Unit - IV

Laminar Flow and Turbulent Flow

= 35 lit/s
35 × (1000 cm³)/s
1 m³ = 1000 lt

→ laminar flow:-

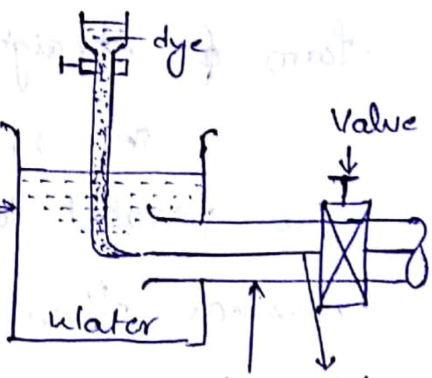
fluid Particles moves along defined path.

Turbulent flow:-

fluid particles moves in a zig-zag way.

→ Reynolds Experiment:

The type of flow is determined from the Reynolds no i.e., $\frac{\rho v \times d}{\mu}$



1. laminar flow.
2. Transion flow.
3. Turbulant flow.

} Study. measure flow loss energy

The Apparatus Consists of

1. A tank Containing water at Consistant head.
2. A Small tank Containing Some dye.
3. A Small glass tube having a bell-mounted entrance one end rectangular other end.

The water is made to flow from the tank through the glass tube into the atmosphere and the Velocity of flow was varied by Adjusting the rectangular valve.

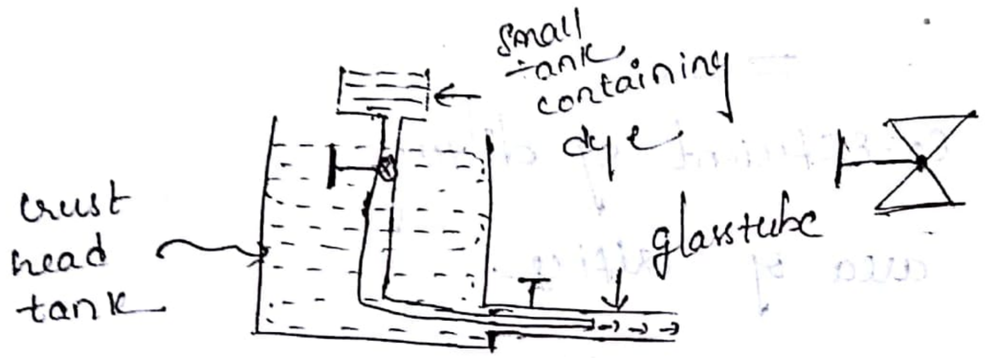
A liquid die have same specific weight as water was introduce into glass tube.

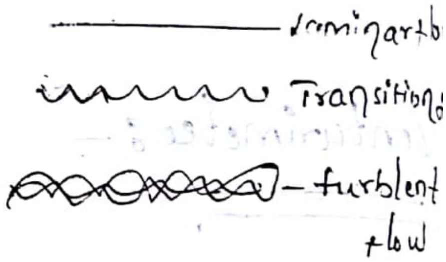
UNIT-4

Laminar flow & turblent flow

Flow Regimes

Renold's Experiment:-





mathematically

$$Re\ (\text{Renold Number}) = \frac{\rho V d}{\mu}$$

velocity, density

μ → dynamic viscoity

$R_e < 2000 \rightarrow$ laminar, more than 4000 → turblent flow

$v = \sqrt{2gH}$

Darcy - weisbach Equation:-

$$h_f = \frac{4fLV^2}{2gD}$$

head loss due to friction

f = co-Efficient of Darcy Equation

(or)

Darcy - weisbach friction factor

L = Length of pipe

V = Average velocity ..... m/s

D = Diameter of pipe

losses

major losses – due to friction

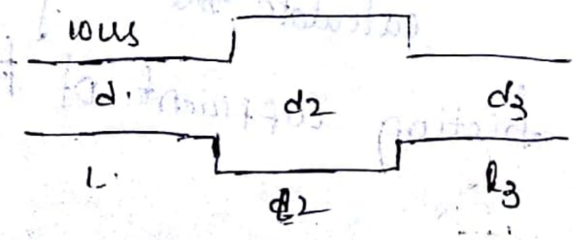
minor losses – Expansion of the pipe line pipe,

Bend, Sudden Contraction / Exit, Entry

$\frac{V^2}{2g}$ $\qquad$ $\frac{1}{2}\frac{V^2}{2g}$ $\qquad$ $0.5\frac{V_1^2}{2g}$

(compound pipe)

pipes in series & pipes in parallel:-



Q = constant, i.e. $Q_1 = Q_2 = Q_3 = Q$

$h_f = h_{f1} + h_{f2} + h_{f3}$

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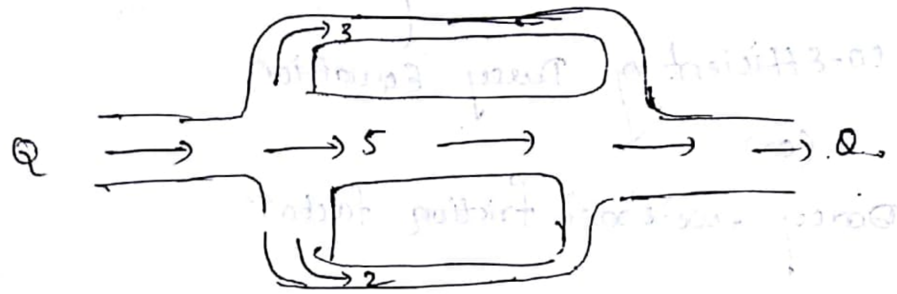
$$= \frac{4f_1 l_1 v_1^2}{2gd_1} + \frac{4f_2 l_2 v_2^2}{2gd_2}$$

pipes in parallels:-



$$Q = Q_1 + Q_2 + Q_3$$

$$h_f = \text{constant}$$

Equivalent pipe: -

$$\frac{\text{length of E.P}}{\text{Diameter of Equival } D_e^5} = \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \cdots\right]$$

problem

1. Two reservoirs are connected by three pipes laid by parallel, the pipe diameters are 10cms 20cms 30cms and they are same length. Same if distance to 20cm pipe is calculate the larger pipe (2) & (3) pipes assume friction cofficient of pipes 'f'

Sol:- given

Condition, h_f = Constant $= \frac{4fLV^2}{2gd}$

$Q_1 + Q_2 + Q_3 = Q.$

$$\frac{4f_1 l_1 V_1^2}{2gd_1} = \frac{4f_2 l_2 V_2^2}{2gd_2} = \frac{4f_3 l_3 V_3^2}{2gd_3}$$

10cm $\Leftarrow Q = 0.1\ m^3/s$

20cm $= A_1V_1 = \frac{\pi d_1^2}{4} \times V_1$

30cm $0.1 = \pi/4 \times (0.1)^2 \times V_1$

L = const $V_1 = 0.1$

$Q_2 = 0.566\ m^3/s$ ⇒

$Q_3 = 1.54\ m^3/s$

(2) A pipe in System Consists of three pipes arranged in Series

pipe AB	length 2000 m l_1	Dia 40cm d_1
BC	1500 m l_2	30cm d_2
CD	1000m l_3	20cm, d_3

transform the System to

(1) Equivalent length of 300 cm.

(2) $D_e = ?$ $L_e = 4500m$.

$$\frac{L_e}{D_e^5} = \left[\frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}\right]$$

$$\frac{L_e}{(0.3)^5} = \left[\frac{2000}{(0.4)^5} + \frac{1500}{(0.3)^5} + \frac{1000}{(0.2)^5}\right]$$

$\Downarrow$ 0.00243 $\quad$ 0.01024 $\quad$ 0.0024 $\quad$ 0.00032

195,312.5 + 625,000 + 3,125,000

3,945,312.5

$\boxed{L_e = 946875}$

Case (ii)

$$\frac{4500}{De^5} = \left[\frac{2000}{(0.4)^5} + \frac{1500}{(0.3)^5} + \frac{1000}{(0.2)^5}\right]$$

$$\Rightarrow$$

$$De = 0.4089\,m$$

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Laminar and Turbulent Flows:

Laminar flow: The fluid particles move in flat or curved unmixing layers or streams and follow a smooth continuous path. There is no transverse displacement of fluid particles which remain in an orderly sequence in each layer.

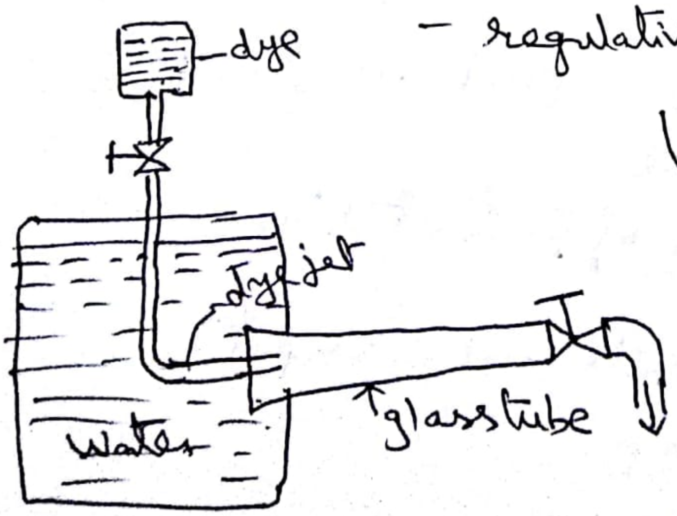
Ex:- Soldiers on a parade

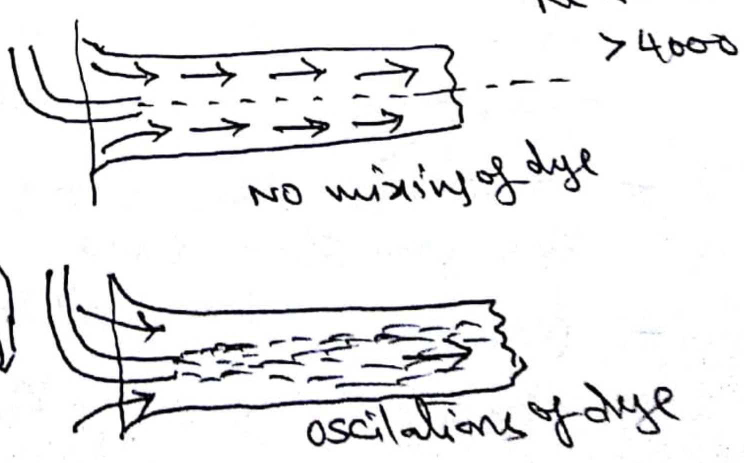
Turbulent flow: motion of fluid particles is irregular and the pathlines are erratic curves. consequently, there is intermingling of fluid particles.

Ex:- it resembles a crowd of passengers in rail/bus station during rush hour.

Osborne Reynolds, an English scientist who confirmed the existence of these two regimes experimentally. apparatus designed by Reynolds experimental setup consists,

- constant head tank filled with ~~tank~~ water
- small tank containing dye
- horizontal glass tube with round entry
- regulative valve.





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characteristics:

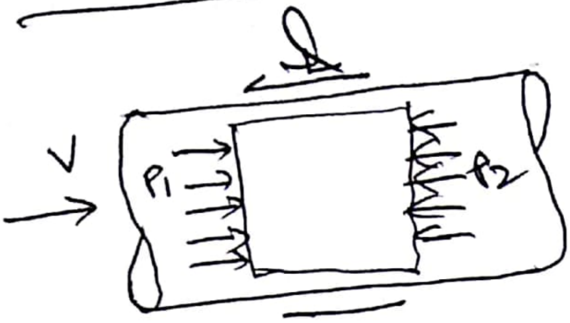
Laminar flow

- smooth streamlines / highly orderly motion.
- The flow in a circular pipe is laminar for $Re < 2000$.
- laminar flow in a straight pipe may be considered as the relative motion of a set of concentric cylinders of fluid.
- laminar flow is common only in cases in which the flow channel is relatively small, the fluid is moving slowly
- oil flow through a thin tube or ~~blow flow~~ blood flow through capillaries is laminar.
- flow is often laminar, especially in a thin layer just adjacent to the surface.

Turbulent flow

- highly unsteady
- flow velocity at a given point is shown to great variance overtime.
- ~~[illegible]~~ Due to intermixing of particles momentum transfer takes place
- process is irreversible and dissipative
- lead to loss of KE.

Darcy–Weisbach eqn



viscous friction effects proportional to :
- pipe length l
- wetted perimeter, P
- v^n, v = average flow velocity

The friction loss for the turbulent flow through a pipe can be evaluated by the force balance on the control volume.

propelling force on the flowing fluid b/w ① – ②,
$= (P_1 - P_2)A$

The frictional resistance force can be written as
$= f' \times P l v^2$

where f' = non-dimensional factor depends on material & nature of pipe surface.

under equilibrium, $(P_1 - P_2)A = f' P L v^2$

$$\frac{P_1 - P_2}{\gamma} = \frac{f'}{\gamma}\frac{P}{A} l v^2$$

$$h_f = \frac{2g f'}{\gamma}\left(\frac{P}{A}\right)\frac{l v^2}{2g}$$

$$= \frac{2g f'}{\gamma} \times \frac{l}{y_m}\frac{v^2}{2g}$$

$\frac{A}{P} = y_m$ = hydraulic mean depth.

let $\frac{2g f'}{\gamma} = f$ (another const)

$$\therefore h_f = f \frac{l}{y_m}\frac{v^2}{2g}$$

For circular pipe running full, $A = \frac{\pi}{4}d^2$, $P = \pi d$.

$\therefore y_m = \frac{d}{4}$ $\therefore h_f = \frac{4 f L v^2}{2 g d}$

f = D – W coefficient of friction.

$Q = \frac{\pi d^2}{4} v$

$v^2 = \frac{16}{\pi^2}\frac{Q^2}{d^4}$

$h_f = \frac{4 f l}{2 g d} \cdot \frac{16}{\pi^2}$

$= \frac{f l Q^2}{3 d^5}$

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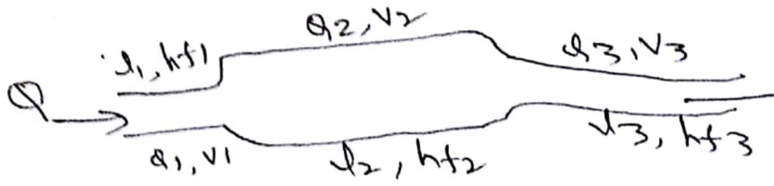
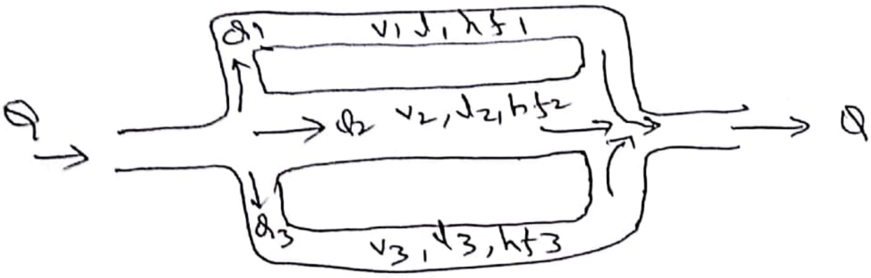
pipes in series and parallel



series

$Q = Q_1 = Q_2 = \cdots$

$hf = hf_1 + hf_2 + \cdots$



parallel

$Q = Q_1 + Q_2 + Q_3 + \cdots$

$hf = hf_1 = hf_2 = \cdots$

① Two reservoirs are connected by three pipes laid by parallel, the pipe diameters are 10 cm, 20 cm, 30 cm and they are of the same length. If discharge through 10 cm pipe is 0.1 m^3/s, calculate the discharge through the larger pipes. Assume the friction coefficient 'f' to be same for the pipes.

sol: for parallel connection,

$$hf = hf_1 = hf_2 = hf_3$$

$$\frac{f_1 l_1 Q_1^2}{3 d_1^5} = \frac{f_2 l_2 Q_2^2}{3 d_2^5} = \frac{f_3 l_3 Q_3^2}{3 d_3^5}$$

$$\therefore \frac{Q_1^2}{d_1^5} = \frac{Q_2^2}{d_2^5} = \frac{Q_3^2}{d_3^5}$$

$Q_2 = 0.566\ m^3/s$

$Q_3 = 1.54\ m^3/s$

$Q = A \cdot V$

$= \frac{\pi d^2}{4} \cdot V$

$hf = \frac{4flv^2}{2gd}$

$= \frac{4fl\left(\frac{Q}{\pi d^2/4}\right)^2}{2gd}$

$= \frac{4flQ^2}{2gd\,\pi^2 d^4/4^2}$

$\frac{32}{64}$

$2 \times 9.81 \times \pi^2$

3.2

3.14×3.14

concept of equivalent pipe:

replacing the series combination by a single pipe of uniform diameter which would have the same head loss and discharge rate, the pipe is called equivalent pipe. uniform dia of equivalent pipe is equivalent diameter of compound pipe.

$$\frac{Le}{De^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5} + \cdots$$

① A piping system consists of three pipes arranged in series

Pipe	Length	Dia
AB	2000 m	40 cm
BC	1500 m	30 cm
CD	1000 m	20 cm

Transform the system to ① equivalent length of 30 cm dia. pipe and ii) an equivalent diameter for the pipe length 4500 m long.

(i) find Le for De = 30 cm

$$\frac{Le}{De^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5} = (0.3)^5 \left[\frac{2000}{(0.4)^5} + \frac{1500}{(0.3)^5} + \frac{10}{(0.}\right.$$

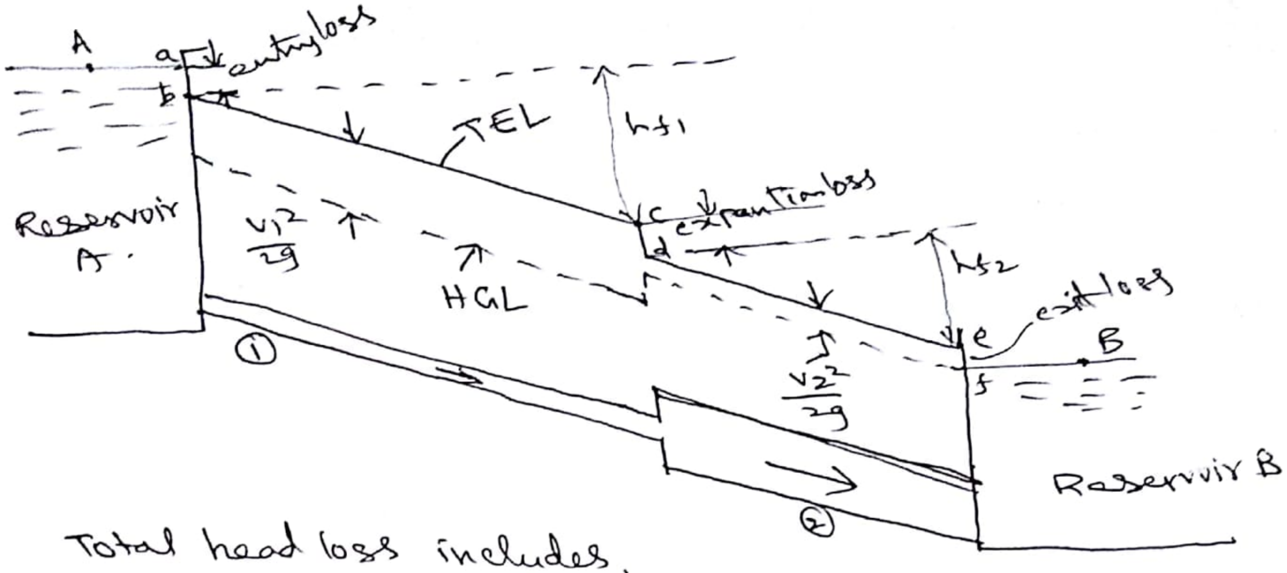
$$= 9568.36 \text{ m}$$

ii) find De for Le = 4500 m

$$\frac{4500}{De^5} = \left[\frac{2000}{(0.4)^5} + \frac{1500}{(0.3)^5} + \frac{1000}{(0.2)^5}\right]$$

$$De = 0.4089 \text{m}.$$

HGL & TEL



Total head loss includes,

$$\underbrace{\frac{0.5 v_1^2}{2g}}_{\text{entry}} + \underbrace{\frac{4 f_1 l_1 v_1^2}{2 g d_1}}_{hf_1} + \underbrace{\frac{(v_1 - v_2)^2}{2g}}_{\text{enlargement}} + \underbrace{\frac{4 f_2 l_2 v_2^2}{2 g d_2}}_{hf_2} + \underbrace{\frac{v_2^2}{2g}}_{\text{exit}}$$

① Two reservoirs are connected by a pipeline which is 15 cm in diameter for the first 5 m and 25 cm in diameter for the remaining 15 m. Entry to and exit from the pipe is sharp, and the water surface in the upper reservoir is 7.5 m above that in the lower reservoir. Represent the layout and tabulate the head losses by assuming that friction coefficient is 0.01 for both the pipes. Further calculate the flowrate through the arrangement and draw the HGL & TEL.

$Q = A_1 V_1 = A_2 V_2 \Rightarrow V_1 = \frac{A_2}{A_1} V_2 = \left(\frac{d_2}{d_1}\right)^2 V_2 \Rightarrow V_1 = 2.78 V_2$

i) entry loss $= 0.5 \frac{v_1^2}{2g} = 3.87 \frac{v_2^2}{2g}$

ii) friction in 15 cm pipe $= \frac{4 f l_1 v_1^2}{2 g d_1} = 10.3 \frac{v_2^2}{2g}$

iii) shock loss at enlargement $= \frac{(v_1 - v_2)^2}{2g} = 3.17 \frac{v_2^2}{2g}$

iv) friction in 25 cm pipe $= \frac{4 f l_2 v_2^2}{2 g d_2} = 2.4 \frac{v_2^2}{2g}$

v) shock loss at exit $= \frac{v_2^2}{2g}$

Total loss $= 20.74 \frac{v_2^2}{2g}$

applying Bernoulli's eqn at A & B.

$$\frac{P_a}{\gamma} + y_a + \frac{V_a^2}{2g} = \frac{P_b}{\gamma} + y_b + \frac{V_b^2}{2g} + \text{losses}$$

$P_a = P_b$ both at atm. pressure only

for large capacity reservoirs, $V_a \cong V_b = 0$

$$y_a - y_b = \text{losses}$$

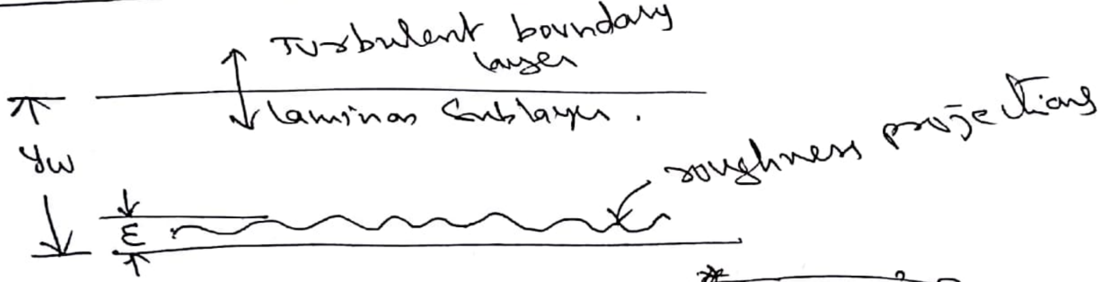
difference in levels of the two reservoirs equals the friction and other losses in the pipe system.

$$7.5 = 20.74 \frac{V_2^2}{2g} \quad \therefore \quad V_2 = 2.67 \text{ m/s}$$

discharge $Q = A_2 V_2 = 0.131 \text{ m}^3/\text{s}$.

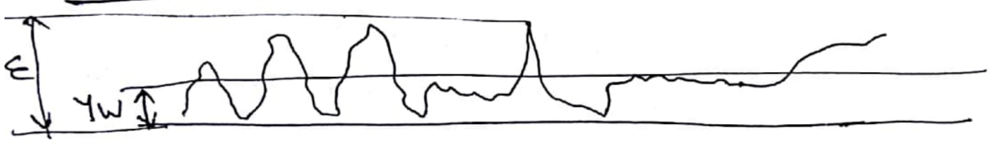
~~Hydro~~..

Hydraulically smooth surfaces:



$$\frac{\epsilon}{y_w} < 0.25$$

Rough:



$$\frac{\epsilon}{y_w} > 6.0$$

when $0.25 < \dfrac{\epsilon}{y_w} < 6.0$ boundary in transition.

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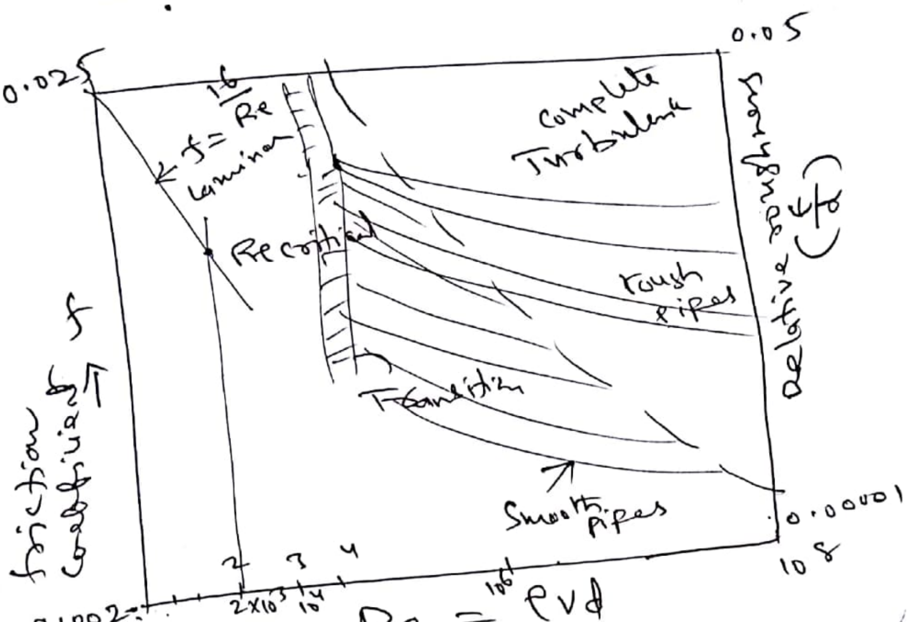
moody diagram for friction factors :



$$Re = \frac{\rho v d}{\mu}$$

A logarithmic plot of friction factor versus Renolds number for values of relative roughness as depicted in the plots called by moody charts.

$$f = 0.001375\left[1 + \left(10000\,\frac{\epsilon}{R} + \frac{10^6}{Re}\right)^{1/3}\right]$$

- roughness of a pipe depends upon the pipe material, its manufacturing process, erosion with time, fluid flowing and the environment etc.
- roughness increases due to rusting and accumulation of sediments on the pipe surface.
- An increase in roughness with age is generally prescribed by the following linear relation.

$$\epsilon = \epsilon_0 + \alpha t$$

ϵ_0 = roughness of the new material

ϵ = roughness after any time t

α = coefficient to be determined by experiments, ① A 20 cm diameter pipe has a relative roughness (R/ε) of 100. After 10 years of service, the relative roughness drops to 80. Determine the magnitude of the rate of roughness increase.

$$\varepsilon_0 = \frac{R}{100} = \frac{20}{100} = 0.2\text{ cm}$$

After 10 years of service $\varepsilon_{10} = \frac{20}{80} = 0.25$ cm

An increase in roughness with time 't' follows the linear law, ie $\varepsilon_t = \varepsilon_0 + \alpha t$

$$\varepsilon_{10} = \varepsilon_0 + 10\alpha$$

$$\alpha = \frac{\varepsilon_{10} - \varepsilon_0}{10} = \frac{0.25 - 0.2}{10} = 0.005 \text{ cm/year}$$

Navier-stokes equations of motion:

$$X - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{du}{dt} - \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$Y - \frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{dv}{dt} - \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$Z - \frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{dw}{dt} - \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

applications:

- Laminar uni-directional flow b/w stationary parallel plates
- " " " " having relative moti
- Laminar flow in circular pipes
- Laminar flow b/w concentric rotating cylinders.

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① Lubricating oil of sp. gravity 0.85 and dynamic viscosity 0.1 NS/m^2 is pumped through a 3cm dia. pipe. If pressure drop per metre length of the pipe is 15 kPa determine ⓘ the mass flow rate in kg/min ⓘⓘ shear stress at the pipe wall iii) the Reynolds number of flow iv) power required per 40 m length of the pipe to maintain the flow.

$$P_1 - P_2 = \frac{32 \mu V L}{d^2}$$

$$15 \times 10^3 = \frac{32 \times 0.1 \times V \times 1}{(0.03)^2} \Rightarrow V_{averge} = 4.218 \text{ m/s}$$

$$\text{flow rate } Q = A \times V = \frac{\pi}{4}(0.03)^2 \times 4.218$$

$$= 0.00298 \text{ m}^3/\text{s}$$

$$\text{mass flow rate} = \rho Q = (0.85 \times 1000) \times 0.00298 \times 60$$

$$= 112.5 \text{ kg/min}$$

shear stress at the pipe wall,

$$\tau_0 = \left(\frac{-dp}{dx}\right) \times \frac{R}{2}$$

$$= \left(\frac{15 \times 10^3}{1}\right) \times \frac{0.015}{2} = 112.5 \frac{N}{m^2}$$

Renolds number $Re = \dfrac{\rho V d}{\mu} = \dfrac{4.218 \times 0.03 \times 0.85 \times 1000}{0.1}$

$$h_f = \frac{P_1 - P_2}{\gamma} = \frac{15 \times 10^3}{0.85 \times 9810} = 1.8$$

$$= 1075.6$$

$Re < 2000$, hence flow is laminar

$$\text{workdone in maintaining flow} = \gamma Q h_f$$

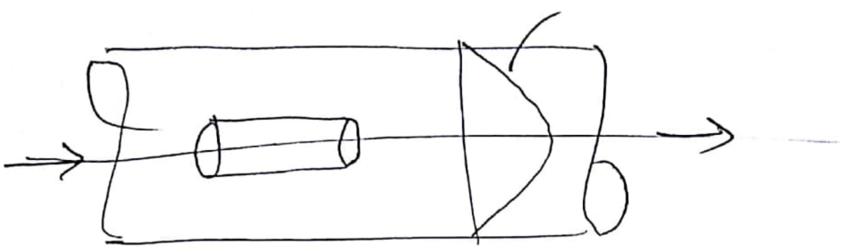
$$= (0.85 \times 9810) \times 0.00298 \times 1.8$$

$$= 44.73 \text{ W}$$

for 40 m length,

power required $= 44.73 \times 40 = 1789 \text{ W} = 1.79 \text{ kW}$.

Hagen poiselle eqn



$$\boxed{\frac{dp}{dx} = 2\frac{\mu}{r}\frac{du}{dr}}$$

$$\frac{dp}{dx} = -\frac{2\tau}{r}$$

$$\frac{du}{dr} = \frac{r}{2\mu}\frac{dp}{dx}$$

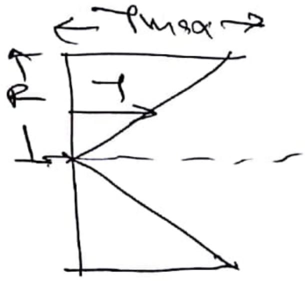
$$u = \frac{1}{4\mu}\left(-\frac{dp}{dx}\right) R^2 - r^2$$

$$u_{max} = \frac{1}{4\mu}\left(-\frac{dp}{dx}\right) R^2$$

$$V_{average} = \frac{1}{8\mu}\left(-\frac{dp}{dx}\right) R^2$$

$$= \frac{1}{2} V_{max}$$

shear stress distribution



$$\tau_{max} = \frac{1}{2}\left(-\frac{dp}{dx}\right) R$$

$$p_1 - p_2 = \frac{32\,\mu\, V_{ave}\, l}{\gamma\, d^2} = \frac{128\,\mu\, Q\, l}{\pi d^2}$$

$$\frac{p_1 - p_2}{\gamma} = \frac{32\,\mu\, V\, l}{\gamma\, d^2}$$

$$f = \frac{16}{Re}$$