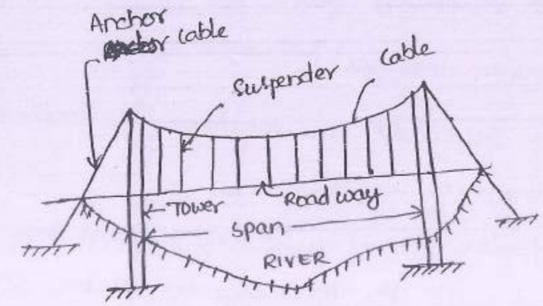
III

CABLESTRUCTURES AND SUSPENSION BRID

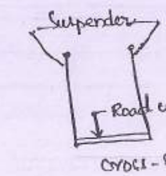
Introduction :-

Cables are used as temporarily guys during erection and as permanent guys for supporting masts and tow
Cables are also used in the suspension bridges.

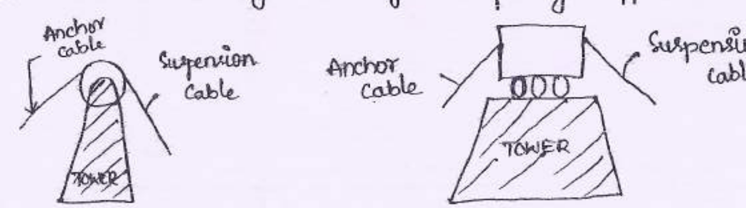
CHARACTERISTICS



ELEVATION

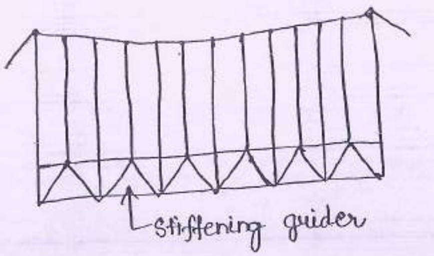


A suspension bridge consists of two cables with the
of suspenders (hangers) which support the roadway. Figure shows a
suspension bridges in which the cable is supported over towers. To
the bending moment in towers anchor cables are provided.
The central sag or dip of the cable varies from $\frac{1}{10}^{th}$ to $\frac{1}{15}^{th}$ of the
The cables will be having either guided pulley support or r



(a) Guided pulley support

If heavy traffic and large spans stiffening girders are provided to support the roadway.



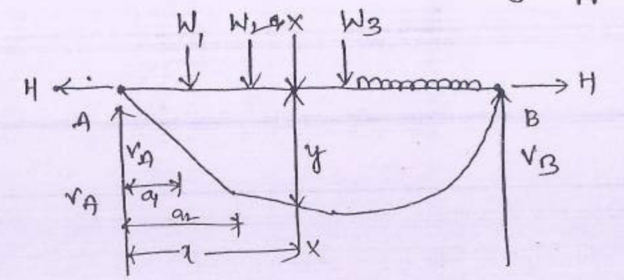
Lakshman Jhula at Rishikesh and Howra bridge are popular examples of suspension bridges

since the number of suspenders are very large, the load on the cable may be taken as uniformly distributed.

Cables being very flexible, do not resist any B.M and they adjust their shape to loads and resist the load only by tension. since, steel is an efficient material in resisting tension, steel finds its application in suspension bridges and hence, they are very economical for larger spans. suspension bridges of span 200 m to 300 m are commonly built.

Equilibrium of cables:-

A cable is a flexible structure which resist bending moment. It deflects so that the bending moment is zero at any point which is achieved by developing horizontal thrust at the support and thus, developing appropriate



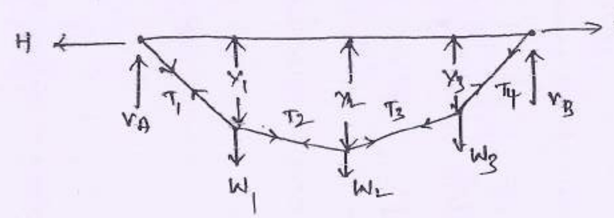
$$M_x = V_A x - W_1(x-a_1) - W_2(x-a_2) - Hy$$

Since, cable is flexible, $M_x = 0$

$$Hy = V_A x - W_1(x-a_1) - W_2(x-a_2) = \text{Beam mom}$$

Considering any segment of cable and using the equation along with usual equations, a loaded cable can be analy

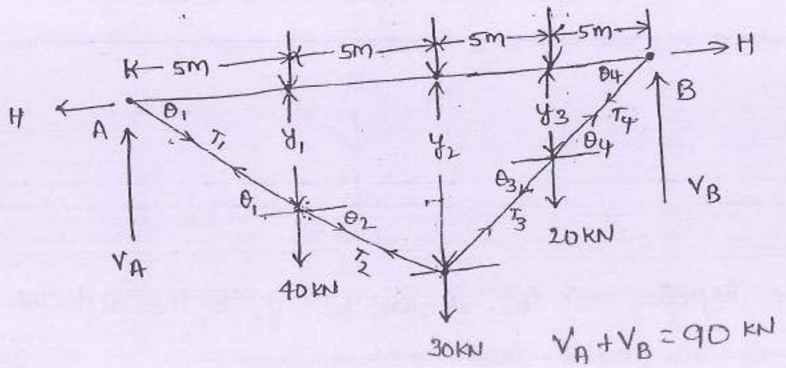
Cable subjected to Concentrated loads:-



$$Hy = M_{beam} \quad \& \quad y = \frac{M_{beam}}{H}$$

$$y_1 = \frac{M_1}{H_1} \quad y_2 = \frac{M_2}{H} \quad y_3 = \frac{M_3}{H}$$

A light cable is supported at two points 20 m apart which are at the same level. The cable supports three concentrated loads as shown in figure. The deflection at the first point is found to be 0.8 m. Determine the tension in different segments and total length of the cable.



$V_A + V_B = 90$ kN

Sol: Taking moment about A, we get

$M_A = 0 = 40\times5 + 30\times10 + 20\times15 - V_B\times20$

$V_B = 40$ kN

$V_A = 50$ kN

The beam moments at point 1, 2 and 3 are

$M_{1,beam} = 50\times5 = 250$ kNm

$M_{2,beam} = 50\times10 - 40\times5 = 300$ kNm

$M_{3,beam} = V_B\times5 = 40\times5 = 200$ kNm

Let H be the horizontal reaction. Since, the cable is flexible, bending moment at all points is zero. Let y_1, y_2 and y_3 be the deflections of points 1, 2 and 3 respectively. Equating bending moments at point 1, 2 and 3 to zero, we get,

$Hy_1 = M_{1,beam} = 250$ kNm

$Hy_2 = M_{2,beam} = 300$ kNm

and $Hy_3 = M_{3,beam} = 200$ kNm

Since, $y_1 = 0.8$ m, we get

$$H = \frac{250}{0.8} = 312.5 \text{ kN}$$

$$y_2 = \frac{300}{312.5} = 0.96 \text{ m}$$

$$y_3 = \frac{200}{312.5} = 0.64 \text{ m}$$

Hence the deflected shape is shown in figure.

Let the inclination of segments A-1, 1-2, 2-3, & 3-B be θ_1 … θ_4 respectively.

Then $\theta_1 = \tan^{-1}\left(\frac{y_1}{5}\right) = \tan^{-1}\left(\frac{0.8}{5}\right) = 9.09^\circ$

$$\theta_2 = \tan^{-1}\left(\frac{y_2 - y_1}{5}\right) = \tan^{-1}\left(\frac{0.16}{5}\right) = 1.833^\circ$$

$$\theta_3 = \tan^{-1}\left(\frac{y_2 - y_3}{5}\right) = \tan^{-1}\left(\frac{0.32}{5}\right) = 3.662^\circ$$

$$\theta_4 = \tan^{-1}\left(\frac{y_3}{5}\right) = 7.294^\circ$$

Let the tension in segment A-1, 1-2, 2-3 and 3-4 be T_1, T_2, T_3 and T_4, respectively. Applying Lami's theorem at … we get the values of T_1, T_2, T_3 and T_4 as

$$\frac{T_1}{\sin(90-\theta_2)} = \frac{T_2}{\sin(90+\theta_1)} = \frac{40}{\sin(180-\theta_1+\theta_2)}$$

$$\therefore T_1 = \frac{40 \times \sin(90-1.833)}{\sin(180-9.09+1.833)} = 316.49 \text{ kN}$$

$$\therefore T_2 = \frac{40 \times (\sin 90+9.09)}{\sin(180-9.09+1.833)} = 312.679 \text{ kN}$$

From the consideration of equilibrium at point 3,

$$\frac{T_3}{\sin(90+\theta_4)} = \frac{T_4}{\sin(90-\theta_3)} = \frac{20}{\sin(180+\theta_3-\theta_4)}$$

$$T_3 = 313.162 \text{ kN}$$

$$T_4 = 315.072 \text{ kN}$$

$\sec\theta = \frac{l}{5}$

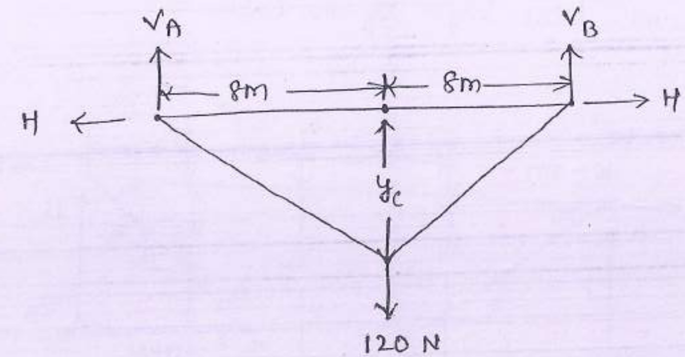
$l = 5\sec\theta$

Length of the cable = sum of length of each segment

$= 5\sec\theta_1 + 5\sec\theta_2 + 5\sec\theta_3 + 5\sec\theta_4$

$= 20.117$ m

* A light cable 18 m long, is supported at two ... the same level. The supports are 16 m apart. The cable supports 120 N load dividing at distance into two parts. Find the shape of the cable and tension in C...



Refering to the figure, let y_c be the deflection ... centre of the cable. Due to symmetry

$$V_A = V_B = \frac{120}{2} = 60 \text{ N}$$

Now, length of cable,

$$L = 2\left[\left(\frac{l}{2}\right)^2 + y_c^2\right]^{1/2}$$

$$18 = 2\left[8^2 + y_c^2\right]^{1/2}$$

$$9 = \left(8^2 + y_c^2\right)^{1/2}$$

$$81 = 8^2 + y_c^2$$

$$y_c = \sqrt{81-64} = 4.123 \text{ m}$$

Taking moment about C, we get, $M_C = 0$

$$H \times y_c = V_A \times 8$$

$H \times 4.123 = 60 \times 8$

$$\boxed{H = 116.42\ N}$$

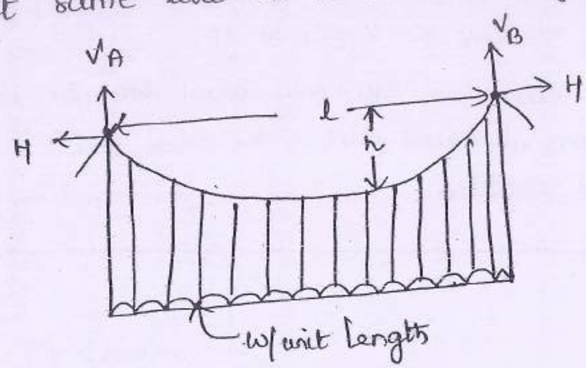
$\therefore$ Tension in Cable $= \sqrt{V_A^2 + H^2}$

$= \sqrt{60^2 + 116.42^2}$

$$\boxed{T = 130.97\ N}$$

* CABLE Subjected TO A . UDL :-

let a cable of length 'L' be supported at points A and B which are at a horizontal distance 'l' and are at same level as shown in figure



The cable is subjected to a UDL w/unit horizontal length

$V_A = V_B = \frac{wl}{2}$

Taking moment about central point and noting B.M is zero at all points in cable, we get,

$$Hh - \frac{wl}{2} \times \frac{l}{2} - \frac{wl}{2} \times \frac{l}{2} \times \frac{1}{2} = 0$$

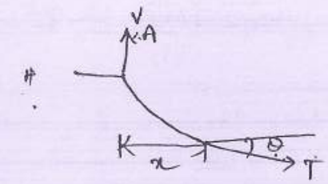
$$Hh - \frac{wl^2}{4} - \frac{wl^2}{8} = 0$$

$Hh = \frac{wl^2}{8}$

$$\boxed{H = \frac{wl^2}{8h}}$$

If V is the shear at any section x-x distance x f

refer the figure



$$T = \sqrt{V^2 + H^2}$$

$V_{max} = \frac{wl}{2}$ at support

$$T_{max} = \sqrt{\left(\frac{WL}{2}\right)^2 + \left(\frac{WL^2}{8h}\right)^2} = \frac{WL}{2}\sqrt{1 + \frac{L^2}{16h^2}}$$

$V_{min} = 0$, at centre

$T_{min} = \sqrt{0 + H^2} = H$

Hence,

At any point, since cable cannot resist shear,

$$V = T\sin\theta$$

Now to find the slope of the cable, consider the portion of the section x-x. let θ be the slope. Then,

$\Sigma_H = 0 \rightarrow T\cos\theta = H$

$\Sigma_V = 0 \rightarrow T\sin\theta = V_A - wx = \frac{wl}{2} - wx$

$$\therefore \tan\theta = \left[\frac{wl}{2} - wx\right] \times \frac{1}{H}$$

i.e $\dfrac{dy}{dx} = \left[\dfrac{wL}{2} - wx\right] \times \dfrac{1}{H}$

$\therefore y = \left[\dfrac{wL}{2}x - \dfrac{wx^2}{2}\right] \times \dfrac{1}{H}$

$\therefore y = \dfrac{wx(l-x)}{2H}$

Substituting the value of $H = \dfrac{wl^2}{8h}$, we get

$$y = \frac{wx(l-x)}{2} \times \frac{8h}{wl^2}$$

$$y = \frac{4hx(l-x)}{l^2}$$

which is a parabola. Thus, the shape of cable is a parabola

To find the length of the cable in any curve

$$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \simeq 1 + \frac{1}{2}\left[\frac{4hx(l-x)}{l^2}\right]^2$$

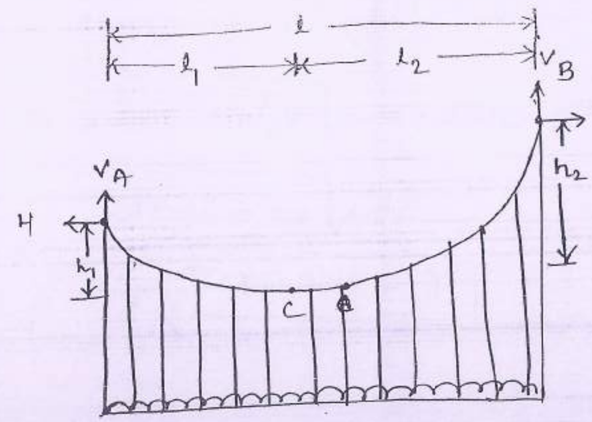
Therefore, the length of the cable is given by

$$L = \int_0^l ds = \int_0^l \left[1 + \frac{1}{2}\left(\frac{16h^2}{l^4}\right)\left(l^2 - 4lx + 4x^2\right)\right] dx$$

$$= \left[x + \frac{8h^2}{l^4}\left(l^2x - \frac{4lx^2}{2} + \frac{4x^3}{3}\right)\right]_0^l$$

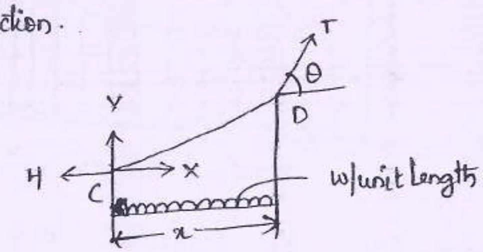
$$= l + \left(\frac{8h^2}{l^4}\right) l^3 \left(1 - 2 + \frac{4}{3}\right)$$

$$\boxed{L = l + \frac{8h^2}{3l}}$$



Consider the cable shown in figure which is at A and B. A is h_1 metres above the lowest point C and B is h_2 met... This is subjected to uniformly distributed load w per unit horizontal len... entire span l. Let the horizontal distance between AC is l_1 and betwee... Let the horizontal reactions at supports be H.

A cable can take only axial force, i.e it cannot ... and shear force. Hence at lowest point C, the axial force = H and there ... force in vertical direction.



Let D be a point where slope is θ. Taking the lowest po... origin and considering the equilibrium of portion CD, we get

$$T\cos\theta = H$$

and $$T\sin\theta = wx$$

$$\therefore \tan\theta = \frac{wx}{H}$$

i.e $dy/dx = \dfrac{wx}{H}$

$$\frac{dy}{dx} = \frac{wx}{H}$$

$$y = w\left(\frac{x^2}{2H}\right) + C_1$$, where C_1 is constant of integration

at C_1 $x = y = 0$

$$0 = 0 + C_1 \quad \& \quad C_1 = 0$$

$$\therefore y = w\left(\frac{x^2}{2H}\right) \quad — (1)$$

This is the equation of parabola. Hence, the cable is having a parabolic shape.

Applying equation (1) to points A and B, we get

$$h_1 = \frac{wl_1^2}{2H} \quad \text{and} \quad h_2 = \frac{wl_2^2}{2H}$$

$$\therefore \frac{h_1}{h_2} = \frac{l_1^2}{l_2^2} \quad \& \quad \frac{\sqrt{h_1}}{\sqrt{h_2}} = \frac{l_1}{l_2}$$

$$\therefore \frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} = \frac{l_1}{l_1+l_2} = \frac{l_1}{l}$$

$$\therefore l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}$$

$$\text{and} \quad l_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$$

To find H, calculate moment at C. If it is calculated from left hand side

$$V_A l_1 - Hh_1 - \frac{wl_1^2}{2} = 0$$

$$V_A = \frac{wl_1}{2} + \left(\frac{h_1}{l_1}\right)H \quad — (2)$$

If we calculated from right hand side

$$V_B l_2 - Hh_2 - \frac{wl_2^2}{2} = 0$$

$$V_B = \frac{wl_2}{2} + \left(\frac{h_2}{l_2}\right)H \quad — (3)$$

Adding (2) & (3)

$$V_A + V_B = \frac{w}{2}(l_1+l_2) + \left(\frac{h_1}{l_1} + \frac{h_2}{l_2}\right)H$$

$$= \left(\frac{w}{2}\right)l + \left(\frac{h_1}{l_1} + \frac{h_2}{l_2}\right)H$$

But $V_A + V_B = wL$, total downward load

$$\therefore wL = \left(\frac{w}{2}\right)l + \left[\frac{h_1}{l_1} + \frac{h_2}{l_2}\right]H$$

$$\& \quad \frac{wl}{2} = \left[\frac{h_1}{l_1} + \frac{h_2}{l_2}\right]H$$

sub. the values of l_1 and l_2 ~~from~~ in above equa

$$H = \frac{wl}{2\left[\frac{h_1(\sqrt{h_1}+\sqrt{h_2})}{l\sqrt{h_1}} + \frac{h_2(\sqrt{h_1}+\sqrt{h_2})}{l\sqrt{h_2}}\right]}$$

$$= \frac{wl^2}{2\left[\sqrt{h_1}(\sqrt{h_1}+\sqrt{h_2}) + \sqrt{h_2}(\sqrt{h_1}+\sqrt{h_2})\right]}$$

$$H = \frac{wl^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}$$

$$\therefore H = \frac{wl}{2(\sqrt{h_1}+\sqrt{h_2})^2}$$

with value of we can found V_A & V_B by eq^{ns} (2) & (3)

To find the length of the Cable,

length of cable ACB = $\frac{1}{2} \times$ Sum of length of ACB' and A'CB

$$= \frac{1}{2} \times \left[2l_1 + \frac{8}{3} \times \frac{h_1^2}{2l_1} + 2l_2 + \frac{8}{3} \times \frac{h_2^2}{2l_2} \right]$$

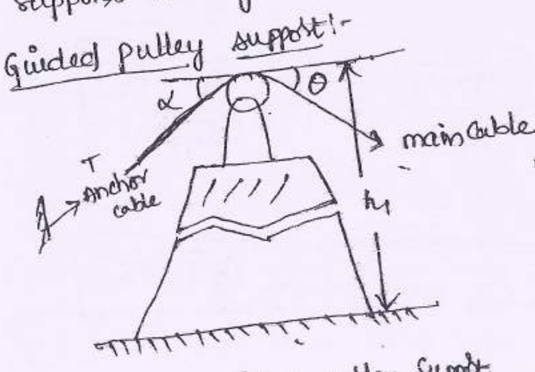
$$= l_1 + l_2 + \frac{2}{3}\frac{h_1^2}{l_1} + \frac{2}{3}\frac{h_2^2}{l_2}$$

$$L = l + \frac{2}{3} \times \frac{h_1^2}{l_1} + \frac{2}{3}\frac{h_2^2}{l_2} \quad //$$

FORCES ON ANCHOR CABLES AND TOWERS :-

The forces on anchor cables and towers depend upon the type of support given to cables. There are two types of supports namely guided pulley and roller support

+ Guided pulley support :-



Guided pulley support

Let the inclination of main cable to horizontal be θ and that of anchor cable be α

Assuming pulley as frictionless, tension in anchor cable is taken as same as tension in the main cable. Let this tension be T.

Therefore, Vertical load transmitted to tower

$$= T\sin\theta + T\sin\alpha$$
$$= T(\sin\theta + \sin\alpha)$$

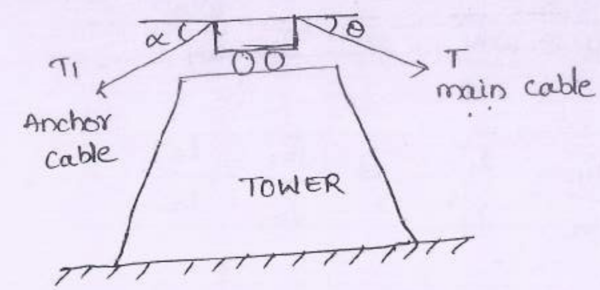
Horizontal load transmitted to Tower

$$= T\cos\theta - T\cos\alpha = T(\cos\theta -$$

Bending moment on the tower

$$= \text{Horizontal force on tower} \times \text{Height of}$$
$$= T(\cos\theta - \cos\alpha) \times h_1$$

+ Roller Support :-



In this case, the suspension cable and anchor cables are connected to a saddle resting on [...] In this arrangement, the two cables need not ha[ve] same tension. Let T be the main cable and T_1 is the a[nchor] cable.

Assuming saddle to have frictionless rollers,

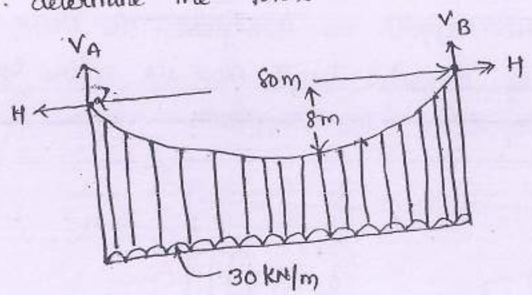
$$T_1 \cos\alpha = T\cos\theta$$

$$\therefore T_1 = T\left(\frac{\cos\theta}{\cos\alpha}\right)$$

Since, saddle is having frictionless rollers, there is no ho[rizontal force] and hence no B.M on tower.

$\therefore$ Vertical force on the Tower $= T_1 \sin\alpha +$

A bridge cable is suspended from towers 80m apart and carries a load of 30 kN/m on the entire span. If the maximum sag is 8m, calculate the maximum tension in the cable. If the cable is supported by saddles which are stayed by wires inclined at 30° to the horizontal, determine the forces acting on the towers. If the same inclination of back stay passes over pulley, determine the forces on the towers



$\frac{wl}{2} \times \frac{l}{4}$

$V_A = \frac{wL}{2} = \frac{30 \times 80}{2} = 1200$ kN

Taking moment about central point C

$$H \times 8 - \frac{wl}{2} \times \frac{l}{2} + \frac{wl}{2} \times \frac{l}{4} = 0$$

$$\boxed{H = \frac{wl^2}{64}} = \frac{30 \times 80^2}{64} = 3000 \text{ kN}$$

Maximum tension occurs at support

$$\boxed{T_{max} = \sqrt{V^2 + H^2}} = \sqrt{1200^2 + 3000^2} = 3231.1 \text{ kN}$$

$\boxed{H = T_{max} \cos\theta}$ To find inclination 'θ'.

$$\theta = \cos^{-1}\left(\frac{H}{T}\right) = \cos^{-1}\left(\frac{3000}{3231.1}\right)$$

$$\theta = 21.80^\circ$$

i) If the cable is supported by saddle, the [illegible] tension T_1 is given by,

$$\boxed{T_1 \cos\alpha = T_{max} \cos\theta}$$

$$T_1 \cos 30^\circ = 3231.80 \times \cos 21.80^\circ$$

$$T_1 = 3464.1 \text{ kN}$$

There is no horizontal force on the tower. The vertical force [on the] tower is given by,

$$= \boxed{T_1 \sin\alpha + T_{max} \sin\theta}$$

$$= 3464.1 \sin 30^\circ + 3231.1 \sin 21.80^\circ$$

$$= 2931.98 \text{ kN}$$

ii) If the cable is on pulley, the vertical force on tower is

$$= \boxed{T_{max} (\sin\alpha + \sin\theta)}$$

$$= 3231.1 (\sin 30^\circ + \sin 21.80^\circ)$$

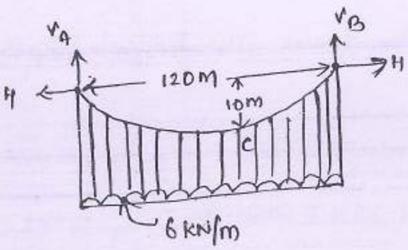
$$= 2815.48 \text{ kN}$$

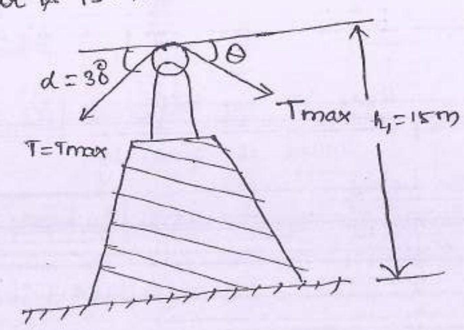
Horizontal force on the tower $= \boxed{T_{max} (\cos\theta - \cos\alpha)}$

$$= 3231.1 (\cos 21.8^\circ - \cos 3[0^\circ])$$

$$= 201.82 \text{ kN}$$

A cable of span 120 m and dip 10 m carries a load of 6 kN/m of horizontal span. Find the maximum tension in the cable and the inclination of the cable at the support. Find the forces transmitted to the supporting pier if the cable passes over smooth pulleys on top of the pier. The anchor cable is at 30° to the horizontal. Determine the maximum B.M for the pier if the height of the pier is 15 m.





Due to symmetry

$$V_A + V_B = \frac{wl}{2} = \frac{6\times120}{2} = 360 \text{ kN}$$

Taking moment about central point C,

$$(H\times h) - \frac{wl}{2}\times\frac{l}{2} - \frac{wl}{2}\times\frac{l}{2}\times\frac{1}{2}$$

$$\therefore \boxed{H = \frac{wl^2}{8h}} = \frac{6\times120^2}{8\times10} = 1080 \text{ kN}$$

$$\boxed{T_{max} = \sqrt{V^2+H^2}} = \sqrt{360^2+1080^2} = 1138.42 \text{ kN}$$

$$\boxed{\cos\theta = \frac{H}{T_{max}}} = \frac{1080}{1138.42}$$

$$\theta = 18.435°$$

$$\boxed{T_{max}(\cos\theta - \cos\alpha)}$$

Horizontal force transferred to pier

$$= T_{max}(\cos 18.435° - \cos 30°)$$
$$= 1138.42(\cos 18.435° - \cos 30°)$$
$$= 94.099 \text{ kN}$$

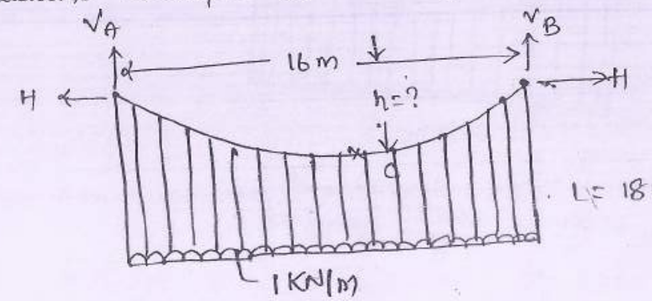
Maximum bending moment to the pier

$$= H h_1$$
$$= 94.099\times15 = 1411.49 \text{ kNm}$$

Vertical force on the pier $= \boxed{T_{max}(\sin\theta + \sin\alpha)}$

$$= 1138.42(\sin 18.435° + \sin 30°)$$
$$= 929.21 \text{ kN}$$

* A light flexible cable 18 m long is supported at two at the same level. The supports are 16 m part. The cable is to UDL of 1 kN/m of horizontal length over its entire span. the reactions developed at the support.



$$L = l + \frac{8}{3}\times\frac{h^2}{l}$$

$$18 = 16 + \frac{8}{3}\times\frac{h^2}{16}$$

$$h = 3.464 \text{ m}$$

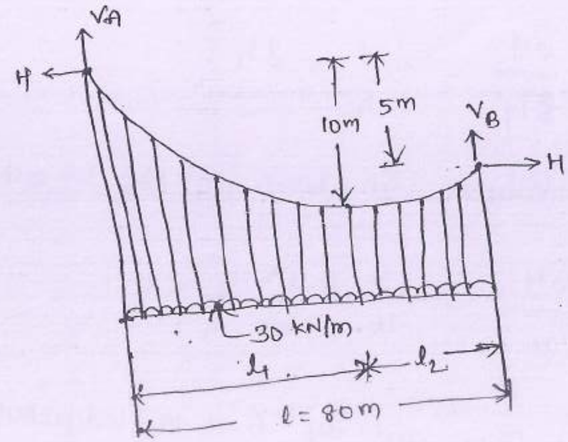
$$\boxed{V_A = \frac{wl}{2}} = \frac{1\times16}{2} = 8 \text{ kN}$$

$$\boxed{H = \frac{wl^2}{8h}} = \frac{1\times16^2}{8\times3.464} = 9.237 \text{ kN}$$

$$\boxed{T_{max} = \sqrt{V^2+H^2}} = \sqrt{8^2+9.237^2} = 12.220 \text{ kN}$$

$$\boxed{H = T_{max}\cos\theta} \quad \therefore \theta = \cos^{-1}\left(\frac{H}{T_{max}}\right) = \cos^{-1}\left(\frac{9.237}{12.220}\right) = 4$$

A cable is suspended from the points A and B which are 80 m apart horizontally and are at different levels, the point A being 5m vertically higher than the point B and the lowest point in the cable is 10m below A. The cable is subjected to UDL of 30 kN/m over the horizontal span. Determine the horizontal and vertical reactions at each end and also the maximum tension in the cable.



<u>unequal hts</u>

$$\boxed{l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}} = \frac{80\sqrt{10}}{\sqrt{10}+\sqrt{5}} = 46.863\ m$$

$$l_2 = 80 - 46.863 = 33.137$$

$$H\times 10 - \frac{wl_1^2}{2} = 0$$

$$H = \frac{wl_1^2}{20}$$

$$H\times 5 - \frac{wl_2^2}{2} = 0$$

$$H = \frac{wl^2}{2(\sqrt{h_1}+\sqrt{h_2})^2} =$$

$$\boxed{H = 3294.2\ kN}$$

<u>Equilibrium on AC</u>

$$V_A = wl_1 = 30\times 46.863 = 1405.89\ kN$$

<u>CB</u>

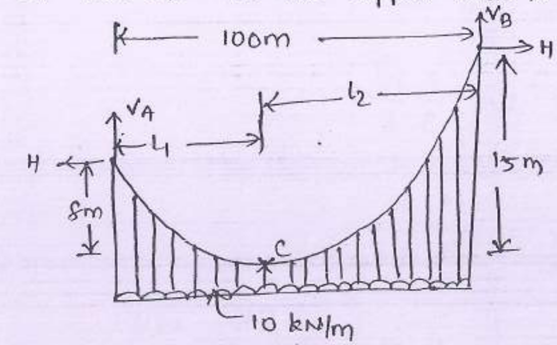
$$V_B = wl_2 = 30\times 33.137 = 994.11\ kN$$

$\therefore V_A > V_B$, T_{max} occur at A

$$\therefore \boxed{T_{max} = \sqrt{V_A^2 + H^2}}$$

$$= \sqrt{1405.89^2 + 3294.2^2} = 3581.66\ kN$$

* A cable of span 100 m has its ends at height 8m and 15 m, above the lowest point of the cable. It carries a 10 kN/m per unit horizontal run of the span. Determine the horizontal and vertical reactions at the support. What is the length of the cable?



$$\therefore \boxed{l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}} = 42.214\ m$$

$$l_2 = 100 - 42.214 = 57.786\ m$$

$$\boxed{H = \frac{wl^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}}$$

$$= 1113.76\ kN$$

$$V_A = wl_1 = 422.14$$

$$V_B = wl_2 = 577.8$$

Length of the cable

$$\boxed{L = l + \frac{2}{3}\times\frac{h_1^2}{l_1} + \frac{2}{3}\times\frac{h_2^2}{l_2}}$$

$$= 103.607\ m.$$

* Effect of Temperature on Cable

Due to rise in temp, the length of cable increases since, the supports are rigid, increase in length of cable, increase the dip of the cable and hence, reduces the horizontal thrust.

$$L = l + \frac{8}{3}\frac{h^2}{l}$$

$$\therefore \frac{dL}{dh} = \frac{16}{3}\frac{h}{l}$$

& $$\delta L = \frac{16}{3} \times \frac{h}{l} \times \delta h \quad -(1)$$

If α is the coefficient of thermal expansion and t is the rise in temperature, then increase in length is given by

$$\delta L = L\alpha t$$

$$= \alpha t\left[l + \frac{8h^2}{3l}\right]$$

Since, $\alpha t\left(\frac{8h^2}{3l}\right)$ is small quantity, it may be neglected,

$$\delta L = l\alpha t \quad -(2)$$

Eq^n (1) & (2)

$$\frac{16}{3} \times \frac{h}{l} \times \delta h = l\alpha t$$

& $$\boxed{\delta h = \frac{3}{16} \times \frac{l^2 \alpha t}{h}} \quad -(3)$$

The above eq^n gives increase in dip due to rise in temp.

M_c, we get, $\quad Hh = M_c$, where M_c is beam moment

$$\boxed{H = \frac{M_c}{h}}$$

$$\frac{dH}{dh} = -\frac{M_c}{h^2} = -\left(\frac{M_c}{h}\right)\frac{1}{h} = -\frac{H}{h}$$

$$\therefore \frac{\delta H}{H} = -\frac{1}{h} \times \delta h$$

Sub. the values δh from eq^n (3), we get

$$\frac{\delta H}{H} = -\frac{3}{16} \times \frac{l^2}{h^2} \times \alpha t$$

* A cable of span 80m and dip 6m is subjected to [rise] in temp of 20°C. If the coefficient of thermal expansion of cable material is $12 \times 10^{-6}/°C$, determine the increase [in] dip of the cable. What are the changes in reactions [and] maximum tension, if the cable carries a load of 15 kN[/m].

Sol:

$l = 80m \qquad h = 6m$

$t = 20°C$ and $\alpha = 12 \times 10^{-6}/°C$

$$\therefore \delta h = \frac{3l^2\alpha t}{16h} = 0.048m$$

If cable carries a udl 15 kN/m, then,

$$Hh = \frac{wl^2}{8}$$

$$H = \frac{wl^2}{8h} = \frac{15 \times 80^2}{8 \times 6} = 2000 \text{ kN}$$

$\therefore$ From eq^n $\frac{\delta H}{H} = -\frac{3}{16} \times \frac{l^2}{h^2}\, \alpha t$

$$\frac{\delta H}{2000} = -\frac{3}{16} \times \frac{80^2}{6^2} \times 12 \times 10^{-6} \times 20$$

$$\delta H = -16\,kN$$

$$\therefore \quad H = 2000 - 16 = 1984\,kN$$

$$V_A = \frac{wl^2}{2} = \frac{15 \times 80}{2} = 600\,N$$

Due to T, there will be no change in V,

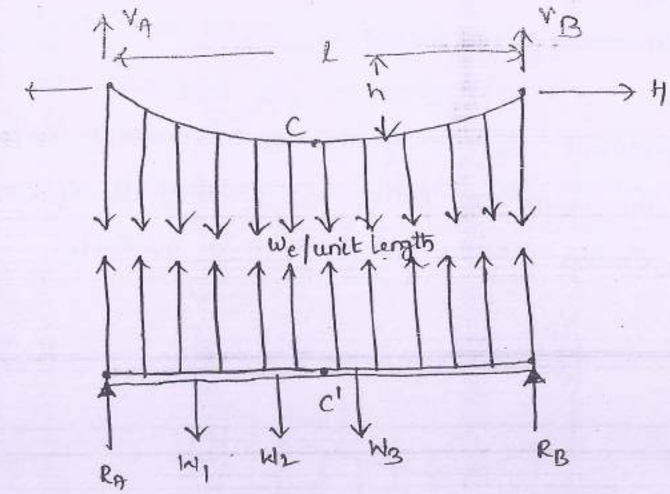
Before rise of temperature,

$$T = \sqrt{2000^2 + 600^2} = 2088.06\,kN$$

and after rise in temperature,

$$T = \sqrt{(1984)^2 + 600^2} = 2072.74\,kN$$

change in max. tension $= 2088.06 - 2072.74 = 15.32\,kN$

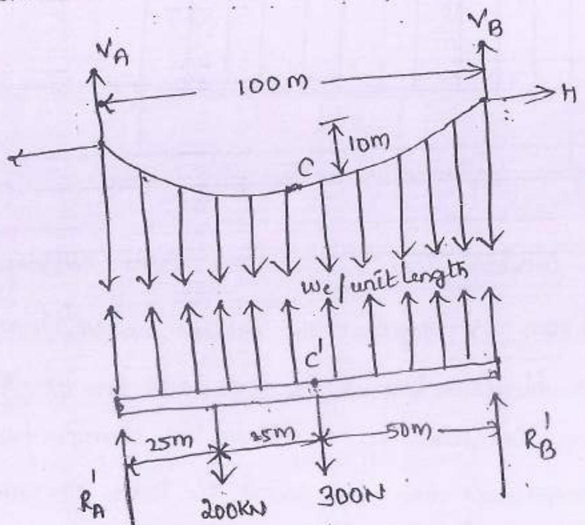


Consider the suspension cable stiffened wi

hinged grider as shown in figure. The grider can be a heavy be
truss which has three hinges • two at the ends and one at the ce
cable and grider are connected by a number of hangers/suspen
since, the number of suspenders are very large, the load on cable &
due to forces in the suspenders, may be taken as UDL, let it be
unit horizontal length.

Considering the moment at central hinge of g
w_e, the UDL exerted by suspenders can be determined. Then, the
be analysed for the given load along with w_e. Note that due to w_e
there is a bending moment of $\frac{w_e x}{8}(l-x)$ at a section x-x and a
moment, i.e $\frac{w_e l^2}{8}$ at C. This moment is a hogging moment. Th
force at that section due to w_e is $-w_e\left(\frac{l}{2}-x\right)$. The cable c
analysed for the UDL w_e. The analysis is illustrated wi
problems below.

... hinged stiffening girder of a suspension bridge of span ... is subjected to two point loads of 200 kN and 300 kN at the distance of 25 m and 50 m from the left end. Find the shear force and bending moment for the girder at a distance 30 m from the left end. The supporting cable has a central dip of 10 m. Find also the maximum tension and its slope in the cable.



Let the suspenders exert a UDL of w_e/unit horizontal length as shown in figure

Reactions at A and B due to given load be R_A' & R_B'

$\therefore \Sigma V = 0, \quad R_A' + R_B' = 300 + 200$

$\boxed{R_A' + R_B' = 500\,kN}$ — (1)

$\therefore \Sigma M_A' = 0$, gives

$R_B' \times 100 = 200 \times 25 + 300 \times 50$

$\boxed{R_B' = 200\ kN}$

Sub in (1) we get $\boxed{R_A' = 300\,kN}$

Considering $M_C' = 0$, gives

Bending moment due to given loading + B.M due to w_e

$\therefore R_B' \times 50 - \frac{w_e l^2}{8} = 0$

$200 \times 50 = \frac{w_e \times 100^2}{8}$

$\boxed{w_e = 8\ kN/m}$

S.F at $x = 30\,m$, from left end

$\boxed{- w_e\left(\frac{l}{2}\right.}$

S.F = S.F due to given loading + S.F due to

$= R_A' - 200 - w_e\left(\frac{100}{2} - 30\right)$

$= 300 - 200 - 8(50-30)$

$= -60\ kN$

$= 60\ kN\,(\downarrow)$

$\boxed{w_e x}$

B.M = M due to given loading + M due to

$= 300 \times 30 - 200 \times 5 - \frac{8 \times 30(100-30)}{8}$

$= -400\ kNm = 400\ kNm$, hogging.

For analysis of Cable,

$V_A = \frac{w_e l}{2} = \frac{8 \times 100}{2} = 400\ kN$

$H = \frac{w_e l^2}{8h} = \frac{8 \times 100^2}{8 \times 10} = 10000$

$T = H * \sqrt{}$ $\quad \boxed{\therefore H = 10000\ kN}$

$T_{max} = \sqrt{H^2 + V_A^2}$

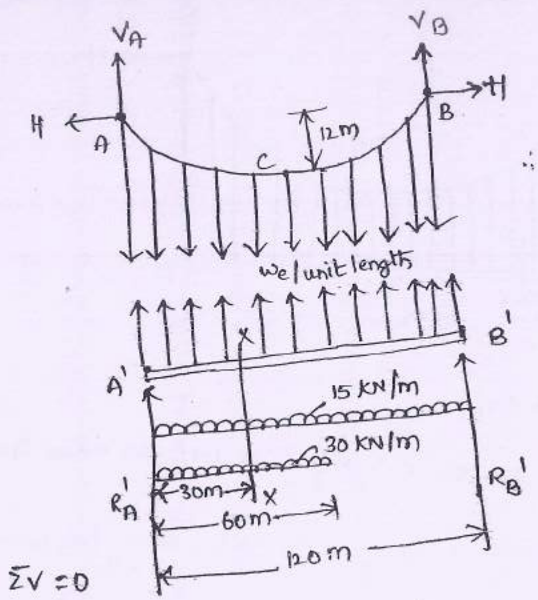
$= \sqrt{1000^2 + 400^2} = 1077.03\ kN$

21-Hope,

$H = T_{max} \cos\theta$

$\theta = \cos^{-1}\left(\frac{H}{T_{max}}\right) = 21.80^\circ$

A suspension bridge of 12m span has two three-hinged stiffening girders supported by two cables having a central dip of 12m. The roadway has a width of 6m. The dead load on bridge is 5 kN/m² while the live load is 10 kN/m² which acts on the left-half of the span. Determine the S.F and B.M in the girder at 30m from the left end. Find also the maximum tension in the cable for position of live load.



width of bridge = 6m

Dead load on bridge = 5 kN/m²

∴ Deadload on each girder

$= 5 \times \frac{6}{2} = 15\ kN/m$

live load on bridge = 10 kN/m²

∴ live load on each girder

$= 10 \times \frac{6}{2} = 30\ kN/m$

$\Sigma V = 0$

$R_A' + R_B' = 15 \times 120 + 30 \times 60$

$\boxed{R_A' + R_B' = 3600\ kN}$ — (1)

$\Sigma M_A' = 0, \quad R_B' \times 120 = 30 \times 60 \times 30 + 15 \times 120 \times 60$

$\boxed{R_B' = 1350\ kN}$

Sub. in eqⁿ (1) we get

$\boxed{R_A' = 2250\ kN}$

Considering,

$\Sigma M_C = 0$, gives

Moment due to given loading + Moment due to w_e = 0

$R_B' \times 60 - 15 \times 60 \times 30 - w_e \times \frac{120^2}{8} = 0$

$1350 \times 60 - 15 \times 60 \times 30 = w_e \times \frac{120^2}{8}$

$w_e = 30\ kN/m$

S.F at $x = 30m$ from left support,

S.F = S.F due to given loading + ~~Moment~~ S.F due to w_e

$= R_A' - 15 \times 30 - 30 \times 30 - w_e\left(\frac{l}{2} - x\right)$

$= 2250 - 15 \times 30 - 30 \times 30 - 30\left(\frac{120}{2} - 30\right)$

$= 0\ kN$

B.M = M due to given loading + M due to w_e

$= R_A' \times 30 - 15 \times 30 \times \frac{30}{2} - 30 \times 30 \times \frac{30}{2} - \frac{w_e x(l-x)}{2}$

$= 2250 \times 30 - 15 \times 30 \times 15 - 30 \times 30 \times 15 - \frac{30 \times 30(120-30)}{2}$

$= 6750\ kNm$

Considering the equilibrium of cable,

$V_A = \frac{w_e l}{2} = \frac{30 \times 120}{2} = 1800\ kN$

$H = \frac{w_e l^2}{8h} = 4500\ kN$

$T_{max} = \sqrt{V_A^2 + H^2}$

$= \sqrt{1800^2 + 4500^2} = 4846.65$ kN

$H = T_{max} \cos\theta$

$\theta = \cos^{-1}\left(\frac{H}{T_{max}}\right) = 21.80^\circ$ //

Suspension cable with 2-hinged stiffening Grider:-

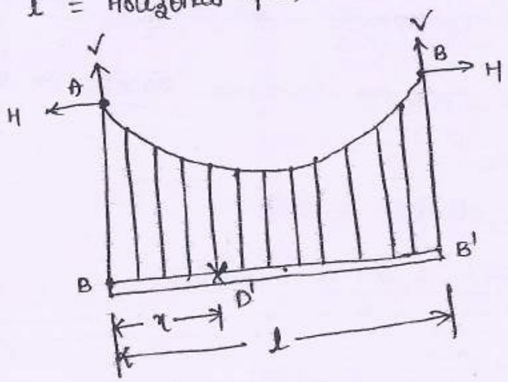
In this case, as the name suggest the stiffening griders are having two hinges. They are provided at the end supports as shown in figure. This is a statically indeterminate structure. However, the analysis is made by assuming that the girder is very stiff and the stiffners are equally stressed. hence, what ever is the load on the grider the stiffners are equally stressed. i.e Equivalent udl due to forces in suspenders is equal to the average load on the stiffening grider. Thus

$$w_e = \frac{w}{L}$$

where w = Total load on the girder

w_e = Equivalent UDL

l = Horizontal span



$$H = \frac{w_e l^2}{8h} \qquad V = \frac{w_e l}{2}$$

$T_{max} = \sqrt{V_A^2 + H^2}$

For grider,

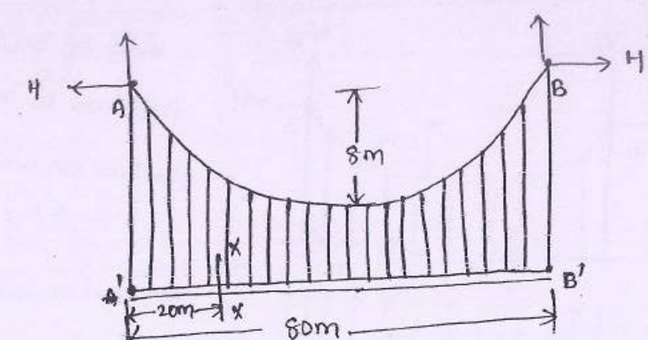
$M_D' =$ Beam moment + Moment due to W_e

$= \text{Beam moment} - \frac{w_e x(l-x)}{2}$

$F_D' =$ Beam shear + shear due to W_e

$= \text{Beam shear} - w_e\left(\frac{l}{2} - x\right)$

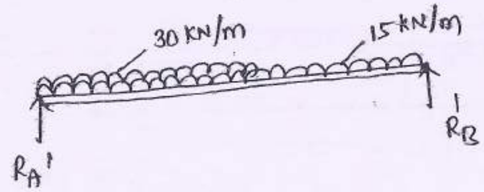
* A suspension bridge of span 80 m and width 6m is having two stiffened with two, hinged griders. The central dip of cables is 8m, load on the bridge is 5 KN/m² and live load is 10 KN/m² which covers left-half of the span. Determine the S.F and B.M at 20m from left and find also the maximum tension in the cable.



$l = 80$ m, $h = 8$ m

Dead load on each grider $= 5 \times \frac{6}{2} = 15$ KN/m on entire span

live load on each grider $= 10 \times \frac{6}{2} = 30$ KN/m over left half portion



$\sum V = 0,\quad R_A' + R_B' = 15 \times 80 + 30 \times 40$

$$\boxed{R_A' + R_B' = 2400\ kN} \quad \text{--- (1)}$$

$\sum M_A = 0,\quad R_B' \times 80 = 15 \times 80 \times 40 + 30 \times 40 \times 20$

$$\boxed{R_B' = 900\ kN}$$

Sub in eq^n (1), we get

$$\boxed{R_A' = 1500\ kN}$$

$\therefore$ equivalent UDL, $W_e = \dfrac{\text{Total load}}{\text{Span}} = \dfrac{W}{l}$

$\therefore$ Total load on girder, $W = 15 \times 80 + 30 \times 40 = 2400$

$\therefore W_e = \dfrac{2400}{80} = 30\ kN/m$

$H = \dfrac{W_e l^2}{8h} = \dfrac{30 \times 80^2}{8 \times 8} = 3000\ kN$

$V = \dfrac{W_e \times l}{2} = \dfrac{30 \times 80}{2} = 1200\ kN$

$T_{max} = \sqrt{V^2 + H^2} = 3231.10\ kN$

At $x = 20m$, from left support

$M_D' =$ Beam moment + Moment due to W_e

$= 1500 \times 20 - 15 \times 20 \times 10 - 30 \times 20 \times 10 - \dfrac{W_e x (l-x)}{2}$

$= 30000 - 3000 - 6000 - \dfrac{30 \times 20 (80-20)}{2}$

$= 2000\ kNm$

$= 1500 - 15 \times 20 - 30 \times 20 - W_e \left(\dfrac{l}{2} - x\right)$

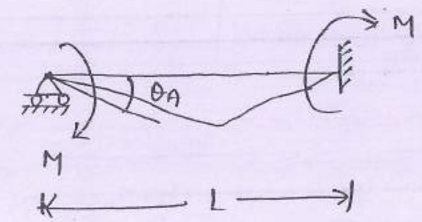
$= 1500 - 300 - 600 - 30 \left(\dfrac{80}{2} - 20\right)$

$= 0$

IV MOMENT DISTRIBUTION METHOD

- This method is widely used for analysis of indetermina
- This was suggested by prof. Hardy Cross in early 1930's
- In this method, solution of simultaneous equations of deflection method is replaced by an iterative distribution pro

Carry over moment :- (COM)



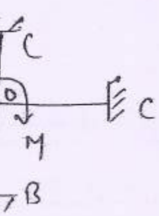
Carryover factor :-

$$\text{Carryover factor (COF)} = \frac{M'}{M}$$

Stiffness (K) :-

$$K = \frac{M}{\theta_A}$$

Distribution factor :-

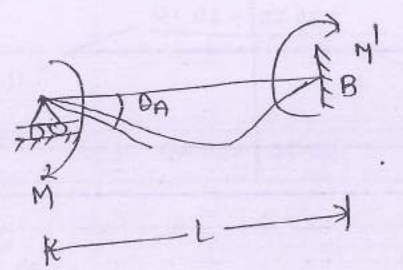


$$d_{OA} = \frac{M_{OA}}{M}$$

Sign conventions :-

- All clockwise moments are positive
- All anticlockwise moments are negative.

Expressions for carry over factor and stiffness :-



Carry over factor $= \frac{M'}{M}$ stiffness of Beam $= \frac{M}{\theta_A}$

$$K = \frac{M}{\theta_A} = \frac{4EI}{L}$$

$$\boxed{K = \frac{4EI}{L}}$$

* Expression for distribution factor :-

$$\text{distribution factor} = \frac{K_i}{\sum_{i=1}^{4} K_i}$$

$\frac{K_i}{\Sigma K}$ is distribution factor of a member

ΣK is joint stiffness

Application of Moment Distribution method to Continuous

Beam with fixed ends :-

In the analysis of continous beams by moment method, the following steps may be taken.

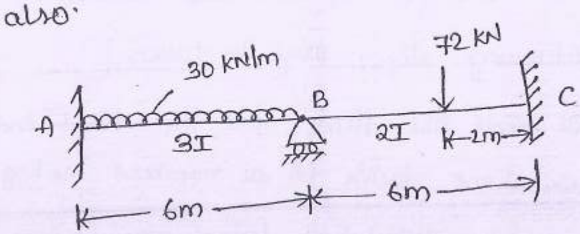
1. Assuming all ends are fixed, find the fixed end moments
2. Calculate distributions factors for all members meeting at a joint
3. Balance a joint by distributing balance moment (negative of moment at the joint) to various members meeting at the joint proportional to their distribution factors. Do similar exercise for joints
4. Carry over half the distributed moment to the far end of This upsets the balance of the joint.
5. Repeat 3 and 4 steps till distributed moments are
6. Sum up all the members moments at a particular the member to get final moment.

NOTE:- A fixed end is a self-balancing joint, since, support equal and opposite moment to keep the end balanced.

Above procedure may be carried out syste in a moment distribution table which is illustrated with ~~above~~ of a example problems.

01

Analyse the continuous beam shown in figure by moment distribution method and draw S.F.D and B.M.D. Draw the elastic curve also.



Fixed end moments :-

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{30\times 6^2}{12} = -90 \text{ KNm}$$

$$M_{FBA} = +\frac{wl^2}{12} = 90 \text{ kNm}$$

$$M_{FBC} = -\frac{wab^2}{l^2} = -\frac{72\times 4\times 2^2}{6^2} = -32 \text{ kNm}$$

$$M_{FCB} = +\frac{wa^2 b}{l^2} = \frac{72\times 4^2\times 2}{6^2} = 64 \text{ kNm}$$

Distribution factors

Joints	MEMBERS	K $\left[\frac{4EI}{L}\right]$	ΣK	Distribution Factors
B	BA	$\frac{4E(3I)}{6} = 2EI$	3.33EI	0.6
	BC	$\frac{4E(2I)}{6} = 1.33EI$		0.4

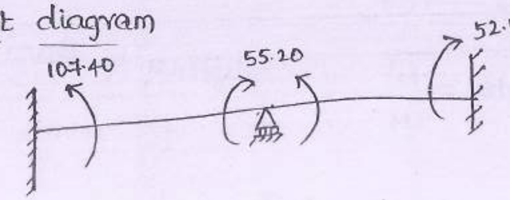
$\boxed{K = \frac{4EI}{L}}$ $\boxed{D.F = \frac{K}{\Sigma K}}$

Distribution Table

Moment distribution

A	B (0.6)	B (0.4)	C
-90	90	-32	64
	-34.80	-23.20	
-17.40			-11.60
-107.40	55.20	-55.20	52.40

4. Bending moment diagram



5. Bending moment :

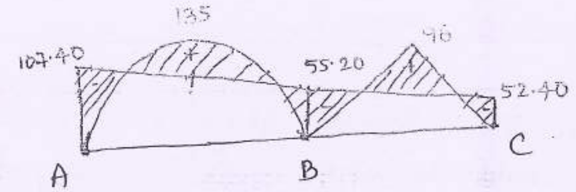
Free moment diagram for AB is a symmetric para with maximum ordinate under the load

$$= \frac{wl^2}{8} = \frac{30\times 6^2}{8} = 135 \text{ kNm}$$

Free moment diagram for BC is a triangle with maxi ordinate under the load

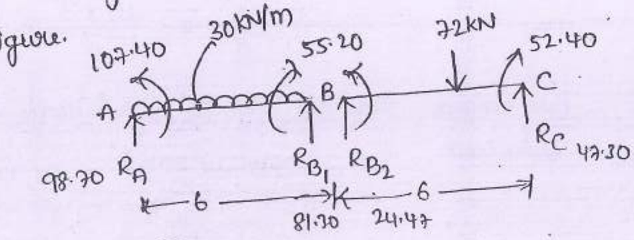
$$= \frac{wab}{l} = \frac{72\times 4\times 2}{6} = 96 \text{ k}$$

Since, we need the difference of free moment a moment diagrams to get final moment diagram, free mome drawn on compression side and end moment diagram on tensi



shear force diagram

Free body diagrams of beam AB and BC are shown in figure.



Beam AB: $\Sigma V = 0$,

$R_A + R_{B_1} = 30 \times 6$

$\boxed{R_A + R_{B_1} = 180\ KN}$ — (1)

$\Sigma M_B = 0$, gives

$R_A \times 6 + 55.20 - 107.40 - 30 \times 6 \times 3 = 0$

$\boxed{R_A = 98.7\ KN}$

Sub in eq^n (1) we get R_{B_1}

$\therefore R_{B_1} = 180 - 98.7 = 81.3\ kN$

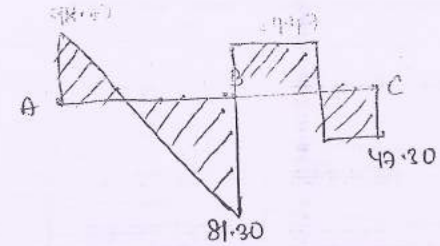
Beam BC: $\Sigma M_C = 0$, gives

$R_{B_2} \times 6 + 52.40 - 55.20 - 72 \times 2 = 0$

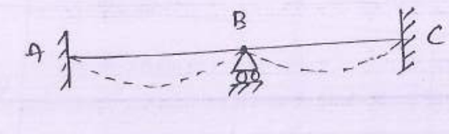
$R_{B_2} = 24.47\ kN$

$\therefore \Sigma V = 0, \quad R_{B_2} + R_C = 72$

$\therefore R_C = 72 - 24.47 = 47.53\ kN$

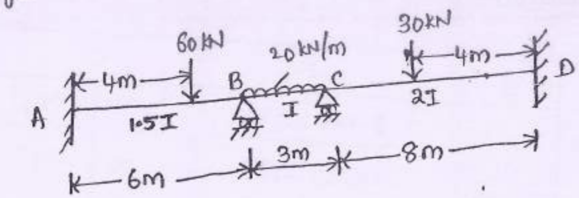


SFD



Elastic curve

* Analyse the continuous beam shown in figure and draw B



1) Fixed end moments:-

$$M_{FAB} = -\frac{wab^2}{l^2} = -\frac{60 \times 4 \times 2^2}{6^2} = -26.6$$

$$M_{FBA} = +\frac{wa^2 b}{l^2} = \frac{60 \times 4^2 \times 2}{6^2} = 53.33\ K$$

$$M_{FBC} = -\frac{wl^2}{12} = \frac{-20 \times 3^2}{12} = -15\ KNm$$

$$M_{FCB} = \frac{wl^2}{12} = 15\ KNm$$

$$M_{FCD} = -\frac{wL}{8} = \frac{30 \times 8}{8} = -30\ KNm$$

$$M_{FDC} = +30\ KNm$$

Distribution Factors

Joints	MEMBERS	K	ΣK	DISTRIBUTION FACTORS
B	BA	$\frac{4E\times1.5I}{6} = EI$	2.33EI	0.429
	BC	$\frac{4EI}{3} = 1.33EI$		0.571
C	CB	1.33EI	2.33EI	0.571
	CD	$\frac{4E2I}{8} = EI$		0.429

$4.29 \times 0.429 = 1.840$
$\times 0.571 = 2.45$

Moment Distribution

Joints	A	B		C		D
Dist. factors		0.429	0.571	0.571	0.429	
FEM	-26.67	53.33	-15	15	-30	30
Balancing		-16.44	-21.88	8.57	6.43	
COM	-8.22		4.29	-10.95		3.22
Balancing		-1.84	-2.45	6.25	4.70	
COM	-0.92		3.13	-1.23		2.35
Balancing		-1.34	-1.79	0.70	0.53	
COM	-0.67		0.35	-0.90		0.27
Balancing		-0.15	-0.20	0.51	0.39	
COM	0.8 0.075		0.26	0.1		0.20
Balancing		-0.11	-0.15	0.06	0.04	
Final moments	36.56	33.45	-33.45	17.91	-17.91	36.04

Bending Moment Diagram

Free moment Daigram for AB is Δ^{le} with maximum under the load

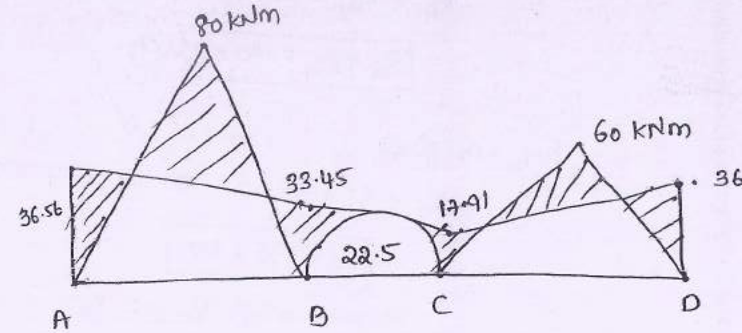
$$= \frac{wab}{l} = \frac{60\times4\times2}{6} = 80 \text{ kNm}$$

Free moment Daigram for BC is symmetric parabola maximum ordinate

$$= \frac{wl^2}{8} = \frac{20\times3^2}{8} = 22.5 \text{ kNm}$$

Free moment Diagram for CD is a triangle with maximum under load

$$= \frac{WL}{4} = \frac{30\times8}{4} = 60 \text{ kNm}$$



BMD

Continuous Beams with Simply Supported [illegible]

Two methods are available for the analysis of such beams by moment distribution procedure

METHOD-I:- To start with, simply supported end is also treated as fixed end and fixed end moments are calculated. In successive iterations also, COM's are coming to this end. If moment exists at this end, it means end is not balanced. Hence, joint balancing is required here. Since, this joint only one member exists, distribution factor is 1. Rest of the procedure is same as discussed for the fixed end continuous beams.

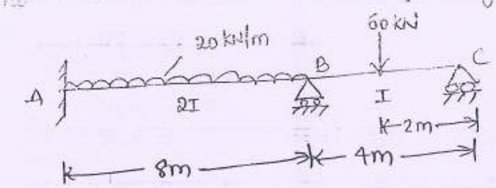
METHOD II

In this method also, fixed end moments are calculate at simply supported end. Then, joint balancing is made for this joint by taking d.f as 1. For further calculations, modified stiffness COM is calculated at the other end. For further calculations, modified stiffness is used for the span which is having simply supported end and COF is taken as zero. For example, let CD be the last span with end D simply supported as shown in figure.

Stiffness for such beam $= \frac{3EI}{L}$ and carry over moment is zero.

$K = \frac{3EI}{L}$

* Analyse the continuous beam shown in figure by moment distribution [illegible]



Fixed end moment :-

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{20\times 8^2}{12} = -106.67 \text{ kNm}$$

$$M_{FBA} = +\frac{wl^2}{12} = +106.67 \text{ kNm}$$

$$M_{FBC} = -\frac{WL}{8} = -\frac{60\times 4}{8} = -30 \text{ kNm}$$

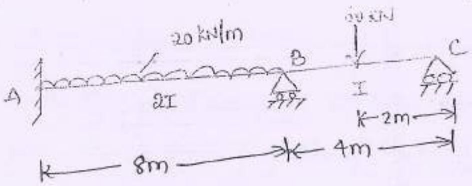
$$M_{FCB} = +\frac{WL}{8} = \frac{60\times 4}{8} = +30 \text{ kNm}$$

Method I :- Distribution factors

Joints	members	K	ΣK	Distribution f
B	BA	$\frac{4E\times 2I}{8} = EI$	2EI	0.5
	BC	$\frac{4E\times I}{4} = EI$		0.5
C	CB	$\frac{4EI}{4} = EI$	EI	1

Moment distribution :-

A	B		C
	0.5	0.5	
-106.67	106.67	-30	30
-19.17	-38.34	-38.34	-30
		-15	-19.17
3.75	7.5	7.5	19.17
		9.58	3.75
-2.40	-4.79	4.79	-3.75
		-1.86	-2.40
0.47	0.93	0.93	+2.40
		1.20	0.47
	-0.60	0.60	-0.47



Fixed end moment :-

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{20\times 8^2}{12} = -106.67 \text{ kNm}$$

$$M_{FBA} = +\frac{wl^2}{12} = +106.67 \text{ kNm}$$

$$M_{FBC} = -\frac{WL}{8} = -\frac{60\times 4}{8} = -30 \text{ kNm}$$

$$M_{FCB} = +\frac{WL}{8} = \frac{60\times 4}{8} = +30 \text{ kNm}$$

Method I :- Distribution factors

Joints	members	K	ΣK	Distribution f
B	BA	$\frac{4E\times 2I}{8} = EI$	2EI	0.5
	BC	$\frac{4E\times I}{4} = EI$		0.5
C	CB	$\frac{4EI}{4} = EI$	EI	1

Moment distribution :-

	A	B	B	C
		0.5	0.5	
				30
	-106.67	106.67	-30	-30
		-38.34	-38.34	
	-19.17		-15	-19.17
		7.5	7.5	19.17
	3.75		9.58	3.75
		-4.79	4.79	-3.75
	-2.40		-1.86	-2.40
		0.93	0.93	+2.40
	0.47		1.20	0.47
		-0.60	0.60	-0.47
Final moments	= -124.02	71.37	71.38	0

Step II: Distribution factors

Joints	Members	K	ΣK	D.F
B	BA	$\frac{4E(2I)}{8} = EI$	1.75EI	0.571
	BC	$\frac{3EI}{4} = 0.75EI$		0.429
	CB	$\frac{4EI}{4} = EI$	EI	1.0

Note:- Stiffness of CB = $\frac{4EI}{4}$, since, end B is continuous

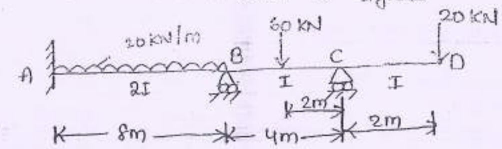
Distribution Table

	A	B	B	C
		0.571	0.429	1.0
				30
	-106.67	106.67	-30	-30
			-15	
		-35.21	-26.46	
	-17.61			
Final moments	124.28	71.46	-71.46	

(106.67 - 45)
× 0.571
× 0.429

NOTE:- method I takes more time and it needs some more iterations to come to the values obtained by method II. method II will be used for further problems. In this method, after balancing simply supported end first time, no values come under this point, since, carryover factor is zero, if modified stiffness of $\frac{3EI}{l}$ is used for member BC.

* Analyse the beam shown in figure



Fixed end moments:- $M_{FAB} = \frac{-20 \times 8^2}{12} = -106.67$ kNm

$M_{FBA} = 106.67$ kNm

$M_{FBC} = -\frac{60 \times 4}{8} = -30$ kNm

$M_{FCB} = +30$ kNm

$M_{FCD} = -20 \times 2 = -40$ kNm (cantilever moment)

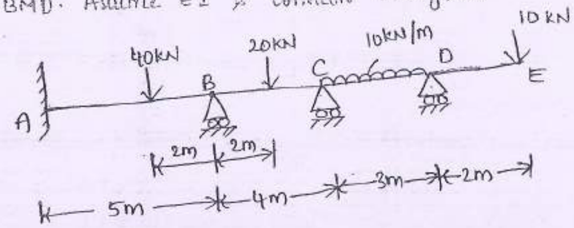
Distribution factors :-

Joints	members	K	ΣK	Distribu
B	BA	$\frac{4E2I}{8} = EI$	1.75EI	0
	BC	$\frac{3EI}{4} = 0.75EI$		0
C	CB	$\frac{4EI}{4} = EI$	EI	
	CD	0		

Moment Distribution Table

	A	B	B	C	C
		0.571	0.429	1.0	0.0
	-106.67	106.67	-30	30	-40
				10	
			-5.0		
		-40.92	-30.75		
	-20.46				
Final moments	-127.13	65.75	-65.75	40	-40

Analyse the continuous beam shown in figure by moment distribution method. Draw BMD. Assume EI is constant throughout.



Fixed end moments :-

$M_{FAB} = -\frac{40\times3\times2^2}{5^2} = -19.2$ kNm

$M_{FBA} = \frac{40\times3^2\times2}{5^2} = 28.8$ kNm

$M_{FBC} = -\frac{WL}{8} = \frac{-20\times4}{8} = -10$ kNm

$M_{FCB} = 10$ kNm

$M_{FCD} = -\frac{10\times3^2}{12} = -7.5$ kNm

$M_{FDC} = 7.5$ kNm

$M_{FDE} = -10\times2 = -20$ kNm (Cantilever moment)

Distribution factors :-

Joints	Members	K	ΣK	Distribution Factors
B	BA	$\frac{4EI}{5} = 0.8EI$	1.8EI	0.444
	BC	$\frac{4EI}{4} = EI$		0.556
C	CB	EI	2EI	0.5
	CD	$\frac{3EI}{3} = EI$		0.5
D	DC	$\frac{4EI}{3} = 1.33EI$	1.33EI	1.0
	DE	0		0.0

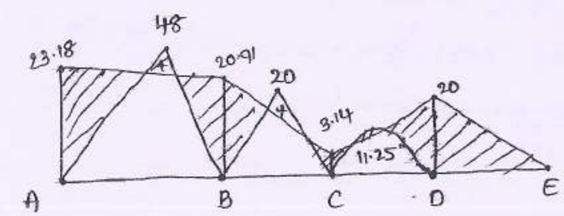
A	B		C		D		E
	0.444	0.556	0.5	0.5	1.0	1.0	0
-19.2	28.8	-10	10	-7.5	7.5	-20	
					12.5		
				6.25			
	-8.35	-10.45	-4.38	-4.38			
-4.18		-2.19	-5.23				
	0.97	1.22	2.62	2.62			
0.49		1.31	0.61				
	-0.58	-0.73	-0.31	-0.30			
-0.29		-0.16	-0.37				
	0.07	0.09	0.19	0.18			
-23.18	20.91	-20.91	3.14	-3.14	20	-20	

Bending moment diagram :-

Freemoment of AB is Δ^{le} with max. ordinate $= \frac{wab}{l} = \frac{40\times3\times2}{5} = 48$

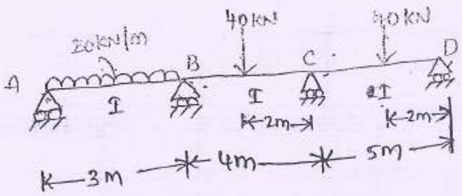
BC is Δ^{le} $= \frac{WL}{4} = \frac{20\times4}{4} = 20$ kN

CD is parabola $= \frac{wl^2}{8} = \frac{10\times3^2}{8} = 11.25$



BMD

method



Fixed end moments:

$M_{FAB} = -\frac{20\times3^2}{12} = -15$ kNm

$M_{FBA} = +15$ kNm

$M_{FBC} = -\frac{40\times2}{8} = -20$ kNm

$M_{FCB} = 20$ kNm

$M_{CD} = -\frac{40\times3\times2^2}{5^2} = -19.2$ kNm

$M_{DC} = -\frac{40\times3^2\times2}{5^2} = 28.8$ kNm

Distribution factors

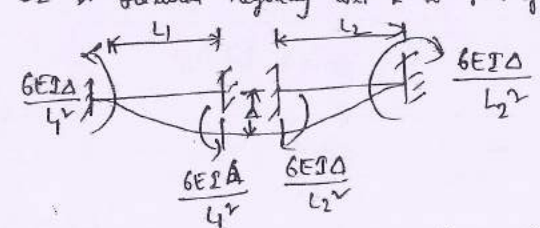
Joints	MEMBERS	K	ΣK	DISTRIBUTION FACTORS
B	BA	$\frac{3EI}{3} = EI$	2EI	0.5
	BC	$\frac{4EI}{4} = EI$		0.5
C	CB	EI	1.6EI	0.625
	CD	$\frac{3E(2I)}{5} = 0.6EI$		0.375
A	AB	—	—	1.0
D	DC	—	—	1.0

Moment distribution:

	A	B		C		D
	1.0	0.5	0.5	0.625	0.375	0.5
	−15	15	−20	20	−19.2	28.8
	15					−28.8
		7.5			−14.4	
		−1.25	−1.25	8.5	−5.1	
			4.25	−0.63		
		−2.13	−2.13	0.39	0.24	
			0.20	−1.06		
		−0.1	−0.1	0.66	0.40	
Final moment	0	19.02	−19.02	27.86	27.86	0

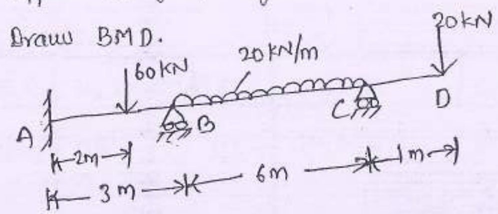
* Continuous Beams without Sinking Supports :-

In the analysis of the fixed beams, That, if the right hand side support settles down, the fixed end mo Produced are given by expression $\frac{6EI\Delta}{L^2}$, anticlockwise, where Δ settlement EI is flexural rigidity and L is span of fixed beam



lly, if the left hand side support yields, fixed end mo developed are given by the expression $\frac{6EI\Delta}{L^2}$, clockwise. In mo distribution method, fixed end moments due to settlement of supp added to fixed end moments due to loadings. Rest of the analy Procedure remains same.

Analyse the continuous beam shown in figure by moment distribution method, if support B yields by 9 mm. Take $EI = 1\times10^{12}\ Nmm^2$ throughout. Draw BMD.



$EI = 1\times10^{12}\ Nmm^2 = 1\times10^3\ kNm^2$

$\Delta = 9mm = 0.009\ m.$

Fixed end moments :-

$$M_{FAB} = -\frac{wab^2}{l^2} - \frac{6EI\Delta}{l^2}$$

$$= -\frac{60\times2\times1^2}{3^2} - \frac{6\times1000\times0.009}{3^2} = -13.33-6$$

$$= -19.33\ kNm$$

$$M_{FBA} = \frac{wa^2b}{l^2} - \frac{6EI\Delta}{l^2}$$

$$= \frac{60\times2^2\times1}{3^2} - \left(\frac{6\times1000\times0.009}{3^2}\right) = 26.67-6$$

$$= 20.67\ kNm$$

$$M_{FBC} = -\frac{wl^2}{12} + \frac{6EI\Delta}{l^2} = -\frac{20\times6^2}{12} + \left(\frac{6\times1000\times0.009}{6^2}\right)$$

$$= -60+1.5 = -58.5\ kNm$$

$$M_{FCB} = \frac{wl^2}{12} + \frac{6EI\Delta}{l^2} = 61.5\ kNm$$

$$M_{CD} = -20\times1 = -20\ kNm$$

Distribution factors:

Joints	members	K	ΣK
B	BA	$\frac{4EI}{3} = 1.33EI$	1.833EI
	BC	$\frac{3EI}{6} = 0.5EI$	
C	CB	$\frac{4EI}{6} = 0.66EI$	0.66EI
	CD	0	

Distribution table :-

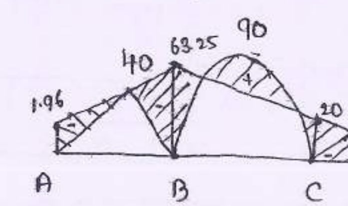
	A	B		C		D
		0.727	0.273	1.0	0.0	0
	-19.33	20.67	-58.75	61.5	-20	
				-41.5		
			-20.75			
		42.58	15.99			
	21.29					
Final moments	1.96	63.25	-63.25	20	-20	0

BMD

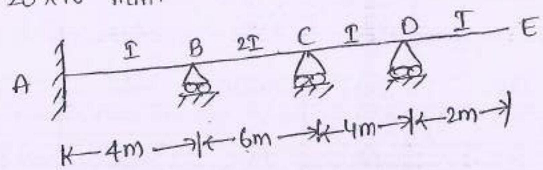
Freemoment of

$$AB = \frac{wab}{l} = \frac{60\times2\times1}{3} = 40\ kNm$$

$$BC = \frac{wl^2}{8} = \frac{20\times6^2}{8} = 90\ kNm$$



Analyse the continuous beam shown in figure by moment distribution method if support B sinks by 12 mm. Given $E = 200\ kN/mm^2$ and $I = 20 \times 10^6\ mm^4$



$\Delta = 12mm = 0.012m$

$EI = 200 \times 20 \times 10^6\ kNmm^2$

$= 200 \times 20\ kNm^2 = 4000\ kNm^2$

Fixed end moments

$M_{FAB} = -\dfrac{6EI\Delta}{4^2} = \dfrac{-6 \times 4000 \times 0.012}{4^2} = -18\ kNm$

$M_{FBA} = -18\ kNm$

$M_{FBC} = \dfrac{6E(2I)\Delta}{6^2} = \dfrac{6 \times 4000 \times 2 \times 0.012}{6^2} = 16\ kNm$

$M_{FCB} = 16\ kNm$

$M_{FCD} = M_{FDC} = \cancel{\dfrac{4EI\Delta}{4^2}} = \cancel{4\Delta}\ 0$

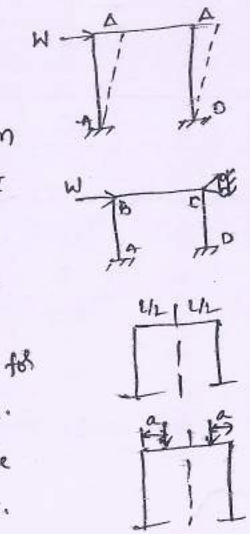
Distribution factors

Joints	members	K	ΣK	Distribution Factors
B	BA	$\frac{4EI}{4} = EI$	2.33EI	0.429
	BC	$\frac{4E(2I)}{6} = 1.33EI$		0.571
C	CB	$\frac{4E(2I)}{6} = 1.33EI$	2.083EI	0.64
	CD	$\frac{3EI}{4} = 0.75EI$		0.36
D	DC	$\frac{4EI}{4} = EI$	EI	1.0
	DE	—	—	

Moment Distribution

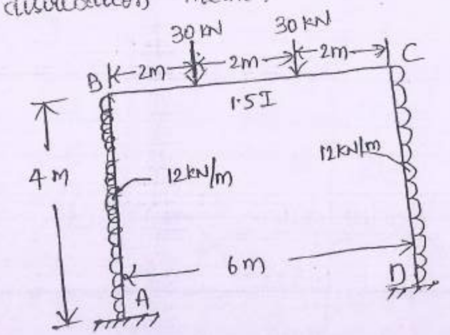
	A	B		C		D	
		0.429	0.571	0.64	0.36	1.0	0.0
	-18	-18	16	16	0	0	
		0.86	1.14	-10.24	-5.76		
	0.43		-5.12	-0.57			
		2.20	2.92	-0.36	-0.21		
	1.10		-0.18	1.46			
		0.08	0.10	-0.93	-0.53		
	0.04		-0.47	0.5			
		0.02	0.27	-0.32	-0.18		
Final moments	-16.43	-14.66	+14.66	6.68	-6.68	0	

Analysis of FRAMES WITHOUT SWAY



There is no sway in symmetrical portal frames and in frames supported at beam levels. Analysis of such frames by moment distribution method is similar to analysis of continuous beams. If the number of members meeting at a joint is not more than two, the distribution table is similar to the one used for the analysis of continuous beams may be used. If the number of members at a joint is more than two, that type of table is not convenient. In such cases, slightly different table is suggested illustrated with

Analyse the symmetric portal frame shown in figure by moment distribution method.



As the frame is symmetric with respect to loading and geometry, there is no sway.

Fixed end moments :-

$M_{FAB} = \frac{-wl^2}{12} = \frac{-12 \times 4^2}{12} = -16$ kNm

$M_{FBA} = 16$ kNm

$M_{FBC} = -\left(\frac{30 \times 2 \times 4^2}{6^2} + \frac{30 \times 4 \times 2^2}{6^2}\right) = -40$ kNm

$M_{FCB} = 40$ kNm

$M_{FCD} = -16$ kNm

$M_{FDC} = 16$ kNm

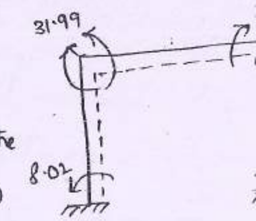
Distribution factors :-

Joints	members	K	ΣK	Distribution factors
B	BA	$\frac{4EI}{4} = EI$	2EI	0.5
	BC	$\frac{4E(1.5I)}{6} = EI$		0.5
C	CB	$\frac{4E(1.5I)}{6} = EI$	2EI	0.5
	CD	$\frac{4EI}{4} = EI$		0.5

Moment Distribution:

A	B		C		D
	0.5	0.5	0.5	0.5	
−16	16	−40	40	−16	16
	12	12	−12	−12	
6		−6	6		−6
	3	3	−3	−3	
1.5		−1.5	1.5		−1.5
	0.75	0.75	−0.75	−0.75	
0.38		−0.38	0.38		−0.38
	0.19	0.19	−0.19	−0.19	
0.10		−0.10	−0.10		−0.1
	0.05	0.05	−0.05	−0.05	
8.02	31.99	−31.99	31.99	−31.99	−8

Bending moment Diagram

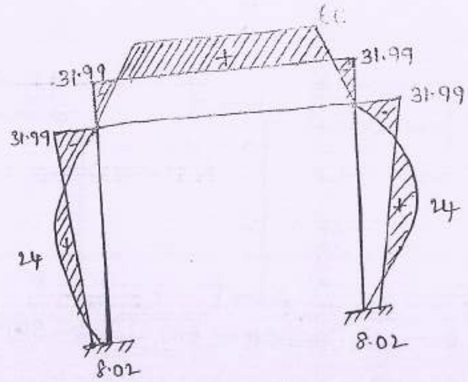


The end moments are marked on the sketch of the portal frame and tension side of the members is identified. The end moment diagram is drawn on the tension side.

For columns, free moment diagrams are symmetric parabolas with ordinate $= \frac{12 \times 4^2}{8} = 24$ kNm

For beam, free end reactions $= \frac{30+30}{2} = 30$ kN

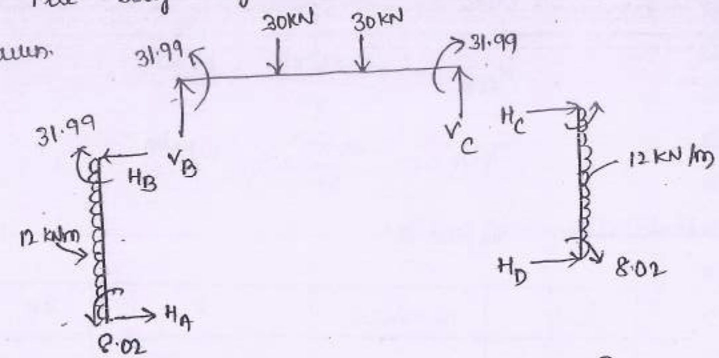
Free moment diagram for beam is having maximum ordinate of 30×



These diagrams are drawn on the compression side. The difference diagram is hatched and is marked positive if it gives tension on the dotted side.

<u>shear force diagram</u>

Free body diagrams for every member of the frames are drawn.



In column AB, $\sum H=0$, $H_B + H_A = 12\times4$. —①

$\sum M_B=0$, $H_A\times4 - 12\times4\times2 + 31.99 - 8.02 = 0$

$H_A = 18$ KN

Sub in ①, $H_B = 30$ kN

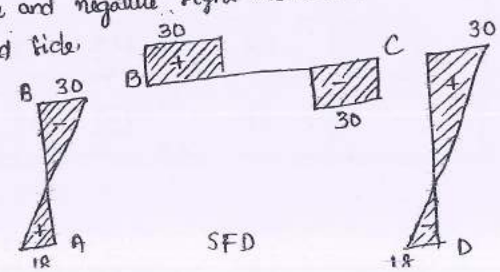
In beam, due to symmetry

$V_B = V_C = 30$ kN

$H_D = H_A = 18$ KN

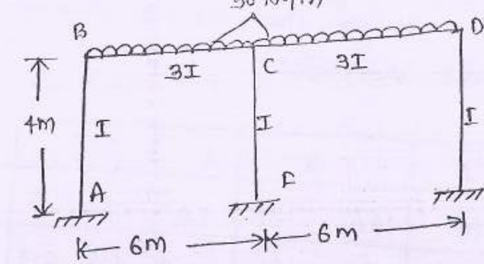
$H_B = H_C = 30$ KN

and Hence, shear force diagram is as shown in figure. In this diagram, positive and negative signs are marked by looking from dotted side.



SFD

analyse the rigid jointed frame shown in figure using moment distribution method and draw bending moment diagram.



There is symmetry in geometry as well as in loading. Hence no sway.

<u>Fixed End Moments</u> :-

$M_{FAB} = M_{FBA} = M_{FCF} = M_{FFC} = M_{FDE} = M_{FED}$

$M_{FBC} = \frac{-36\times6^2}{12} = -108$ KNm

$M_{FCB} = 108$ KNm

$M_{FCD} = -108$ KNm

$M_{FDC} = 108$ KNm

<u>Distribution Factors</u> :

Joints	members	K	ΣK	Distr...
B	BA	$\frac{4E(I)}{4} = EI$	3EI	$\frac{1}{3}$
	BC	$\frac{4E(3I)}{6} = 2EI$		$\frac{2}{3}$
C	CB	$\frac{4E(3I)}{6} = 2EI$	5EI	0.
	CF	$\frac{4EI}{4} = EI$		0
	CD	$\frac{4E(3I)}{6} = 2EI$		0
D	DC	$\frac{4E(3I)}{6} = 2EI$	3EI	[illegible]
	DE	$\frac{4EI}{4} = EI$		[illegible]

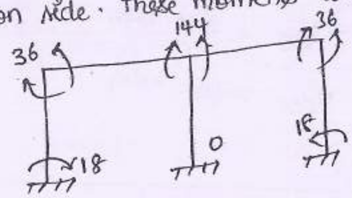
Distribution procedure:-

As more than two members meet joint C, table for moment distribution is modified.

Joints	A	B		C			D		E	F
members	AB	BA	BC	CB	CF	CD	DC	DE	ED	FC
IST. FACTORS		1/3	2/3	0.4	0.2	0.4	2/3	1/3	-	-
FEM			-108	108		-108	108	-36	0	0
Balancing	0	36	72	0	0	0	-72	-36		
COM	18	0	0	36	0	-36	0	0	-18	
Final	18	36	-36	144	0	-144	36	-36	-18	0

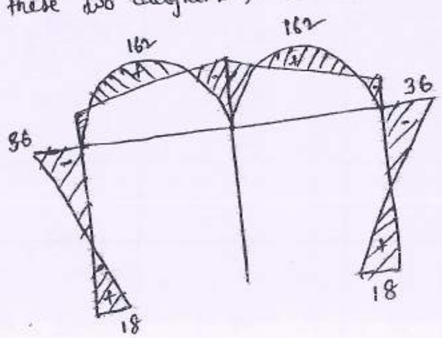
Bending moment diagram:-

The final end moments are drawn on a sketch of a frame to identify tension side. These moments are drawn on tension side.



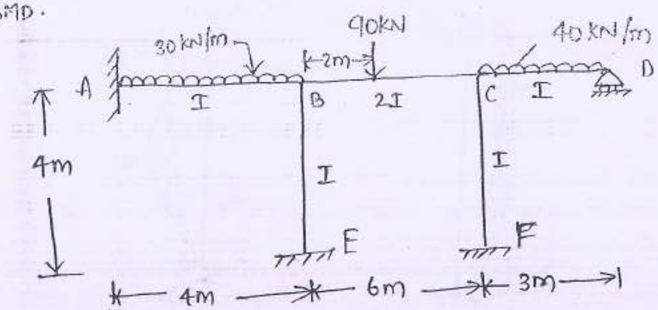
Free moment diagram in BC and CD are symmetric parabola with maximum ordinates $= \frac{30 \times 6^2}{8} = 162$ KNm

This is drawn on compression side of the members. The difference of these two diagrams is the B.M.D.



In this diagram, +ve & -ve signs are marked depending upon whether it causes tension or compression on dotted side.

Draw BMD.



Fixed end moments:-

$M_{FAB} = \frac{-30 \times 4^2}{12} = -40$ KNm

$M_{FBA} = 40$ KNm

$M_{FBC} = \frac{-90 \times 2 \times 4^2}{6^2} = -80$ KNm

$M_{FCB} = \frac{+90 \times 2^2 \times 4}{6^2} = 40$ KNm

$M_{FCD} = \frac{-40 \times 3^2}{12} = -30$ KNm

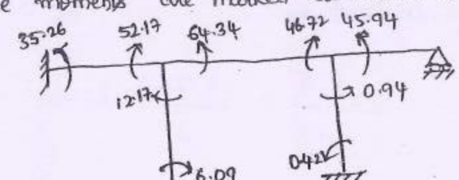
Distribution factors:-

Joints	members	K	ΣK	Distribution
B	BA	$\frac{4EI}{4} = EI$	$\frac{10}{3}EI$	0.3
	BE	$\frac{4EI}{4} = EI$		0.3
	BC	$\frac{4E(2I)}{6} = \frac{4}{3}EI$		0.4
C	CB	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	10 EI	0.4
	CF	$\frac{4EI}{4} = EI$		0.3
	CD	$\frac{3EI}{3} = EI$		0.3

Solution Table

Joints	A	B			C			D	E	F
Members	AB	BA	BE	BC	CB	CE	CD	DC	EC	FB
Dist. factors	-	0.3	0.3	0.4	0.4	0.3	0.3	-	-	
FEM	-40	40		-80	40		-30	30		
							-15	-30		
Bal. D					2.0	1.5	1.5			
Bal, B E C		12	12	16						
COM Bal.	6			1.0	8.0				0.75	6
Bal		-0.3	-0.3	-0.4	-3.2	-2.4	-2.4			
COM	~~-105~~ -0.15			-1.6	-0.2				-1.2	-0.15
Bal		0.48	0.48	0.64	0.08	0.06	0.06		0.03	0.24
COM	0.24			0.04	0.32					
Bal.		-0.01	-0.01	-0.02	-0.12	-0.10	-0.10			
Final	-35.26	52.17	12.17	-64.34	46.72	-0.94	-45.94	0	-0.42	6.09

These moments are marked as shown in figure



Free moment diagram for AB is symmetric parabola with maximum ordinate

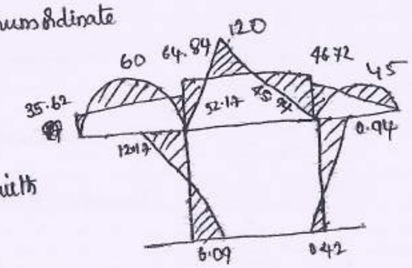
$$= \frac{30 \times 4^2}{8} = 60 \text{ KNm}$$

Free moment diagram for BC is a Δ^{le} with maximum ordinate under the load with value

$$= \frac{90 \times 2 \times 4}{6} = 120 \text{ KNm}$$

Free moment diagram for CD is symmetric parabola with its maximum ordinate

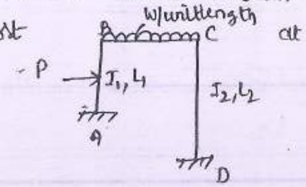
$$= \frac{40 \times 3^2}{8} = 45 \text{ KNm}$$



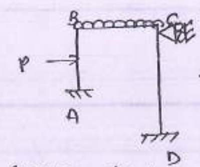
Analysis of FRAMES WITH SWAY

If there is no symmetry in geometry and loading and if there is no support at the beam level, there will be movement of the columns, which is called sway. The procedure for the analysis of frames with sway is given below.

(a) Assume the sway in the frame shown in figure is prevented giving external support at beam level.



Typical frame with sway



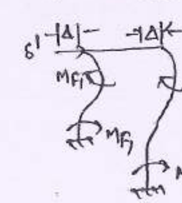
sway prevented by external support

carryout analysis as explained before.

This is called non-sway analysis. Considering the free-body of columns, find horizontal forces developed at supports. Then, the horizontal equilibrium of the entire system, to get force 'S' due to additional support assumed at beam level.

(b) Actually, there is no support at beam level and hence 'S' sway force moving the beam laterally as shown in figure. For given sway force, it is difficult to find the end moments due. Hence the following procedure is followed.

Assume arbitrary sway Δ. Then, fixed end moments in column, AB and CD are



$$M_{F_1} = \frac{-6EI_1\Delta}{L_1^2} \quad \text{and} \quad M_{F_2} = -\frac{6EI_2\Delta}{L_2^2}$$

Thus,

$$\frac{M_{F_1}}{M_{F_2}} = \left(\frac{I_1}{L_1^2}\right) \div \left(\frac{I_2}{L_2^2}\right)$$

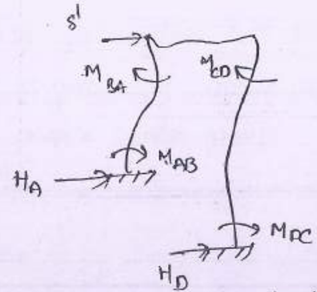
$$M_{F_1} : M_{F_2} = \left(\frac{I_1}{L_1^2}\right) : \left(\frac{I_2}{L_2^2}\right)$$

Now, arbitrary but proportionate values may be assumed for M_1 and M_2.
Then, moment distribution is carried out to get final moments

let M_{AB}, M_{BA}, M_{CD} and M_{DC} be the final values

$$H_A = \frac{M_{AB} + M_{BA}}{L_1}$$

$$H_D = \frac{M_{CD} + M_{DC}}{L_2}$$



$\therefore$ sway force 'S'' acting in this case is obtained by considering horizontal equilibrium of the frame as

$$S' + H_A + H_D = 0$$

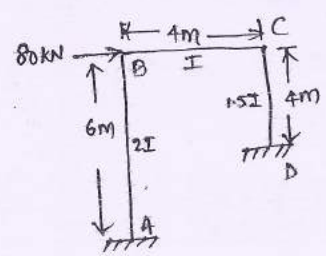
$$K = \frac{S}{S'}$$

The term K is called sway correction factor, Hence,

Final moments = Non-sway moments + K × sway moments

The procedure is illustrated with the examples given below

* Analyse the rigid frame shown in figure by moment distribution method



- Non-sway Analysis:-

Fixed end moments

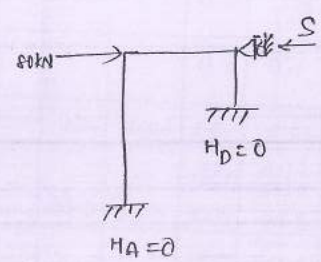
Fixed end moment in all the members = 0

Consider free body diagram of columns and taking moments about top joints, we find

$$H_A = H_D = 0$$

Considering horizontal equilibrium of the frame we find

$$S = 80kN$$



2. Sway analysis

If the structure sway by Δ as shown in figure fixed end are developed are

$$\frac{6EI_1\Delta}{l_1^2} = \frac{6E(2I)\Delta}{6^2} \text{ in colum AB}$$

and $\frac{6EI_2\Delta}{l_2^2} = \frac{6E(1.5)I\Delta}{4^2}$ in column DC

$$\therefore \frac{\text{FEM in AB}}{\text{FEM in DC}} = \left(\frac{2I}{36}\right) / \left(\frac{1.5I}{16}\right) = \frac{16}{27}.$$

let us assume fixed end moments developed in AB = -16 kNm

Therefore moments developed in DC = -27 kNm

For these arbitrary fixed end moments, moment distribution is out.

Joints	members	K	ΣK	D.F
B	BA	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	2.33 EI	0.571
	BC	$\frac{4EI}{4} = EI$		0.429
C	CB	$\frac{4EI}{4} = EI$	2.5 EI	0.4
	CD	$\frac{4E(1.5I)}{4} = 1.5EI$		0.6

Distribution table

A	B		C		D
	0.571	0.429	0.4	0.6	
				-27	27
-16	-16				
	9.14	6.86	10.8	16.2	8.1
4.57		5.4	3.43		
	-3.08	-2.32	-1.37	-2.06	1.03
-1.54		-0.69	-1.16		
	0.39	0.30	0.46	0.70	0.35
0.20		0.23	0.15		
	-0.13	-0.10	-0.06	-0.09	-19.58
-12.77	-9.68	9.68	12.25	-12.25	

If $\vec{H_A}$ & $\vec{H_D}$ are horizontal forces developed, then

$$H_A = \frac{M_{AB} + M_{BA}}{6} \quad \text{and} \quad H_D = \frac{M_{CD} + M_{DC}}{4}$$

If S' is the sway force in such case,

$\Sigma H = 0$ gives $\quad S' + H_A + H_D = 0$

$S' = 11.70$

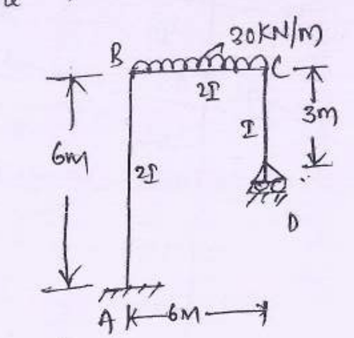
Therefore, sway correction factor $K = \frac{S}{S'} = \frac{80}{11.70} = 6.837$

Final moments may be calculated in the following table

Final moments = Non sway moments + K x sway m

Assumed sway	-12.77	-9.68	9.68	12.25	-12.25
Actual sway	-87.32	-66.19	66.19	83.76	-83.76
Non-sway	0	0	0	0	0
Final	-87.32	-66.19	66.19	83.76	-83.76

* Analyse the frame shown in figure by moment dis method



6.8376 x

(1) Non-sway analysis

$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$M_{FBC} = -\frac{30 \times 6^2}{12} = -90 \text{ kNm}$

$M_{FCB} = 90 \text{ kNm}$

istribution factors

Joints	members	K	ΣK	D.F
B	BA	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	$\frac{8}{3}EI$	0.5
	BC	$\frac{4E(2I)}{6} = \frac{4}{3}EI$		0.5
C	CB	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	$\frac{7}{3}EI$	0.571
	CD	$\frac{3EI}{3} = EI$		0.429

loment Distribution

A	B		C		D
	$\boxed{0.5}$	$\boxed{0.5}$	$\boxed{0.571}$	$\boxed{0.429}$	
		-90	90		
	45	45	-51.39	-38.61	
22.5		-25.70	22.5		
	12.85	12.85	-12.85	-9.65	
6.43		-6.43	6.43		
	3.22	3.22	-3.67	-2.76	
1.61		-1.84	1.61		
	0.92	0.92	-0.92	-0.69	
0.46		-0.46	0.46		
	0.23	0.23	-0.26	-0.20	
31.00	62.22	-62.22	51.91	-51.91	0

$$\vec{H_A} = \frac{M_{AB} + M_{BA}}{L} = \frac{31.0 + 62.22}{6} = 15.$$

$$\vec{H_D} = \frac{M_{CD} + M_{DC}}{L} = \frac{-51.91 + 0}{3} = -17.$$

$\therefore\ S = H_A + H_D = 15.54 - 17.30 = -1.76$

$$\boxed{\therefore S = -1.76}$$

Sway Analysis :-

let M_{F_1} and M_{F_2} be fixed end moments and CD respectively and Δ be the sway

$$\frac{M_{F1}}{M_{F2}} = \left(\frac{6E(2I)\Delta}{6^2}\right) \div \left(\frac{6EI\Delta}{3^2}\right) = \frac{1}{2}$$

let us take $M_{F1} = -50$ KN-m and $M_{F2} = -100$ KNm

i.e $M_{FAB} = M_{FBA} = -50$ KN-m and $M_{FCD} = M_{FDC} = -10$

Distribution factors have been already found. So distribution is carried out

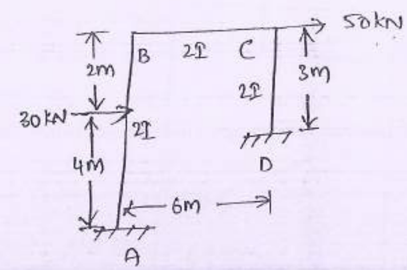
A	B		C		D
	$\boxed{0.5}$	$\boxed{0.5}$	$\boxed{0.571}$	$\boxed{0.429}$	
				-100	-10
-50	-50		50		10
	25	25	28.55	21.45	
12.5		14.28	12.5		
	-7.14	-7.14	-7.14	-5.36	

AB	BA	BC	CB	CD	DC
-3.57		-3.57	-3.57		
	1.78	1.78	2.04	1.53	
0.89		1.02	0.89		
	-0.51	-0.51	-0.51	-0.39	
-0.26		-0.26	0.26		
	0.13	0.13	-0.15	-0.11	
-40.44	-30.74	30.74	32.88	-32.88	0

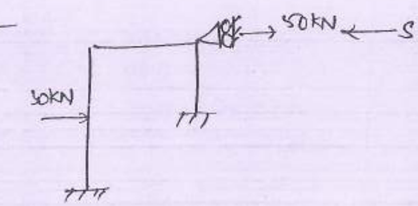
Final moment Calculation

Assumed sway	-40.44	-30.74	30.74	32.88	-32.88	0
Actual sway	3.12	-2.37	2.37	-2.58	2.54	0
non-sway	31.0	62.22	-62.22	51.91	-51.91	0
Final moments	34.20	59.85	-59.85	49.39	-49.37	0

Analyse the frame shown in figure by moment distribution method and sketch bending moment diagram



1. Non-sway Analysis



Fixed end moments :-

$$M_{FAB} = \frac{-30 \times 2 \times 4^2}{6^2} = -13.33 \text{ kNm}$$

$$M_{FBA} = \frac{30 \times 4 \times 2^2}{6^2} = 26.67 \text{ kNm}$$

Distribution factors :-

joints	members	K	ΣK	Distribu
B	BA	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	$\frac{8}{3}EI$	0.5
	BC	$\frac{4E(2I)}{6} = \frac{4}{3}EI$		0.5
C	CB	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	$\frac{8}{3}EI$	0.
	CD	$\frac{4E(2I)}{6} = \frac{4}{3}EI$		0.

Distribution Table

	A	B		C		D
		0.5	0.5	0.5	0.5	
	-13.33	26.67				
		-13.34	-13.33			
	-6.67			-6.67		
				3.33	3.34	
			1.67			1.67
		-0.84	-0.84			
	-0.42			-0.42		
				0.21	0.21	
			0.10			0.10
		-0.05	-0.05	-3.55	3.55	-1.77
	-20.42	12.44	-12.44			

Sway force

Considering equilibrium of columns, we get

$$\vec{H_A} = \frac{M_{AB} + M_{BA} - 30\times 2}{6} = \frac{-20.42 - 12.44 - 30\times 2}{6} = -11.33\text{ kN}$$

$$\vec{H_D} = \frac{M_{CD} + M_{DC}}{3} = \frac{3.55 + 1.77}{3} = 1.773\text{ kN}$$

Considering the horizontal equilibrium of the frame, we get

$$S = 30 + H_A + H_D + 50$$

$$= 30 - 11.33 + 1.773 + 50 = 70.443\text{ kN}$$

Sway Analysis:-

Let the frame sway by Δ towards right & moments developed in column AB be M_{F_1} and that in CD be M_{F_2}

Then $M_{F_1} = \frac{-6E(2I)\Delta}{6^2} = \frac{-EI\Delta}{3}$

$$M_{F_2} = \frac{-6EI\Delta}{3^2} = -\frac{2}{3}EI$$

$$\therefore \frac{M_{F1}}{M_{F2}} = \frac{+1/\cancel{3}}{+2/\cancel{3}} = \frac{1}{2}$$

Let $M_{F_1} = -10$ KNm and $M_{F2} = -20$ kNm

Now, for this case, moment distribution may be carried out:

	A	B		C		D
	-	0.5	0.5	0.5	0.5	
	-10	-10			-20	-20
		5	5	10	10	
	2.5		5.0	2.5		5
		-2.5	-2.5	-1.25	-1.25	
	-1.25		-0.63	-1.25		-0.63
		0.31	0.32	0.62	0.63	
	0.16		0.31	0.16		0.32
		-0.15	-0.16	-0.08	-0.08	
	-0.08		-0.04	-0.08		-0.04
		0.02	0.02	0.04	0.04	
Final moments	-8.67	-7.32	7.32	10.66	-10.66	15.3

$H_A = \frac{M_{AB}+M_{BA}}{6}$ and $H_D = \frac{M_{CD}+M_{DC}}{3}$

$$S' + H_A + H_D = 0$$

$$S' - \left(\frac{8.67+7.32}{6}\right) - \left(\frac{10.66+15.35}{3}\right) = 0$$

$$S' = 11.335 \text{ kN}$$

therefore, sway correction factor $k = \frac{S}{S'} = \frac{70.443}{11.335} = 6.2146$

Final moment calculations

Arbitary sway	-8.67	-7.32	7.32	10.66	-10.66	-15.35
Actual sway	-53.88	-45.49	45.49	66.25	-66.25	-95.39
non-sway	-20.42	12.44	-12.44	-3.55	3.55	1.77
final	74.30	-33.05	33.05	62.70	-62.70	-93.62

$AB = \frac{wab}{L}$

$= \frac{30 \times 2 \times 4}{6}$

$= 40$

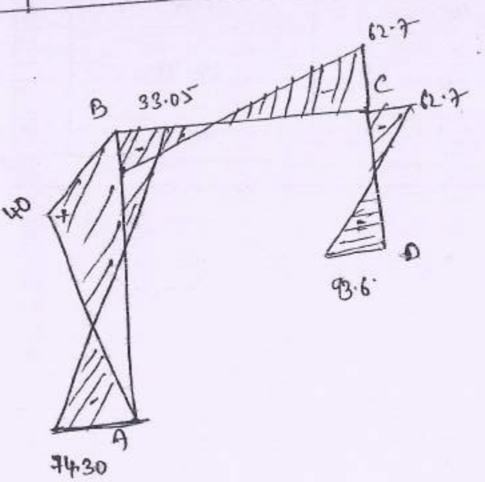
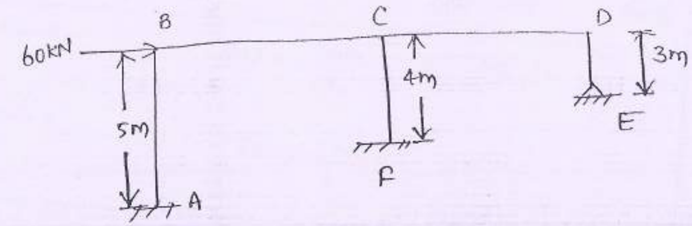


figure and draw bending moment of inertia is same for all



Non - sway analysis

Fixed end ments

In all members FEM = 0

Hence, no horizontal force develops at supports if sway is

$\therefore$ Sway force = 60kN

Sway Analysis: let the beam sway to the right by Δ. Fixed end

In columns AB, CF and DE be M_{F_1}, M_{F2} & M_{F3}. Then,

$$\therefore M_{F_1} = \frac{-6EI\Delta}{5^2} = -\frac{6EI\Delta}{25}$$

$$M_{F_2} = \frac{-6EI\Delta}{4^2} = -\frac{3}{8}EI\Delta$$

$$M_{F_3} = \frac{-6EI\Delta}{3^2} = -\frac{2}{3}EI\Delta$$

$$\therefore M_{F_1} : M_{F_2} : M_{F_3} = \frac{6}{25} : \frac{3}{8} : \frac{2}{3}$$

let $M_{F_1} = M_{FAB} = M_{FBA} = -48$ kNm

$M_{F_2} = M_{FCF} = M_{FFC} = -75$ kNm

$M_{F_3} = M_{FDE} = M_{FED} = -133.33$ kNm

Joints	members	K	Σk	D.F
B	BA	$\frac{4EI}{5} = 0.8EI$	1.8EI	0.444
	BC	$\frac{4EI}{4} = EI$		0.556
C	CB	$\frac{4EI}{4} = EI$	3EI	1/3
	CF	$\frac{4EI}{4} = EI$		1/3
	CD	$\frac{4EI}{4} = EI$		1/3
D	DC	$\frac{4EI}{4} = EI$	2EI	0.5
	DE	$\frac{3EI}{3} = EI$		0.5
E	ED			1.0

Moment Distribution TABLE :-

Joints	A	B		C			D		E	F
MEMBERS	AB	BA	BC	CB	CF	CD	DC	DF	ED	FC
DISTRIBUTION FACTORS		0.444	0.556	1/3	1/3	1/3	0.5	0.5	1.0	
FEM	-48	-48	-	-	-75	-	-	-133.33	-133.33	-75
								66.67	133.33	
		21.31	26.69	25	25	25	33.33	33.33	-	12.5
	10.66		12.5	13.25		16.67	12.5			
		-5.55	-6.95	-10.01	-10.00	-10.01	-6.25	-6.25		-5.00
	-2.78		-5.00	-3.48		-3.13	-5.00			
		-2.22	2.78	2.20	2.21	2.20	2.50	2.50	-	1.11
	1.11		1.1	1.39		1.29	1.10	-	-	
		-0.49	-0.61	-0.88	-0.88	-0.88	-0.55	-0.55		
	-39.01	-30.51	30.51	27.57	-58.87	31.10	37.63	-37.63	0.0	-65.99

$$s' + \frac{M_{AB}+M_{BA}}{5} + \frac{M_{CF}+M_{FC}}{4} + \frac{M_{DE}+M_{ED}}{3} = 0$$

$$s' - \left(\frac{39.01+30.51}{5}\right) - \left(\frac{58.67+65.99}{4}\right) - \left(\frac{37.63+0}{3}\right) = 0$$

$$s' = 57.612$$

$$\therefore K = \frac{S}{S'} = \frac{60}{57.612} = 1.04144$$

As non-sway moments are zero, final moments = 0 + kx s

Final moments

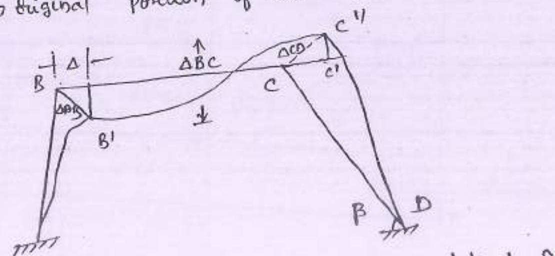
End moment	moment in KNm
AB	-40.62
BA	-31.77
BC	31.77
CB	28.71
CF	-61.10

End moment	Mom
CD	
DC	
DE	
ED	
FC	

A typical skew frame is shown in figure

When there is sway in frame, there will be relative displacements (Δ terms) in all the members of the frame. These terms, which are required for finding fixed end moments, may be found from the geometry of the frame and remembering the following two points:

1. Since, deflections are small, the length of all members, in the deflected conditions, remains equal to the original undeflected lengths
2. Since, axial deflections are neglected, the joint movement is only in the direction $\perp^r$ to original position of the member.



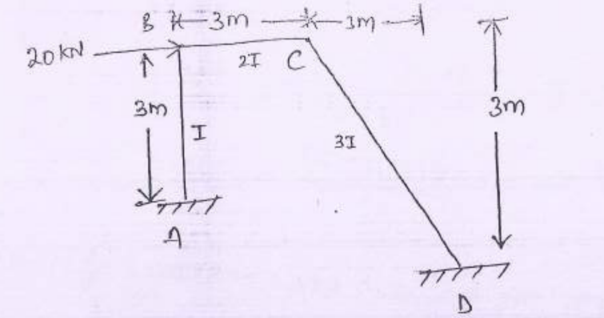
Thus, in the frame, shown in figure, if the lateral sway is Δ, the end B moves B' and relative displacement of the AB is $\Delta_{AB} = \dfrac{\Delta}{\sin\alpha}$

End 'C' has to move at right angles to BC and DC. Hence, its final position is C' as shown in figure.

$$\Delta_{BC} = -(\Delta\cot\alpha + \Delta\cot\beta)$$, negative, since, right side end moves up.

$$\Delta_{CD} = \Delta\cot\beta$$

method



Non-Sway Analysis

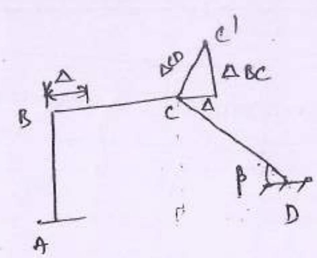
No fixed end moments in any member

Hence, non-sway moments in all members are zero

$\therefore H_A = H_D = 0$

$\therefore$ sway force $S = 20$ kN

Sway Analysis:-



Relative displacements of various members are

$\Delta_{AB} = \Delta$

$\Delta BC = -\Delta\cot\beta = -0.75\Delta$

$\Delta CD = \dfrac{\Delta}{\sin\beta} = \dfrac{\Delta}{0.8} = 1.25\Delta$

let M_{f_1} be fixed end moment in AB

M_{f_2} " BC

M_{f_3} " CD

$$M_{F_1} = \frac{-6EI\Delta}{6^2} = -\frac{2}{3}EI\Delta$$

$$M_{F_2} = \frac{-6E(2I)(-0.75\Delta)}{3^2} = EI\Delta$$

$$M_{F_3} = -\frac{6E(3I)(1.25\Delta)}{5^2} = -0.9EI\Delta$$

$$M_{F_1} : M_{F_2} : M_{F_3} = -\frac{2}{3}EI\Delta : EI\Delta : -0.9EI\Delta = -\frac{2}{3} : 1 : -0.9$$

$$= -20 : 30 : -27$$

let $M_{F_1} = M_{FAB} = M_{FBA} = -20$ kNm

$M_{F_2} = M_{FBC} = M_{FCB} = 30$ kNm

$M_{F_3} = M_{FCD} = M_{FDC} = -27$ kNm

D.F's

Joints	members	K	ΣK	D.F
B	BA	$\frac{4EI}{3} = \frac{4}{3}EI$	4EI	1/3
	BC	$\frac{4E(2I)}{3} = \frac{8EI}{3}$		2/3
C	CB	$\frac{4E(2I)}{3} = \frac{8EI}{3}$	5.067 EI	0.526
	CD	$\frac{4E(3I)}{5} = 2.4EI$		0.474

Moment distribution

A	B		C		D
	1/3	2/3	0.526	0.474	
-20	-20	30	30	-27	-27
	-3.33	-6.67	-1.58	-1.42	
-1.67		-0.79	-3.34		-0.71
	0.25	0.49	1.65	1.69	
0.13		0.88	0.27		0.79
	-0.29	-0.57	-0.14	-0.13	
-21.80	-23.26	23.26	26.97	-26.97	-26.92

Final moments

Sway force

$$S' - \frac{21.80 + 23.26}{3} - \frac{26.97 + 26.92}{4} = 0$$

$$S' = 28.45 \text{ kN}$$

$\therefore$ Sway correction factor $K = \frac{S}{S'} = \frac{20}{28.45} = 0.703$

Final moments

	A	B		C	
Arbitrary sway	-21.80	-23.36	23.36	26.97	-26.97
Actual sway	-15.33	-16.56	16.43	18.95	-18.96
Non-sway	0	0	0	0	0
Final	-15.33	-16.56	16.43	18.95	-18.96