V

KANI'S METHOD

* Gasper Kani, a German engineer developed another distribution pro
based on slope deflection equations
* This method is very useful for the analysis of multistorey f
(3 to 4 storeys) even today.
* This method is first explained for structures with fixed e
later modified for simply and overhanging supports.

* Sign conventions:-
- All clockwise end moments are positive
- clockwise rotations are positive

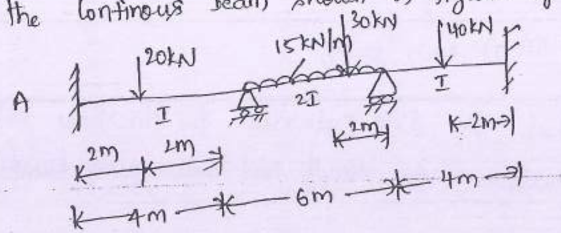
* Rotation factor (RF) :-

$$\boxed{R.F = -\frac{1}{2}\frac{k}{\Sigma K}}$$

Application of Kani's method to continuous beams with fixed ends:-

The above procedure may be applied to continuous beams with fixed ends noting that the rotation contribution at the end is zero, since the end is being fixed, its rotation is zero.

Analyse the continuous beam shown in figure by Kani's method.



Fixed end moments :-

$$M_{FAB} = -\frac{20\times4}{8} = -10\,kNm$$

$$M_{FBA} = +10\,kNm$$

$$M_{FBC} = -\frac{15\times6^2}{12} - \frac{30\times4\times2^2}{6^2} = -58.33\,kNm$$

$$M_{FCB} = \frac{15\times6^2}{12} + \frac{30\times4^2\times2}{6^2} = 71.67\,kNm$$

$$M_{FCD} = -\frac{40\times4}{8} = -20\,kNm$$

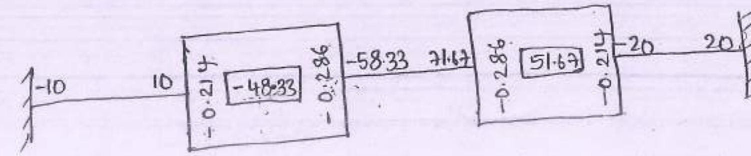
$$M_{FDC} = 20\,kNm$$

∴ Rotation factors :-

$$R.F = -\frac{1}{2}\frac{K}{\Sigma K}$$

Joint	Members			
B	BA	$\frac{4EI}{4} = EI$	2.33EI	− 0.
	BC	$\frac{4E(2I)}{6} = \frac{4}{3}EI$		− 0.
C	CB	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	2.33 EI	−0
	CD	$\frac{4EI}{4} = EI$		−0

* Rotation Contribution table :-



10.34	13.84	−18.73	−14.01
14.35	19.18	−20.26	−15.16
14.68	19.62	−20.39	−15.26
$\boxed{14.70}$	$\boxed{19.65}$	$\boxed{-20.40}$	$\boxed{-15.26}$

M'_{AB} ① $-0.214[-48.33+0] = 10.34$ ② $-0.286[-48.33+0] = \underline{13.84}$

③ $-0.286[51.67+\underline{13.84}] = \underline{-18.73}$ ④ $0.214[51.67+\underline{13.84}] = -14.0$

① $-0.214[-48.33-18.73] = 14.35$ ② $-0.286[-48.33+18.73] = \underline{19.1}$

③ $-0.286[51.67+19.18] = \underline{-20.26}$ ④ $0.214[51.67+19.18] = -15.$

① $-0.214[-48.33-20.26] = 14.68$ ② $0.286[-48.33-20.26] = \underline{19.62}$

$-0.286[51.67+19.62] = \underline{-20.39}$ $-0.214[51.67+19.62] = -15$

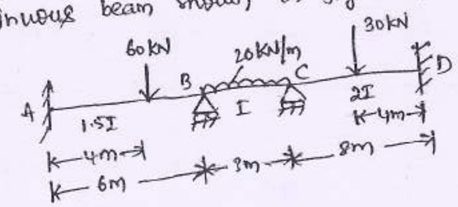
$-0.286\left[51.67+19.65\right] = -20.40$ | $-0.214\left\{51.67+19.65\right\} = -15.26$

Final moment Calculation :-

FM = FEM + 2 (near end contribution) + far end contribution

Joints	A	B		C		D
FEM	-10	10	-58.33	71.67	-20	20
NEC	2x0	2x14.7	2x19.65	2x-20.40	2x-15.16	2x0
FEC	14.70	0	-20.40	19.65	0	-15.26
Final	4.70	39.4	-39.4	50.52	-50.52	4.74

* Analyse the continuous beam shown in figure by kani's method



Fixed end moments

$M_{FAB} = \frac{-60\times4\times2^2}{6^2} = -26.67\,kNm$

$M_{FBA} = \frac{60\times4^2\times2}{6^2} = 53.33\,kNm$

$M_{FBC} = \frac{-20\times3^2}{12} = -15\,kNm$

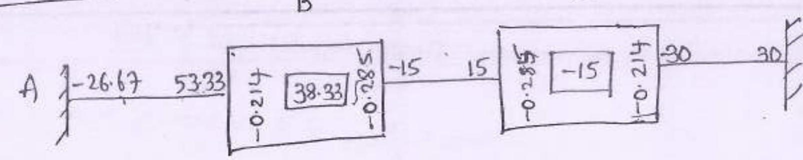
$M_{FCB} = 15\,kNm$

$M_{FCD} = \frac{-30\times8}{8} = -30\,kNm$ $M_{FDC} = +30\,kNm$

R. F.

Joints	members	k	Σk	R.
B	BA	$\frac{4E(1.5I)}{6} = EI$	2.33EI	-
	BC	$\frac{4EI}{3} = 1.33EI$		-
C	CB	1.33EI	2.33EI	-
	CD	$\frac{4E(2I)}{8} = EI$		-

Rotation table



-8.20	-10.92	7.38	5.54
-9.78	-13.02	8.00	6.00
-9.91	-13.20	8.03	6.03
$\boxed{-9.92}$	$\boxed{-13.21}$	$\boxed{8.04}$	$\boxed{6.04}$

Final moment Calculation:-

FM = FEM + 2x near end Contribution + far end

	A	B		C		D
FEM	-26.67	53.33	-15	15	-30	30
2X NEC	2x0	2x-9.92	2x-13.21	2x8.04	2x6.04	2x0
FEC	-9.92	0.	8.04	-13.21	0	6.04
Final moment	-36.60	33.49	-33.4	17.87	-17.92	36.04

Application to continuous beams with simply supported and overhanging ends:-

METHOD I:- If the end of the continuous beam is simply supported & has overhang, the last support also rotates. Hence, rotation contribution of that joint also should be found. Stiffness of overhanging portion may be taken as zero, since the moment in the overhanging portion does not depend upon the loading in the other portion. Then, rotation factor for interior member at such joint is −0.5.

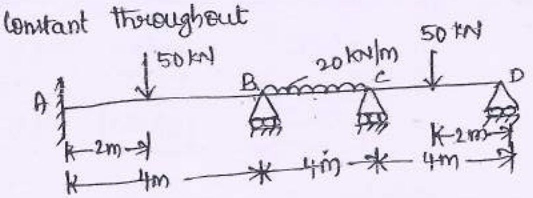
METHOD - II:-

$$K = \frac{3EI}{L}$$

FEM at the simply supported end is zero

$$M_{FAB} = M_{FAB} + 0.5 \times \text{balancing moment at B}$$
$$= M_{FAB} - 0.5 \times \text{unbalancing moment at B}$$

* (7) Analyse the continuous beam shown by Kani's method. Flexural rigidity is constant throughout



Fixed end moments:-

$$M_{FAB} = -\frac{50\times4}{8} = -25 \text{ KNm}$$
$$M_{FBA} = 25 \text{ KNm}$$
$$M_{FBC} = -\frac{20\times4^2}{12} = -26.6 \text{ KNm}$$
$$M_{FCB} = 26.67 \text{KNm}$$
$$M_{FCD} = -\frac{50\times4}{8} = -25 \text{ KNm}$$
$$M_{FDC} = 25 \text{ KNm}$$

METHOD - II

modification of FEM in the last span is required

$$M_{FCD} = M_{FCD} - 0.5\times \text{unbalanced moment}$$
$$= -25 - 0.5(25) = -37.5 \text{ KNm}$$

$$M_{FDC} = 0$$

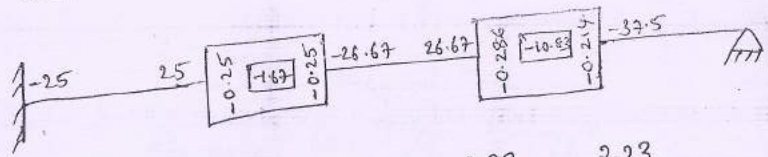
$$R.F = -\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$$

$$K \text{ in span } CD = \frac{3EI}{4}$$

Rotation factors:-

Joints	members	K	ΣK	R.
B	BA	$\frac{4EI}{4} = EI$	2EI	–
	BC	$\frac{4EI}{4} = EI$		–
C	CB	$\frac{4EI}{4} = EI$	1.75EI	–
	CD	$\frac{3EI}{4} = 0.75EI$		–

Iteration Contributions

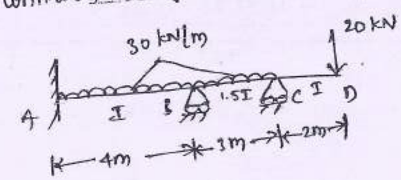


0.42	0.42	2.98	2.23
-0.33	-0.33	3.19	2.19
$\boxed{-0.38}$	$\boxed{-0.38}$	$\boxed{3.21}$	$\boxed{2.39}$

Final moment Calculations

	A	B		C		D
FEM	-25	25	-26.67	26.67	-37.5	0
NEC	2x0	2(-0.38)	2(-0.38)	2(3.21)	2(2.39)	2x0
FEC	-0.38	0	3.21	-0.38	0	0
Final	-25.38	24.24	-24.24	32.71	-32.71	0

* Analyse the continuous beam shown in figure



Fixed end moments

$M_{FAB} = -\frac{30 \times 4^2}{12} = -40$ kNm

$M_{FBA} = 40$ kNm

$M_{FBC} = -\frac{30 \times 3^2}{12} = -22.5$ kNm

$M_{FCB} = 22.5$ kNm

$M_{FCD} = -20 \times 2 = -40$ kNm

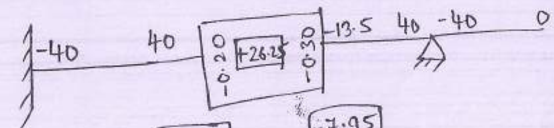
$M_{FBC} = -22.5 - 0.5(22.5 - 40) = -13.5$ kNm

$M_{FCB} = 40$ kN

Rotation factor $R_f = -\frac{1}{2}\left(\frac{k}{\Sigma k}\right)$

R.F.S

Joint	members	K	Σk	R.F
B	BA	$\frac{4EI}{4} = EI$	2.5EI	-0.2
	BC	$\frac{3E(1.5)}{3} = 1.5EI$		-0.3

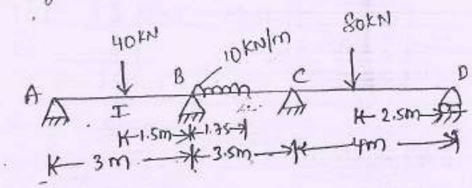


$\boxed{-5.3}$ $\boxed{-7.95}$

Final moment calculation :-

	A	B		C		D
FEM	-40	~~40~~	-13.5	40	-40	0
NEC	2x0	2(-5.3)	2(-7.95)	0	0	0
FEC	-5.33	0	0	0	0	0
FM	-45.33	35.66	-29.40	40	-40	0

Analyse the continuous beam shown in figure by [illegible]
constant throughout



Fixed end moments

$M_{FAB} = -\frac{40\times3}{8} = -15$ KNm

$M_{FBA} = +15$ KNm

$M_{FBC} = -\int_0^{1.75} \frac{10\,dx\; x(3.5-x)^2}{3.5^2}$ $\qquad \int \frac{wx(l-x)^2}{l^2}$

$= \frac{-10}{3.5^2}\int_0^{1.75} (3.5^2 x - 7x^2 + x^3)\,dx$

$= \frac{-10}{3.5^2}\left[3.5^2\left(\frac{x^2}{2}\right) - 7\left(\frac{x^3}{3}\right) + \frac{x^4}{4}\right]_0^{1.75}$ $\qquad$ (18 ... -1 ... 2

$= -22.33$ KNm $\quad = -\cancel{7}$

$M_{FCB} = \int_0^{1.75} \frac{10x^2(3.5-x)}{3.5^2}\,dx$ $\qquad \frac{wx^2(l-x)}{l^2}$

$= \frac{10}{3.5^2}\int_0^{1.75}(3.5x^2 - x^3)\,dx$

$= \frac{10}{3.5^2}\left[3.5\left(\frac{x^3}{3}\right) - \frac{x^4}{4}\right]_0^{1.75}$ $\qquad M_{FDC} = +\frac{80\times1.5^2\times2.5}{4^2} =$

$= 3.19$ KNm

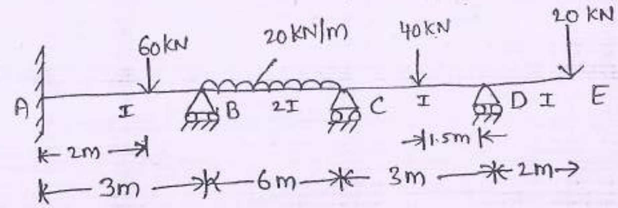
$M_{FCD} = -\frac{80\times1.5\times2.5^2}{4^2} = -46.88$ kNm

Modification to account for rotation A & D

$M_{FAB} = 0 \qquad M_{FDC} = 0$ $\qquad M_{FCD} = -46.88 - 0.$
$\qquad\qquad = -60.95$ KN

$M_{FBA} = 15 - 0.5(-15) = 22.5$ KNm

* Analyse the continuous beam shown in figure by Kani's method



Fixed end moments :-

$M_{FAB} = \frac{-60 \times 2 \times 1^2}{3^2} = -13.33$ kNm

$M_{FBA} = \frac{60 \times 2^2 \times 1}{3^2} = 26.67$ kNm

$M_{FBC} = \frac{-20 \times 6^2}{12} = -60$ kNm

$M_{FCB} = \frac{20 \times 6^2}{12} = 60$ kNm

$M_{FCD} = -\frac{40 \times 3}{8} = -15$ kNm

$M_{FDC} = 15$ kNm

$M_{FDE} = -2 \times 20 = -40$ kNm

Modification in FEM to account for rotation of D

$M_{FDC} = +40$ kNm (to balance joint C)

$M_{FCD} = -15 - 0.5(15 - 40) = -2.5$ kNm

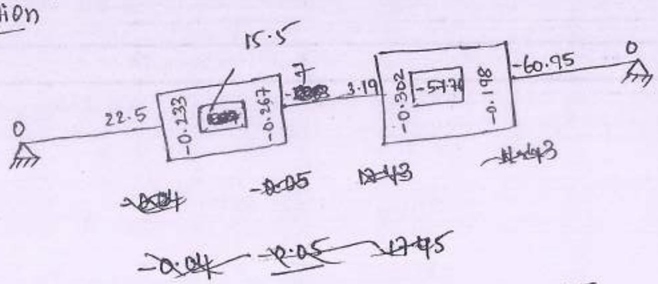
Rotation factor

$RF = -\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$

Rotation factor (RF):

Joints	members	K	ΣK	R.F
B	BA	$\frac{3EI}{3} = EI$	2.143EI	-0.233
	BC	$\frac{4EI}{3.5} = 1.143EI$		-0.267
C	CB	1.143 EI	1.893EI	-0.302
	CD	$\frac{3EI}{4} = 0.75EI$		-0.198

Rotation contribution

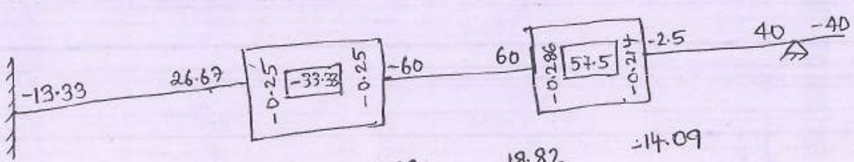


-3.61	-4.13	18.70	12.25
-7.96	-9.13	20.2	13.24
-8.31	-9.53	20.3	13.32
$\boxed{-8.34}$	$\boxed{-9.55}$	$\boxed{20.32}$	$\boxed{13.32}$

Final moments = FEM + 2 x NEC + FEC

Joints	A	B		C		D
FEM	0	22.5	-7	3.19	-60.95	0
2xNEC	2x0	2(-8.34)	2(-9.55)	2(20.32)	2(13.32)	2x0
FEC	0	0	20.32	-9.55	0	0
FM	0	5.82	-5.78	34.28	-34.31	0

Joint	members	K	ΣK	R.F
B	BA	$\frac{4EI}{3} = 1.33EI$	$\frac{8}{3}EI$ 2.66EI	-0.25
	BC	$\frac{4E(2I)}{6} = \frac{4}{3}EI = 1.33EI$		-0.25
C	CB	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	2.33EI	-0.286
	CD	$\frac{3EI}{3} = EI$		-0.214

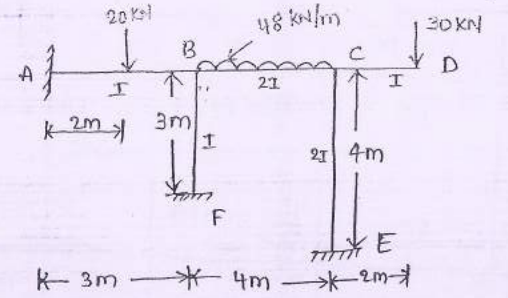


		-18.82	-14.09
8.33	8.33	-20.17	-15.10
13.04	13.04	-20.27	-15.17
13.38	13.38		
$\boxed{13.40}$	$\boxed{13.40}$	$\boxed{-20.27}$	$\boxed{-15.17}$

Final moment Calculations

	A	B		C		D		E
FEM	-13.33	26.67	-60	60	-2.5	40	-40	-
2×NEC	2×0	2×(13.40)	2×(13.40)	2(-20.27)	2(-15.17)	0	-	-
FEC	13.40	0	-20.28	13.40	0	-	-	-
Final	0.07	53.47	-53.48	32.86	-32.84	40	-40	

* Analyse the rigid frame shown in figure by Kani's method



Fixed end moments :-

$$M_{FAB} = \frac{-20 \times 2 \times 1^2}{3^2} = -4.44 \text{ kNm}$$

$$M_{FBA} = \frac{20 \times 2^2 \times 1}{3^2} = 8.89 \text{ kNm}$$

$$M_{FBC} = -\frac{48 \times 4^2}{12} = -64 \text{ kNm}$$

$$M_{FCB} = 64 \text{ kNm}$$

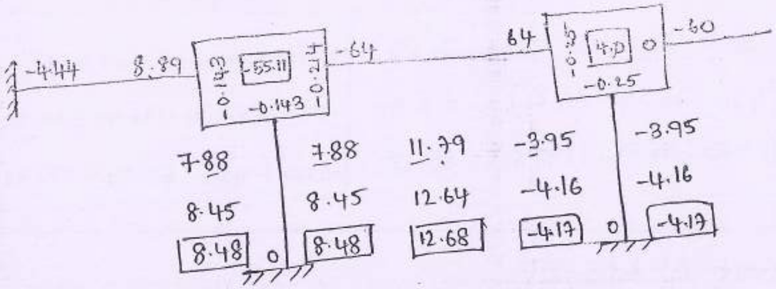
$$M_{FCD} = -30 \times 2 = -60 \text{ kNm}$$

$$M_{FBF} = M_{FFB} = M_{FCE} = M_{FEC} = 0$$

Rotation factor (RF) $= -\frac{1}{2}\left(\frac{k}{\Sigma K}\right)$

Rotations factors :-

Joint	members	K	ΣK	R.F
B	BA	$\frac{4EI}{3} = 1.33EI$	4.67EI	-0.143
	BF	$\frac{4EI}{3} = 1.33EI$		-0.143
	BC	$\frac{4E(2I)}{4} = 2EI$		-0.214
C	CB	2EI	4EI	-0.25
	CD	0		0
	CE	$\frac{4E(2I)}{4} = 2EI$		-0.25



inal moments

Joints	A	B			C			E	F
~~[illegible]~~	AB	BA	BF	BC	CB	CE	CD	EC	FB
FEM	-4.44	8.89	0	-64	64	0	-60	0	0
2xNEC	2x0	2x8.48	2x8.48	(2x12.68)	2(-4.17)	2(-4.17)	2x0	2x0	2x0
FEC	8.48	0	0	-4.17	12.68	0	0	-4.17	8.48
inal	4.04	25.85	16.96	-42.81	68.34	-8.34	-60	-4.17	8.48

$M_{FAB} = -4.44 + 2\times0 + 8.48 = 4.04$ KNm

$M_{FBA} = 8.89 + 2\times8.48 + 0 = 25.85$ KNm

$M_{BF} = 0 + 2\times8.48 + 0 = 16.96$ KNm

$M_{BC} = -64 + 2\times12.68 - 4.17 = -42.81$ KNm

$M_{CB} = 64 + 2(-4.17) + 12.68 = 68.34$ KNm

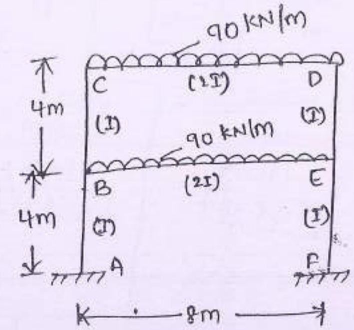
$M_{CD} = -60$ KNm

$M_{CE} = 0 + 2(-4.17) + \overset{0}{\cancel{8.48}} = -8.34$ KNm

$M_{EC} = 0 + 2\times0 - 4.17 = -4.17$ KNm

$M_{FB} = 0 + 2\times0 + 8.48 = 8.48$ KNm

* Analyse the symmetric frame shown in figure by Kani and indicate the final end moments on the sketch of the



$M_{FBE} = -\frac{90\times8^2}{12} = -480$ KNm

$M_{FEB} = 480$ KNm

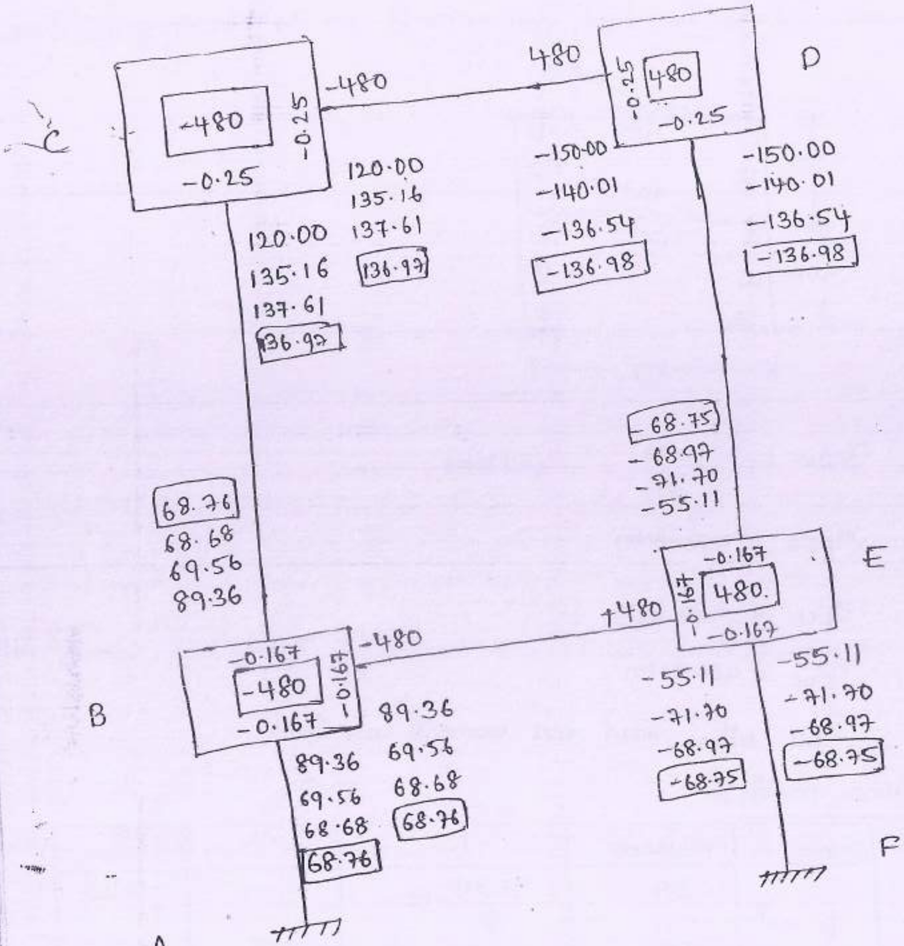
$M_{FCD} = -480$ KNm

$M_{FDC} = 480$ KNm

All other fixed end moments are zero

Rotations factors:-

Joints	members	K	ΣK	R.F
B	BA	$\frac{4E(I)}{4} = EI$	3EI	-0.167
	BE	$\frac{4E(2I)}{8} = EI$		-0.167
	BC	$\frac{4EI}{4} = EI$		-0.167
C	CB	$\frac{4EI}{4} = EI$	2EI	-0.25
	CD	$\frac{4E(2I)}{8} = EI$		-0.25
D	DC	$\frac{4E(2I)}{8} = EI$	2EI	-0.25
	DE	$\frac{4EI}{4} = EI$		-0.25
E	ED	$\frac{4EI}{4} = EI$	3EI	-0.167
	EB	$\frac{4E(2I)}{8} = EI$		-0.167
	EF	$\frac{4EI}{..} = EI$		-0.167



$-0.25(-480+0) = 120$

$-0.25(480+120) = -150$

$-0.167(480-150) = -55.11$

$-0.167(-480-55.11) = 89.36$

$-0.25(-480+\overset{-150}{89.36}) = 135.16$

$-0.25(480+\underset{-55.11}{135.16}) = -140.01$

$-0.167(480+140.01+89.36) = -71.70$

$-0.167(-480-71.70+135.16) = 69.56$

$-0.25(480+137.61-71.70) = -136.54$

$-0.167(480-136.54+69.56) = -68.97$

$-0.167(-480-68.97+137.61) = 68.68$

$-0.25(480+136.97-68.97$

$-0.167(480-136.98+6$

$-0.167(-480-68.75+13$

final moment Calculations :-

FM = FEM + 2×NEC + FEC

Joints	A	B			C		D		E	
members	AB	BA	BE	BC	CB	CD	DC	DE	ED	EB
FEM	0	0	-480	0	0	-480	480	0	0	480
2×NEC	0	2×68.76	2×68.76	2×68.76	2×136.97	2× 136.97	2× -136.98	2× -136.98	2× -68.75	2× -68.[illegible]
FEC	68.76	0	-68.75	136.97	68.76	-136.98	136.97	-68.75	-136.98	68.[illegible]
Final	68.76	137.52	-411.23	274.49	342.7	-343.04	343.01	-342.71	-274.48	411.[illegible]

Analysis of symmetric frames

Frames with symmetry do not have side sway

Such a case occurs

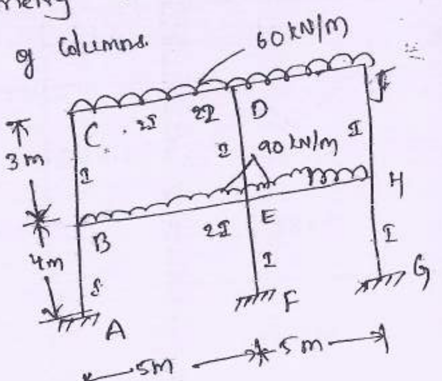
(a) line of symmetry passes through the columns. when the number of bays are even

(b) line of symmetry passes through the mid-span of the beams. Such a case occurs when the number of bays are odd.

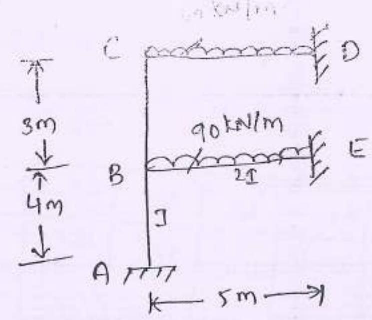
Analysis of symmetric frames when line of symmetry passes through columns.

Because of symmetry, the joints on the line of symmetry do not rotate. Hence they may be treated as fixed ends and only one half may be analysed. using symmetry, the bending moment in the columns through which the line of symmetry passes.

Analyse the symmetric frame shown in figure. Make use of the symmetry for the analysis given that the I of beams are twice that of columns.



Since, the line of symmetry passes through the columns DEF, joints D and E also will not rotate. only one half of frame may be considered for the analysis as shown in figure.



Fixed end moments

$M_{FCD} = \frac{-60 \times 5^2}{12} = -125$

$M_{FDC} = 125$ kNm

$M_{FBE} = -\frac{90 \times 5^2}{12} = -187$

$M_{FEB} = 187.5$ kNm

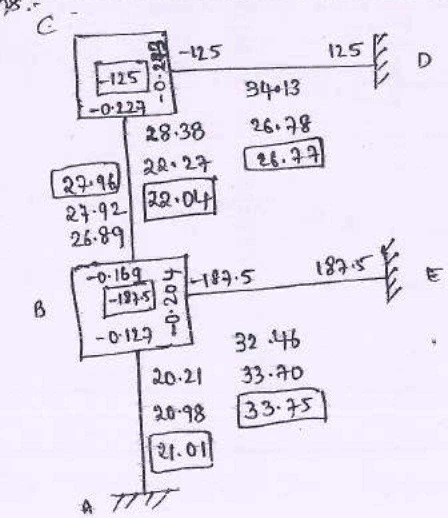
Fixed end moment at other ends = 0

Rotation factor (RF) $= -\frac{1}{2}\left(\frac{k}{\Sigma k}\right)$

R.F's :-

Joint	Members	K	Σk	R.F
B	BA	$\frac{4EI}{4} = EI$	3.93 EI	-0.127
	BE	$\frac{4E(2I)}{5} = 1.6EI$		-0.204
	BC	$\frac{4EI}{3} = 1.33EI$		-0.169
C	CB	$\frac{4E(I)}{3} = 1.33EI$	2.93EI	-0.227
	CD	$\frac{4E(2I)}{5} = 1.6EI$		-0.273

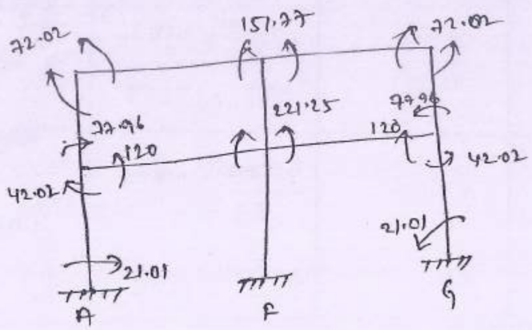
Rotation Contributions :-



inal moments

Joints	A	B			C		D	E
members	AB	BA	BE	BC	CB	CD	DC	EB
FEM	0	0	-187.5	0	0	-125	125	187.5
2×NEC	2x 0	2x21.01	2x 33.75	2x27.96	2x22.04	2x26.77	0	0
FEC	21.01	0	0	22.04	27.96	0	26.77	33.75
	21.01	42.02	-120	77.96	72.04	-71.46	151.77	221.25

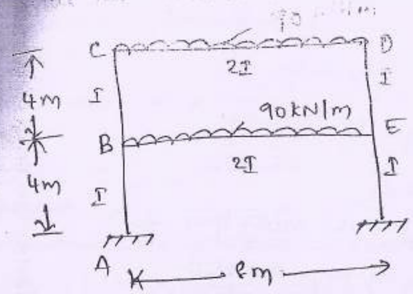
using symmetry, these values are marked on other part of frame also



* Analysis of Symmetric frames when line of Symmetry passes through mid span of Beams.:-

If k is the stiffness of original beam, this beam may be replaced by another beam having its stiffness $\frac{k}{2}$, with fixed end at mid span

$\therefore k' = \frac{k}{2}$



$M_{FBE} = -\frac{90\times8^2}{12} = -$

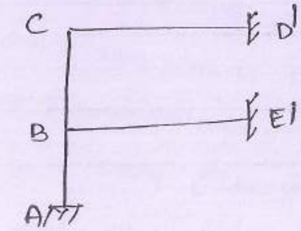
$M_{FEB} = 480 \text{ kNm}$

$M_{FCD} = -480 \text{ kNm}$

$M_{FDC} = 480 \text{ kNm}$

Fixed end moments of all other beam ends are zero

Now using symmetry, one half of the frame,



$k' = \frac{k}{2} = \frac{\frac{4E(2}{8}}{2}$

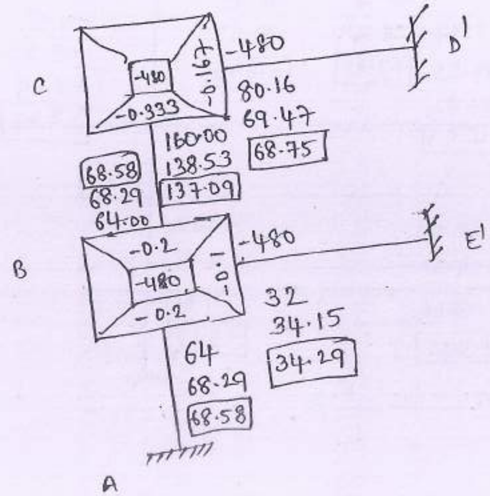
$\therefore k' = 0.5EI$

Rotation factors:-

Joint	members	K	ΣK	R
C	CD'	0.5EI	1.5EI	-0.
	CB	$\frac{4EI}{4} = EI$		-0.33
B	BC	$\frac{4EI}{4} = EI$	2.5EI	-0.2
	BE'	0.5EI		-0.1
	BA	$\frac{4EI}{4} = EI$		-0.2

ation Contributions

Rotations are started considering Joint first following which is Joint B.



Final moments

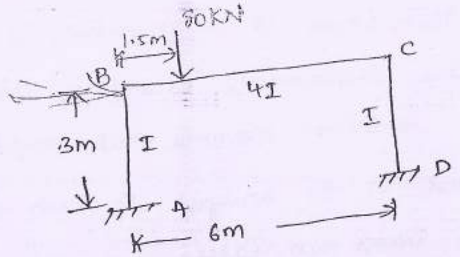
Joints	A	B			C		D'	E'
members	AB	BA	BE'	BC	CB	CD'	D'C	E'B
FEM	0	0	-480	0	0	-480	0	0
2xNEC	0	2x68.58	2x34.29	2x68.58	2x137.09	2x68.75	0	0
FEC	68.58	0	0	137.09	68.58	0	68.75	34.29
FM	68.58	137.16	411.42	274.25	342.76	-342.76	68.75	34.29

Analysis of frames with sway

If the frame is unsymmetric, it wi laterally, causing relative displacements at ends of columns lateral sway causes additional moments, which may be ca displacement contributions. In the analysis of such frame have sway, we come across two cases:

1. Heights of all columns in a storey are the same.
2. Heights of columns in storey are different.

Using Kani's rotational contribution method, analyse the frame shown in figure



Fixed end moments :-

$$M_{FBC} = -\frac{80 \times 1.5 \times 4.5^2}{6^2} = -67.5 \text{ kNm}$$

$$M_{FCB} = \frac{80 \times 1.5^2 \times 4.5}{6^2} = 22.5 \text{ kNm}$$

Fixed end moments in both the columns are zero

Rotation factor (RF) $= \frac{1}{2}\frac{K}{\Sigma K}$

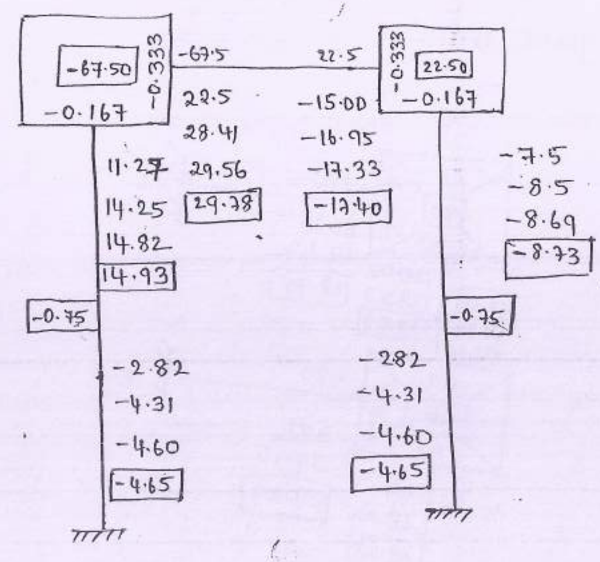
Joints	members	K	ΣK	RF
B	BA	$\frac{4EI}{3} = 1.33EI$	4EI	−0.167
	BC	$\frac{4E(4I)}{6} = 2.67EI$		−0.333
C	CB	$\frac{4E(4I)}{6} = 2.67EI$	4EI	−0.333
	CD	$\frac{4EI}{3} = 1.33EI$		−0.167

Displacement factors (DF) $= -\frac{3}{2}\left(\frac{K}{\Sigma K}\right)$

Storey	members	K	ΣK	DF
I	AB	$\frac{4EI}{3} = 1.33EI$	$\frac{8EI}{3} = 2.66EI$	−0.75
	DC	$\frac{4EI}{3} = 1.33EI$		−0.75

Shear, $S_r = 0$, then moment $= \frac{1}{3} S_r h_r = 0$

Rotation and displacement contributions :-



I

$-0.167(-67.50) = 11.27$
$-0.333(-67.50) = 22.5$
$-0.333(22.50+22.50) = -15.00$
$-0.167(22.50+22.50) = -7.5$

$-0.75(11.27-7.5) = -2.82$

II

$-0.167(-67.50-15-2.82) = 14.25$
$-0.333(-67.50-15-2.82) = 28.41$
$-0.333(22.50+28.41) = -16.95$
$-0.167(22.50+28.41) = -8.5$

$-0.75(14.25-8.5) = -4.31$

III

$-0.167(-67.50-16.95-4.31) = 14.82$
$-0.333(-67.50-16.95-4.31) = 29.56$
$-0.333(22.50+29.56) = -17.33$
$-0.167(22.50+29.56) = -8.69$

$-0.75(14.82-8.69) = -4.60$

$-0.167(-67.50-17.33-4.60) = 14.93$

$-0.333(-67.50-17.33-4.60) = 29.78$

$-0.333(22.50+29.78) = -17.40$

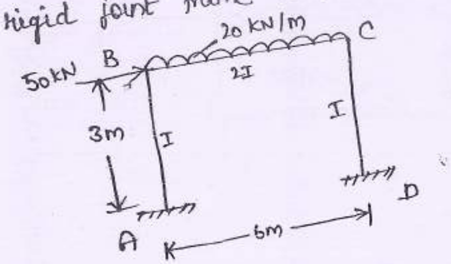
$-0.167(22.50+29.78) = -8.73$

$-0.75(14.93-8.73) = -4.65$

Final moment

Joint	A	B		C		D
members	AB	BA	BC	CB	CD	DC
FEM	0	0	-67.5	22.5	0	0
2xNEC	2x0	2x14.93	2x29.78	2(-17.40)	2(-8.73)	2x0
FEC	14.93-4.65	-4.65	-17.40	29.78	-4.65	-8.73-4.65
FM	10.28	25.21	-25.34	17.48	-22.11	-13.38

* Analyse the rigid joint frame shown in figure by Kani's method



Fixed end moments:

$$M_{FBC} = -\frac{20\times 6^2}{12} = -60 \text{ kNm}$$

$$M_{FCB} = \frac{20\times 6^2}{12} = 60 \text{ kNm}$$

other FEMs are zero

Rotation factors (RF) $= -\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$

Joint	members	K	ΣK	RF
B	BA	$\frac{4EI}{3} = 1.33EI$	2.66EI	-0.25
	BC	$\frac{4E(2I)}{6} = 1.33EI$		-0.25
C	CB	$\frac{4E(2I)}{6} = 1.33EI$	2.66EI	-0.25
	CD	$\frac{4EI}{3} = 1.33EI$		-0.25

Displacement factors (DF) $= \frac{-3}{2}\left(\frac{K}{\Sigma K}\right)$

Storey	members	K	ΣK	DF
1	AB	$\frac{4EI}{3} = 1.33EI$	2.66EI	-0.75
	DC	$\frac{4EI}{3} = 1.33EI$		-0.75

Storey Shear $= 50$ KN

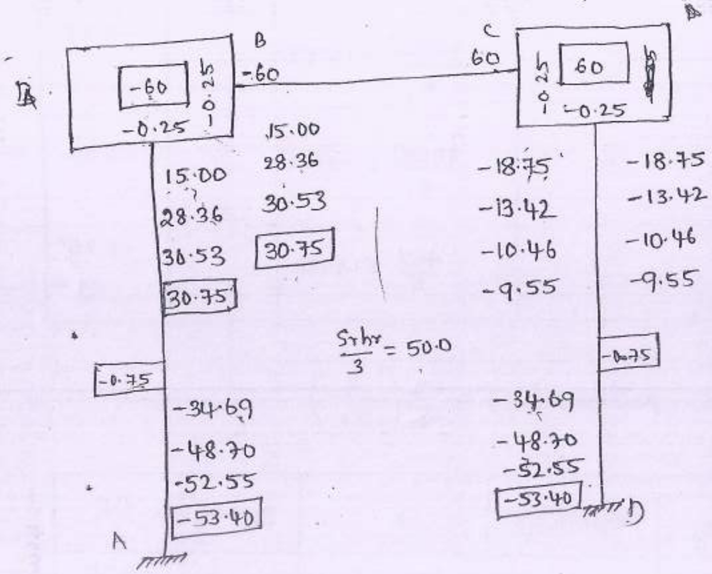
$\therefore$ storey moment $= \frac{S_r h_r}{3} = \frac{50\times 3}{3} = 50$ kNm

Distribution procedure

It first starts from end B and then proce[eds to] end C. After obtaining the rotation contributions are to be foun[d]

Displacement contribution = DF (storey moment + ΣRotation at top and [bottom] of the col[umn]

Rotation and displacement contributions:-



I. $-0.25(-60+0) = 15.00$ | $-0.75(50+15-18.75) = -34.69$

$-0.25(60+15) = -18.75$

II. $-0.25(-60 - 18.75 - 34.69) = 28.36$ | $-0.75(50+28.36-13.42) = -48.70$

$-0.25(60+28.36-34.69) = -13.42$

III. $-0.25(-60-13.42-48.70) = 30.53$ | $-0.75(50+30.53-10.46) = -52.55$

$-0.25(60+30.53-48.70) = -10.46$

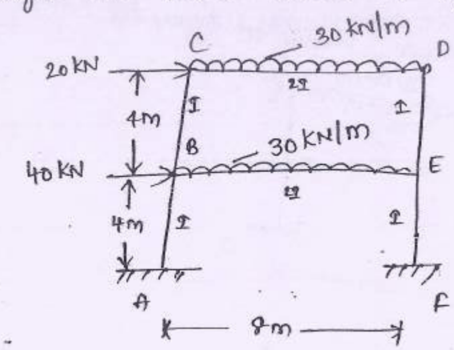
$-0.25(-60-10.46-52.55) = 30.75$ | $-0.75(50+30.75-9.5$

$-0.25(60+30.75-52.55) = -9.55$ | $=$

Final moments

	AB	BA	BC	CB	CD	DC
FEM	0	0	-60	60	0	0
2xNEC	2x0	2x30.75	2x30.75	-2x9.55	-2x9.55	2x0
FEC	30.75 -53.40	-53.40	-9.55	30.75	-53.40	-9.55 -53.40
Final moment	-22.65	8.1	-8.05	71.65	-72.5	-62.95

* Analyse the frame shown in figure by Kani's method



FEM:-

FEM in columns = 0

$M_{FCD} = -\frac{30 \times 8^2}{12} = -160$ KNm

$M_{FDC} = 160$ KNm

$M_{FBE} = -160$ KNm

$M_{FEB} = 160$ KNm

Rotation factors (RF) = $-\frac{1}{2}\frac{K}{\Sigma K}$

Joint	members	K	ΣK	R.F
B	BA	$\frac{4EI}{4} = EI$	3EI	−0.167
	BE	$\frac{4E(2I)}{8} = EI$		−0.167
	BC	$\frac{4EI}{4} = EI$		−0.167
C	CB	$\frac{4EI}{4} = EI$	2EI	−0.25
	CD	$\frac{4E(2I)}{8} = EI$		−0.25
D	DC	$\frac{4E(2I)}{8} = EI$	2EI	−0.25
	DE	$\frac{4E(I)}{4} = EI$		−0.25
E	ED	EI	3EI	−0.167
	EB	EI		−0.167
	EF	EI		−0.167

Displacement factors (DF):-

$$DF = -\frac{3}{2}\frac{K}{\Sigma K}$$

Storey	members	K	ΣK	DF
I	AB	EI	2EI	−0.75
	FE	EI		−0.75
II	BC	EI	2EI	−0.75
	ED	EI		−0.75

Storey Moments

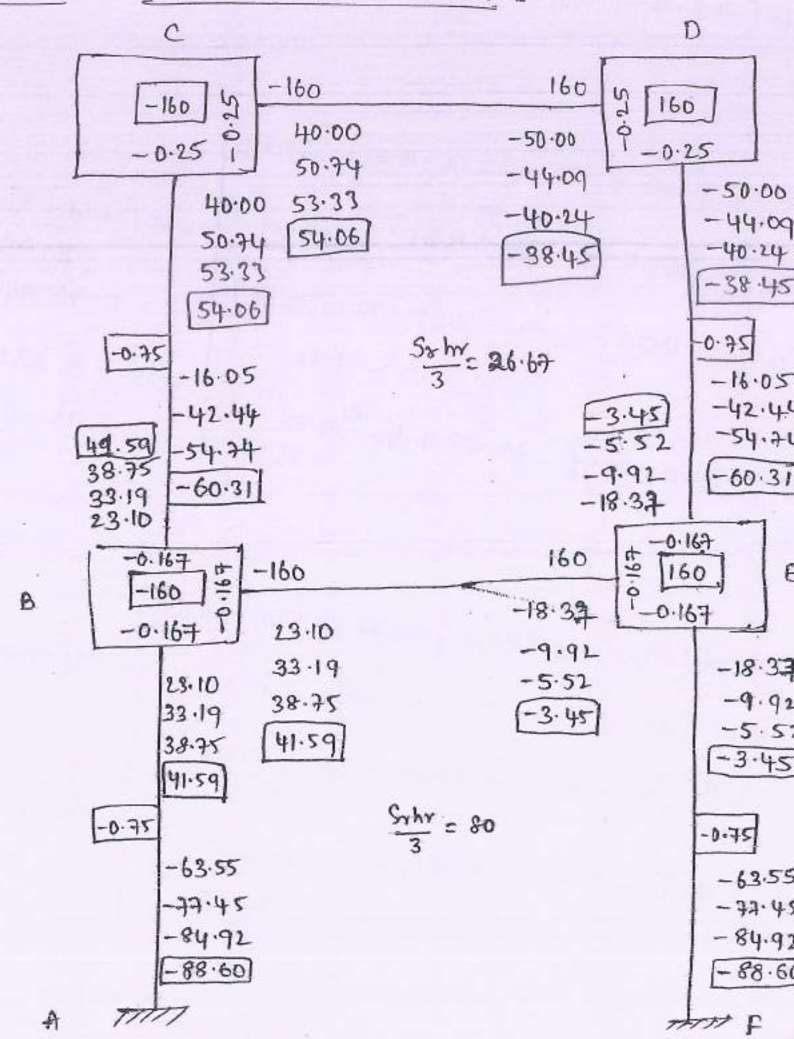
Storey Shear $S_{r_1} = 20 + 40 = 60\text{ kN}$

$S_{r_2} = 20\text{ kN}$

Shear moment in storey I = $\frac{S_{r_1} h_r}{3} = \frac{60 \times 4}{3} = 80$

Store II $= \frac{1}{3} S_{r_2} h_r = \frac{20 \times 4}{3} = 26$

Rotation and Displacement Contributions:-



$-0.25(-160+0) = 40.00$

$-0.25(-160+0) = 40$

$-0.25(160+40) = -50$

$-0.25(160+40) = -50$

$-0.167(160-50) = -18.37$

$-0.167(-160-18.37+40) = 23.10$

$-0.75\left(\overset{26.67+}{\cancel{40}}_{-18} - \cancel{50.00}\right)$

$-0.75(26.67+40-50-18.37+23.10)$
$= -16.05$

$-0.75(80+23.10-18.37) = -63.55$

$-0.25(-160-50+23.10-16.05) = 50.74$

$-0.25(160+50.74-18.33-16.05) = -44.09$

$-0.167(160-44.09-16.05+23.10-63.55)$
$= -9.92$

$-0.167(-160-9.92-63.55+50.74-16.05)$
$= 33.19$

$-0.75(26.67+50.74-9.92-44.09+33.19)$
$= -42.44$

$-0.75(80+33.19-9.92)$
$= -77.45$

---

ii

$-0.25(-160-44.09+33.19-42.44) = 53.33$

$-0.25(160+53.33-9.92-42.44) = -40.24$

$-0.167(160-40.24-42.44+33.19-77.45)$
$= -5.52$

$-0.167(-160-5.52-77.45+53.33-42.44)$
$= 38.75$

$-0.75(26.67+53.33-5.52-40.24+38.75)$
$= -54.74$

$-0.75(80+38.75-5.52)$
$= -84.92$

$-0.25(-160-40.24+38.75-54.74) = 54.06$

$-0.25(160+54.06-5.52-54.74) = -38.45$

$-0.167(160-38.45-54.74+38.75-84.92) = 3.45$

$-0.167(-160-3.45-84.92+54.06-54.74) = 41.59$

$-0.75(26.67\ldots$
$-3.4\ldots$
$+\ldots$
$= -\ldots$

$-0.75(80\ldots$

$= -88\ldots$

Final moments $= FEM + 2 \times NEC + FEC$

$M_{AB} = 0 + 2\times 0 + 41.59 - 88.60 = -47.01$ KNm

$M_{BA} = 0 + 2\times 41.59 + 0 - 88.60 = -5.42$ KNm

$M_{BE} = -160 + 2\times 41.59 - 3.45 + 0 = -80.27$ KNm

$M_{BC} = 0 + 2\times \overset{41.59}{\cancel{54.07}} + 54.06 - 60.31 = 76.93$ KNm

$M_{CB} = 0 + 2\times 54.06 + 41.59 - 60.31 = 89.4$ KNm

$M_{CD} = -160 + 2\times 54.06 - 38.45 + 0 = -90.33$ KNm

$M_{DC} = 160 - 2\times 38.45 + 54.06 + 0 = 137.16$ KNm

$M_{DE} = 0 - 2\times 38.45 - 60.31 - 3.45 = -140.66$ KNm

$M_{ED} = 0 - 2\times 3.45 - 38.45 - 60.31 = -105.66$ KNm

$M_{EB} = 160 - 2\times 3.45 + 41.59 + 0 = 194.7$ KNm

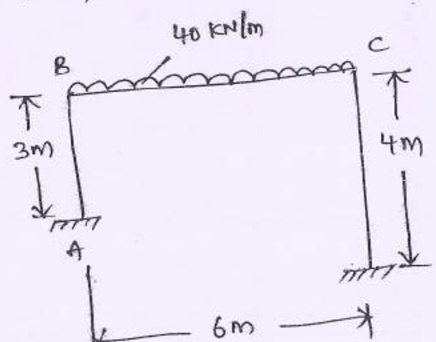
$M_{EF} = 0 - 2\times 3.45 - 88.60 = -95.5$ KNm

$M_{FE} = 0 + 2\times 0 - 3.45 - 88.60 = -92.05$ KNm nal moments = FEM + 2xNEC + FEC

Joints	A	B			C		D	
members	AB	BA	BE	BC	CB	CD	DC	DE
FEM	0	0	-160	0	0	-160	160	0
2x NEC	2x0	2x41.59	2x 41.59	2x 41.59	2x 54.06	2x 54.06	-2x38.45	-2x38.45
FEC	41.59	0	-3.45	54.06	41.59	-38.45	54.06	-3.45
DEC	-88.60	-88.60	0	-60.31	-60.31	0	0	-60.31
	-47.01	-5.42	-80.27	76.93	89.4	~~[illegible]~~ -90.33	137.16	-140.66

E			F
ED	EB	EF	FE
0	160	0	0
-2x3.45	-2x3.45	-2x3.45	2x0
-38.45	41.59	0	-3.45
-60.31	0	-88.60	-88.60
-105.66	194.7	-95.5	-92.05

* Analysis of frames with sway in figure by Kani's method.



Fixed end moments

Fixed end moments in columns = 0

$$M_{FBC} = \frac{-40\times6^2}{12} = -120 \text{ kNm}$$

$$M_{FCB} = 120 \text{ KNm}$$

Rotation factor (RF): $R\cdot F = -\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$

Joint	members	K	ΣK	R·F
B	BA	$\frac{4EI}{3} = 1.33EI$	2.66EI	-0.25
	BC	$\frac{4E(2I)}{6} = 1.33EI$		-0.25
C	CB	$\frac{4E(2I)}{6} = 1.33EI$	2.33EI	-0.286
	CD	$\frac{4EI}{4} = EI$		-0.214

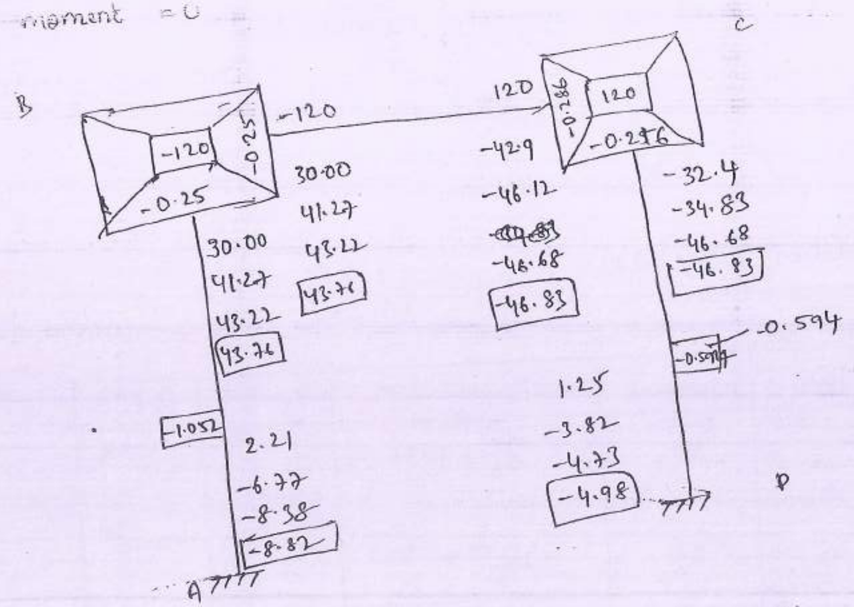
Displacement factor (DF) $= -\frac{3}{2}\left(\frac{KC_{pq}}{\Sigma C_{pq}^r K}\right)$

Let the storey height be h_r be 3m

$C_{AB} = \frac{3}{3} = 1.0$ $\qquad C_{CD} = \frac{3}{4} = 0.75$

Storey	members	K	C_{pq}	$C_{pq}^r K$	$\Sigma C_{pq}^r K$
I	AB	$\frac{4EI}{3} = 1.33EI$	1.0	1.33	1.8925
	DC	$\frac{4EI}{4} = EI$	0.75	0.56	

moment = 0



I
$-0.25(-120) = 30.00$ | $-1.052(30-32.1) = 2.21$

$-0.286(120+30) = -42.9$ | $-0.594(30-32.1) = \cancel{-1.25}$

$-0.216(120+30) = -32.4$

II
$-0.25(-120 - 42.9 - 2.21) = 41.27$ | $-1.052(41.27-34.83) = -6.77$

$-0.286(120+41.27) = -46.12$ | $-0.594(41.27-34.83) = -3.82$

$-0.216(120+41.27) = -34.83$

III
$-0.25(-120 - 46.12 - 6.77) = 43.22$ | $-1.052(43.22-35.25) = -8.38$

$-0.286(120+43.22) = -46.68$ | $-0.594(43.22-35.25) = -4.73$

$-0.216(120+43.22) = -35.25$

IV
$-0.25(-120 - 46.68 - 8.38) = 43.76$ | $-1.052(43.76-35.37) = -8.8$

$-0.286(120+43.76) = -46.83$ | $-0.594(43.76-35.37) = -4.$

$-0.216(120+43.76) = 35.37$

Final moments = FEM + 2xNEC + FEC

Joints	A	B		C		D
members	AB	BA	BC	CB	CD	DC
FEM	0	0	-120	120	0	0
2xNEC	2x0	2x43.76	2x43.76	2x-46.83	2x-46.83	2x0
FEC	43.76	2x0	-46.83	43.76	0	-46.83
DEC	-8.82	-8.82	0	0	-4.98	-4.98
Final	34.94	78.7	-79.31	70.01	-98.64	-51.81

UNIT 4

FLEXIBILITY MATRIX METHOD

- Flexibility matrix method is also known as force method 'Compatability' or method of 'Consistant displacements' 'Released structure Method' or Base structure method.

- In this method, Basic unknowns are Redudant forces (
Hence the analysis has to first identify basic static in
of the structure. The no. of Redundant forces is equal to De
Static indeterminacy. From the principle of super positio
displacement at any point in a statically determinate
is the sum of the displacements in the basic indeterm
structure due to applied loads & Redundant forces

$$\therefore \quad \Delta = [\delta][P] + [\Delta L]$$

This equation is called as Compatibility equati

Flexibility matrix :-

If a structure has 'n' no. of Co-ordi
the displacement matrix is represent as

$$[\delta] = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \delta_{23} & \cdots & \delta_{2n} \\ \vdots \\ \delta_{n1} & \delta_{n2} & \delta_{n3} & \cdots & \delta_{nn} \end{bmatrix}$$

If $n = 2$

$$[\delta] = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}_{2\times 2}$$

If $n = 3$,

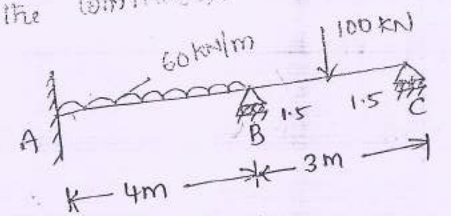
$$[\delta] = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix}_{3\times3}$$

Procedure:- 1. Determine the degree of static indeterminacy n

- Choose the Redundants
- Assign the coordinates to the redundant force directions & draw released beam diagram
- Find the deflection matrix $[\Delta_L]$
- Determine the flexibility matrix by applying a unit force to the co-ordinates
- Applying compatability conditions to the compute

$$[P] = [\delta^{-1}][\Delta - \Delta_L]$$

Analyse the continuous beam shown below ...



Fixed - 3
Roller - 1
Hinged - 2

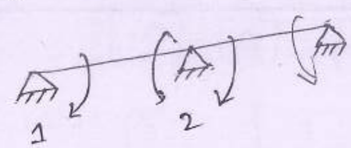
STEP 1:- <u>Degree of static indeterminacy</u>:-

Number of reaction components = 3+1+1 = 5

Number of independent equilibrium equations = 3

$\therefore$ Degree of static indeterminacy = 5-3 = 2

STEP 2:- Assign the co-ordinates & Draw released beam diagram



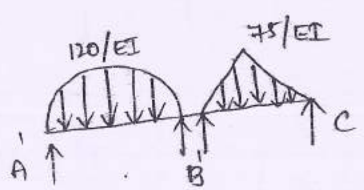
STEP 3:- <u>Conjugate Beam diagram</u>:-

Free moment diagram in AB = $\frac{wl^2}{8} = \frac{60\times4^2}{8} = 120$ kNm

" — in BC = $\frac{WL}{4} = \frac{100\times3}{4} = 75$ kNm

$\therefore$ conjugate beam $= \frac{M}{EI}$

$\therefore AB = \frac{120}{EI}$ and $BC = \frac{75}{EI}$



2/64
3

Step 4:- Displacement matrix $[\Delta_L]$

Δ_L depends on no. of redundant forces. In this case $n=2$

$$\therefore \Delta_L = \begin{bmatrix} \Delta_{L1} \\ \Delta_{L2} \end{bmatrix}$$

Δ_{L1} = Shear in conjugate beam at A

$\Delta_{L1} = \frac{1}{2}\left[\text{area of conjugate beam diagram}\right]$

$$= \frac{1}{2}\left[\frac{2}{3}\times4\times\frac{120}{EI}\right] = \frac{160}{EI}$$

Δ_{L2} = Rotation at B in A'B' + Rotation at B in B'C'

= Shear at B in A'B' + Shear at B in B'C'

$$= \frac{1}{2}\left(\frac{2}{3}\times4\times\frac{120}{EI}\right) + \frac{1}{2}\left[\frac{1}{2}\times3\times\frac{75}{EI}\right]$$

$$= \frac{160}{EI} +$$

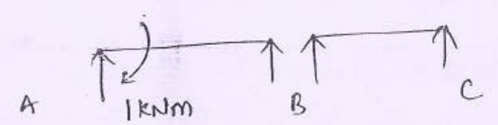
$$\Delta_{L2} = \frac{216.25}{EI} \qquad \therefore \Delta_L = \begin{bmatrix} 160/EI \\ 216.25/EI \end{bmatrix}$$

Step 5:- Determine the flexibility matrix by applying a unit force to the coordinates.

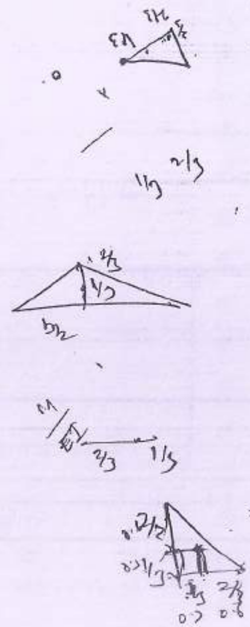
if $n=2$, $\quad \delta = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}_{2\times2}$

Applying a unit force at the coordinate at po[int] ...

then



then released structure & conjugate beam

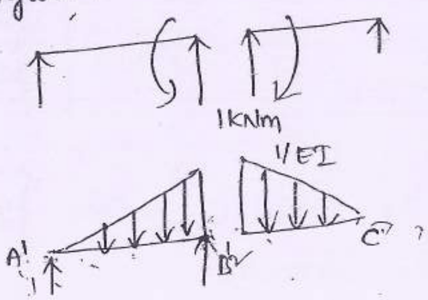


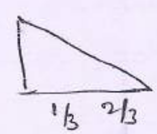
$\delta_{11} = ?$ $\delta_{21} = ?$

$$\delta_{11} = \frac{2}{3}\left(\frac{1}{2} \times 4 \times \frac{1}{EI}\right) = \frac{4}{3EI}$$

$$\delta_{21} = \frac{1}{3}\left(\frac{1}{2} \times 4 \times \frac{1}{EI}\right) = \frac{2}{3EI}$$

III'ly, a unit load is applied in coordinate direction 2 as shown in figure.





$\delta_{22} = ?$ $\delta_{12} = ?$

$$\delta_{12} = \frac{1}{3}\left(\frac{1}{2} \times 4 \times \frac{1}{EI}\right) = \frac{2}{3EI}$$

and

$$\delta_{22} = \frac{2}{3}(A_1) + \frac{2}{3}(A_2)$$

$$= \frac{4}{3EI} + \frac{3}{3EI}$$

$$\delta_{22} = \frac{7}{3EI}$$

Step 6 :-

Let redundant forces matrix, if $n=2$ $[P] = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$

For consistency conditions, final displacements

$$\Delta_1 = 0 \text{ \& } \Delta_2 = 0$$

$\therefore$ The matrix equation is

$$\Delta = [\delta][P] + [\Delta_L]$$

$$[\delta][P] = [\Delta] - [\Delta_L]$$

$$\begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \Delta_{L_1} \\ \Delta_{L_2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{3EI} & \frac{2}{3EI} \\ \frac{2}{3EI} & \frac{7}{3EI} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} -\Delta_{L_1} \\ -\Delta_{L_2} \end{bmatrix}$$

$$\frac{1}{3EI} \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} -160/EI \\ -216.5/EI \end{bmatrix}$$

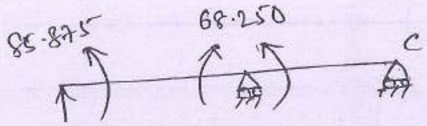
$$\frac{1}{3EI} \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -160 \\ -216.5 \end{bmatrix}$$

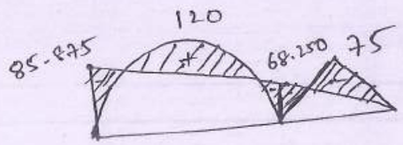
$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = 3 \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}^{-1} \begin{bmatrix} -160 \\ -216.5 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{3}{4\times7-2\times2}\begin{bmatrix} 7 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -160 \\ -216.5 \end{bmatrix}$$

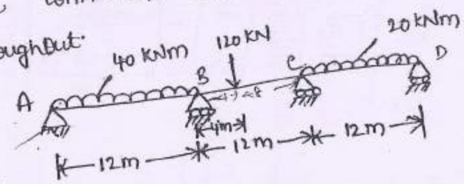
$$= \frac{1}{8}\begin{bmatrix} -687 \\ -546 \end{bmatrix} = \begin{bmatrix} -85.875 \\ -68.250 \end{bmatrix}$$

Final moments and BMD's are





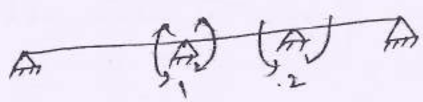
Analyse the Continuous Beam shown in figure by FMM. Take EI constant throughout.



Step 1:- $5-3=2$

The moment at B and C are taken as redundant forces.

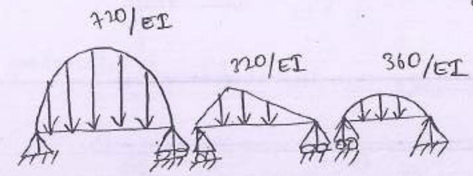
Step 2:- Assign the co-ordinates & Draw released beam diagram



Free moment diagram in AB = $\frac{40\times12^2}{8}$ = ~~480~~ 720 kN

Free moment diagram in BC = $\frac{120\times4\times8}{12}$ = 320 kNm

Free moment diagram in CD = $\frac{20\times12^2}{8}$ = 360 kNm



STEP 4:- Displacement matrix $[\Delta_L]$

Δ_L depends upon no. of redundant forces. $n=2$

$$\Delta_L = \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$$

Δ_1 = Displacement in co-ordinate direction 1

= Rotation of AB at B + Rotation of BC at B

= Shear force in conjugate beam in AB at B

+ Shear force in conjugate beam in BC a[t B]

$$= \frac{1}{2}\left[\frac{2}{3}\times 12\times\frac{720}{EI}\right] + \frac{1}{2}\left[\frac{12+8}{3}\times\frac{320}{EI}\right]$$

Area o[f] loading = [...]

$$= \frac{2880}{EI} + \frac{1066.66}{EI} = \frac{3946.6}{EI}$$

Δ_2 = Displacement in co-ordinate direction 2

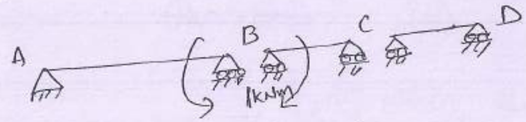
$$= \frac{1}{2}\left[\frac{12+4}{3}\times\frac{320}{EI}\right] + \frac{1}{2}\left[\frac{2}{3}\times12\times\frac{360}{EI}\right] = \frac{2293.33}{EI}$$

$$\Delta_L = \begin{bmatrix} \Delta_{L_1} \\ \Delta_{L_2} \end{bmatrix} = \begin{bmatrix} \frac{3946.67}{EI} \\ \frac{2293.3}{EI} \end{bmatrix}$$

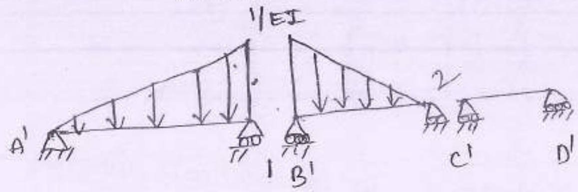
Step 5 :- ~~Step~~ Determine the flexibility matrix applying unit force for the co-ordinates

if $n = 2$, $\delta = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}_{2\times2}$

Applying a unit force at the coordinate direction 1



Released structure

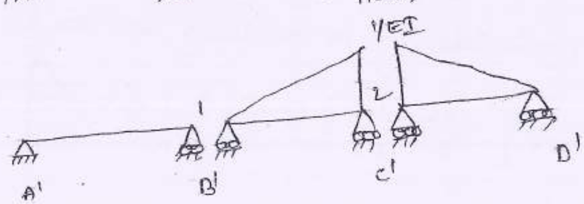


$$\delta_{11} = \frac{2}{3}(A_L) + \frac{2}{3}(A_R)$$

$$= \frac{2}{3}\left(\frac{1}{2} \times 12 \times \frac{1}{EI}\right) + \frac{2}{3}\left(\frac{1}{2} \times 12 \times \frac{1}{EI}\right) = \frac{8}{EI}$$

$$\delta_{21} = \frac{1}{3}\left(\frac{1}{2} \times 12 \times \frac{1}{EI}\right) = \frac{2}{EI}$$

Applying a unit force at the coordinate direction 2



$$\delta_{12} = \frac{1}{3}\left(\frac{1}{2} \times 12 \times \frac{1}{EI}\right) = \frac{2}{EI}$$

&

$$\delta_{22} = \frac{2}{3}\left(\frac{1}{2} \times 12 \times \frac{1}{EI}\right) + \frac{2}{3}\left(\frac{1}{2} \times 12 \times \frac{1}{EI}\right)$$

$$= \frac{8}{EI}$$

STEP 6 :-

$n = 2$, $P = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}$

$$\Delta = \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

∴ The matrix equation is

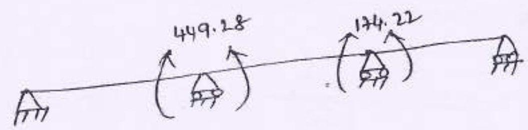
$$\Delta = [\delta][P] + [\Delta_L]$$

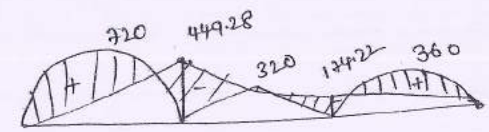
$$[\delta][P] = [\Delta] - [\Delta_L]$$

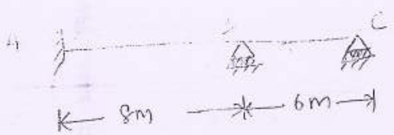
$$\begin{bmatrix} \frac{8}{EI} & \frac{2}{EI} \\ \frac{2}{EI} & \frac{8}{EI} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{3946.67}{EI} \\ \frac{2293.33}{EI} \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = -\begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 3946.67 \\ 2293.33 \end{bmatrix}$$

$$= -\frac{1}{64-4}\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix}\begin{bmatrix} 3946.67 \\ 2293.33 \end{bmatrix} = \begin{bmatrix} -449. \\ -174 \end{bmatrix}$$







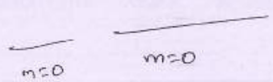
'B' sinks by 10 mm

$\leftarrow 8m \rightarrow \leftarrow 6m \rightarrow$

Step 1: $D.S = 5 - 3 = 2$

Step 2: Release beam diagram

Step 3: Conjugate Beam diagram

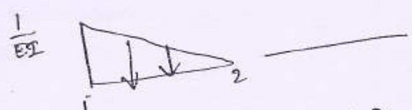


Step 4: Displacement matrix $[\Delta L]$

$n = 2$

$$\delta = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

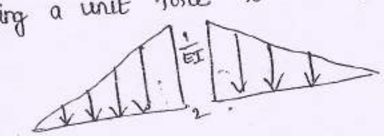
applying a unit force to the point 1



$$\delta_{11} = \frac{2}{3}\left(\frac{1}{2} \times 8 \times \frac{1}{EI}\right) = \frac{8}{3EI}$$

$$\delta_{21} = \frac{1}{3}\left(\frac{1}{2} \times 8 \times \frac{1}{EI}\right) = \frac{4}{3EI}$$

applying a unit force to the point 2



$$\delta_{12} = \frac{1}{3}\left(\frac{1}{2} \times 8 \times \frac{1}{EI}\right) = \frac{4}{3EI}$$

$$\delta_{22} = \frac{2}{3}\left(\frac{1}{2} \times 6 \times \frac{1}{EI}\right) + \frac{2}{3}\left(\frac{1}{2} \times 8 \times \frac{1}{EI}\right) = \frac{14}{3EI}$$

Step 5

$[P] = [\delta]^{-1}[\Delta - \Delta L]$

$$\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0.01 \end{bmatrix} - \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8/3EI & 4/3EI \\ 4/3EI & 14/3EI \end{bmatrix}^{-1} \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8/3 & 4 \\ 4/3 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} -1.25 \times 1 \\ 2.5 \times 10^{-3} \end{bmatrix}$$

$M_A = -1.25 \times 10^{-3}$ kN-m

$M_B = 2.5 \times 10^{-3}$ kN-m

STIFFNESS MATRIX METHOD

In this method, the basic unknowns are to d
In the analysis are the rotation contribution values. then b
slope deflection method equations we can find Redundant f
(ie moments). In this method the stiffness matrix depend. o
is $K = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & \cdots & & \\ \vdots & & & \\ K_{n1} & \cdots & & K_{nn} \end{bmatrix}$

By using compatability conditions final moments at th
supports will be determined.

$$[K][\theta] = [\Delta] - [\Delta_L]$$

$$\theta = [K^{-1}]\left[[\Delta] - [\Delta_L]\right]$$

where $[K]$ - stiffness matrix

$[\theta] = \begin{bmatrix} \theta_A \\ \theta_B \\ \vdots \\ \theta_n \end{bmatrix}$ - θ = rotation contribution valu
support

$[\Delta]$ = settlement matrix

$[\Delta_L]$ = fixed end moment matrix

Procedure :-

1) $D_K = ?$
Degree of kinematic indeterminacy of structure

2) Assign co-ordinate values

3) Draw fully restrained structure

4) Find out fixed end moment

5) Determine Δ_L

6) Determine stiffness matrix by giving unit displacement (Rotation) to the Restrained structure.

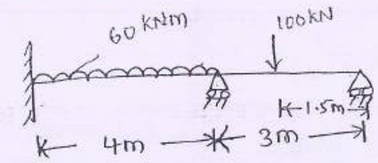
7) ~~From~~ Form & solve the stiffness eqn's

$$[\theta] = [k]^{-1}\left[[\Delta] - [\Delta_L]\right]$$

to get the displacement (Rotation contribution values) at various co-ordinate points.

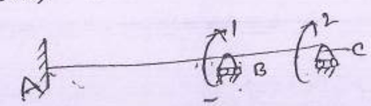
8) By using slope deflection method equn final moments will be determined.

* Analyse the continuous beam shown in figure by displacement method

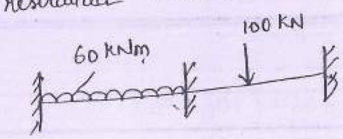


Sol :- 1) $D_K = 5 - 3 = 2$

2) co-ordinates selection



3) fully restrained structure



4) Fixed end moments

$M_{FAB} = -\frac{60 \times 4^2}{12} = -80$ KNm

$M_{FBA} = 80$ KNm

$M_{FBC} = \frac{-100 \times 3}{8}$

$M_{FCB} = 37.5$ KN

5) $\Delta_L = \begin{bmatrix} \Delta_{L1} \\ \Delta_{L2} \end{bmatrix}$

$\Delta_{L1} = \cancel{M_{FBA}} + M_{FBA} + M_{FBC} = 80 - 37.5 = 42.5$

$\Delta_{L2} = M_{FCB} = 37.5$

$\therefore \Delta_L = \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix}$

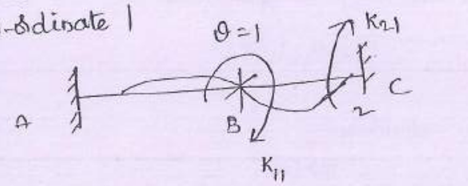
For n=2

$$[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}_{2\times2}$$

→ To determine stiffness matrix unit displacement is applied at the coordinates to restrained structure

→ use slope-deflection eqn's at the co-ordinates

unit displacement
→ at co-ordinate 1



$\theta_B = 1$
$\theta_A = \theta_C = 0$

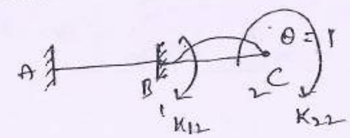
$$K_{11} = \frac{2EI}{l}(2\theta_B + \theta_A) + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= \frac{2EI}{4}(2(1)) + \frac{2EI}{3}(2(1))$$

$$= EI + \frac{4}{3}EI = \frac{7EI}{3}$$

$$K_{21} = \frac{2EI}{l}(2\theta_C + \theta_B) = \frac{2EI}{3}(0+1) = \frac{2EI}{3}$$

→ unit displacement at co-ordinate 2



$$K_{22} = \frac{2EI}{L}(2\theta_C + \theta_B) = \frac{2EI}{3}(2) = \frac{4EI}{3}$$

$$K_{12} = \frac{2EI}{L}(2\theta_B + \theta_C) = \frac{2EI}{3}$$

(7) $[\theta] = [K]^{-1}\{[\Delta] - [\Delta_L]\}$

$$= \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}^{-1} \left[0 - \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix}\right]$$

$$= -\begin{bmatrix} 7/3\,EI & 2/3\,EI \\ 2/3\,EI & 4/3\,EI \end{bmatrix}^{-1} \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix}$$

$$= -\frac{1}{EI}\begin{bmatrix} 7/3 & 2/3 \\ 2/3 & 4/3 \end{bmatrix}^{-1} \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = -\begin{bmatrix} 11.875 \\ 22.187 \end{bmatrix} 1/EI$$

$\theta_B = -11.875/EI$ $\quad \theta_C = -22.187/EI$

(8) Final moments

$$M_{AB} = M_{FAB} + \frac{2EI}{L}(2\theta_A + \theta_B) = -80 + \frac{2EI}{4}\left(2(0) - \frac{11.}{E}\right.$$

$= -85.93$ kNm

$$M_{BA} = M_{FBA} + \frac{2EI}{L}(2\theta_B + \theta_A) = 80 + \frac{2EI}{4}\left(2\left(\frac{-11.875}{EI}\right) + 0\right) = 6$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}(2\theta_B + \theta_C) = -37.5 + \frac{2EI}{3}\left(2\left(\frac{-11.875}{EI}\right) - \frac{22.187}{EI}\right)$$

$\therefore M_B = -68.125$ kN-m

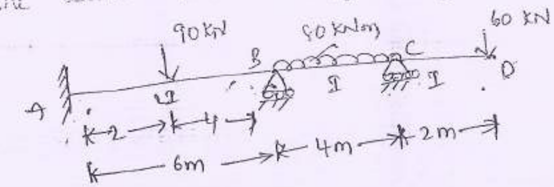
$$M_{CB} = M_{FCB} + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$= 37.5 + \frac{2EI}{3}\left(2\left(\frac{-22.187}{EI}\right) - \frac{11.875}{EI}\right) = 37.5 - 37.5 = 0$$

$\therefore M_{CB} = 0$

$\therefore M_C = 0$

Analyse the beam shown in figure by stiffness matrix method



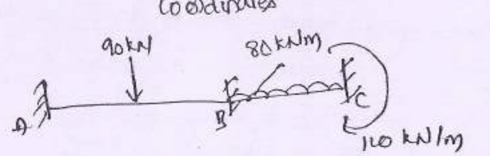
STEP-1 :- $5-3=2$,

Overhanging portion is a determinate portion,

It is taken as cantilever moment, final moment at C

Step 2 :- Released beam Diagram

coordinates selected



Full restrained structure

Step 3 : Fixed end moments

$$M_{FAB} = \frac{-90\times2\times4^2}{6^2} = -80 \text{ kNm}$$

$$M_{FBA} = \frac{90\times2^2\times4}{6^2} = 40 \text{ kNm}$$

$$M_{FBC} = -\frac{80\times4^2}{12} = -106.67 \text{ kNm}$$

$$M_{FCB} = 106.67 \text{ kNm}$$

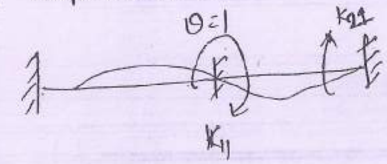
$$[A_L] = \begin{pmatrix} M_{FBA} + M_{FBC} \\ M_{FCB} \end{pmatrix} = \begin{pmatrix} 40-106.67 \\ 106.67 \end{pmatrix} = \begin{pmatrix} -66.6 \\ 106. \end{pmatrix}$$

6) For $n=2$,

$$[K] = \left[\begin{array}{c|c} K_{11} & K_{12} \\ K_{21} & K_{22} \end{array}\right]$$

- Stiffness matrix, k,

a) Unit matrix displalement in coordinate direction 1



K_{11} = moment required to rotate BA and BC by 1

$$= \frac{2EI}{L}\left[2\theta_B + \theta_A - 0\right] + \frac{2EI}{L}\left(2\theta_B + \theta_C - 0\right)$$

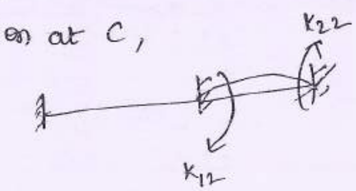
$$= \frac{2E(2I)}{6}\left[2\times1+0\right] + \frac{2EI}{4}\left[2\times1+0\right]$$

$$= \frac{8EI}{6} + EI = \frac{7}{3}EI$$

$$k_{21} = \frac{2EI}{4}\left[2\theta_C\right.$$

$k_{21} =$

b) unit rotation at C,



$$k_{12} = \frac{2EI}{4}\left(2\theta_B + \theta_C - 0\right) = 0.5EI$$

$$k_{22} = \frac{2EI}{4}\left(2\theta_C + 0\right) = EI$$

∴ The stiffness matrix $K = \begin{bmatrix} 7/3EI & 0.5EI \\ 0.5EI & EI \end{bmatrix}$

∴ The stiffness matrix equation is

$$[K][\theta] = [A - A] \qquad \therefore \Delta = \begin{bmatrix} 0 \\ 120 \end{bmatrix}$$

$$\begin{bmatrix} \frac{7EI}{3} & 0.5EI \\ 0.5EI & EI \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 - (-66.7) \\ 120 - 106.67 \end{bmatrix}$$

$$\frac{EI}{3} \begin{bmatrix} 7 & 1.5 \\ 1.5 & 3 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 66.67 \\ 13.33 \end{bmatrix}$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \frac{3}{EI} \begin{bmatrix} 7 & 1.5 \\ 1.5 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 66.67 \\ 13.33 \end{bmatrix}$$

$$= \frac{3}{EI} \left(\frac{1}{7 \times 3 - 1.5^2} \right) \begin{bmatrix} 3 & -1.5 \\ -1.5 & 7 \end{bmatrix} \begin{bmatrix} 66.67 \\ 13.33 \end{bmatrix}$$

$$= \frac{3}{EI} \times \frac{1}{18.75} \begin{bmatrix} 180.015 \\ -66.95 \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 28.802 \\ -1.017 \end{bmatrix}$$

$$\theta_B = \frac{28.802}{EI} \quad \& \quad \theta_C = \frac{-1.017}{EI}$$

From slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l} (2\theta_A + \theta_B - 0)$$

$$= -60.80 \text{ KNm}$$

$$M_{BA} = 40 + \frac{2E(2I)}{6} \left(0 + 2 \times \frac{28.802}{EI} - 0 \right) = 78.403 \text{ KNm}$$

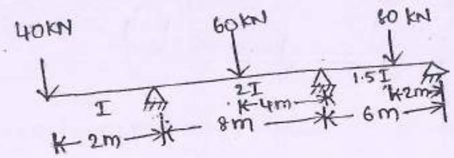
$$M_{BC} = -106.67 + \frac{2EI}{4} (2\theta_B + \theta_C - 0)$$

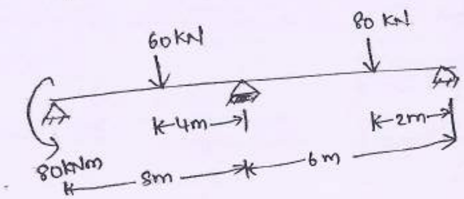
$$= -106.67 + \frac{2EI}{4} \left(\frac{2 \times 28.02}{EI} - \frac{1.071}{EI} \right) = -78.403 \text{ kNm}$$

$$M_{CB} = -106.67 + \frac{2EI}{4} (\theta_B + 2\theta_C - 0)$$

$$= -106.67 + \frac{2EI}{4} \left(\frac{28.802}{EI} - \frac{2 \times 1.071}{EI} \right) = 120 \text{ KNm}$$

* Analyse the continuous beam shown in figure by displacemen





Equivalent beam

Co-ordinates selected

Fully restrained Structure

Final force vector $[\Delta] = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} = \begin{bmatrix} -80 \\ 0 \\ 0 \end{bmatrix}$

Fixed end moments :-

$M_{FBC} = -\dfrac{60 \times 8}{8} = -60$ kNm

$M_{FCB} = 60$ kNm

$M_{FCD} = -\dfrac{80 \times 4 \times 2^2}{6^2} = -35.55$ kNm

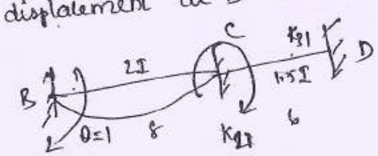
$M_{FDC} = \dfrac{80 \times 4^2 \times 2}{6^2} = 71.11$ kNm

Displacement matrix

$\therefore [\Delta_L] = \begin{bmatrix} \Delta_{L1} \\ \Delta_{L2} \\ \Delta_{L3} \end{bmatrix} = \begin{bmatrix} M_{FBC} \\ M_{FCB} + M_{FCD} \\ M_{FDC} \end{bmatrix} = \begin{bmatrix} -60 \\ 60 - 35.55 \\ 71.11 \end{bmatrix}$

Stiffness matrix :- $n = 3$, $k = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$

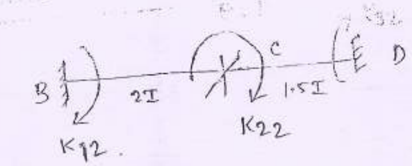
(a) unit displacement at B



$\therefore k_{11} = k_{BC} = \dfrac{2E(2I)}{8}(2\theta_B + \theta_C - 0) = EI$

$\therefore k_{21} = k_{CB} = \dfrac{2E(2I)}{8}(2\theta_C + \theta_B - 0) = 0.5\,EI$

$k_{31} = 0$

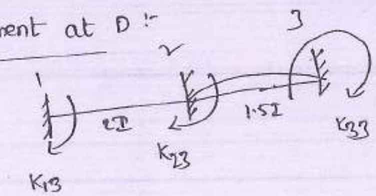


$k_{12} = \dfrac{2E(2I)}{8} = 0.5\,EI$

$k_{22} = \dfrac{2E(2I)}{8}(2) + \dfrac{2E(1.5I)}{8} = 2EI$

$k_{32} = \dfrac{2E(1.5I)}{6} = 0.5\,EI$

(c) unit displacement at D :-



$k_{13} = 0$

$k_{23} = \dfrac{2E(1.5I)}{6} = 0.5\,EI$

$k_{33} = \dfrac{2E(1.5I)}{6}(2) = EI$

Stiffness matrix equation is

$[K][\theta] = [[\Delta] - [\Delta L]]$

$EI \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 2 & 0.5 \\ 0 & 0.5 & 1.0 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} -80 + 60 \\ 0 - 24.45 \\ -71.11 \end{bmatrix}$

$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \dfrac{1}{EI} \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 2 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -20 \\ -24.45 \\ -71.11 \end{bmatrix}$

$$= \frac{1}{EI}\left(\frac{1}{2\times1-0.5^2-0.5(0.5)}\right)\begin{bmatrix}1.75 & -0.5 & 0.25\\ -0.5 & 1.0 & -0.5\\ 0.25 & -0.5 & 1.75\end{bmatrix}\begin{bmatrix}-20\\ -24.45\\ -71.11\end{bmatrix}$$

$$= \frac{1}{EI}\times\frac{1}{1.5}\underset{3\times3}{\begin{bmatrix}1.75 & -0.5 & 0.25\\ -0.5 & 1.0 & -0.5\\ 0.25 & -0.5 & 1.75\end{bmatrix}}\underset{3\times1}{\begin{bmatrix}-20\\ -24.45\\ -71.11\end{bmatrix}}$$

$$= -\frac{1}{EI}\begin{bmatrix}-27.035\\ 14.07\\ -78.145\end{bmatrix}$$

$$\therefore \theta_B = \frac{-27.035}{EI},\quad \theta_C = \frac{14.07}{EI},\quad \theta_D = \frac{-78.145}{EI}$$

$\therefore M_{AB} = -80$ KNm

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C\right]$$

$$= -60 + \frac{2E(2I)}{8}\left[2\left(\frac{-27.035}{EI}\right) + \frac{14.07}{EI}\right] = -80 \text{ kNm}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B\right]$$

$$= 60 + \frac{2E(2I)}{8}\left[2\left(\frac{14.07}{EI}\right) - \frac{27.035}{EI}\right] = 60.55 \text{ kNm}$$

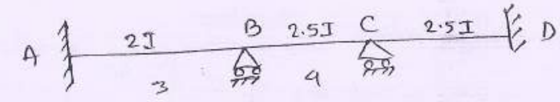
$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \theta_D\right]$$

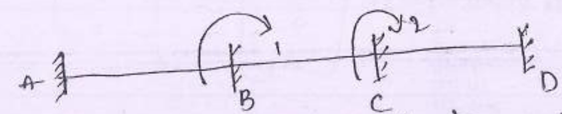
$$= -35.55 + \frac{2E(1.5I)}{6}\left[\frac{2\times14.07 - 78.145}{EI}\right] = -60.55 \text{ kNm}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left[2\theta_D + \theta_C\right] = 71.11 + \frac{2E(1.5I)}{6}\left[\frac{14.07 - 2\times78.145}{EI}\right]$$

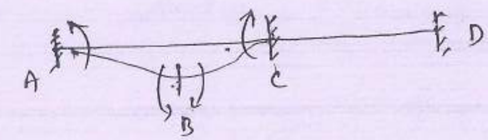
$$= 0$$

Analyse the continuous beam shown in figure. If the [illegible] sinks by 10 mm. Use displacement method. Take EI = 600[illegible]





Full restrained structure and coordinat[illegible]



$$M_{FAB} = -\frac{6EI\Delta}{l^2} = -\frac{6\times6000\times2\times0.01}{3^2} = -80 \text{ KNm}$$

$M_{FBA} = -80$ KNm

$$M_{FBC} = \frac{6EI\Delta}{l^2} = \frac{6\times6000\times(2.5)\times0.01}{4^2} = 56.25 \text{ KNm}$$

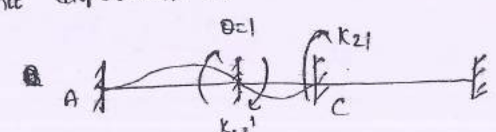
$M_{FCB} = 56.25$ KNm

Now $\quad \Delta = \begin{bmatrix}0\\0\end{bmatrix} \qquad \Delta_L = \begin{bmatrix}\Delta_{L1}\\ \Delta_{L2}\end{bmatrix} = \begin{bmatrix}M_{FBA} + M_{FBC}\\ M_{FCB}\end{bmatrix}$

$$= \begin{bmatrix}-80+56.25\\ 56.25\end{bmatrix} = [\text{illegible}]$$

<u>Stiffness matrix</u>:

(a) unit displacement in coordinate direction 1

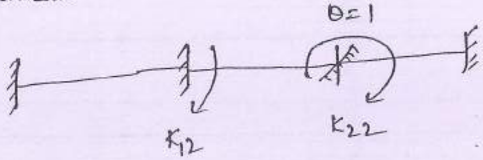


$$K_{11} = \frac{2EI}{l}\left(2\theta_B + \theta_A\right) + \frac{2EI}{l}\left(2\theta_B + \theta_C\right)$$

$$k_{11} = \frac{4E(2I)}{3} + \frac{4E(2.5I)}{4} = 5.167\,EI$$

$$k_{21} = \frac{2EI}{l}(2\theta_C + \theta_B) = \frac{2E(2.5I)}{4} = 1.25\,EI$$

(b) unit displacement in co-ordinate direction 2



$$k_{12} = \frac{2EI}{l}(2\theta_B + \theta_C) = \frac{2E(2.5I)}{4} = 1.25\,EI$$

$$k_{22} = \frac{2EI}{l}(2\theta_C + \theta_B) + \frac{2EI}{l}(2\theta_C + \theta_D)$$

$$= \frac{2E(2.5I)}{4}(2) + \frac{2E(2.5I)}{4}(2)$$

$$\doteq 5EI$$

∴ Therefore stiffness equation is given by

$$[K][\theta] = [[\Delta] - [\Delta_L]]$$

$$EI\begin{bmatrix} 5.167 & 1.25 \\ 1.25 & 5 \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0 - (-23.75) \\ 0 - 56.25 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_B \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 5.167 & 1.25 \\ 1.25 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 23.75 \\ -56.25 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \left(\frac{1}{EI}\right)\begin{bmatrix} 5.167 & 1.25 \\ 1.25 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 23.75 \\ -56.25 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{pmatrix} 7.79 \\ -13.198 \end{pmatrix}$$

$$\theta_B = \frac{7.79}{EI} \quad \text{and} \quad \theta_C = -\frac{13.198}{EI}$$

$$\therefore M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -80 + \frac{2E(2I)}{3}\left(0 + \frac{7.79}{EI}\right) = -69.613 \text{ kNm}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

$$= -80 + \frac{2E(2I)}{3}\left(0 + 2\times\frac{7.79}{EI}\right) = -59.227 \text{ kNm}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= 56.25 + \frac{2E(2.5I)}{4}\left(\frac{2\times 7.79}{EI} - \frac{13.198}{EI}\right) = 5$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B)$$

$$= 56.25 + \frac{2E(2.5I)}{4}\left(\frac{2\times -13.198}{EI} + \frac{7.79}{EI}\right)$$

$$= 33 \text{ kNm}$$