BL Theory

- Hydraulics studies the phenomenon that can be observed but not proved.
- Hydrodynamics (mathematical counterpart of hydraulics) deals with phenomenon that can be proved but not observed.

Ludwig Prandtl → when a fluid flows around an object or when the object moves through a body of fluid there exists a thin layer of fluid close to the solid surface within which shear stresses significantly influence the velocity distribution.

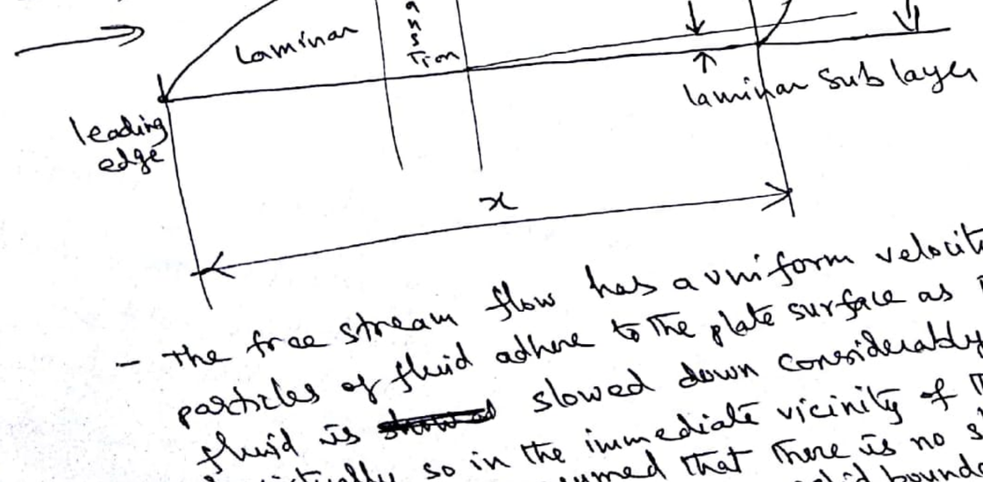
The fluid velocity varies from zero at the solid surface to the velocity of free stream flow at a certain distance away from the solid surface. This layer of stagnated fluid has been called the hydrodynamic boundary layer.

Outside the boundary layer, the flow behaves as it would for an ideal fluid. ~~and, therefore~~, ~~the solution can be sought by the~~

The concept of boundary layer eventually permitted analysis of lifting vanes, control surfaces and propellers.

Indeed, the enunciation of boundary layer theory provided the key that unlocked the door for much of the progress during this century in the fields of both fluid mechanics and convective heat transfer.

Flat plate:



- The free stream flow has a uniform velocity u_0 in the x-direction. The particles of fluid adhere to the plate surface as they approach it and the fluid is ~~slowed~~ slowed down considerably. The fluid becomes stagnant or virtually so in the immediate vicinity of the plate surface, it is presumed that there is no slip b/w the fluid and the solid boundary.

Thus there exists a region where the flow velocity changes from that solid boundary to that of the mainstream fluid, and in this region the velocity gradient exists in the fluid. consequently the flow is rotational and shear stresses are present. This region of changing velocity has been called the hydrodynamic boundary layer suggested by pradtt

ii) the condition $\frac{\partial u}{\partial y} \neq 0$ is true for ~~zone~~ within the boundary layer whilst the conditions for flow beyond the boundary layer and at its outer border are, $\frac{\partial u}{\partial y} = 0$; $u = U_0$

Thus all the variation in the fluid velocity is concentrated in a comparatively thin layer in immediate vicinity of the plate surface.

iii) ~~For some distance fr~~ The concepts of boundary layer thickness and outer edge of the boundary layer are quite ficticious as there is no abrupt transition from the boundary layer to the flow beyond or outside it.

velocity within the boundary layer approaches the free stream velocity usually the BL thickness δ is taken to the distance from the plate surface to a point at which the velocity $u = 0.99\, U_0$.

δ then becomes a nominal measure of the thickness of the boundary layer ie region in which the major portion of the velocity deformation takes place.

The thickness is measured normal to the plate surface.

iv) The thickness of the boundary layer is variable along the flow direction; it is zero at leading edge of the plate and increases as the distance 'x' from the leading edge is increased. This aspect may be attributed to the viscous forces which dissipate more and more energy of fluid stream as the flow proceeds.

The boundary layer growth is also governed by other parameters such as magnitude of the incoming velocity and kinematic viscosity of flowing fluid. For higher incoming velocities, there would be less time for viscous forces to act and accordingly there would be less quantum of boundary layer thickness at a particular distance from the leading edge. Further the BL thickness is greater for the fluids with greater kinematic viscosity.

Lamin

Laminar & Turbulent flows:

Reynold's experiment:

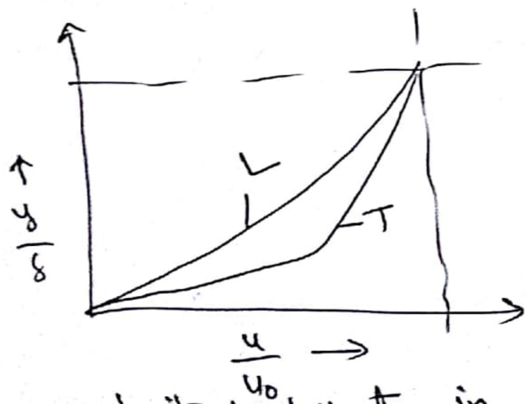
- For some distance from the leading edge, the boundary layer is laminar and the velocity profile is parabolic. Flow within the laminar boundary layer is smooth and the streamlines are essentially parallel to the plate. Laminar BL becomes unstable and the laminar flow undergoes a change in its flow structure at a certain point, called transition point in the flow field. After going through a transition zone of finite length, the boundary layer entirely changes to turbulent BL.

- The turbulent BL does not extend the solid surface. Underlying it, an extremely thin layer called laminar sublayer, is formed where in the flow is essentially of laminar character.

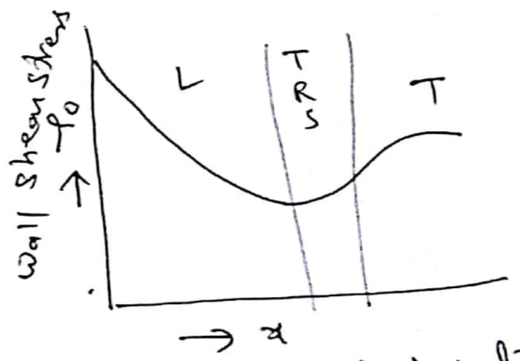
- The pattern of flow in the boundary layer is judged by the Renolds number

$$Re = \frac{x\,U_0}{\nu}$$

$Re = 3 \times 10^5$ to 5×10^5 Transition zone.



velocity distribution in laminar & turbulent BL on a flat plate.



shear stress distribution on either side of plate

BL parameters :

Boundary layer Thickness (δ)

distance 'δ' at which the velocity $u = 0.99 U_0$ becomes nominal
measure of the BLt

Displacement thickness (δ^*)

the Thickness of flow (measured $\perp r$ to the Boundary of the Solid) moving at the free stream velocity and having flow rate equal to the loss in flowrate on account of the boundary layer formation.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_0}\right) dy$$

Momentum thickness : (θ)

the Thickness of flow moving at the free stream velocity and having momentum flux equal to the deficiency of momentum flux in the region of boundary layer.

$$\theta = \int_0^\delta \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right) dy$$

Energy thickness : δ^{**}

the Thickness of flow moving at the free stream velocity and having the energy equal to deficiency of energy in the boundary layer region

~~Shape factors~~

prandtl BL equations :

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \quad \text{or} \quad \frac{\partial p}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

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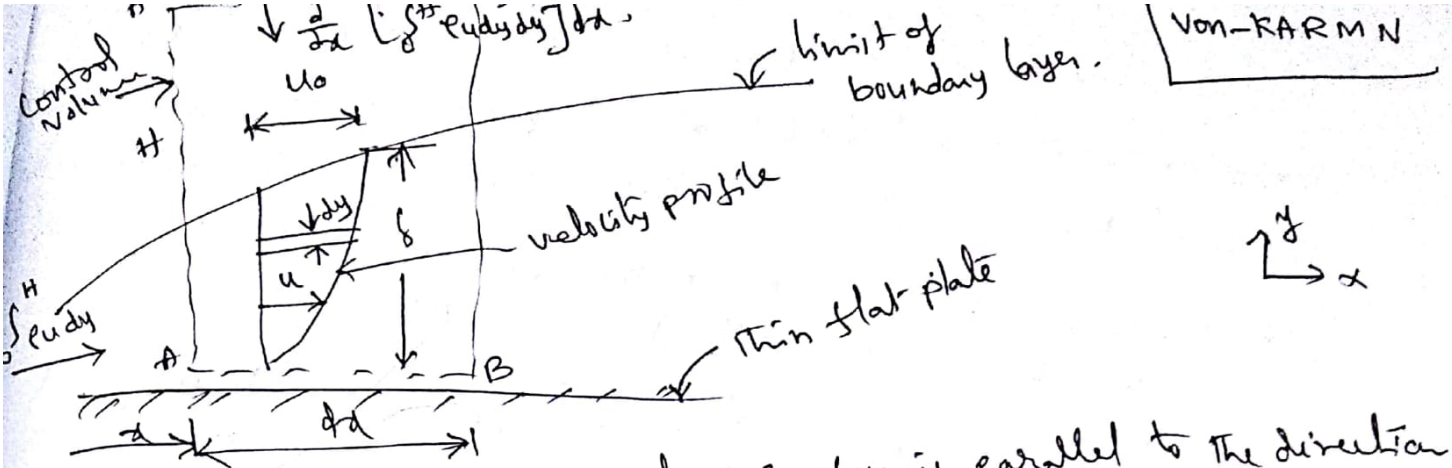
Von-KARMN



- select a thin flat plate whose surface is parallel to the direction of free stream velocity :
- Consider a differential element of boundary layer located at a distance x and $(x + dx)$ from the the leading edge.
- the control volume is sufficiently high and it encloses the edge of boundary layer i.e $H > \delta$.

For unit width of plate,

the flow rate entering through AA' is, $m = \int_0^H \rho u dy$

~~leaving~~ " leaving $BB' =$

$$m + \frac{\partial m}{\partial x} dx = \int_0^H \rho u dy + \frac{\partial}{\partial x}\left[\int_0^H \rho u dy\right] dx.$$

No mass can enter the control volume through its solid wall AB,

the continuity requirement then stipulates that the mass increment $\frac{\partial}{\partial x}\left[\int_0^H \rho u dy\right] dx$ must represent the mass flow that enters the control volume through plane $A'B'$ with free stream velocity u_0.

The corresponding x-momentum fluxes are.

momentum through ~~[illegible]~~ $AA' = \int_0^H \rho u^2 dy$

" " $BB' = \int_0^H \rho u^2 dy + \frac{\partial}{\partial x}\left[\int_0^H \rho u^2 dy\right] dx$

" upper face $A'B' = u_0 \frac{\partial}{\partial x}\left[\int_0^H \rho u dy\right] dx.$

In the absence of any pressure and gravity forces, the ~~drag~~ shear force ($\tau_0 dx$) at the plate surface must be balanced by the net momentum change for the control vol.

(There is no shear force at the upper face which is outside the boundary layer)

$$\therefore \tau_0 dx = u_0 \frac{\partial}{\partial x}\left[\int_0^H \rho u\,dy\right]dx - \frac{\partial}{\partial x}\left[\int_0^H \rho u^2 dy\right]dx$$

$y > \delta \Rightarrow$ at $u = u_0 = \text{const}$ for $y > \delta$ ie $u_0 - u$ is zero

$$= u_0 \frac{\partial}{\partial x}\left[\int_0^\delta \rho u\,dy\right]dx - \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 dy\right]dx$$

$$\therefore \tau_0 = \frac{\partial}{\partial x}\int_0^\delta \rho\,(u_0 - u)\,u\,dy$$

$$= \rho u_0^2 \frac{\partial}{\partial x}\left[\int_0^\delta \left(1 - \frac{u}{u_0}\right)\frac{u}{u_0}\,dy\right] = \rho u_0^2 \frac{\partial \theta}{\partial x}$$

where θ = momentum thickness

$$\boxed{\frac{\tau_0}{\frac{1}{2}\rho u_0^2} = 2\frac{\partial \theta}{\partial x}}$$

$\leftarrow$ von-Karman integral eqn for hydrodynamic boundary

Laminar Boundary layer;

- For laminar boundary layer, the velocity profile is parabolic
- velocity profile at different locations along the plate are geometrically similar.
- This means that the dimensionless velocity $\frac{u}{u_0}$ can be expressed at any location x as a function of the dimensionless distance from the wall, y/δ.

$$\frac{u}{u_0} = f\left(\frac{y}{\delta}\right) = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2 + d\left(\frac{y}{\delta}\right)^3$$

The constants can be evaluated subject to the following compatibility (boundary) conditions:

ie At the wall surface ($y=0$); $u=0$ and $\frac{d^2u}{dy^2}=0$

outer edge ($y=\delta$); $u=u_0$ & $\frac{du}{dy}=0$

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Examine whether or not the following velocity profiles satisfy the essential boundary conditions for velocity distribution in laminar boundary layer on a flat plate.

a) $\frac{u}{u_0} = 1 + \left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2$

b) $\frac{u}{u_0}$ sin $\left(\frac{\pi y}{2\delta}\right)$

where u is velocity at height y above the surface, u_0 is the free stream velocity and δ is the nominal Boundary layer thickness.

Sol: BC's are $u = 0,\ \frac{\partial^2 u}{\partial y^2}$ at $y = 0$

$u = u_0$ at $y = \delta$

a) $\frac{u}{u_0} = 1 + \left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2$

$\frac{\partial u}{\partial y} = u_0 \left[\frac{1}{\delta} - \frac{4y}{\delta^2}\right]$

$\frac{\partial^2 u}{\partial y^2} = u_0\left(-\frac{4}{\delta^2}\right)$

At $y = 0$; $u = u_0 \neq 0$; $\frac{\partial^2 u}{\partial y^2} = \frac{-4u_0}{\delta^2} \neq 0$

At $y = \delta$ $u = 0 \neq u_0$

BC's are not satisfied; hence the given velocity profile is not a proper distribution in a laminar boundary layer.

b) $\frac{u}{u_0} = \sin\left(\frac{\pi y}{2\delta}\right)$

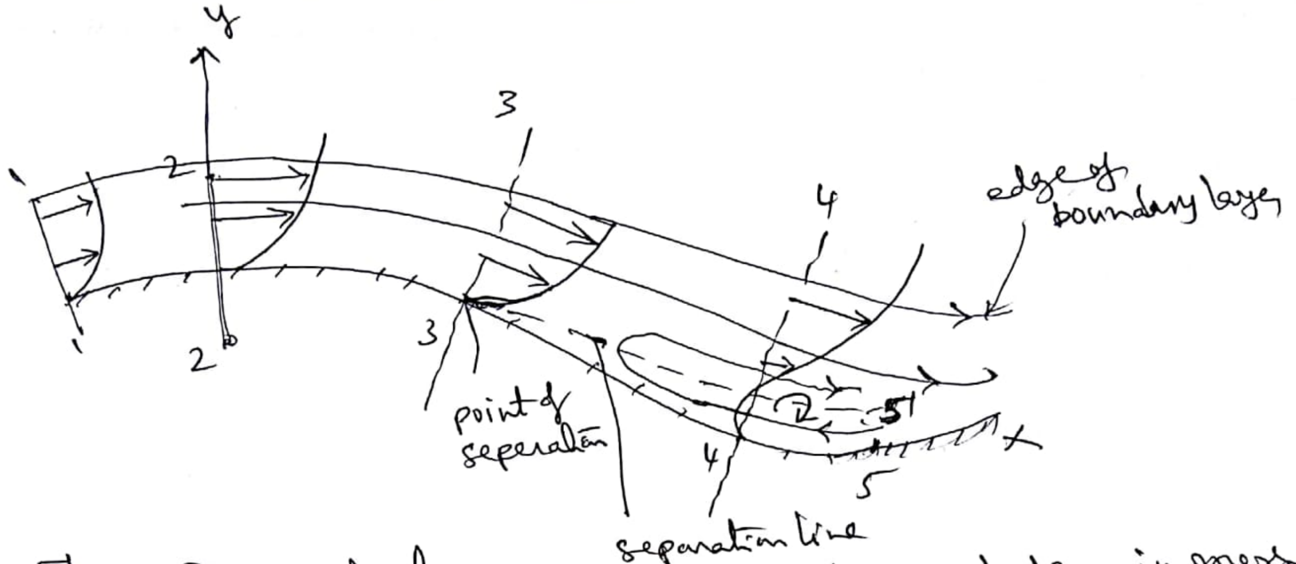
$\frac{\partial u}{\partial y} = u_0 \frac{\pi}{2\delta} \cos\left(\frac{\pi y}{2\delta}\right)$; $\frac{\partial^2 u}{\partial y^2} = \frac{-\pi^2 u_0}{4\delta^2}\sin\left(\frac{\pi y}{2\delta}\right)$

at $y = 0$; $u = u_0$ and $\frac{\partial^2 u}{\partial y^2} = 0$

At $y = \delta$ $u = u_0 \sin\frac{\pi}{2}$ ', $= u_0$

Hence BC's are satisfied.

Boundary Layer Separation :



- There is gradual increase in velocity and drop in pressure as flow proceeds from section 1-1 to section 2-2.
- Elements of the BL are drawn forward with pressure force behind them. This serves to diminish the effectiveness of viscous action in slowing down the fluid near the surface.
- The flow decelerates and pressure increases as flow proceeds downstream from section 2-2. pressure growth and shear forces act together to bring about a continuous reduction in the flow momentum. If these effects persist over a sufficient distance, the retarded fluid mass is unable to proceed downstream in the region of increased pressure.
- A separation line 3-5' is obtained when all the zero velocity points are joined by a smooth curve. Above the separation line fluid moves in the forward direction and below it the fluid moves in the opposite. Because of flow reversal, large irregular eddies are formed and a considerable amount of energy is dissipated as heat.
- Position of point of separation depends on the roughness of surface, Re, geometry of curved surface and the nature of boundary layer.
- Separation occurs with both laminar & turbulent boundary layers. However, turbulent BL resist separation better than do the laminar BL's. This stems from fact that turbulent boundary layers possess more energy than the corresponding laminar ones against normal to the flow.

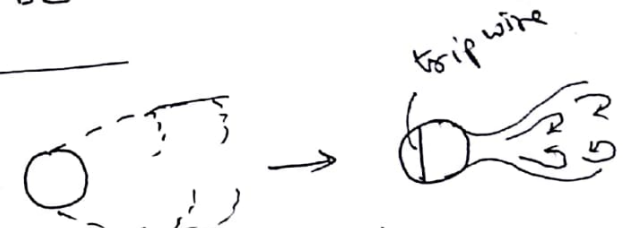
$\left.\frac{\partial u}{\partial y}\right|_{y=0}$ is +ve attached flow

$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$ flow is at the verge of separation

$\left.\frac{\partial u}{\partial y}\right|_{y=0} = -ve$ separated flow

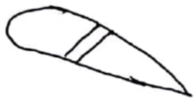
<u>methods to control separation of BL</u>

- stream line design bodies →

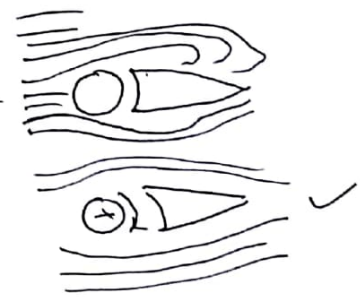


- aerofoil sections

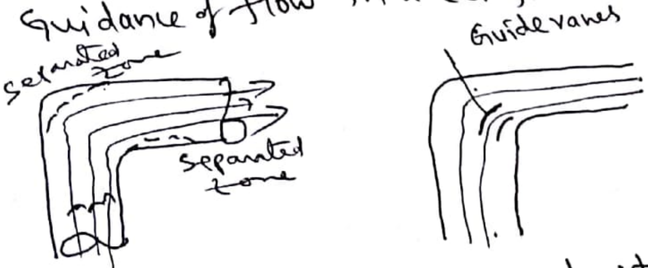
- creating narrow holes / slots
- slotted wings



- rotation of cylinder at the leading edge



- Guidance of flow in a confined passage



(2) Examine the following velocity profiles to state whether the flow is attached or detached.

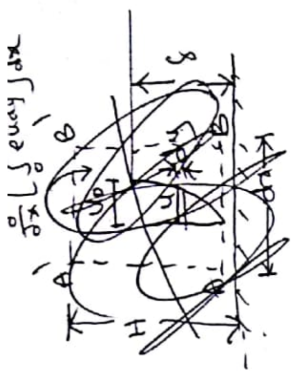
(1) $\frac{u}{u_0} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ (2) $\frac{u}{u_0} = 2\left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^3 - 2\left(\frac{y}{\delta}\right)^4$

Ans: $\frac{du}{dy} = \frac{2}{\delta} - 2\left(\frac{y}{\delta}\right)\frac{1}{\delta} = \frac{2}{\delta}$ at $y=0$ → +ve attached

(2) Ans: $\frac{du}{dy} = -\frac{2}{\delta} + 3\left(\frac{y}{\delta}\right)^2 \times \frac{1}{\delta} + 8\left(\frac{y}{\delta}\right)^3 \times \frac{1}{\delta} = -\frac{2}{\delta}$ at $y=0$

-ve separated

Flow around submerged objects :



- Design of vehicles such as aircrafts, trains and automobiles that move in the atmospheric environment, and the design of submarine and torpedoes that are completely immersed in water
- Design of buildings, bridges and other structures where the whole or a part of the structure is immersed in air or water.

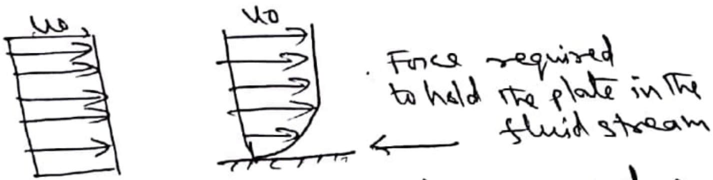
These units have to be designed to withstand the dynamic forces imposed on them by the surrounding fluid

- The resistance due to a body moving through a large mass of stationary fluid, or due to a fluid flowing around a stationary submerged body is of great technical importance in the field of hydrodynamics and aerodynamics

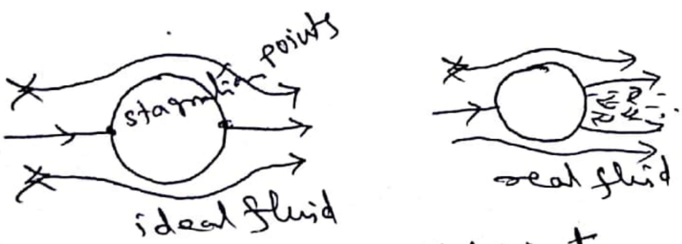
laminar

- deals with pressure and shear stress distribution around the body
- separation zones
- resultant drag and lift on the body

Drag force :



Drag is an energy loss and for a body immersed in a fluid stream its value depends upon the shape and orientation of body. ie force required to balance the resistance offered by the fluid, is called as drag force.



Lift and drag coefficients

When a body is either unsymmetrical or has its axis not aligned with the flow direction, both the pressure and shear forces exist. Their combined effect yields a resultant force F which can be resolved into lift and drag forces.

- Lift is the component of resultant force which acts normal to the free stream direction

- Drag is the component of resultant force which acts parallel to the free stream direction.

Lift coefficient $C_L = \dfrac{F_L}{\frac{1}{2} \rho V_0^2 \times A}$

Drag coefficient $C_D = \dfrac{F_D}{\frac{1}{2} \rho V_0^2 A}$

(1) A flat plate, 1m x 1m, moves at 6.5 m/s normal to its plane. Compute the resistance of the plate when the surrounding fluid is (i) air with mass density 1.2 kg/m³ (ii) water with mass density 1000 kg/m³.
Assume $C_D = 1.15$ for both.

sol: The resistance of drag force, $F_D = C_D \times \frac{1}{2} \rho V_0^2 \times A$

(i) $F_D = 1.15 \times \frac{1}{2} \times 1.2 \times 6.5^2 \times (1 \times 1) = 29.13\,N$

(ii) $F_D = 1.15 \times \frac{1}{2} \times 1000 \times 6.5^2 \times (1 \times 1) = 24293.75\,N$

(5) A passenger car with frontal projected area of $1.5\,m^2$ travels at 56 Km/hr. Determine the power required to overcome wind resistance. If the drag coefficient of the car is 0.4. For the same power expended in overcoming resistance, what percentage change in speed of the car is possible if drag coefficient is reduced to 0.32 by streamlining the car body?. Take density of air $\rho = 1.2\,kg/m^3$

Sol: speed of car, $U_0 = 56\,Kmph = \frac{56 \times 1000}{3600} = 15.56\,m/s$

(i) Drag force $F_D = C_D \times \frac{1}{2}\rho U_0^2 \times A = 0.4\left(\frac{1}{2} \times 1.2 \times 15.56^2\right)1.5$

$= 87.16\,N$

power $= F_D \times U_0 = 87.16 \times 15.56 = 1356\,W = 1.356\,KW$

(ii) let U be the speed of streamlined car,

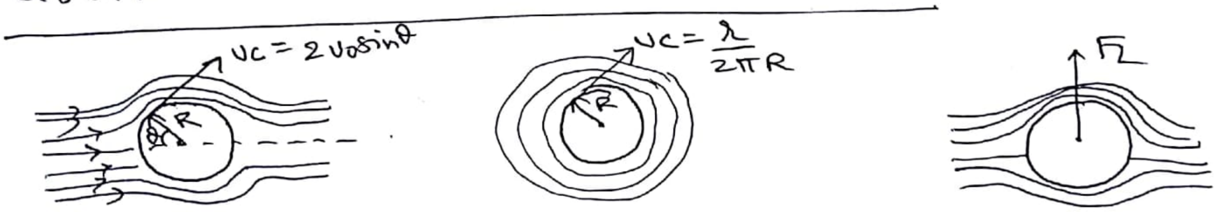
power $= \left(C_D \times \frac{1}{2}\rho U^2 \times A\right)U = C_D \times \frac{1}{2}\rho U^3 \times A$

$\therefore 1356 = 0.32 \times \frac{1}{2}\, 1.2 \times U^3 \times 1.5 = 0.288\,U^3$

$U = \frac{1356}{0.288} = 16.17\,m/s = 58.21\,Km/hr$

% increase in speed $= \frac{58.21 - 56}{56} = 3.95$

circulation and lift on a circular cylinder:



When an ideal fluid with an uniform velocity U_0 flows past a circular cylinder the circumferential velocity at any point on the cylinder is given as $V_c = 2U_0 \sin\theta$

- The magnitude of the transverse force i.e lift may be changed by altering either the speed of rotation or the stream velocity. ~~the general~~
- The generation of lift by spinning cylinder in a fluid stream is

* magnus effect

- This aspect has been successfully employed in the propulsion of ships by turning round and round the ship propeller by means of electric power. The ship gets propelled in the flowing water or wind with a force equal to the lift force.