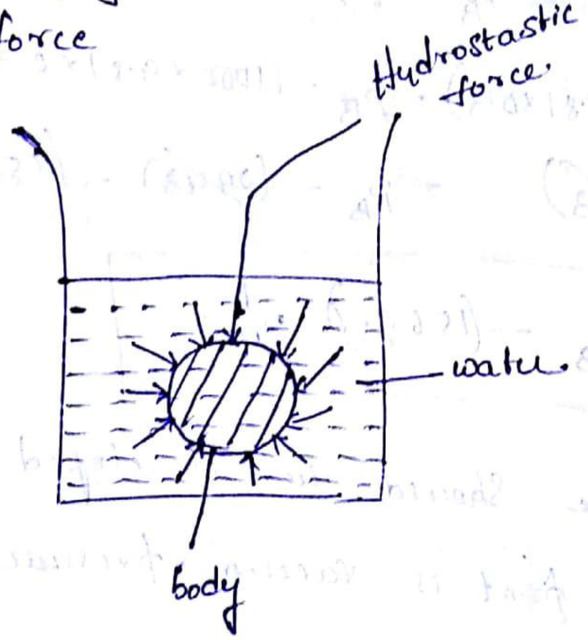
Chaphter - 2

HYDRO.STASTIC FORCE ON IMMERGED SURFACE

fluid at rest position

Hydro stastic forces on Surface :-

Hydrostastic force means the force is Exacted by the fluid at rest position for the hydrostastic force the fluid is must be rest. The weight of the fluid will act on the body son of the pressure forces or acting on the particul body. we studying on that particular forces is cal hydro stastic force

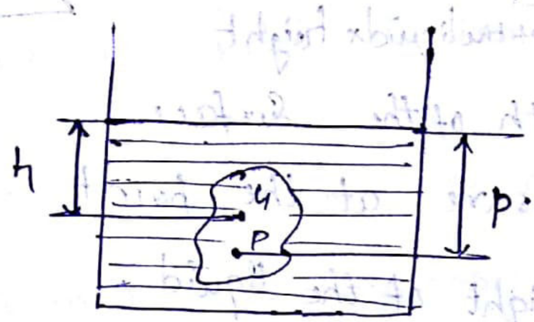


Total pressure :-

It is defined as the force Exacted by the static fluid on a Surface when the fluid comes in contanct with the Surface these forces is always at right angle to the Surface

CENTRE OF PRESSURE :-

It is defined as the point of application of the total pressure on the surface the total pressure is exacted by a liquid on Immerged surface



$$p = \frac{I_G}{A \cdot \bar{h}} + \bar{h}$$

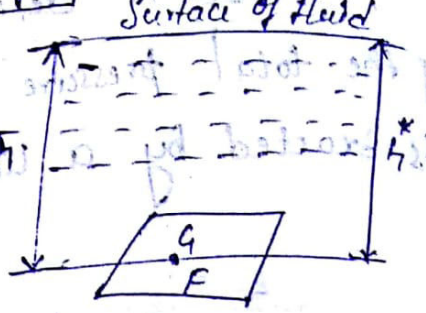
The Immerged Surfaces are classified in to

1. Horizontal plane Surfaces.
2. Vertical plane Surface
3. Inclined plane Surface.
4. Curved Surface.

Hydroctatic force on Horizantal Immerged Surface:-

Consider a plane Horizontal Surface Immerged in a static fluid Every point of the surface is at the same depth from the fre-Surface of the liquid

The pressure instensity will be Equal on the Entire Surface at the point $\boxed{p = \rho g h}$



$\therefore \rho g = \text{constant}$

So $\boxed{p = w \cdot h}$

↳ of the liquid height

Where h = depth of the Surface.

p = pressure at the point.

w = weight of the liquid.

Then the total Pressure force is applied on the horizontal of plain Surface is

$f = p \times A$

↳ the pressure at single point × Area

$F = w \times \bar{h} \times A \qquad \therefore p = w \times \bar{h}$

$F = \rho g h \times A$

$\boxed{F = \rho g \bar{h} \times A}$

where:

F = force Total hydrostatic pressure force

A = Total Area of the plane surface

$\bar{h}$ = depth (or height) from Surface to horizontal plane

ρ = density of the plane

Hydrostatic force on im-merged vertical plane:-

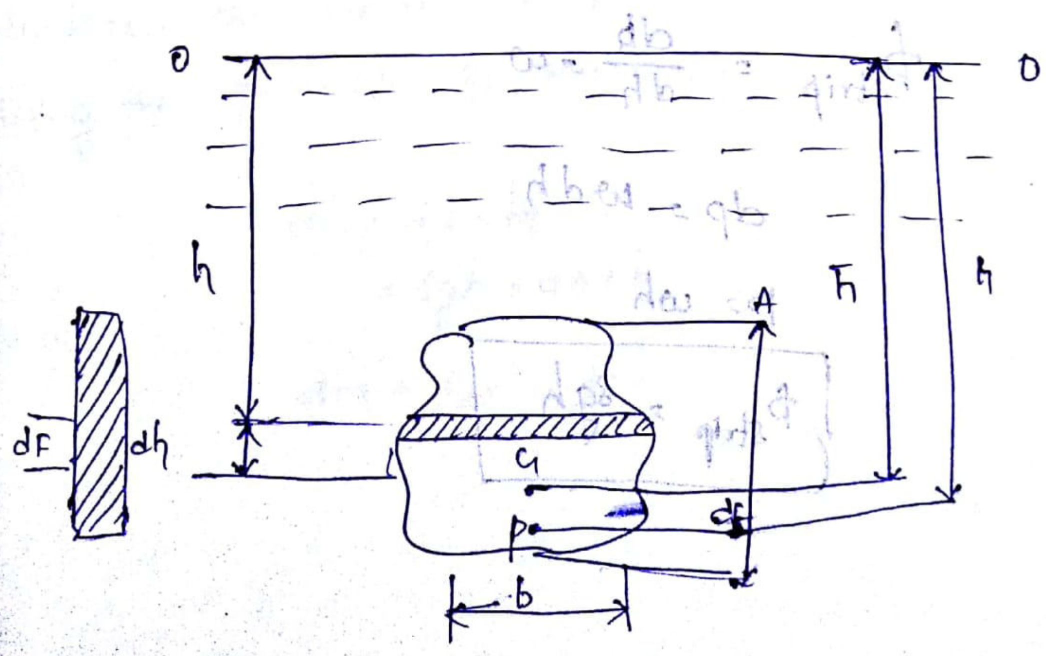
→ Consider a plane surface of arbitary shape is im-merged vertically in a static mass of fluid

→ The depth of liquid varied from point to point

→ The pressure intensity is not constant entire over the surface of the arbitary

→ Analysis for the total pressure is than made by dividing the entire surface in to the No: of small parallel shapes strips

→ The force on small strip is calculated and the total pressure force on the whole area is calculated by integrating the force on small strip



Where

A = total area of the surface

$\bar{h}$ = distance of centre of gravity of the area form free of the ~~f~~ liquid

G = centre of gravity of plane surface

P = centre of pressure

h^* = distance of centre of pressure form free surface of liquid

b = breadth of the strip

dh = thickness of the strip

Total pressure, F =

Consider a small strip of thickness dh and breadth (b) on the depth (h) form the surface then the pressure instensity of a strip

$$p_{strip} = \frac{dp}{dh} = \omega$$

$$dp = \omega dh$$

$$p = \omega h$$

$$\boxed{p_{strip} = \rho g h}$$

Area of the Small Strip $\boxed{dA = b \times dh}$

pressure force of a small Strip

$$dF = p \times dA$$
$$dF = \rho g h \times dA$$

$$F = \int dF = \int \rho g h \cdot dA$$

$$F = \rho g \cdot \int h \cdot d\theta$$

$$\boxed{F = \rho g \cdot \bar{h} A}$$

Centre of pressure :-

The position of Centre of pressure is always diffuent form the poiton of centre of gravity In case for a Emerged Surface vertical or inclined because the instensity of pressure $p = \rho g h$ with increasing the height measured form the free Surface the centre of pressure is calculated by using the principle of momemtum. Equation

$$dm = dF \times h$$
$$= \rho g h \times dA \times h$$

$$dm = \rho g h^2 \times dA$$

The resultant force (F) is acting at 'p' at a distance of h^* form the free-Surface of the liquid for resolving these Equation applying the integuals

$$F \times h^* = \rho g \frac{\int h^2 \times dA}{I_0}$$

$$F \times h^* = \rho g . I_0$$

$$\boxed{h^* = \frac{\rho g I_0}{F}}$$

$$h^* = \frac{\rho g I_0}{\rho g \bar{h} \times A}$$

$$h^* = \frac{I_0}{\bar{h} \times A}$$

By applying the parallel axis theorem

$$I_0 = I_G + A\bar{h}^2$$

$$I_0 = I_G + A\bar{h}^2$$

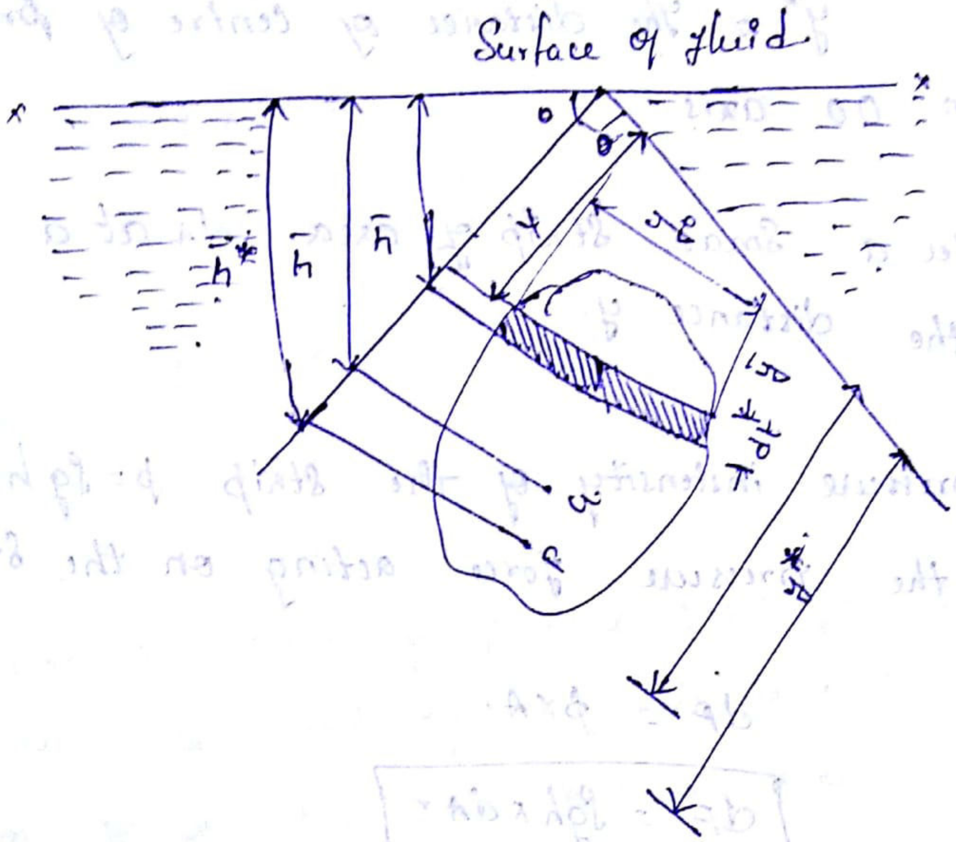
$$h^* = \frac{I_G + A\bar{h}^2}{A \cdot \bar{h}}$$

$$\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$

Hydrostatic force on immerged Inclined plane Surface :-

(Imp.)

Consider a plane of Surface of orbitary shape It is immerged in liquid in a such way that the plane of surface makes an angle 'θ' with the free Surface of the liquid



Let A = Total area of the inclined Surface.

$\bar{h}$ = depth of centre of gravity of Inclined area from free Surface

h^* = distance of centre of pressure form free Surface of liquid

θ = Angle made by the plain

Let the plain Surface produces the free liquid Surface at a point of O then O'O axis is ter to the plain of the Surface.

Than assume $\bar{y}$ = distance of centre of Gravity of inclined Surface Surface from O'O axis

y^* = The distance of centre of pressure from OO axis

consider a Small Strip of area dA at a depth 'h' the distance y.

The pressure intensity of the strip $p = \rho g h$
Then the pressure force acting on the strip

$$dF = p \times A.$$

$$\boxed{dF = \rho g h \times dA.}$$

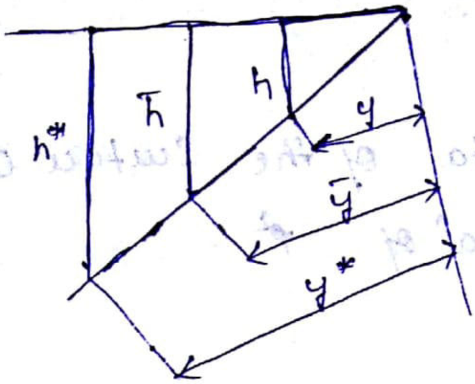
The pressure force acting on the total area

$$P = \int dF = \int \rho g h . dA.$$

$$P = \int \rho g h . dA.$$

$$P = \rho g \int h . dA.$$

$$F = \rho g \bar{h} . A$$



$$\left[\sin\theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}\right]$$

$$\sin\theta = \frac{h}{y}$$

$$F = \int \rho g h_2 \, dA$$

$$\boxed{h = y\sin\theta}$$

$$= \int \rho g \, y \sin\theta$$

$$\frac{h^2}{2}$$

$$= \rho g \sin\theta \int y \, dA$$

$$= \rho g \sin\theta \, \bar{y} \cdot A$$

$$\boxed{F = \rho g \bar{h} \cdot A}$$

The poiston of centre of prencure from free surface of liquid it is calculated by using the principle of momentum It st which states that the moment of the resultant force at axis is equal to the sum of Moment of components the resultant Force 'F' is acting at point 'B' at a distance of (Y) from axis of 0 to 0 Then the moment of force

$$dF \times y = \rho g h \times dA \times y$$

$$= \rho g \, y\sin\theta \, dA \times y$$

$$= \rho g \, y^2 \sin\theta \, dA$$

$$= \int \rho g \, y^2 \sin\theta \cdot dA$$

$$= \int g \sin\theta \cdot \int y^2 \, dA$$

Then the moment of inercia of the surface O'O at a distance of y then integral of y^2

$$\boxed{\int y^2 \cdot dA = I_0}$$

$$= \int g \sin\theta \int y^2 \cdot dA$$

$$= \int g \sin\theta \cdot I_0$$

The Sum of the all momentum of the all forces about O to O axis we will get $f \times y^*$

$$f \times y^* = \int g \sin\theta \cdot y \, I_0$$

$$y^* = \frac{\int g \sin\theta \times I_0}{F}$$

$$\frac{h^*}{\sin\theta} = \frac{\int g \sin\theta \times I_0}{\int g \, \bar{h} \cdot A}$$

By applying the parallel axis theorm

$$\boxed{I_0 = I_g + A\bar{h}^2}$$

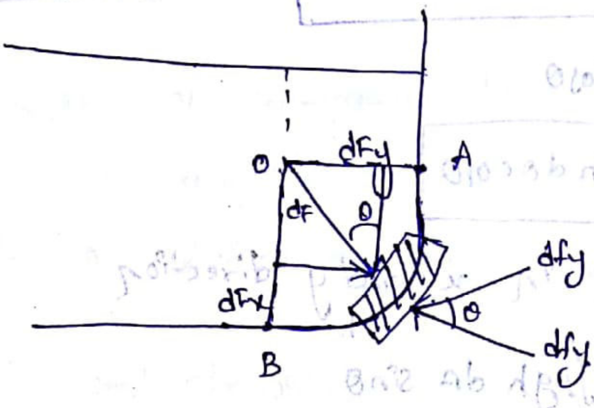
$$\frac{h^*}{\sin\theta} = \frac{\int g \sin\theta \times [I_g + A\bar{h}^2]}{\int g \, \bar{h} A}$$

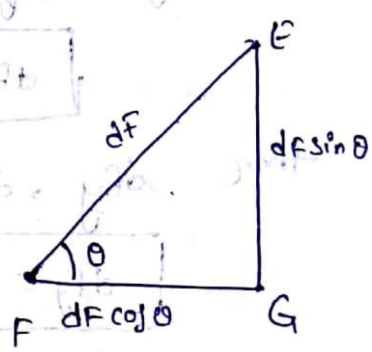
$$h^* = \frac{\cancel{\rho g} \sin^2\theta \,[I_G + A\bar{h}^2]}{\cancel{\rho g} \cdot \bar{h} \cdot A}$$

$$h^* = \frac{\sin^2\theta}{\bar{h} \cdot A}\left[I_G + A\bar{h}^2\right]$$

Hydrostatic force on curved surface on im-merged liquid:-

Consider a curved surface A, B submerged in a static fluid, let dA is the area of small strip the depth of strip from surface of liquid 'h'





The intensity of pressure on the area of

$$\boxed{dA = P = \rho g h}$$

$$dF = P \times dA$$
$$= \rho g h \times dA$$

Applying integration on the both sides.

$$\int dF = \int \rho g h \cdot dA$$

$$F = \rho g \int h \cdot dA$$

$$\boxed{F = \rho g \int h \cdot dA}$$

Hence the force (dF) is resolved in two components dF_x and dF_y.

the total force in the x & y directions then F_x and F_y obtained integration of dF_x and dF_y.

The total force on curved surface.

$$\boxed{F = \sqrt{F_x^2 + F_y^2}}$$

inclination of Resultant force with horizontal

Resolving the forces dF in the direction of x and y $dF_x = dF \sin\theta$

$$\boxed{dF_x = \rho g h \, d\theta \sin\theta}$$

$$\boxed{\tan\theta = \frac{F_y}{F_x}}$$

then, $dF_y = dF \cos\theta$

$$\boxed{dF_y = \rho g h \, d\theta \cos\theta}$$

the total force in x and y direction

$$F_x = \int \rho g h \, dA \sin\theta$$

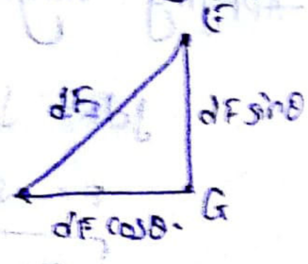
$$\boxed{F_y = \int \rho g h \, dA \cos\theta}$$

$$F_x = \rho g \int h \, dA \sin\theta$$

$$F_y = \rho g \int h \, dA \cos\theta$$

from right angle triangle we anlong the ~~st~~

from right angle triangle EFG



$EF = dF$

$EG = dF \sin\theta$

$FG = dF \cos\theta$

Scanned by CamScanner

The Equation $EG = dF\sin\theta$, vertical projection of Area dA.

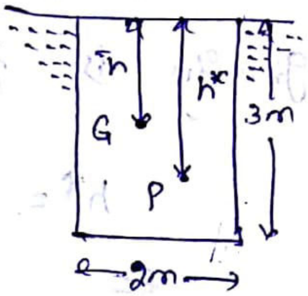
The Eq. $FG = dF\cos\theta$ = Horizontal axis of Area dA.

Then f_x = Total pressure forces on the projected area of curved surface.

*Problems on vertical plane in immerged of in liquid:

*** (1) A rectangular plane surface 2m wide and 3m depth it is lies on vertical plane in a water determine the total pressure and poistion of centre of pressure on the plane surface when a upper is horizontal coincide with the water surface and 2.5m below the water surface.

Soln:- given data



depth of rectangular plain in vertical direction = 3m

width of Rectangular channel = 2m

find (i) Total pressure

(ii) centre pressure

Formula used:

total pressure: $\boxed{p = \rho g \bar{h} A}$

centre of pressure

$$\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$

total pressure $(P) = \rho g \bar{h} A$

$c.G = d/_2 = 3/_2 = 1.5m$
$(\bar{h})$

$A = w \times d = 3 \times 2 = 6m^2$

$P = \rho g \bar{h} \cdot A$

$= 1000 \times 9.81 \times 1.5 \times 6$

$$\boxed{P = 88290 \, N}$$

Centre of pressure

$I_G = \frac{bd^3}{12}$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

I_G = moment of interia at centre of gravity is passing through an axis

$$h^* = \frac{\frac{bd^3}{12}}{A\bar{h}} + \bar{h}$$

$$\frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 mm^3$$

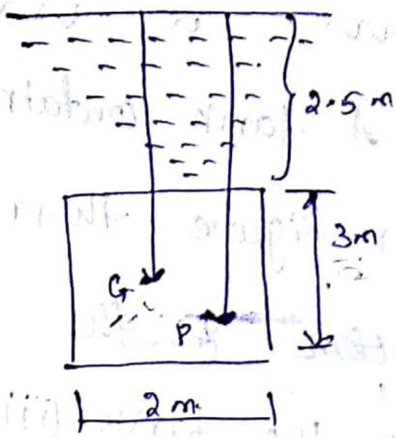
$$h^* = \frac{4.5}{6 \times 1.5} + 1.5$$

$$= \frac{4.5}{9} + 1.5$$

$$\boxed{h^* = 2m}$$

$F = \rho g \bar{h} A$

$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$



$F = 1000 \times 9.81 \times 4 \times 6$

$\boxed{F = 235440 \, N}$

$I_G = \frac{2 \times (5.5)^3}{12}$

$\boxed{I_G = 27.729 \, m^2}$

$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

$= \frac{27.729}{6 \times 4} + 4$

$\boxed{h^* = 5.15 \, m}$

Result :

A rectangular plane of Edge conntact (or) coinside with the surface of the liquid.

4

2 (i) Total pressure force (P) = 86890 N

(ii) Centre of gravity (h^*) = 2m.

II A Rectangular plane of Edge below 2.5 m from the surface of the liquid.

(i) Total pressure force (F) = 235440 N

(ii) centre of gravity (h^*) = 5.15 M.

Chaphter - II(b)

Kinematics

fluid Kinematics defition :-

fluid kinematics is defined as It is the brarch of the Science which deals with motion of the particle without considering the pressure forces

Types of fluid flows :-

1. Steady flow (2) unsteady flow
2. uniform flow (2) Non-uniform flow
3. laminar flow (2) Turblant flow
4. Rotational flow (2) Irrotational flow
6. compressible flow (2) in compressible flow

1. **Steady flow :-**

It is defined as the type of fluid flow in which fluid characteristics like (density velocity pressure) at a point doesn't changes with respective to time

Mathematically $\left\{ \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0 \right\}$

Un steady flow :-

It is defined as the type of fluid flow in which the fluid characteristics like. (d, v, p) at a point changes with respective to time

$$\left\{ \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \right\}$$

uniform flow :-

It is defined as the type of flow in which the velocity of a fluid at any given point with respective to time doesn't changes with in a confined Space

$$\left(\frac{\partial v}{\partial s}\right)_t = 0$$

Non- uniform flow :-

It is defined as the type of flow in which the velocity at any given time changes with in a confined Space

$$\left(\frac{\partial v}{\partial s}\right)_t \neq 0$$

Scanned by CamScanner

Laminar flow :-

It is defined as the type of flow in which the fluid particles moving along well defined path → particular sequence

Turbulent flow :-

It is defined as the type of flow in which the fluid particles are moving in zig zag way

Rotational flow :-

It is defined as the fluid particles flowing along stream line also rotate with their own axis.

Irrotational flow :-

The fluid particles flowing along stream line also not rotate their own axis

Compressible flow :-

The density of fluid changes point to point

$$\rho \neq \text{constant}$$

$$0 \neq \left(\frac{\partial \rho}{\partial t}\right)$$

Types of flow lines

The flow lines are classified based on the moving position of a fluid particles

(1) Stream line
(2) Streak line
(3) path line

Stream line

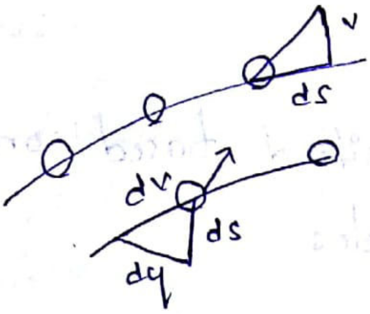
A stream line is an imaginary line drawn in a flow of fluid such that a tangent drawn at any point on it indicate the direction of the velocity vector of that particular point.

Let 'T' is the time taken by fluid particle to move a distance ds along the stream line with velocity 'v'

$$\boxed{\text{Time } (t) = \frac{ds}{v}}$$

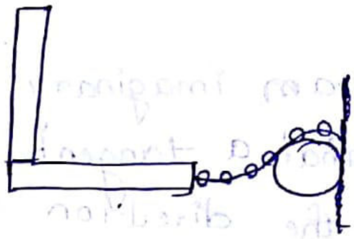
Properties of stream line

* There cannot be any moment of fluid mass across the stream line
* The stream line neither intersect itself two streams line can across the fluid particle



Streak line:-

A streak line is the locus of the fluid particle that have passed sequentially through a prescribed point in a flow



path line:-

A path line is a line (or) path traced by a single fluid particle during its motion over same time period

Note:- * These 3 lines are identical only steady flow

The rate of discharge:-

It is defined as the Quantity of fluid flowing per a second through a section of pipe or channel for an incompressible fluid the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second for compressible fluid the ratio of flow is usually expressed as the weight of fluid flowing across the section

$$\boxed{Q = A \times V}$$

Where Q = discharge (or) rate of flow

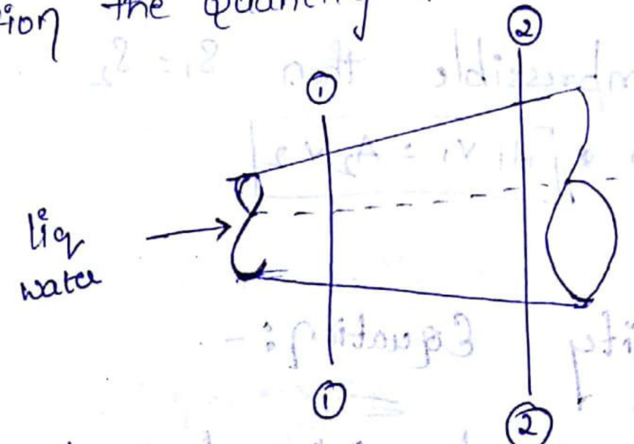
A = ~~Area~~ cross sectional area of the channel or pipe line

V = ~~velu~~ velocity of the fluid

$\boxed{\text{units for discharge} = m^3/s.}$

* Cotinuity ~~Cavity~~ Equation :- (Q)

The equation based upon the principle of conversation of mass is called continuity ~~Cavity~~ Equation for the fluid flowing through a pipe at all cross section the quantity of fluid per second = constant.



AT section 1-1

d_1 = Intial dia of pipe at 1-1

A_1 = Area of pipe at 1-1

S_1 = density of fluid pipe at sec 1-1 ①

V_1 = Average velocity at sec 1-1

Similarly

V_2 = Average velocity at sec 2-2

A_2 = Area of pipe at sec 2-2.

ρ_2 = density of the fluid at sec 2-2 then

the rate of flow at Sec 1-1 = $A_1 V_1 \rho_1$

The rate of flow at Sec 2-2 = $A_2 V_2 \rho_2$.

According the mass conservation of law the rate of flow at Sec 1-1 = The rate of flow at sec 2-2

$$\boxed{A_1 V_1 \rho_1 = A_2 V_2 \rho_2}$$

If the fluid is incompressible then $\rho_1 = \rho_2$

Then the rate of flow = $\boxed{A_1 V_1 = A_2 V_2}$

Problems on Continuity Equation:-

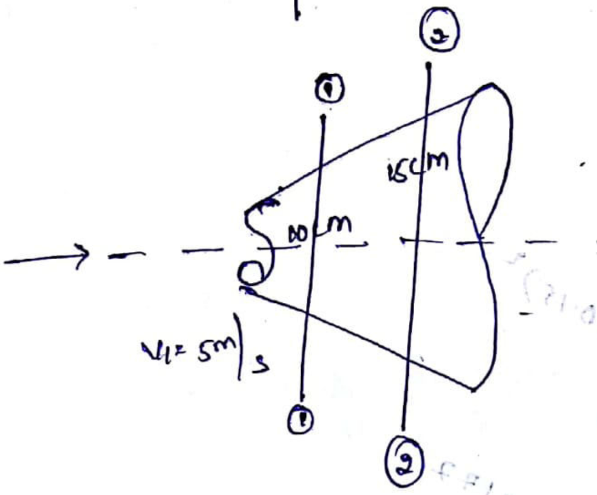
1. The diameter of the pipe at sec 1 and 2 (or) 10cms (or) 15cms respectively. Find the discharge through a pipe with velocity of the water flowing through a pipe at sec 1 is 5m/s determine the velocity at sec 2

Sol:- given :-

diameter at section (d_1) = 10 cm = 0.01 m

diameter at section (d_2) = 15 cm = 0.15 m

velocity at section (v_1) = 5 m/s



Find

velocity at section (v_2) = ?

Formula

$$Q = A_1 v_1 = A_2 v_2$$

$$Q = A_1 \times V_1 \quad m^2$$

$$Q = 0.785 \times 5 \ m/s$$

$$\boxed{Q = 4 \ m^3/s}$$

Q2:

Solution

$d_1 = 10cm$

$A_1 = \pi/4 \times (d)^2 \rightarrow (0.1)^2$

$= 78.5\ cm^2$

$$\boxed{A_1 = 78.5\ cm^2}$$

$d_2 = 15cm$

$A_2 = \pi/4\ (15)^2 \rightarrow (0.15)^2$

$= 176.62\ cm^2$

$$\boxed{A_2 = 1.76 m^2} = 0.0177$$

$Q = A_1 V_1 = A_2 V_2$

$Q = A_2 V_2$ (or) $A_1 V_1 = A_2 V_2$

$$\boxed{V_2 = \frac{A_1 V_1}{A_2}}$$

$$V_2 = \frac{0.785 \times 5}{1.76}$$

$$\boxed{V_2 = 2.23\ m/s}$$

Result

Velocity at sec $(V_2) = 2.23\ m/s$

2. A 30 cm diameter pipe line conveying the water in to two branches in two pipes of diameter 20cm and 15cm respectively. If the average velocity in 30cm diameter pipe is 2.5 m/s find the discharge of the pipe and also determine the velocity in 15cm pipe. If the average velocity in 20cm diameter pipe is 2m/s

Sol:- given :-

diameter of the pipe (d_1) = 30cm ⇒ $\frac{3}{100}$ = 0.03 m

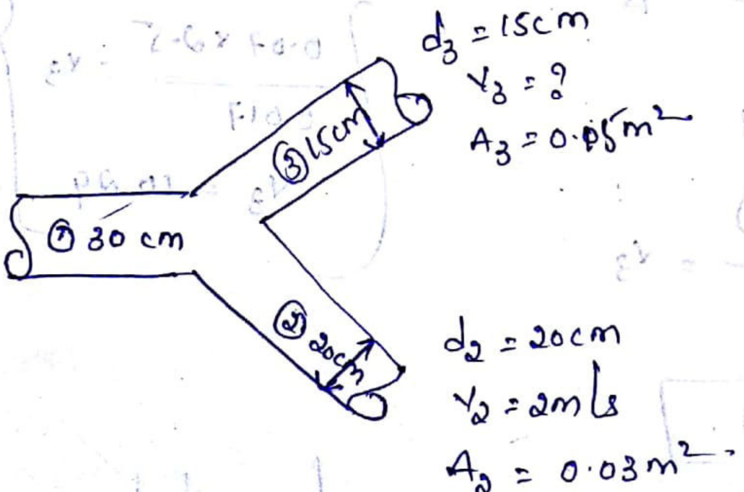
diameter of the another pipe (d_2) } = 20cm ⇒ $\frac{20}{100}$ = 0.02m

" " " " (d_3) = 15cm = $\frac{15}{100}$ = 0.15 m

Average velocity of at pipe 1 = 2.5m/s

" " " pipe 2 = 2m/s

Diagram



find

The average velocity of pipe ③ V_3 = ?

formula

$Q = A \times V$

$A_1 = \pi/4 \, (d_1)^2$

$= \pi/4 \, (0.3)^2$

$= 0.07 \, m^2$

$V_1 = 2.5 \, m/s$

$A_2 = \pi/4 \times (0.2)^2$

$= 0.03 \, m^2$

$V_2 = 0.2 \, m/s$

$A_3 = \pi/4 \times (0.15)^2$

$= 0.017 \, m^2$

$Q = A_1 V_1$

$Q = A_1 V_1 = (0.07)(2.5)$

$= 0.175 \, m^3/s$

$Q = A_1 V_1$

$= (0.07)(2.5)$

$= 0.175 \, m^3/s$

$A_1 V_1 = A_3 V_3$

$\frac{0.07 \times 2.5}{0.017} = V_3$

$V_3 = 10.29$

$A_2 V_2 = A_3 V_3$

$V_3 = \frac{A_2 V_2}{A_3}$

$= \frac{(0.03)(0.\ldots)}{0.017}$

$V_3 = 3.52$

$A_2 V_2 = A_3 V_3$

$\frac{(0.03)(0.2)}{0.017} = V_3$

$\Rightarrow \boxed{V_3 = 3.52.}$

which one is the nearest value that one only the should be Correct