay constant, but reduces with time. Even during prestressing of the tendons and the transfer of prestress to the concrete member, there is a drop of the prestressing force from the recorded value in the jack gauge. The various reductions of the prestressing force are termed as the losses in prestress.

The losses are broadly classified into two groups, immediate and time-dependent. The immediate losses occur during prestressing of the tendons and the transfer of prestress to the concrete member. The time-dependent losses occur during the service life of the prestressed member. The losses due to elastic shortening of the member, friction at the tendon-concrete interface and slip of the anchorage are the immediate losses. The losses due to the shrinkage and creep of the concrete and relaxation of the steel are the time-dependent losses. The causes of the various losses in prestress are shown in the following chart.

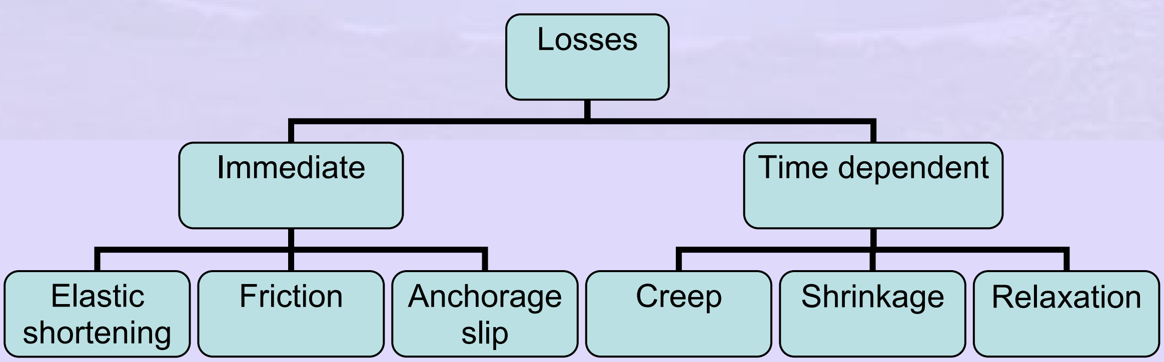


Figure 2-1.3 Causes of the various losses in prestress

2.1.2 Elastic Shortening

Pre-tensioned Members

When the tendons are cut and the prestressing force is transferred to the member, the concrete undergoes immediate shortening due to the prestress. The tendon also shortens by the same amount, which leads to the loss of prestress.

Post-tensioned Members

If there is only one tendon, there is no loss because the applied prestress is recorded after the elastic shortening of the member. For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons.

The elastic shortening loss is quantified by the drop in prestress (Δf_p) in a tendon due to the change in strain in the tendon ($\Delta\varepsilon_p$). It is assumed that the change in strain in the tendon is equal to the strain in concrete (ε_c) at the level of the tendon due to the prestressing force. This assumption is called **strain compatibility** between concrete and steel. The strain in concrete at the level of the tendon is calculated from the stress in concrete (f_c) at the same level due to the prestressing force. A linear elastic relationship is used to calculate the strain from the stress.

The quantification of the losses is explained below.

$$\begin{aligned}\Delta f_p &= E_p \Delta\varepsilon_p \\ &= E_p \varepsilon_c \\ &= E_p \left(\frac{f_c}{E_c}\right) \\ \Delta f_p &= m f_c\end{aligned} \tag{2-1.1}$$

For simplicity, the loss in all the tendons can be calculated based on the stress in concrete at the level of CGS. This simplification cannot be used when tendons are stretched sequentially in a post-tensioned member. The calculation is illustrated for the following types of members separately.

- Pre-tensioned Axial Members
- Pre-tensioned Bending Members
- Post-tensioned Axial Members

- Post-tensioned Bending Members

Pre-tensioned Axial Members

The following figure shows the changes in length and the prestressing force due to elastic shortening of a pre-tensioned axial member.

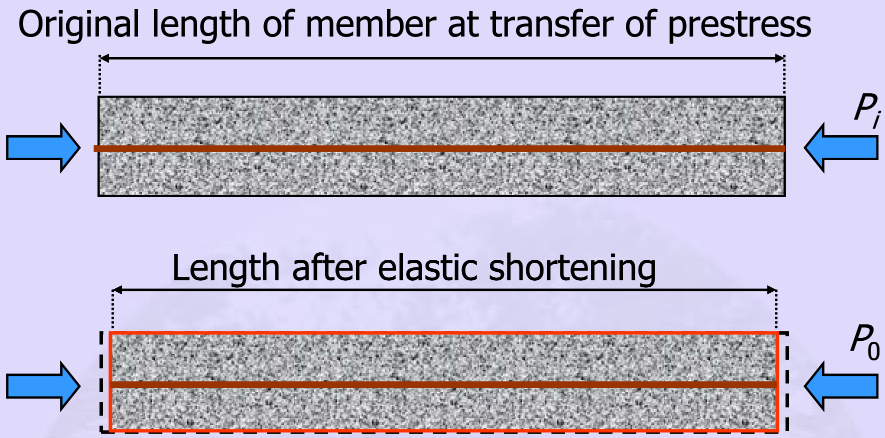


Figure 2-1.4 Elastic shortening of a pre-tensioned axial member

The loss can be calculated as per Eqn. (2-1.1) by expressing the stress in concrete in terms of the prestressing force and area of the section as follows.

$$\Delta f_p = m f_c$$

$$= m\left(\frac{P_0}{A_c}\right)$$

$$\Delta f_p = m\left(\frac{P_i}{A_t}\right) \approx m\left(\frac{P_i}{A}\right) \qquad \textbf{(2-1.2)}$$

Note that the stress in concrete due to the prestressing force after immediate losses (P_0/A_c) can be equated to the stress in the transformed section due to the initial prestress (P_i/A_t). This is derived below. Further, the transformed area A_t of the prestressed member can be approximated to the gross area A.

The following figure shows that the strain in concrete due to elastic shortening (ε_c) is the difference between the initial strain in steel (ε_{pi}) and the residual strain in steel (ε_{p0}).

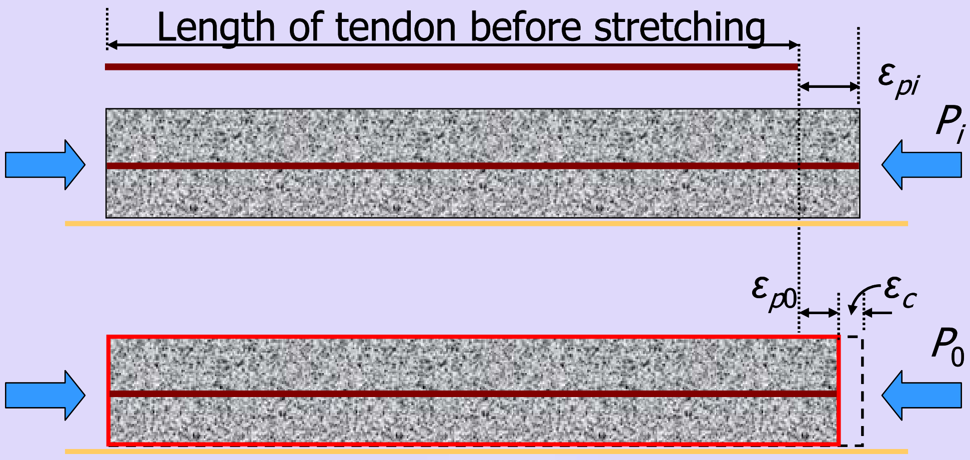


Figure 2-1.5 Strain variables in elastic shortening

The following equation relates the strain variables.

$$\varepsilon_c = \varepsilon_{pi} - \varepsilon_{p0} \tag{2-1.3}$$

The strains can be expressed in terms of the prestressing forces as follows.

$$\varepsilon_c = \frac{P_0}{A_c E_c} \tag{2-1.4}$$

$$\varepsilon_{pi} = \frac{P_i}{A_p E_p} \tag{2-1.5}$$

$$\varepsilon_{p0} = \frac{P_0}{A_p E_p} \tag{2-1.6}$$

Substituting the expressions of the strains in Eqn. (2-1.3)

$$\frac{P_0}{A_c E_c} = \frac{P_i}{A_p E_p} - \frac{P_0}{A_p E_p}$$

$$\text{or,} \quad P_0\left(\frac{1}{A_c E_c} + \frac{1}{A_p E_p}\right) = \frac{P_i}{A_p E_p}$$

$$\text{or,} \quad P_0\left(\frac{m}{A_c} + \frac{1}{A_p}\right) = \frac{P_i}{A_p}$$

$$\text{or,} \quad \frac{P_0}{A_c} = \frac{P_i}{mA_p + A_c}$$

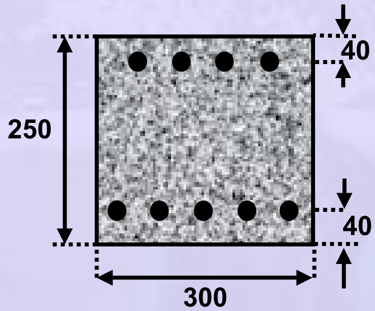
$$\text{or} \quad \frac{P_0}{A_c} = \frac{P_i}{A_t} \tag{2-1.7}$$

Thus, the stress in concrete due to the prestressing force after immediate losses (P_0/A_c) can be equated to the stress in the transformed section due to the initial prestress (P_i/A_t).

The following problem illustrates the calculation of loss due to elastic shortening in an idealised pre-tensioned railway sleeper.

Example 2-1.1

A prestressed concrete sleeper produced by pre-tensioning method has a rectangular cross-section of 300mm × 250 mm (*b* × *h*). It is prestressed with 9 numbers of straight 7mm diameter wires at 0.8 times the ultimate strength of 1570 N/mm^2. Estimate the percentage loss of stress due to elastic shortening of concrete. Consider *m* = 6.



Solution

a) Approximate solution considering gross section

The sectional properties are calculated as follows.

Area of a single wire, $A_w = \pi/4 \times 7^2$

$= 38.48\ \text{mm}^2$

Area of total prestressing steel, $A_p = 9 \times 38.48$

$= 346.32\ \text{mm}^2$

Area of concrete section, $A = 300 \times 250$

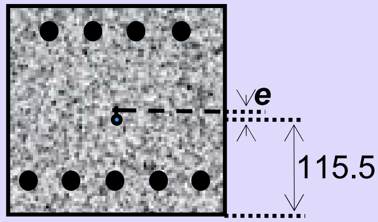
$= 75 \times 10^3\ \text{mm}^2$

Moment of inertia of section, $I = 300 \times 250^3/12$

$= 3.91 \times 10^8\ \text{mm}^4$

Distance of centroid of steel area (CGS) from the soffit,

$$\bar{y} = \frac{4 \times 38.48 \times (250 - 40) + 5 \times 38.48 \times 40}{9 \times 38.48}$$
$$= 115.5 \text{ mm}$$



Prestressing force, P_i = 0.8 × 1570 × 346.32 N

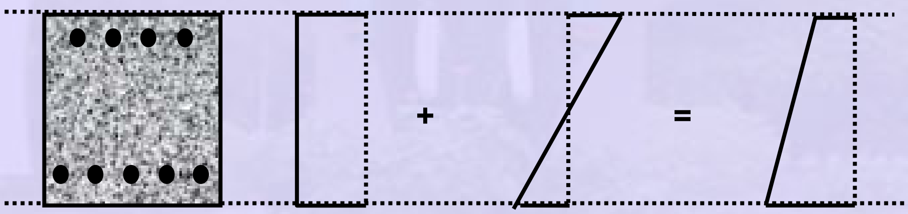
= 435 kN

Eccentricity of prestressing force,

e = (250/2) – 115.5

= 9.5 mm

The stress diagrams due to P_i are shown.



$$-\frac{P_i}{A} \qquad \pm\frac{P_i.e}{I}y \qquad -\frac{P_i}{A} \pm \frac{P_i.e}{I}y$$

Since the wires are distributed above and below the CGC, the losses are calculated for the top and bottom wires separately.

Stress at level of top wires ($y = y_t$ = 125 – 40)

$$(f_c)_t = -\frac{P_i}{A} + \frac{P_i.e}{I}y_t$$
$$= -\frac{435 \times 10^3}{75 \times 10^3} + \frac{435 \times 10^3 \times 9.5}{3.91 \times 10^8} \times (125 - 40)$$
$$= -5.8 + 0.9$$
$$= -4.9 \text{ N/mm}^2$$

Stress at level of bottom wires ($y = y_b = 125 – 40$),

$$\begin{aligned}(f_c)_b &= -\frac{P_i}{A} - \frac{P_i.e}{I}y_b \\ &= -\frac{435\times10^3}{75\times10^3} - \frac{435\times10^3\times9.5}{3.91\times10^8}\times(125-40) \\ &= -5.8-0.9 \\ &= -6.7\ \text{N/mm}^2\end{aligned}$$

Loss of prestress in top wires = mf_cA_p
(in terms of force) = 6 × 4.9 × (4 × 38.48)
= 4525.25 N

Loss of prestress in bottom wires = 6 × 6.7 × (5 × 38.48)
= 7734.48 N

Total loss of prestress = 4525 + 7735
= 12259.73 N
≈ 12.3 kN

Percentage loss = (12.3 / 435) × 100%
= 2.83%

b) Accurate solution considering transformed section.

Transformed area of top steel,

A_1 = (6 – 1) 4 × 38.48
= 769.6 mm^2

Transformed area of bottom steel,

A_2 = (6 – 1) 5 × 38.48
= 962.0 mm^2

Total area of transformed section,

A_T = $A + A_1 + A_2$
= 75000.0 + 769.6 + 962.0
= 76731.6 mm^2

Centroid of the section (CGC)

$$\bar{y} = \frac{A \times 125 + A_1 \times (250 - 40) + A_2 \times 40}{A}$$

$$= 124.8 \text{ mm from soffit of beam}$$

Moment of inertia of transformed section,

$$I_T = I_g + A(0.2)^2 + A_1(210 - 124.8)^2 + A_2(124.8 - 40)^2$$
$$= 4.02 \times 10^8 \text{mm}^4$$

Eccentricity of prestressing force,

$$e = 124.8 - 115.5$$
$$= 9.3 \text{ mm}$$

Stress at the level of bottom wires,

$$(f_c)_b = -\frac{435 \times 10^3}{76.73 \times 10^3} - \frac{(435 \times 10^3 \times 9.3)84.8}{4.02 \times 10^8}$$
$$= -5.67 - 0.85$$
$$= -6.52 \text{ N/mm}^2$$

Stress at the level of top wires,

$$(f_c)_t = -\frac{435 \times 10^3}{76.73 \times 10^3} + \frac{(435 \times 10^3 \times 9.3)85.2}{4.02 \times 10^8}$$
$$= -5.67 + 0.86$$
$$= -4.81 \text{N/mm}^2$$

Loss of prestress in top wires $= 6 \times 4.81 \times (4 \times 38.48)$
$= 4442$ N

Loss of prestress in bottom wires $= 6 \times 6.52 \times (5 \times 38.48)$
$= 7527$ N

Total loss $= 4442 + 7527$
$= 11969$ N
≈ 12 kN

Percentage loss $= (12 / 435) \times 100\%$
$= 2.75$ %

It can be observed that the accurate and approximate solutions are close. Hence, the simpler calculations based on A and I is acceptable.

Pre-tensioned Bending Members

The following figure shows the changes in length and the prestressing force due to elastic shortening of a pre-tensioned bending member.

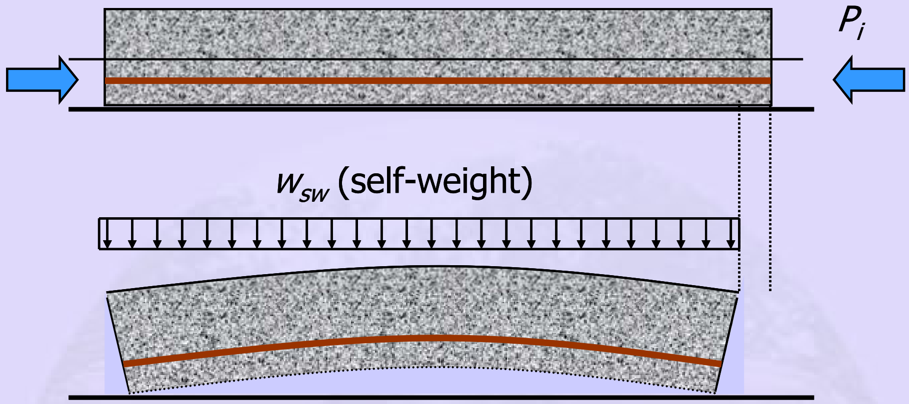


Figure 2-1.6 Elastic shortening of a pre-tensioned bending member

Due to the effect of self-weight, the stress in concrete varies along length (Figure 2-1.6). The loss can be calculated by Eqn. (2-1.1) with a suitable evaluation of the stress in concrete. To have a conservative estimate of the loss, the maximum stress at the level of CGS at the mid-span is considered.

$$f_c = -\frac{P_i}{A} - \frac{P_i e.e}{I} + \frac{M_{sw} e}{I} \qquad \textbf{(2-1.8)}$$

Here, M_{sw} is the moment at mid-span due to self-weight. Precise result using A_t and I_t in place of A and I, respectively, is not computationally warranted. In the above expression, the eccentricity of the CGS (e) was assumed to be constant.

For a large member, the calculation of the loss can be refined by evaluating the strain in concrete at the level of the CGS accurately from the definition of strain. This is demonstrated later for post-tensioned bending members.

Post-tensioned Axial Members

For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons. The loss in each tendon can be calculated in progressive sequence. Else, an approximation can be used to calculate the losses.

The loss in the first tendon is evaluated precisely and half of that value is used as an average loss for all the tendons.

$$\begin{aligned}\Delta f_p &= \frac{1}{2}\Delta f_{p1}\\ &= \frac{1}{2} m f_{c1}\\ &= \frac{1}{2} m \sum_{j=2}^{n} \frac{P_{i,j}}{A}\end{aligned} \qquad \textbf{(2-1.9)}$$

Here,

$P_{i,j}$ = initial prestressing force in tendon j

n = number of tendons

The eccentricity of individual tendon is neglected.

Post-tensioned Bending Members

The calculation of loss for tendons stretched sequentially, is similar to post-tensioned axial members. For curved profiles, the eccentricity of the CGS and hence, the stress in concrete at the level of CGS vary along the length. An average stress in concrete can be considered.

For a parabolic tendon, the average stress ($f_{c,avg}$) is given by the following equation.

$$f_{c,avg} = f_{c1} + \frac{2}{3}\left(f_{c2} - f_{c1}\right) \qquad \textbf{(2-1.10)}$$

Here,

f_{c1} = stress in concrete at the end of the member

f_{c2} = stress in concrete at the mid-span of the member.

A more rigorous analysis of the loss can be done by evaluating the strain in concrete at the level of the CGS accurately from the definition of strain. This is demonstrated for a beam with two parabolic tendons post-tensioned sequentially. In Figure 2-1.7, Tendon B is stretched after Tendon A. The loss in Tendon A due to elastic shortening during tensioning of Tendon B is given as follows.

$$\begin{aligned}\Delta f_p &= E_p \varepsilon_c\\ &= E_p\left[\varepsilon_{c1} + \varepsilon_{c2}\right]\end{aligned} \qquad \textbf{(2-1.11)}$$

Here, ε_c is the strain at the level of Tendon A. The component of ε_c due to pure compression is represented as ε_{c1}. The component of ε_c due to bending is represented as ε_{c2}. The two components are calculated as follows.

$$\begin{aligned}
\varepsilon_{c1} &= \frac{P_B}{AE_c} \\
\varepsilon_{c2} &= \frac{\delta L}{L} \\
&= \frac{1}{L}\int_0^L \frac{P_B.e_B(x).e_A(x)}{IE_c}dx \\
&= \frac{P_B}{E_c LI}\int_0^L e_B(x).e_A(x)\,dx
\end{aligned} \quad \textbf{(2-1.12)}$$

Here,

A = cross-sectional area of beam

P_B = prestressing force in Tendon B

E_c = modulus of concrete

L = length of beam

$e_A(x)$, $e_B(x)$ = eccentricities of Tendons A and B, respectively, at distance x from left end

I = moment of inertia of beam

δL = change in length of beam

The variations of the eccentricities of the tendons can be expressed as follows.

$$e_A(x) = e_{A1} + 4\Delta e_A \frac{x}{L}\left(1-\frac{x}{L}\right) \quad \textbf{(2-1.13)}$$

$$e_B(x) = e_{B1} + 4\Delta e_B \frac{x}{L}\left(1-\frac{x}{L}\right) \quad \textbf{(2-1.14)}$$

Where, $\Delta e_A = e_{A2} - e_{A1}$

$\Delta e_B = e_{B2} - e_{B1}$

e_{A1}, e_{A2} = eccentricities of Tendon A at 1 (end) and 2 (centre), respectively.

e_{B1}, e_{B2} = eccentricities of Tendon B at 1 and 2, respectively.

Substituting the expressions of the eccentricities in Eqn. (2-1.12), the second component of the strain is given as follows.

$$\frac{P_B}{E_c I} = \left[\frac{1}{5}e_{A1}e_{B1} + \frac{2}{15}\left(e_{A1}e_{B2} + e_{A2}e_{B1}\right) + \frac{8}{15}e_{A2}e_{B2}\right] \qquad \textbf{(2-1.15)}$$

2.2 Losses in Prestress (Part II)

This section covers the following topics

- Friction
- Anchorage Slip
- Force Variation Diagram

2.2.1 Friction

The friction generated at the interface of concrete and steel during the stretching of a curved tendon in a post-tensioned member, leads to a drop in the prestress along the member from the stretching end. The loss due to friction does not occur in pre-tensioned members because there is no concrete during the stretching of the tendons.

The friction is generated due to the curvature of the tendon and the vertical component of the prestressing force. The following figure shows a typical profile (laying pattern) of the tendon in a continuous beam.

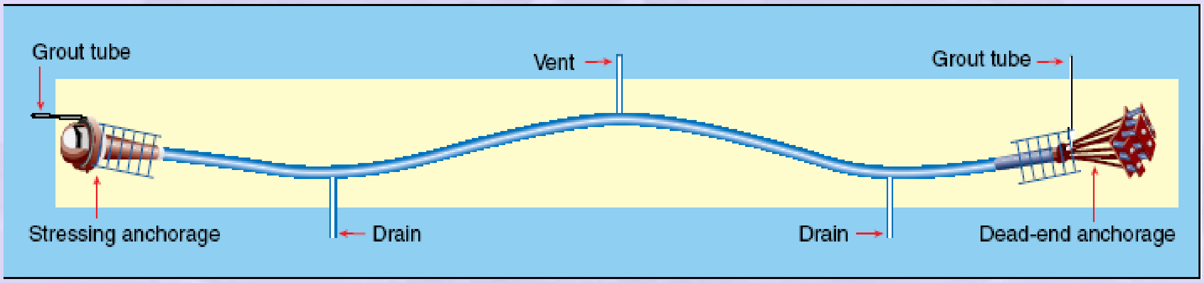


Figure 2-2.1 A typical continuous post-tensioned member

(Reference: VSL International Ltd.)

In addition to friction, the stretching has to overcome the **wobble** of the tendon. The wobble refers to the change in position of the tendon along the duct. The losses due to friction and wobble are grouped together under friction.

The formulation of the loss due to friction is similar to the problem of belt friction. The sketch below (Figure 2-2.2) shows the forces acting on the tendon of infinitesimal length *dx*.

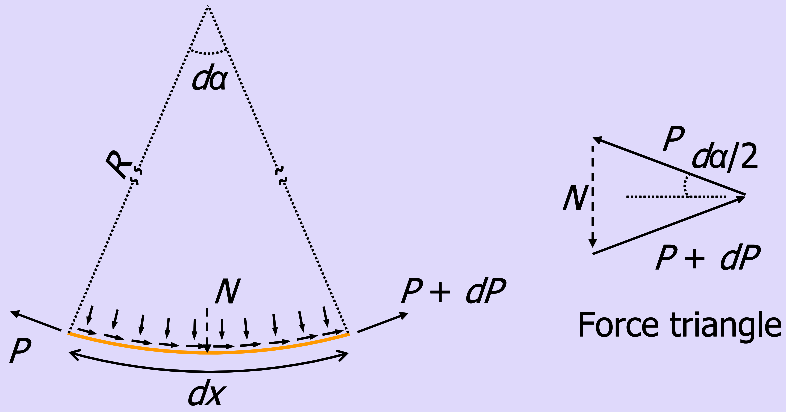


Figure 2-2.2 Force acting in a tendon of infinitesimal length

In the above sketch,

P = prestressing force at a distance x from the stretching end

R = radius of curvature

$d\alpha$ = subtended angle.

The derivation of the expression of P is based on a circular profile. Although a cable profile is parabolic based on the bending moment diagram, the error induced is insignificant.

The friction is proportional to the following variables.

- Coefficient of friction (μ) between concrete and steel.
- The resultant of the vertical reaction from the concrete on the tendon (N) generated due to curvature.

From the equilibrium of forces in the force triangle, N is given as follows.

$$N = 2P\sin\frac{d\alpha}{2}$$

$$\approx 2P\frac{d\alpha}{2} = Pd\alpha \qquad \textbf{(2-2.1)}$$

The friction over the length dx is equal to $\mu N = \mu P d\alpha$.

Thus the friction (dP) depends on the following variables.

- Coefficient of friction (μ)
- Curvature of the tendon ($d\alpha$)
- The amount of prestressing force (P)

The wobble in the tendon is effected by the following variables.

- Rigidity of sheathing
- Diameter of sheathing
- Spacing of sheath supports
- Type of tendon
- Type of construction

The friction due to wobble is assumed to be proportional to the following.

- Length of the tendon
- Prestressing force

For a tendon of length dx, the friction due to wobble is expressed as $kPdx$, where k is the wobble coefficient or coefficient for wave effect.

Based on the equilibrium of forces in the tendon for the horizontal direction, the following equation can be written.

$$P = P + dP + (\mu P d\alpha + kPdx)$$

$$\text{or, } dP = -(\mu P d\alpha + kPdx) \qquad \textbf{(2-2.2)}$$

Thus, the total drop in prestress (dP) over length dx is equal to $-(\mu Pd\alpha + kPdx)$. The above differential equation can be solved to express P in terms of x.

$$\int_{P_0}^{P_x} \frac{dP}{P} = -\left(\mu \int_0^{\alpha} d\alpha + k \int_0^{x} dx\right)$$

$$\text{or, } lnP\Big|_{P_0}^{P_x} = -(\mu\alpha + kx)$$

$$\text{or, } ln\frac{P_x}{P_0} = -(\mu\alpha + kx)$$

$$\text{or, } \quad P_x = P_0 e^{-(\mu\alpha + kx)} \qquad \textbf{(2-2.3)}$$

Here,

P_0 = the prestress at the stretching end after any loss due to elastic shortening.

For small values of $\mu\alpha + kx$, the above expression can be simplified by the Taylor series expansion.

$$P_x = P_0\,(1 - \mu\alpha - kx) \qquad \textbf{(2-2.4)}$$

Thus, for a tendon with single curvature, the variation of the prestressing force is linear with the distance from the stretching end. The following figure shows the variation of prestressing force after stretching. The left side is the stretching end and the right side is the anchored end.

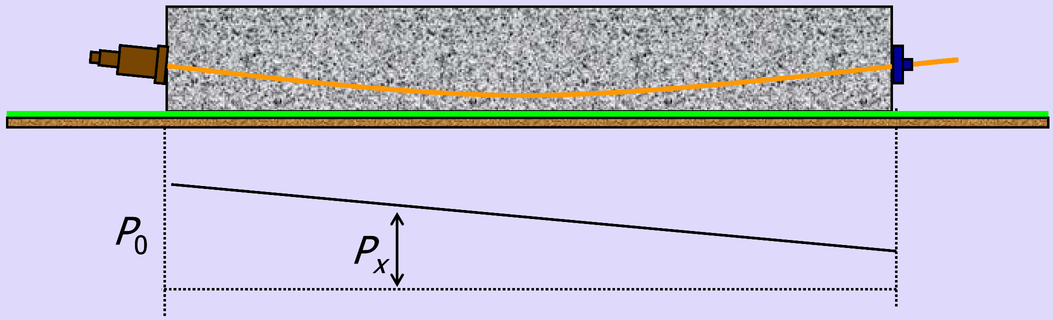


Figure 2-2.3 Variation of prestressing force after stretching

In the absence of test data, **IS:1343 - 1980** provides guidelines for the values of μ and k.

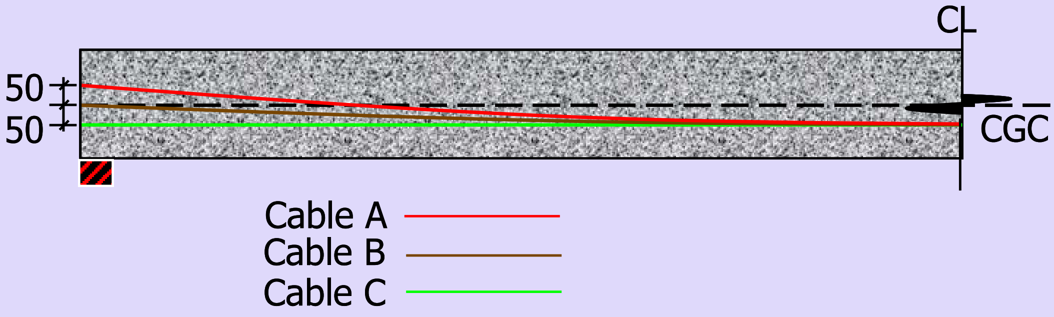
Table 2-2.1 Values of coefficient of friction

Type of interface	μ
For steel moving on smooth concrete	0.55.
For steel moving on steel fixed to duct	0.30.
For steel moving on lead	0.25.

The value of k varies from 0.0015 to 0.0050 per meter length of the tendon depending on the type of tendon. The following problem illustrates the calculation of the loss due to friction in a post-tensioned beam.

Example 2-2.1

A post-tensioned beam 100 mm × 300 mm ($b \times h$) spanning over 10 m is stressed by successive tensioning and anchoring of 3 cables A, B, and C respectively as shown in figure. Each cable has cross section area of 200 mm^2 and has initial stress of 1200 MPa. If the cables are tensioned from one end, estimate the percentage loss in each cable due to friction at the anchored end. Assume μ = 0.35, k = 0.0015 / m.



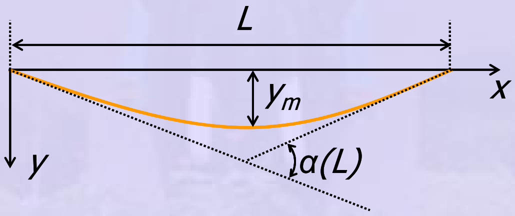
Solution

Prestress in each tendon at stretching end $= 1200 \times 200$

$= 240$ kN.

To know the value of $\alpha(L)$, the equation for a parabolic profile is required.

$$\frac{dy}{dx} = \frac{4y_m}{L^2}(L - 2x)$$



Here,

y_m = displacement of the CGS at the centre of the beam from the ends

L = length of the beam

x = distance from the stretching end

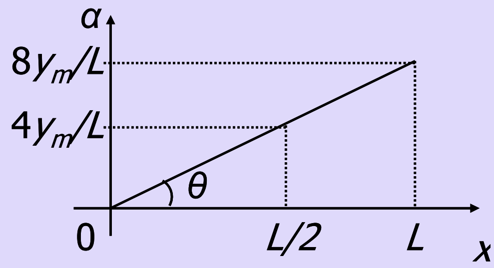
y = displacement of the CGS at distance x from the ends.

An expression of $\alpha(x)$ can be derived from the change in slope of the profile. The slope of the profile is given as follows.

$$\frac{dy}{dx} = \frac{4y_m}{L^2}(L - 2x)$$

At $x = 0$, the slope $dy/dx = 4y_m/L$. The change in slope $\alpha(x)$ is proportional to x.

The expression of $\alpha(x)$ can be written in terms of x as $\alpha(x) = \theta.x$, where, $\theta = 8y_m/L^2$. The variation is shown in the following sketch.



The total subtended angle over the length L is $8y_m/L$.

The prestressing force P_x at a distance x is given by

$$P_x = P_0 e^{-(\mu\alpha + kx)} = P_0 e^{-\eta x}$$

where, $\eta x = \mu\alpha + kx$

For cable A, y_m = 0.1 m.

For cable B, y_m = 0.05 m.

For cable C, y_m = 0.0 m.

For all the cables, L = 10 m.

Substituting the values of y_m and L

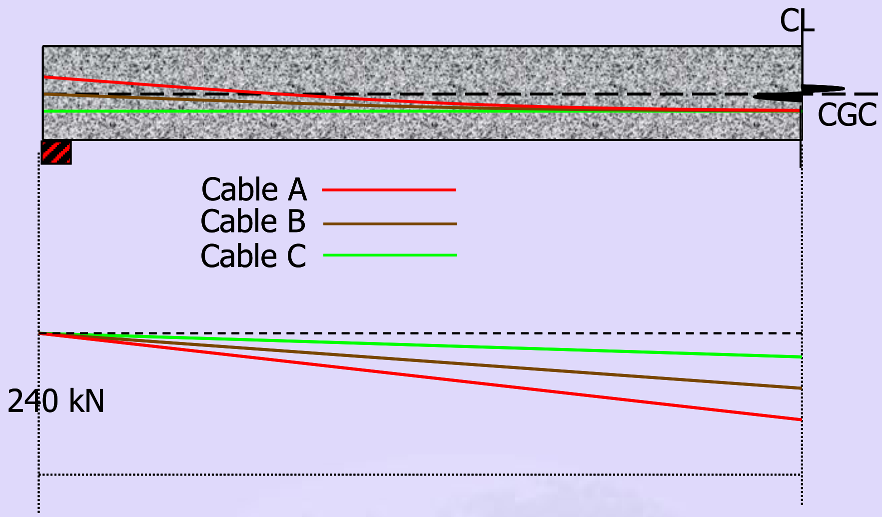
$$\eta x = \begin{cases} 0.0043x & \text{for cable A} \\ 0.0029x & \text{for cable B} \\ 0.0015x & \text{for cable C} \end{cases}$$

The maximum loss for all the cables is at $x = L = 10$, the anchored end.

$$e^{-\eta L} = \begin{cases} 0.958 & \text{for cable A} \\ 0.971 & \text{for cable B} \\ 0.985 & \text{for cable C} \end{cases}$$

Percentage loss due to friction = $(1 - e^{-\eta L}) \times 100\%$

$$= \begin{cases} 4.2\% & \text{for cable A} \\ 2.9\% & \text{for cable B} \\ 1.5\% & \text{for cable C} \end{cases}$$



Variation of prestressing forces

The loss due to friction can be considerable for long tendons in continuous beams with changes in curvature. The drop in the prestress is higher around the intermediate supports where the curvature is high. The remedy to reduce the loss is to apply the stretching force from both ends of the member in stages.

2-2.2 Anchorage Slip

In a post-tensioned member, when the prestress is transferred to the concrete, the wedges slip through a little distance before they get properly seated in the conical space. The anchorage block also moves before it settles on the concrete. There is loss of prestress due to the consequent reduction in the length of the tendon.

The total anchorage slip depends on the type of anchorage system. In absence of manufacturer's data, the following typical values for some systems can be used.

Table 2-2.2 Typical values of anchorage slip

Anchorage System	Anchorage Slip (Δs)
Freyssinet system	
12 - 5mm Φ strands	4 mm
12 - 8mm Φ strands	6 mm
Magnel system	8 mm
Dywidag system	1 mm

(Reference: Rajagopalan, N., *Prestressed Concrete*)

Due to the setting of the anchorage block, as the tendon shortens, there is a reverse friction. Hence, the effect of anchorage slip is present up to a certain length (Figure 2-2.4). Beyond this **setting length**, the effect is absent. This length is denoted as l_{set}.

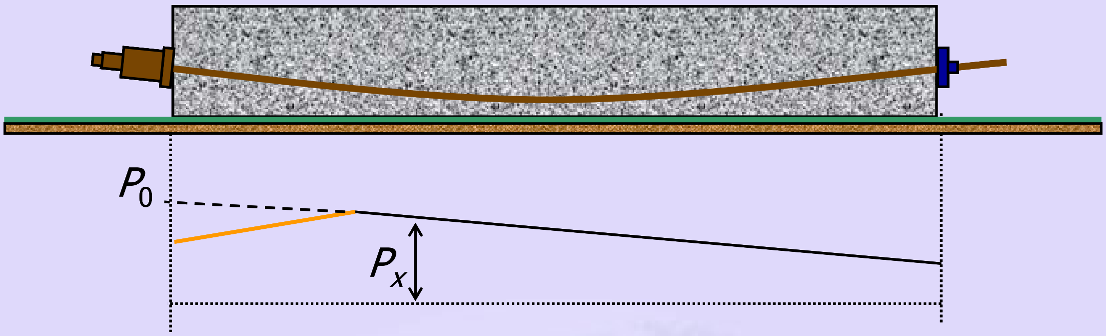


Figure 2-2.4 Variation of prestressing force after anchorage slip

2.2.3 Force Variation Diagram

The magnitude of the prestressing force varies along the length of a post-tensioned member due to friction losses and setting of the anchorage block. The diagram representing the variation of prestressing force is called the force variation diagram.

Considering the effect of friction, the magnitude of the prestressing force at a distance x from the stretching end is given as follows.

$$P_x = P_0 e^{-\eta x} \tag{2-2.5}$$

Here, $\eta x = \mu\alpha + kx$ denotes the total effect of friction and wobble. The plot of P_x gives the force variation diagram.

The initial part of the force variation diagram, up to length l_{set} is influenced by the setting of the anchorage block. Let the drop in the prestressing force at the stretching end be ΔP. The determination of ΔP and l_{set} are necessary to plot the force variation diagram including the effect of the setting of the anchorage block.

Considering the drop in the prestressing force and the effect of reverse friction, the magnitude of the prestressing force at a distance x from the stretching end is given as follows.

$$P_x' = (P_0 - \Delta P)e^{\eta' x} \tag{2-2.6}$$

Here, η' for reverse friction is analogous to η for friction and wobble.

At the end of the setting length ($x = l_{set}$), $P_x = P'_x$

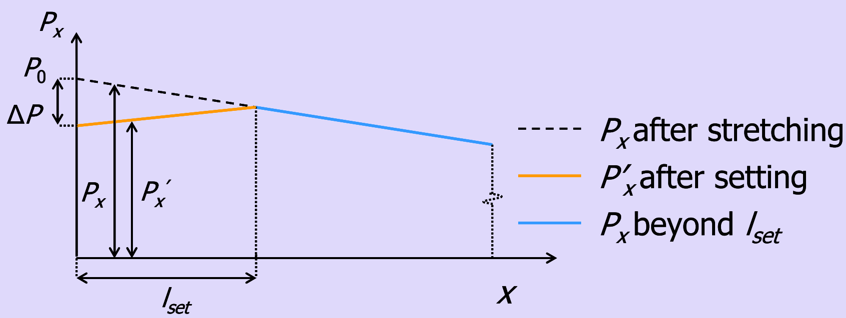


Figure 2-2.5 Force variation diagram near the stretching end

Substituting the expressions of P_x and P_x' for $x = l_{set}$

Since it is difficult to measure η' separately, η' is taken equal to η. The expression of ΔP simplifies to the following.

$$P_0 e^{-\eta l_{set}} = (P_0 - \Delta P)\, e^{\eta' l_{set}}$$

$$P_0 e^{-(\eta+\eta')l_{set}} = P_0 - \Delta P$$

$$P_0\left[1-(\eta+\eta')l_{set}\right] = P_0 - \Delta P$$

$$\Delta P = P_0(\eta+\eta')l_{set} = P_0 \eta l_{set}\left(1+\frac{\eta'}{\eta}\right) \qquad \textbf{(2-2.7)}$$

$$\Delta P = 2P_0\eta l_{set} \qquad \textbf{(2-2.8)}$$

The following equation relates l_{set} with the anchorage slip Δs.

$$\Delta_s = \frac{1}{2}\frac{\Delta P}{A_p E_p} l_{set}$$

$$\Delta_s = \frac{1}{2}\frac{l_{set}}{A_p E_p} P_0 \eta l_{set}\left(1+\frac{\eta'}{\eta}\right) \qquad \textbf{(2-2.9)}$$

Transposing the terms,

$$l_{set}^2 = \Delta_s \frac{2A_p E_p}{P_0 \eta\left(1+\dfrac{\eta'}{\eta}\right)}$$

$$= \frac{\Delta_s A_p E_p}{P_0 \eta} \qquad \text{for } \eta' = \eta$$

Therefore,

$$l_{set} = \sqrt{\frac{\Delta_s A_p E_p}{P_0 \eta}} \qquad \textbf{(2-2.10)}$$

The term $P_0\eta$ represents the loss of prestress per unit length due to friction.

The force variation diagram is used when stretching is done from both the ends. The tendons are overstressed to counter the drop due to anchorage slip. The stretching from both the ends can be done simultaneously or in stages. The final force variation is more uniform than the first stretching.

The following sketch explains the change in the force variation diagram due to stretching from both the ends in stages.

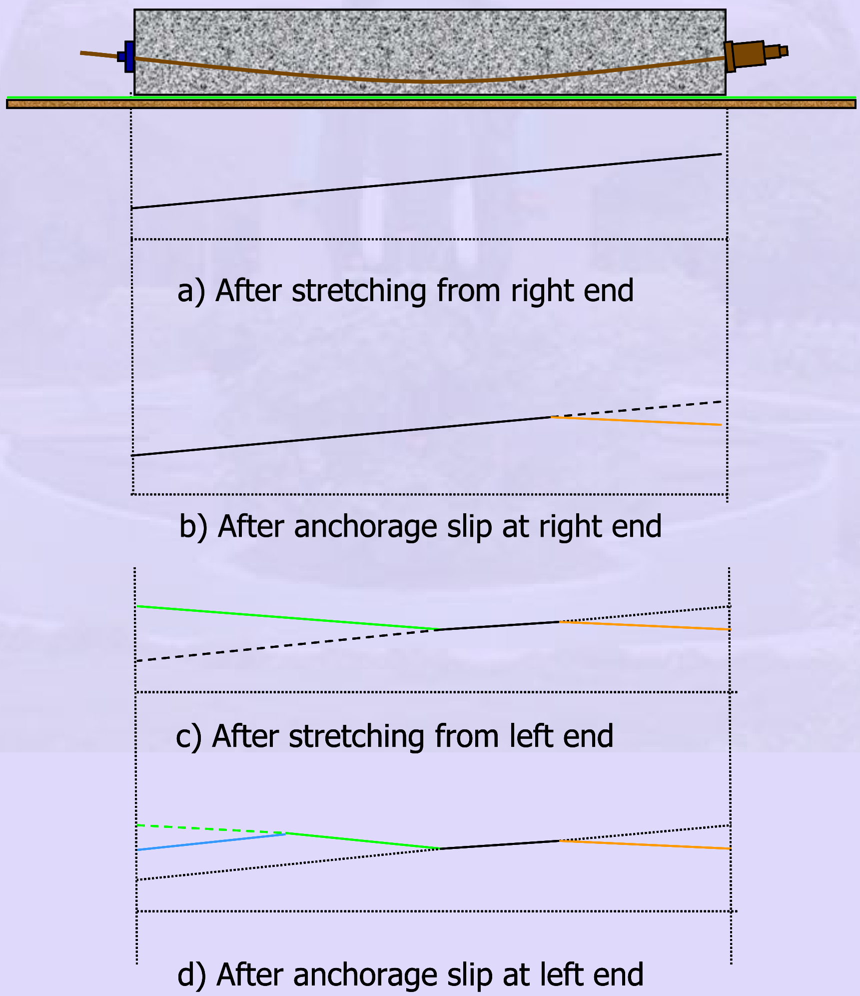


Figure 2-2.6 Force variation diagrams for stretching in stages

The force variation diagrams for the various stages are explained.

a) The initial tension at the right end is high to compensate for the anchorage slip. It corresponds to about 0.8 f_{pk} initial prestress. The force variation diagram (FVD) is linear.

b) After the anchorage slip, the FVD drops near the right end till the length l_{set}.

c) The initial tension at the left end also corresponds to about 0.8 f_{pk} initial prestress. The FVD is linear up to the centre line of the beam.

d) After the anchorage slip, the FVD drops near the left end till the length l_{set}. It is observed that after two stages, the variation of the prestressing force over the length of the beam is less than after the first stage.

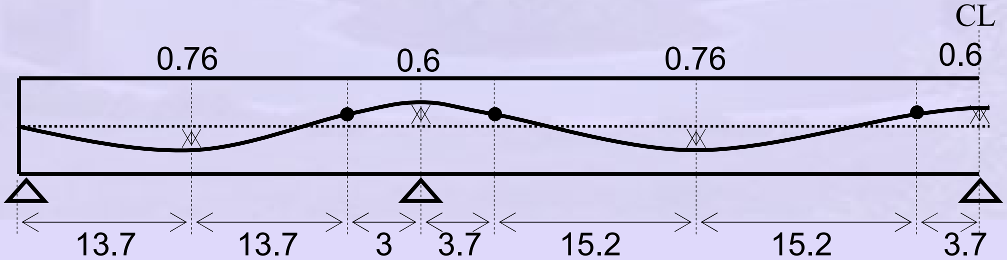
Example 2-2.2

A four span continuous bridge girder is post-tensioned with a tendon consisting of twenty strands with f_{pk} = 1860 MPa. Half of the girder is shown in the figure below. The symmetrical tendon is simultaneously stressed up to 75% f_{pk} from both ends and then anchored. The tendon properties are A_p = 2800 mm^2, E_p = 195,000 MPa, μ = 0.20, K = 0.0020/m. The anchorage slip Δ_s = 6 mm.

Calculate

a) The expected elongation of the tendon after stretching,

b) The force variation diagrams along the tendon before and after anchorage.



All dimensions are in metres

- Inflection points

Solution

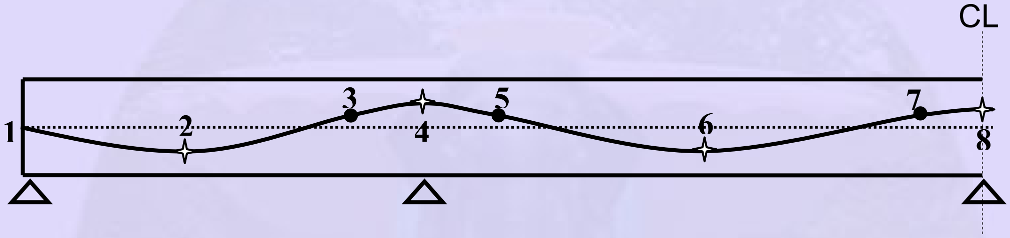
Initial force at stretching end

$$0.75f_{pk} = 1395 \text{ MPa}$$

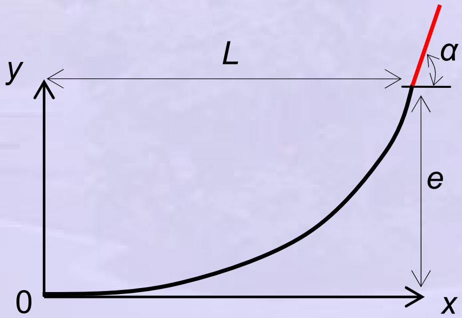
$$P_0 = 0.75f_{pk}\,A_p$$

$$= 3906 \text{ kN}$$

The continuous tendon is analysed as segments of parabola. The segments are identified between the points of maximum eccentricity and inflection points. The inflection points are those where the curvature of the tendon reverses. The different segments are as follows: 1-2, 2-3, 3-4, 4-5, 5-6, 6-7 and 7-8.



The following properties of parabolas are used. For segment 1-2, the parabola in the sketch below is used.

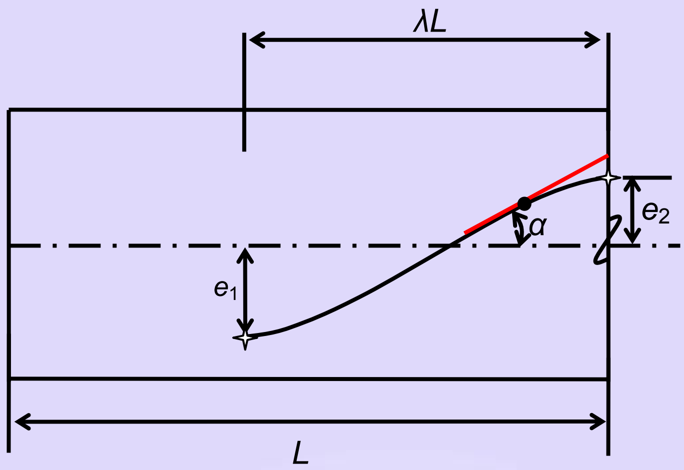


The change in slope from the origin to the end of the parabola is same as the slope at the end of the tendon which is $\alpha = 2e/L$, where

L = length of the segment

e = vertical shift from the origin.

For segments 2-3 and 3-4 and subsequent pairs of segments, the following property is used.



For the two parabolic segments joined at the inflection point as shown in the sketch above, the slope at the inflection point $\alpha = 2(e_1 + e_2)/\lambda L$.

Here,

e_1, e_2 = eccentricities of the CGS at the span and support respectively

L = length of the span

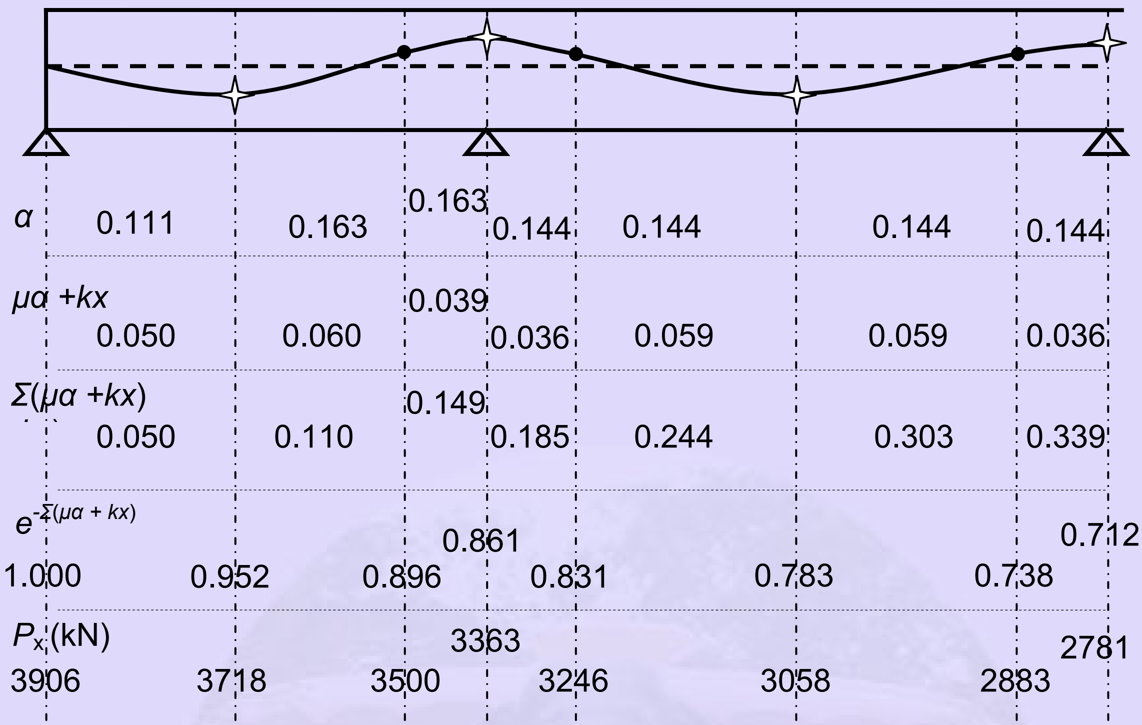
λL = fractional length between the points of maximum eccentricity

The change in slope between a point of maximum eccentricity and inflection point is also equal to α.

The change in slope (α) for each segment of the tendon is calculated using the above expressions. Next the value of $\mu\alpha + kx$ for each segment is calculated using the given values of μ, k and x, the horizontal length of the segment. Since the loss in prestress accrues with each segment, the force at a certain segment is given as follows.

$$P_x = P_0 e^{-\Sigma(\mu\alpha + kx)}$$

The summation $\sum$ is for the segments from the stretching end up to the point in the segment under consideration. Hence, the value of $\sum(\mu\alpha + kx)$ at the end of each segment is calculated to evaluate the prestressing force at that point (P_x, where x denotes the point).



The force variation diagram before anchorage can be plotted with the above values of P_x. A linear variation of the force can be assumed for each segment. Since the stretching is done at both the ends simultaneously, the diagram is symmetric about the central line.

a) The expected elongation of the tendon after stretching

First the product of the average force and the length of each segment is summed up to the centre line.

$$
\begin{aligned}
P_{av}L = &\frac{1}{2}[3906+3718]\times 13.7+\frac{1}{2}[3718+3500]\times 13.7\\
&+\frac{1}{2}[3500+3363]\times 3+\frac{1}{2}[3363+3246]\times 3.7\\
&+\frac{1}{2}[3246+3058]\times 15.2+\frac{1}{2}[3058+2883]\times 15.2\\
&+\frac{1}{2}[2883+2718]\times 3.7\\
&= 227612.2\ \text{kN}
\end{aligned}
$$

The elongation (Δ) at each stretching end is calculated as follows.

$$
\begin{aligned}
\Delta &= \frac{P_{av}L}{A_P E_P} \\
&= \frac{227612 \times 10^3}{2800 \times 195000} \\
&= 0.417\ \text{m}
\end{aligned}
$$

b) The force variation diagrams along the tendon before and after anchorage

After anchorage, the effect of anchorage slip is present up to the setting length l_{set}. The value of l_{set} due to an anchorage slip Δ_s = 6 mm is calculated as follows.

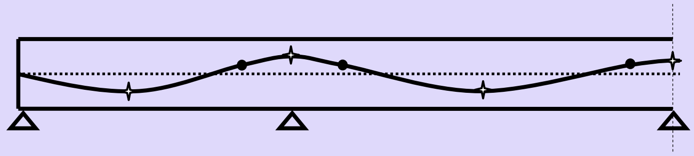
$$
\begin{aligned}
l_{set} &= \sqrt{\frac{\Delta_s A_P E_P}{P_0 \mu}} \\
&= \sqrt{\frac{6 \times 2800 \times 195000}{13.7}} \\
&= 15.46\ \text{m}
\end{aligned}
$$

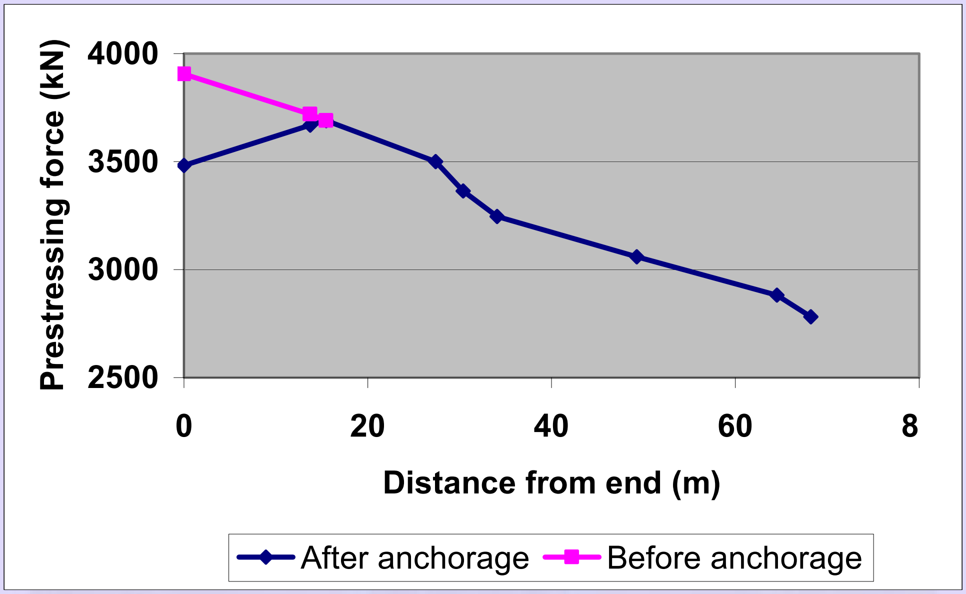
The quantity $P_0\mu$ is calculated from the loss of prestress per unit length in the first segment. $P_0\mu$ = (3906 – 3718) kN /13.7 m = 13.7 N/mm. The drop in the prestressing force (Δ_p) at each stretching end is calculated as follows.

$$
\begin{aligned}
\Delta_p &= 2P_0 \mu l_{set} \\
&= 2 \times 13.7 \times 15464 \\
&= 423.7\ \text{kN}
\end{aligned}
$$

Thus the value of the prestressing force at each stretching end after anchorage slip is 3906 – 424 = 3482 kN. The force variation diagram for l_{set} = 15.46 m is altered to show the drop due to anchorage slip.

The force variation diagrams before and after anchorage are shown below. Note that the drop of force per unit length is more over the supports due to change in curvature over a small distance.




4000
3500
3000
2500
Prestressing force (kN)
0
20
40
60
8
Distance from end (m)
After anchorage
Before anchorage

2.3 Losses in Prestress (Part III)

This section covers the following topics.

- Creep of Concrete
- Shrinkage of Concrete
- Relaxation of Steel
- Total Time Dependent Losses

2.3.1 Creep of Concrete

Creep of concrete is defined as the increase in deformation with time under constant load. Due to the creep of concrete, the prestress in the tendon is reduced with time.

The creep of concrete is explained in Section 1.6, Concrete (Part II). Here, the information is summarised. For stress in concrete less than one-third of the characteristic strength, the ultimate creep strain ($\varepsilon_{cr,ult}$) is found to be proportional to the elastic strain (ε_{el}). The ratio of the ultimate creep strain to the elastic strain is defined as the ultimate creep coefficient or simply creep coefficient θ.

The ultimate creep strain is then given as follows.

$$\varepsilon_{cr,ult} = \theta\varepsilon_{el} \qquad \textbf{(2-3.1)}$$

IS:1343 - 1980 gives guidelines to estimate the ultimate creep strain in **Section 5.2.5**. It is a simplified estimate where only one factor has been considered. The factor is age of loading of the prestressed concrete structure. The creep coefficient θ is provided for three values of age of loading.

Curing the concrete adequately and delaying the application of load provide long term benefits with regards to durability, loss of prestress and deflection. In special situations detailed calculations may be necessary to monitor creep strain with time. Specialised literature or international codes can provide guidelines for such calculations.

The loss in prestress (Δf_p) due to creep is given as follows.

$$\Delta f_p = E_p\, \varepsilon_{cr,\, ult} \qquad \textbf{(2-3.2)}$$

Here, E_p is the modulus of the prestressing steel.

The following considerations are applicable for calculating the loss of prestress due to creep.

1) The creep is due to the sustained (permanently applied) loads only. Temporary loads are not considered in the calculation of creep.
2) Since the prestress may vary along the length of the member, an average value of the prestress can be considered.
3) The prestress changes due to creep and the creep is related to the instantaneous prestress. To consider this interaction, the calculation of creep can be iterated over small time steps.

2.3.2 Shrinkage of Concrete

Shrinkage of concrete is defined as the contraction due to loss of moisture. Due to the shrinkage of concrete, the prestress in the tendon is reduced with time. The shrinkage of concrete was explained in details in the Section 1.6, Concrete (Part II).

IS:1343 - 1980 gives guidelines to estimate the shrinkage strain in **Section 5.2.4**. It is a simplified estimate of the ultimate shrinkage strain (ε_{sh}). Curing the concrete adequately and delaying the application of load provide long term benefits with regards to durability and loss of prestress. In special situations detailed calculations may be necessary to monitor shrinkage strain with time. Specialised literature or international codes can provide guidelines for such calculations.

The loss in prestress (Δf_p) due to shrinkage is given as follows.

$$\Delta f_p = E_p \varepsilon_{sh} \quad \textbf{(2-3.3)}$$

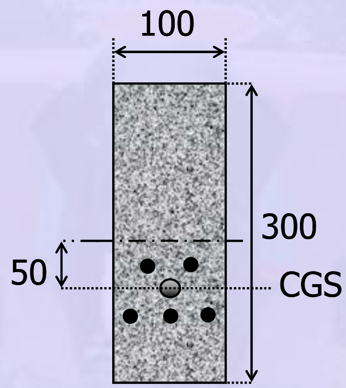
Here, E_p is the modulus of the prestressing steel.

2.3.3 Relaxation of Steel

Relaxation of steel is defined as the decrease in stress with time under constant strain. Due to the relaxation of steel, the prestress in the tendon is reduced with time. The relaxation depends on the type of steel, initial prestress (f_{pi}) and the temperature. To calculate the drop (or loss) in prestress (Δf_p), the recommendations of **IS:1343 - 1980** can be followed in absence of test data.

Example 2-3.1

A concrete beam of dimension 100 mm × 300 mm is post-tensioned with 5 straight wires of 7mm diameter. The average prestress after short-term losses is 0.7f_{pk} = 1200 N/mm^2 and the age of loading is given as 28 days. Given that E_p = 200 × 103 MPa, E_c = 35000 MPa, find out the losses of prestress due to creep, shrinkage and relaxation. Neglect the weight of the beam in the computation of the stresses.



Solution

Area of concrete A = 100 × 300

= 30000 mm^2

Moment of inertia of beam section

I = 100 × 300^3 / 12

= 225 × 10^6 mm^4

Area of prestressing wires A_p = 5 × (π/4) × 7^2

= 192.42 mm^2

Prestressing force after short-term losses

$P_0 = A_p.f_{p0}$

= 192.4 × 1200

= 230880 N

Modular ratio $m = E_p / E_c$

$= 2 \times 10^5 / 35 \times 10^3$

$= 5.71$

Stress in concrete at the level of CGS

$$f_c = -\frac{P_0}{A} - \frac{P_0 e}{I} e$$

$$= -\frac{230880}{3 \times 10^4} - \frac{230880}{225 \times 10^6} \times 50^2$$

$= -7.69 - 2.56$

$= -10.25 \text{ N/mm}^2$

Loss of prestress due to creep

$(\Delta f_p)_{cr} = E_p\, \varepsilon_{cr,\, ult}$

$= E_p\, \theta \varepsilon_{el}$

$= E_p\, \theta\, (f_c/E_c)$

$= m\, \theta\, f_c$

$= 5.71 \times 10.25 \times 1.6$

$= 93.64 \text{ N / mm}^2$

Here, θ = 1.6 for loading at 28 days, from **Table 2c-1 (Clause 5.2.5.1, IS:1343 - 1980).**

Shrinkage strain from **Clause 5.2.4.1, IS:1343 - 1980**

$\varepsilon_{sh} = 0.0002 / \log_{10}(t + 2)$

$= 0.0002 / \log_{10} (28 + 2)$

$= 1.354 \times 10^{-4}$

Loss of prestress due to shrinkage

$(\Delta f_p)_{sh} = E_p \varepsilon_{sh}$

$= 2 \times 10^5 \times 1.354 \times 10^{-4}$

$= 27.08 \text{ N/mm}^2$

From **Table 2c-2 (Table 4, IS:1343 - 1980)**

Loss of prestress due to relaxation

$$(\Delta f_p)_{rl} = 70.0 \text{ N/mm}^2$$

Loss of prestressing force $= \Delta f_p A_p$

Therefore,

Loss of prestressing force due to creep $= 93.64 \times 192.42$

$= 18018$ N

Loss of prestressing force due to shrinkage

$= 27.08 \times 192.42$

$= 5211$ N

Loss of prestressing force due to relaxation

$= 70 \times 192.42$

$= 13469$ N

Total long-term loss of prestressing force (neglecting the interaction of the losses and prestressing force)

$= 18018 + 5211 + 13469$

$= 36698$ N

Percentage loss of prestress $= 36698 / 230880 \times 100\%$

$= 15.9$ %

2.3.4 Total Time-dependent Loss

The losses of prestress due to creep and shrinkage of concrete and the relaxation of the steel are all time-dependent and inter-related to each other. If the losses are calculated separately and added, the calculated total time-dependent loss is over-estimated. To consider the inter-relationship of the cause and effect, the calculation can be done for discrete time steps. The results at the end of each time step are used for the next time step. This step-by-step procedure was suggested by the Precast / Prestressed

Concrete Institute (PCI) committee and is called the **General method** (Reference: *PCI Committee, "Recommendations for Estimating Prestress Losses", PCI Journal, PCI, Vol. 20, No. 4, July-August 1975, pp. 43-75*).

In the PCI step-by-step procedure, a minimum of four time steps are considered in the service life of a prestressed member. The following table provides the definitions of the time steps (Table 2-3.3).

Table 2-3.3 Time steps in the step-by-step procedure

Step	Beginning	End
1	Pre-tension: Anchorage of steel Post-tension: End of curing	Age of prestressing
2	End of Step 1	30 days after prestressing or when subjected to superimposed load
3	End of Step 2	1 year of service
4	End of Step 3	End of service life

The step-by-step procedure can be implemented by a computer program, where the number of time steps can be increased.

There are also approximate methods to calculate lump sum estimates of the total loss. Since these estimates are not given in **IS:1343 - 1980**, they are not mentioned here.