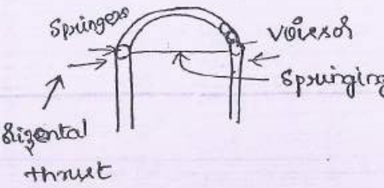
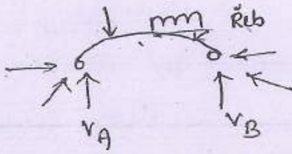
* Individual stone or Brick used the Construction of an arch is ca
voissors.

* Lower most voissors are called Springers

Rib- Bend Steel bars



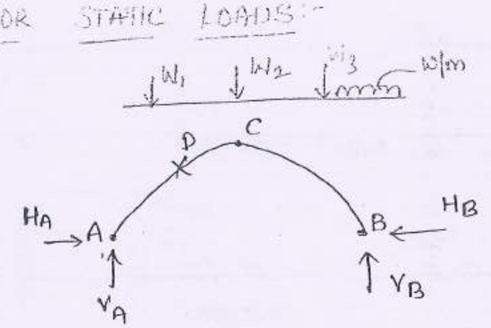
* Force developed along the tangent is called Normal thrust.



* Force developed $\perp^r$ (90°) to the Normal thrust is called Radial shear

* Top most Centre point on the arch is called Crown.

IS <u>FOR STATIC LOADS</u>:-



Consider a 3-hinged arch subjected to loads
own in figure. Since ends are hinged there will
o reaction components at each namely horizontal &
al. Hence there are ~~total~~ four components namely,
H_A, V_B & H_B

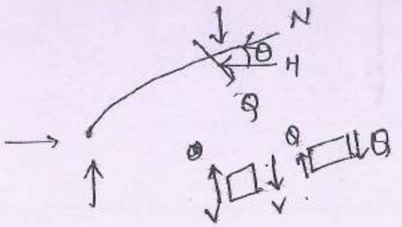
s any plane, $\Sigma F_H = 0$, $\Sigma F_V = 0$, & M_A & $M_B = 0$.

In this case, fourth is solved by $M_c = 0$, since c is hinge.

$\therefore \Sigma F_H = 0, \Rightarrow H_A = H_B$, say H

Since, the loads tend to spread the arch, the horizontal
t is in the inward direction as shown in figure. ✓

consider a section at D,
V, be the vertical shear
Q, be the Radial shear
N, Normal thrust



all these forces are shown in their positive senses
ure. Let the normal to the section make an angle θ with
horizontal

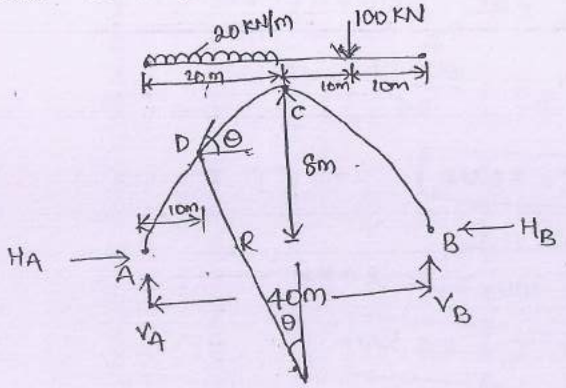
Then,

$$N = V\sin\theta + H\cos\theta$$

$$Q = V\overset{\cos\theta}{\cancel{\sin\theta}} - H\overset{\sin\theta}{\cancel{\cos\theta}}$$

The moment i.e. [illegible] can be obtained by considering all the forces including the reaction on any one part of arch.

A 3-hinged arch hinged at the springing and crown points has a span of 40 m and a central rise of 8m. It carries a UDL of 20 kN/m over the left half of the span together with a concentrated load of 100 kN at right quarter span point. Find the reactions at the supports, normal thrust and shear at a section 10 m from left support



) Given,

$L = 40m$

$h = 8 m$

$V_A, V_B, H_A, H_B = ?$

$N = ?$

$Q = ?$

1. REACTIONS:-

i, $\Sigma F_V = 0$

$V_A + V_B = 20 \times 20 + 100$,

$V_A + V_B = 500$ —①

ii, $\Sigma M_B = 0$, gives

$V_A \times 40 - 20 \times 20 \left(\frac{20}{2} + 20 \right) - 100 \times 10 = 0$

$40 V_A = 20 \times 20 \times 30 + 1000$

$V_A = \frac{13000}{40} = 325$ kN

∴ sub $V_A = 325$, in ①

we get $V_B = 500 - 325 = 175$ kN

$\boxed{\therefore V_A = 325 \text{ kN}}$ & $\boxed{V_B = 175 \text{ kN}}$ //

iii, Since C is hinged, $M_c = 0$ gives [∵ $H_A = H_B =$

$V_B \times 20 - 100 \times 10 - H \times 8 = 0$

$175 \times 20 - 100 \times 10 = 8H$

$\boxed{H = 312.5 \text{ kN}}$

∴ $H_A = H_B = H = 312.5$ kN //

2. finding the value of R & θ

Let D be the point 10m from left support where th normal thrust & shear are to found. Now, from property of

$\frac{L}{2} \times \frac{L}{2} = h(2R - h)$

$\frac{40}{2} \times \frac{40}{2} = 8(2R - 8)$

$\frac{1600}{4} = 16R - 64$

$\frac{1600}{4} + 64 = 16R$

$\frac{1682}{4} = 16R$ $\frac{1600+256}{4} = 16R$

$R = \frac{1682}{64}$ $\boxed{\therefore R = 29m}$

$R =$

$R = \frac{L^2}{8h} + \frac{h}{2}$

$= \frac{40^2}{8 \times 8} + \frac{8}{2}$

$= \frac{1600}{64} + 8$

$= \frac{1600 + 8 \times}{64}$

$= 29m$

∴ slope at $D = \theta = \sin^{-1}\left(\frac{x}{R}\right) = \sin^{-1}\left(\frac{10}{29}\right) = 20.$

Shear V at $x = 10m$:-

$\therefore V = V_A - 20 \times 10$

$= 325 - 200 = 125 \text{ kN}$

$\boxed{\therefore V = 125 \text{ kN}}$

l thrust, $N = V \sin\theta + H \cos\theta$

$= 125 \sin 29.171^\circ + 312.5 \cos 20.171^\circ$

$\boxed{N = 336.437 \text{ kN}}$ //

al shear, $R = V \cos\theta - H \sin\theta$

$= 125 \cos 29.171^\circ - 312.5 \sin 20.171^\circ$

$\boxed{\therefore R = 9.575 \text{ kN}}$ //

circular arch to span 25m with central raise 5m is hinged at the
un and springing. It carries a point load of 100 kN at 6m from the
t support. calculate Reactions at supports, Reactions at crown and
oment at 5m from the left support.

iven,

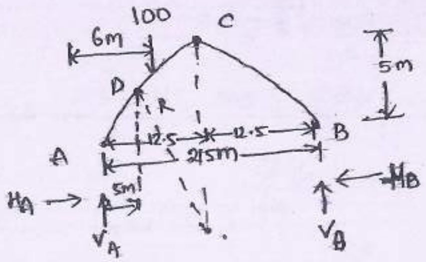
$L = 25m$

$h = 5m$

$V_A, V_B, H_A, H_B = ?$

$H_c, V_c = ?$

M_x at $x = 5 = ?$



i, $\Sigma F_y = 0$

$V_A + V_B = 100 \text{ kN}$ - ①

ii, $\Sigma M_B = 0$,

$V_A \times 25 - 100(25 - 6) = 0$

$V_A = \dfrac{\overset{4}{100} \times 19}{25} = 76 \text{ kN}$

$\boxed{\therefore V_A = 76 \text{ kN}}$

Sub. in ①

$V_B = 100 - 76 = 24 \text{ kN}$

$\boxed{\therefore V_B = 24 \text{ kN}}$

iii, $\Sigma M_c = 0$, since C is hinged

$-V_B \times 12.5 + H \times 5 = 0$

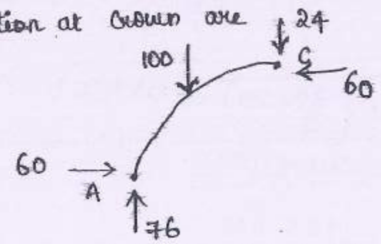
$5H = 24 \times 12.5$

$\boxed{\therefore H = 60 \text{ kN}}$

$\boxed{\therefore H_A = H_B = 60 \text{ kN}}$

2, RECATIONS AT CROWN:-

considering equilibrium of the left half of the arch, the reaction at crown are



$\therefore H = 60 \text{ kN}$

$V_c = 24 \text{ kN}$

[illegible] from the left support.

From property of circle,

i, $\boxed{\therefore R = \frac{L^2}{8h} + \frac{h}{2}}$

$= \frac{25^2}{8\times5} + \frac{5}{2}$

$\boxed{R = 18.125\,m}$

ii, $\boxed{x = \frac{L}{2} - R\sin\theta}$

$5 = \frac{25}{2} - R\sin\theta$

$\therefore R\sin\theta = 12.5 - 5 = 7.5$

$\sin\theta = \frac{7.5}{R} = \frac{7.5}{18.125} = 0.4138$

$\boxed{\theta = 24.443^\circ}$

iii, $\boxed{y_D = h - R(1-\cos\theta)}$

$= 5 - 18.125(1 - \cos 24.443^\circ)$

$\boxed{y_D = 3.375}$

$\therefore M_D = V_A \times 5 - H \times Y_D$

$= 76\times5 - 60\times3.375$

$\boxed{M_D = 177.5\ KNm}$

[illegible] of radius R [illegible] intensity w/unit length over its entire horizontal span. Determine the reactions supports and maximum bending moment in the arch.

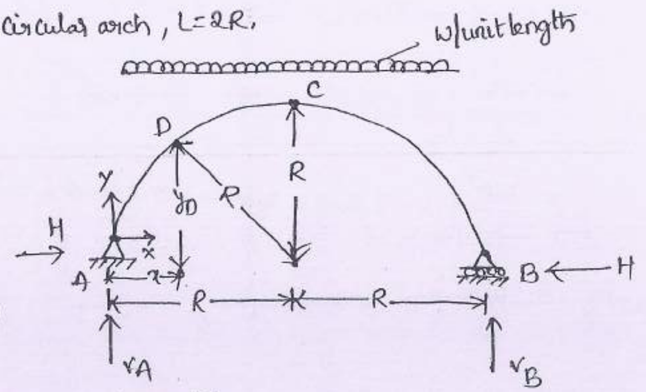
A) Given, it is semi-circular arch, L=2R.

$h = R$

$L = 2R$

$\left.\begin{matrix} V_A, V_B \\ H_A, H_B \end{matrix}\right\} ?$

M_{ax} BM = ?



1. Reactions:- Due to Symmetry

$V_A = V_B = \frac{w\times2R}{2} = WR$

$\boxed{\therefore V_A = V_B = WR}$

Taking momement about C, we get

$\therefore M_c = 0$

$H\times R - V_A\times R + WR\times\frac{R}{2} = 0$

$H\times R - WR\times R + \frac{WR^2}{2} = 0$

$R\times H = WR^2 - \frac{WR^2}{2}$

$\therefore H = WR - \frac{WR}{2}$

$\boxed{\therefore H = \frac{WR}{2}}$

2. Bending moment :-

i, $\boxed{x = \frac{L}{2} - R\sin\theta}$ & ii, $y = \boxed{h - R(1-\cos}$

$\boxed{\therefore x = R - R\sin\theta}$ $\boxed{\therefore y = R\cos\theta}$

$M_x = V_A \times x - H \times y - W \times x \times \frac{x}{2}$

$= WR(R-R\sin\theta) - \frac{WR}{2} \times R\cos\theta - \frac{W}{2}(R-R\sin\theta)^2$

$= \frac{WR^2}{2}\left[2(1-\sin\theta) - \cos\theta - (1-\sin\theta)^2\right]$

$= \frac{WR^2}{2}\left[2 - 2\cancel{\sin\theta} - \cos\theta - 1 + 2\cancel{\sin\theta} - \sin^2\theta\right]$

$M_x = \frac{WR^2}{2}(1 - \cos\theta - \sin^2\theta)$ //

For M_x to be maximum,

$\frac{dM_x}{d\theta} = 0,$

$\Rightarrow \frac{WR^2}{2}(+\sin\theta - 2\sin\theta\cos\theta) = 0$

$\therefore \sin\theta(1 - 2\cos\theta) = 0$

$\theta = 0$ gives the crown point where moment is zero (minimum)

$1 - 2\cos\theta = 0$ should ~~be~~ give maximum point

i.e $\cos\theta = 0.5$

$\Rightarrow \theta = 60^\circ$

∴ therefo x for maximum point, $x = R(1-\cos 60^\circ)$

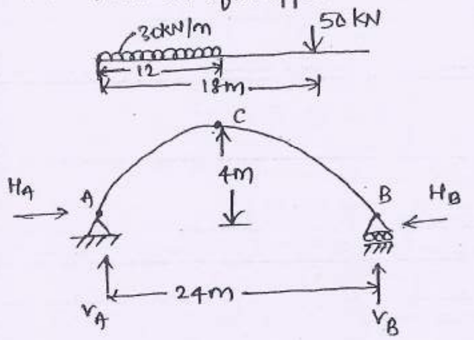
$\therefore M_{max} = \frac{WR^2}{2}(1 - \cos 60^\circ - \sin^2 60^\circ)$

$= \frac{WR^2}{2}\left(1 - \frac{1}{2} - \frac{3}{4}\right) = \frac{WR^2}{2}\left(\cancel{\frac{4-2-3}{4}}\right)$

$= -\frac{WR^2}{8}$

$\boxed{M_{max} = -\frac{WR^2}{8}}$

has a span of 24m and a central rise of 4m. It carries a concentrated load of 50 KN at 18m from left support and a uniformly distributed load of 30 kN over the left-half portion. Determine the Moment, thrust and radial shear at a section 6m from the left support.



A) Given,

$L = 24m$

$h = 4m$

$V_A, H_A, V_B, H_B, N, Q = ?$

1. Reactions at Supports:-

(i) $\Sigma F_y = 0$

$V_A + V_B = 50 + 30 \times 12$

$\boxed{\therefore V_A + V_B = 410 \text{ kN}}$ — ①

(ii) $\Sigma M_B = 0,$

$V_A \times 24 - 50 \times 6 - 30 \times 12\left(\frac{12}{2} + 12\right) = 0$

$\boxed{V_A = 282.50 \text{ KN}}$

Sub. V_A in ①, we get

$V_B = 410 - 282.50 = 127.5$ KN

$\boxed{\therefore V_B = 127.5 \text{ KN}}$

$\sum M_C = 0$,

$$V_B \times 12 - H \times 4 - 50 \times 6 = 0$$

$$127.5 \times 12 - H \times 4 - 50 \times 6 = 0$$

$$\boxed{\therefore H = 307.5\ KN}$$

2. Moment At 6m from left support,

$$M = V_A \times 6 - H \times y_D - \frac{30 \times 6^2}{2}$$

∴ In the parabolic arch,

$$y = \frac{4hx(L-x)}{L^2}$$

∴ at $x = 6$,

$$y_D = \frac{4 \times 4 \times 6(24-6)}{24^2} = 3m$$

$$\therefore M = 282.5 \times 6 - 307.5 \times 3 - \frac{30 \times 6^2}{2}$$

$$\boxed{\therefore M = 232.5\ KNm}$$

3. Vertical shear at D,

$$V = V_A - 30 \times 6$$

$$= 282.5 - 30 \times 6 = 102.5\ KN$$

$$\boxed{\therefore V = 102.5\ KN}$$

4. curve is given by $y = \frac{4hx(L-x)}{L^2}$

$$\therefore \frac{dy}{dx} = \tan\theta = \frac{4h(L-2x)}{L^2}$$

$$\tan\theta = \frac{4 \times 4(24 - 2 \times 6)}{24 \times 24}$$

$$\theta = 18.435^\circ$$

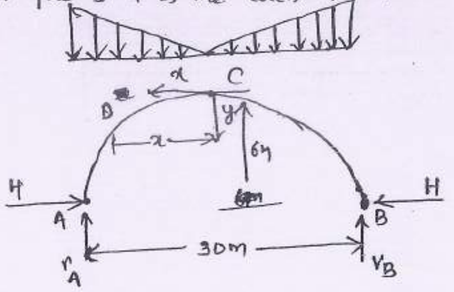
$$N = V\sin\theta + H\cos\theta = 324.133\ kN$$

$$Q = V\cos\theta - H\sin\theta = 0$$

* A Symmetric three-hinged parabolic arch has a span of 30m and of 6m. The arch carries a distributed load which varies uniformly at each abutment to zero at mid-span. determine

(a) The horizontal thrust at the abutments

(b) maximum the B.M in the arch. 40 KN/m



Sol:-

Due to Symmetry

$$V_A = V_B = \frac{1}{2} \times \text{total load}$$

$$= \frac{1}{2} \times \left(\frac{1}{2} \times 15 \times 40\right) = 300\ kN$$

$\sum M_C = 0$, gives

$$V_A \times 15 - H \times 6 - \frac{1}{2} \times 15 \times 40 \times \frac{2}{3}(15) = 0$$

$$H = 250\ KN$$

Taking origin at crown point, C, the equation of Parabola is given be

$$\frac{x^2}{y} = \text{constant} = a$$

∴ at $x = 15$, $y = 6$,

$$\therefore a = \frac{15^2}{6} = 37.5$$

$$\therefore x^2 = 37.5y \quad \& \quad y = x^2/37.5$$

$$\therefore M_x = H \times y - \frac{1}{2} \times x \times \frac{x}{15} \times 40 \times \frac{x}{3}$$

$$= 250 \times \frac{x^2}{3.75} - \frac{1}{2} \times x \times \frac{x}{15} \times 40 \times \frac{x}{3}$$

$$= \frac{250x^2}{37.5} x^2 - \frac{40x^3}{45}$$

$$\therefore M_{x\,max}, \quad \frac{dM_x}{dx} = 0 \Rightarrow \frac{250}{3.75} 2x - \frac{40}{45} . 3x^2 = 0$$

$$\therefore x \left[\frac{500}{3.75} - \frac{40}{15} x \right] = 0$$

$$\therefore x = 0 \text{ and } \frac{500}{3.75} - \frac{40}{15} x = 0$$

$\therefore$ $x=0$, gives crown point, where B.M is zero. Here, the point of maximum moment is given by $\therefore$

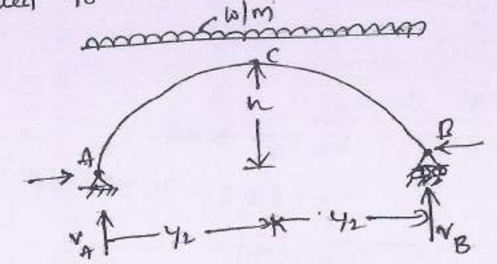
$$x = \frac{15}{40} \times \frac{400}{3.75} = 5$$

$$\therefore M_{max} = \left[\frac{250x^2}{37.5}\right] - \left[\frac{40x^3}{45}\right]$$

$$= 55.555 \text{ kN m}$$

* Show that the parabolic shape is funicular shape for a 3-hinged arch subjected to U.D.L over to its entire span.

A)



Let the span of the arch be L and sketch as shown in figure

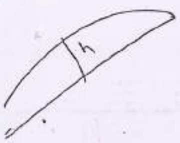
Due to symmetry,

$$V_A = V_B = \frac{1}{2} \times \text{total load}$$

$$\therefore V_A = V_B = \frac{1}{2} \times W \times L = \frac{WL}{2}$$

$$V_A \times \frac{L}{2} - H\times h - w\times\frac{L}{2}\times\frac{L}{4} = 0$$

$$wL\times\frac{L}{2} - H\times h - \frac{wL^2}{8} = 0$$



$$H\times h = \frac{wL^2}{2} - \frac{wL^2}{8}$$

$$\boxed{H = \frac{wL^2}{8h}}$$

$(h = 1)$

$\frac{wL^2}{2}$

$\frac{w(L/2)^2}{4}$

$wxL\times\frac{L}{2}$

$w\times\frac{L}{2}\times\frac{L}{4}$

at any section distance x from A,

$$M = V_A\times x - H\times y - \frac{wL^2}{2}$$

$w\times x\times\frac{x}{2}$

∴ But in a parabolic arch, $y = \frac{4hx(L-x)}{L^2}$

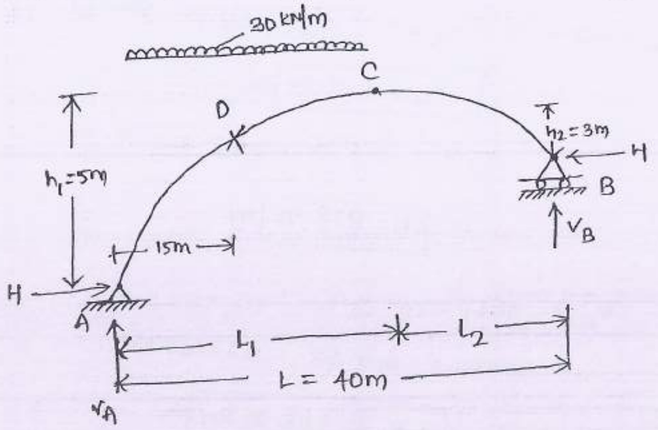
$$M = \frac{wL}{2}x - \frac{wL^2}{8h}\times\frac{4hx(L-x)}{L^2} - \frac{wL^2}{2} = 0$$

$$= \frac{wLx}{2} - \frac{w}{2}(x(L-x)) - \frac{wL^2}{2} = 0$$

$\frac{wLx}{2} - \frac{wx^2}{2}$

Thus, for a parabolic arch subjected to a UDL over its entire span, the B.M at any section is zero. Hence, the parabolic shape is a funicular shape for a 3-hinged arch subjected to UDL over entire span.

Figure carries a UDL of intensity 30 kN/m over the portion left
Determine the horizontal thrust developed. Find also the B.M, normal
radial shear force developed at a section 15m from the left Suppo



Sol:- Taking C as the origin, the eqⁿ of parabola is

$$\frac{x^2}{y} = a, \text{ where } a \text{ is a constant}$$

Let the horizontal distance b/w A and C be L_1 and that of CB

$$\frac{L_1^2}{5} = a = \frac{L_2^2}{3}$$

$$\therefore \frac{L_1}{\sqrt{5}} = \frac{L_2}{\sqrt{3}} = \frac{L_1+L_2}{\sqrt{5}+\sqrt{3}} = \frac{L}{\sqrt{5}+\sqrt{3}}$$

$$\therefore L_1 = \frac{L\sqrt{5}}{\sqrt{5}+\sqrt{3}} = \frac{40\sqrt{5}}{\sqrt{5}+\sqrt{3}} = 22.54\text{ m}$$

$$\therefore L_2 = 40 - 22.54 = 17.46\text{ m}$$

∴ $\Sigma M_C = 0$, gives

$$V_B\times 17.46 = H\times 3$$

$$H = \frac{17.46}{3}V_B$$

$$\boxed{\therefore H = 5.82\,V_B} \quad -①$$

$$V_B \times 40 + H \times 2 = 30 \times 22.54 \times \frac{22.54}{2}$$

$$= 7620.774$$

from ①

$$V_B \times 40 + 5.82 V_B \times 2 = 7620.774$$

$$\boxed{\therefore V_B = 147.58}\ \text{KN}$$

$$H = 5.82 \times 147.58 = 858.92$$

$$\boxed{\therefore H = 858.92}\ \text{KN}$$

and $V_A = 30L_1 - V_B$

$$= 30 \times 22.5 - 147.58 = 528.02 \text{ KN}$$

$$\boxed{\therefore V_A = 528.02 \text{ KN}}$$

The portion left of C may be treated as a parabola of span

$$L' = 2 \times L_1 = 2 \times 22.54 = 45.08 \text{m}$$

refore, equation of parabola is

$$y = \frac{4h_1 x(L' - x)}{L'^2} = \frac{4(5)(x)(45.08 - x)}{45.08^2}$$

$x = 15\text{m}$, $y = 4.44$ m

Therefore, M at this section

$$M = V_A \times 15 - H \times 4.44 - 30 \times 15 \times \frac{15}{2}$$

$$= 528.62 \times 15 - 858.92 \times 4.44 - 30 \times \frac{15 \times 15}{2}$$

$$= 740 \text{ KNm}$$

$$\frac{dy}{dx} = \tan\theta = \frac{4h(L' - 2x)}{L'^2}$$

At $x = 15$m, $\tan\theta = \frac{4 \times 5 \times (45.08 - 2 \times 15)}{45.08^2}$

$$\theta = 8.44^\circ$$

$$N = V \sin\theta + H \cos\theta$$

$$= 78.62 \sin 8.44^\circ - 8858.92 \sin 8.44^\circ$$

$$N = 861.25 \text{ KN}$$

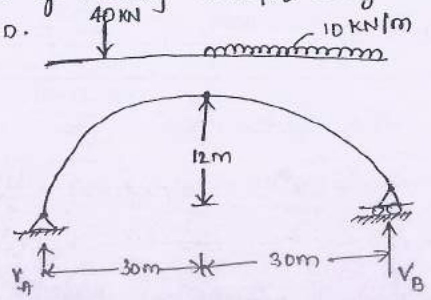
$$Q = V\cos\theta - H\sin\theta$$

$$= 78.62 \cos 8.44^\circ - 858.92 \sin 8.44^\circ$$

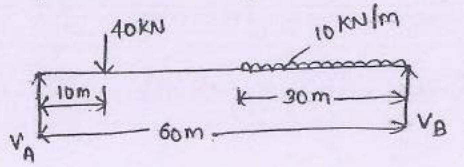
$$Q = -48.30 \text{ KN}$$

* A 3-hinged symmetric parabolic arch of span 60 m and rise 12m is subjected to a concentrated load of 40 KN acting at 10m from its left support & uniformly distributed load of intensity 10 KN/m acting over its entire right half portion. Draw the B.M.D.

Sol :-



The arch with its loading is shown in figure and the equivalent beam is



$$V_A + V_B = 40 + 10 \times 30 = 340 \text{ KN} \quad ①$$

$M_B = 0$, $V_A \times 60 - 40(50) - 10 \times 30 \times 15 = 0$

$$V_A = \frac{40(50) + 10 \times 30 \times 15}{60} = 108.33 \text{ KN}$$

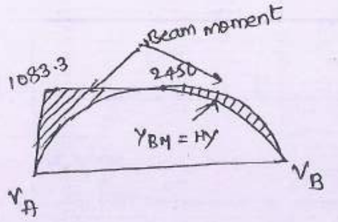
$$\boxed{\therefore V_A = 108.33 \text{ KN}}$$

V_B = 340 108.33 = [illegible]

$\boxed{\therefore V_B = 231.67 \text{ kN}}$

$M_D = 108.33 \times 10 = 1083.3$

$M_C = 108.33 \times 30 - 40 \times 20 = 2450$ KNm



$\therefore$ At the Centre of the Span, B M in the arch should be zero

$H \times 12 = 2450$

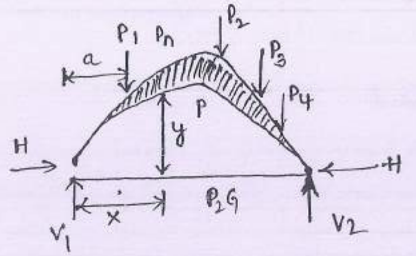
$H = 204.167$ kN

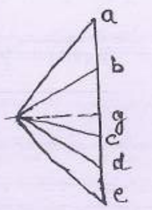
Hence, a parabola with $Y_{MB} = Hy$ is drawn over this diagram. The difference diagram is the B.M. Diagrams for the arch. At the Central hinge, the two diagrams cross line, the B.M at the hinge is zero.

Eddy's Theorem:-

It states that Bending moment at a point (section) of an arch is propstional to the Vertical between the linear arch and the Center line of the actual

Proof:-





Consider a section 'xx' which is at a distanc the point A. The ordinate of arch at the section 'y' represents the polygon potrays the B.M.D to some scale the B.M due to Extern on the section 'xx' is given by the Vertical intercepts 'P_1' & 'P_2'

Let 1 cm = p be the Scale to which arch is drawn
(load diagram - Scale)

1 cm = q be the scale of load diagram.

The Bending moment at section 'p' is given by

$$M_p = V_1 x + W_1 (x-a) - Hy$$

$$= \mu x - Hy$$

where $\mu x = V_x - W(x-a)$ = usual B.M at the section due to load syst

from the figure, we have,

$$\mu x = (P_1 P_2) \times \text{scale of B.M.D}$$

$$= P_1 P_2 \, (p.q.r)$$

The scale will be 'p.q.r' N-m. This is when the distance of from the load line is 'r'.

$$H \times y = (PP_2) \times \text{scale of B.M.D}$$

$M_p = \mu x - Hy$

$= P_1P_2\,(P.q.r) - PP_2\,(P.q.r)$

$= (P.q.r)\,(P_1P_2 - PP_2)$

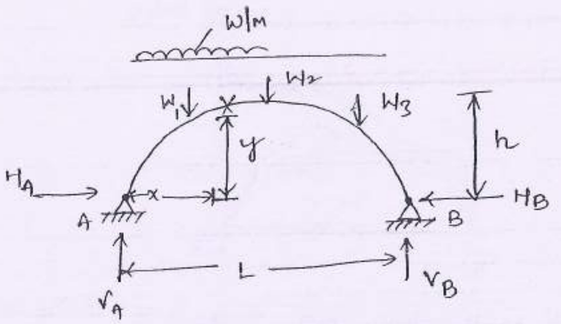
$= (PP_1)(P.q.r)$

where PP_1 is the ordinate b/w the linear arch and the actual arch.

Hence proved.

TWO-HINGED ARCHES

- * Two hinged arches have degree of indeterminacy one.
- * The analysis is restricted to circular & parabolic arches only



* A typical two hinged arch is shown in figure. There are four reaction components H_A, V_A, H_B and V_B where as number of equations of equilibrium are only three. Normally arches are subjected to vertical forces only. Hence, equilibrium of forces in the horizontal directions gives, $H_A = H_B$, say H

The horizontal force H may be taken as the redundant force. The expression for horizontal thrust may be found by any one of the following two methods.

i) first theorem of Castigliano

ii) unit load method

* **first theorem of Castigliano [Strain energy Principle].**

$$M_x = M' - Hy$$

∴ Therefore, strain energy stored in the arch, $U = \int \frac{M_x^2\, ds}{2EI}$

$$= \int (M' - Hy)^2 \left(\frac{ds}{2EI}\right)$$

According to castigliano's first theorem,

$$\frac{dU}{dH} = \text{Horizontal displacement of one end w.r.t to the other}$$

$$= 0 \quad (\text{Since supports and unyielding})$$

$$\therefore \frac{d}{dH}\int (M' - Hy)^2 \frac{ds}{2EI} = 0$$

$$\int 2(M' - Hy)(-y)\frac{ds}{2EI} = 0$$

$$\int (-M'y + Hy^2)\frac{ds}{EI} = 0$$

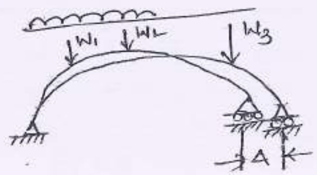
$$\int M'y \frac{ds}{EI} = \int Hy^2 \frac{ds}{EI}$$

$$\int M'y \frac{ds}{EI} = H\int y^2 \frac{ds}{EI}$$

$$\boxed{\therefore H = \frac{\int M'y \times \frac{ds}{EI}}{\int y^2 \frac{ds}{EI}}}$$

* Unit Load Method [Consistant Deformation Method] :-

Replacing hinged end B by rollers, we get the basic determinate structure as shown in figure. In this structure, H=0. Hence, the BM is the same as the beam moment M'.



let the BM due to unit horizontal load at B be 'm'. Hence, at any section P(x,y) the B.M due to unit load is

$$m = y$$

$$= \int M'm\left(\frac{ds}{EI}\right) = \int M'y\left(\frac{ds}{EI}\right)$$

Now, consider the inward displacement of the roller end when force H is applied as shown in figure. The Bending moment due to P(x,y) is Hy, Since the bending moment due to unit load is y.

$\therefore$ Horizontal displacement of roller

$$= \int Hy \times y\left(\frac{ds}{EI}\right)$$

$$= H\int y^2\left(\frac{ds}{EI}\right)$$

$\therefore$ For consistency condition, the support of arch must betwee Equation must be equal. Thus

$$\int M'y\left(\frac{ds}{EI}\right) = H\int y^2\left(\frac{ds}{EI}\right)$$

$$H = \frac{\int M'y\left(\frac{ds}{EI}\right)}{\int y^2\left(\frac{ds}{EI}\right)}$$

Analysis of 2-hinged circular arches:-

$$y = h - (R - R\cos\theta) = h - R(1-\cos\theta)$$

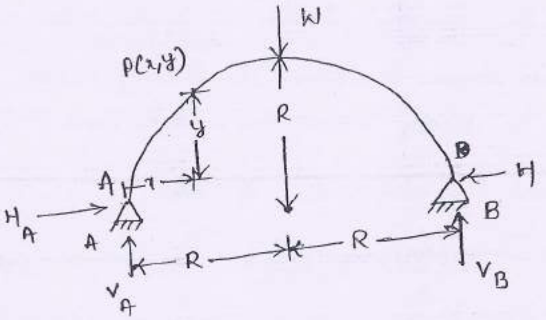
$$ds = R\,d\theta$$

$$N = V\sin\theta + H\cos\theta$$

$$Q = V\cos\theta - H\sin\theta$$

the horizontal thrust developed in a semi circular arch

s R subjected to a concentrated load W at the crown



Consider an arch shown in figure

Due to symmetry,

$$V_A = V_B = \frac{W}{2}$$

h-R(1-cosθ) | $y = R\cos\theta$

-x+R cosθ

$$x = \frac{L}{2} - R\sin\theta = \frac{2R}{2} - R\sin\theta = R(1-\sin\theta)$$

$$\therefore \frac{L}{2} = R$$

$$M' = V_A \times x = \frac{W}{2}x = \frac{W}{2}R(1-\sin\theta)$$

ow,

$$H = \frac{\int M' y \left(\frac{ds}{EI}\right)}{\int y^2 \left(\frac{ds}{EI}\right)} = \frac{\int M' y\, ds}{\int y^2\, ds} \quad (\therefore \text{EI is constant})$$

$$(\because ds = Rd\theta)$$

$$\int M' y\, ds = 2\int_0^{\pi/2} \left(\frac{W}{2}\right) R(1-\sin\theta)\, R\cos\theta\, Rd\theta$$

$$= WR^3 \int_0^{\pi/2} (1-\sin\theta)\cos\theta\, d\theta$$

$$= WR^3 \int_0 (1-\sin\theta)\cos\theta\, d\theta$$

let, $(1-\sin\theta) = t$

then, $dt = -\cos\theta\, d\theta$

$$\therefore \int (1-\sin\theta)\cos\theta\, d\theta = \int t(-dt) = \frac{-t^2}{2}$$

$$= -\frac{1}{2}\left[(1-\sin\theta)^2\right]$$

$$\therefore M' y\, ds = WR^3\left[-\frac{1}{2}(1-\sin\theta)\right]_0^{\pi/2}$$

$$= \frac{WR^3}{2}\left[-\left[(1-1)-(1-0)\right]\right]$$

$$= \frac{WR^3}{2}$$

Now, consider the denominator of the expression for H.

$$\int y^2\, ds = 2\int_0^{\pi/2} R^2 \cos^2\theta\, Rd\theta$$

$$= 2R^3 \int_0^{\pi/2} \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= R^3 \left[\theta + \frac{1+\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= R^3\left[\frac{\pi}{2} + \frac{1}{2} + 0 - 0 - \frac{1}{2} - 0\right]$$

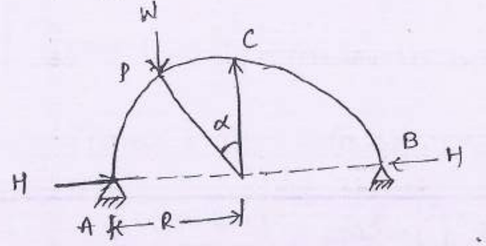
$$= \frac{\pi R^3}{2}$$

$$\therefore H = \frac{\int M' y\, ds}{\int y^2\, ds} = \frac{W\cancel{R^3}}{\cancel{2}} \times \frac{\cancel{2}}{\pi \cancel{R^3}} = \frac{W}{\pi}$$

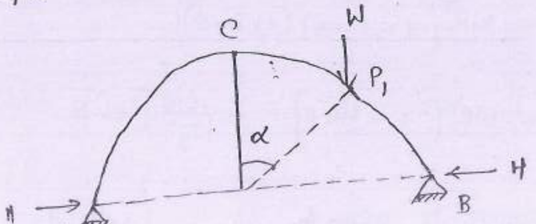
$$\boxed{\therefore H = \frac{W}{\pi}}$$

NOTE:-

The horizontal thrust for a semi-circular arch is indepe of its radius when a load W acts at a point P as shown in figure Assume uniform flexural rigidity.

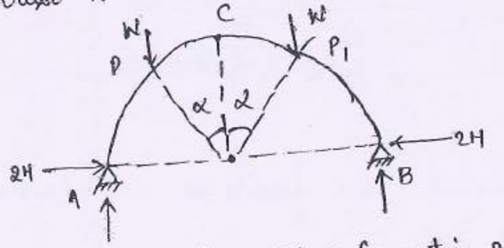


Let the horizontal thrust produced be H. If P_1 is the symmetric point of P and if W acts at this point, naturally the horizontal thrust will be H. ~~Hence, in the~~



Load on Symmetric point

Hence, in the Case shown in below figure, the horizontal thrust will be 2H. This arch will be analysed to get 2H and then horizontal thrust H will be obtained.



Load at the point & symmetric point

$V_A = V_B = W$

$x = \frac{L}{2} - R\sin\theta = R - R\sin\theta$ | $y = h - R(1-\cos\theta)$

$= R - R + R\cos\theta$

$x = R(1-\sin\theta)$

In the portion $\theta = 0$ to α

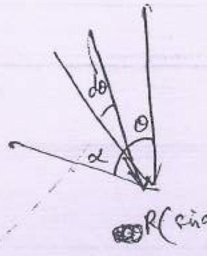
$$M' = Wx - WR(\sin\alpha - \sin\theta)$$

$$= WR(1-\sin\theta) - WR(\sin\alpha - \sin\theta)$$

$$= WR(1-\sin\alpha)$$

For the portion, $\theta = \alpha$ to $\pi/2$,

$$M' = Wx = WR(1-\sin\theta)$$



$\therefore$ Horizontal thrust $= \dfrac{\int M'y\left(\frac{ds}{EI}\right)}{\int y^2\left(\frac{ds}{EI}\right)} = \dfrac{\int M'y\,ds}{\int y^2 ds}$

$$\int M'y\,ds = 2\int_0^{\alpha} WR(1-\sin\alpha)R\cos\theta\,Rd\theta + 2\int_{\alpha}^{\pi/2} WR(1-\sin\theta)R\cos\theta\,Rd$$

$$= 2WR^3(1-\sin\alpha)\int_0^{\alpha}\cos\theta\,d\theta + 2WR^3\int_{\alpha}^{\pi/2}(1-\sin\theta)\cos\theta\,d\theta$$

$$= 2WR^3(1-\sin\alpha)\left[\sin\theta\right]_0^{\alpha} + 2WR^3 - \left[\frac{(1-\sin\theta)^2}{2}\right]_{\alpha}^{\pi/2}$$

$$= 2WR^3(1-\sin\alpha)\sin\alpha - WR^3\left[(1-1)^2 - (1-\sin\alpha)^2\right]$$

$$= 2WR^3(1-\sin\alpha)\sin\alpha + WR^3\left[1-\sin\alpha\right]^2$$

$$= WR^3(1-\sin\alpha)\left[2\sin\alpha + 1 - \sin\alpha\right]$$

$$= WR^3(1-\sin\alpha)(1+\sin\alpha)$$

$$= WR3(1-\sin^2\alpha)$$

$$= WR^3\cos^2\alpha$$

$$\int y^2 ds = 2\int_0^{\pi/2} R^2\cos^2\theta\,Rd\theta = 2R^3\int_0^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)d\theta = R^3\left[\theta + \frac{\sin 2}{2}\right.$$

$$= R^3\left[\pi/2 + 0 - 0 - 0\right) = \frac{\pi R^3}{2}$$

$$\therefore \int y^2 ds = \frac{\pi R^3}{2}$$

ne, Horizontal thrust $= \dfrac{\int M'y\,ds}{\int y^{2} ds}$

$= WR^3 \cos^2\alpha \times \left(\dfrac{2}{\pi R^3}\right)$

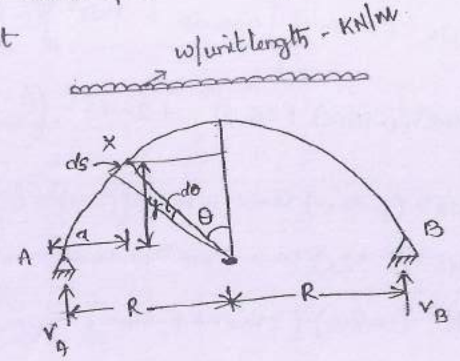
$= \dfrac{2W}{\pi} \cos^2\alpha$

i.e $\quad 2H = \dfrac{2W}{\pi} \cos^2\alpha$

$$\boxed{\therefore\ H = \frac{W}{\pi} \cos^2\alpha}$$

when load is at crown, $\alpha = 0$, $H = \dfrac{W}{\pi}$

semi- circular arch of Radius R is subjected to a U.D.L of
nit length over the entire span. Assuming EI to be constant, determine
Horizontal thrust



$V_A = V_B = \dfrac{1}{2} \times W \times 2R = WR$

$x = \dfrac{L}{2} - R\sin\theta = \dfrac{2R}{2} - R\sin\theta = R(1-\sin\theta)$

$y = h - R(1-\cos\theta) = R - R(1-\cos\theta) = R\cos\theta$

$x = R(1-\sin\theta)$ & $y = R\cos\theta$

$ds = R\,d\theta$

$M' = WRx - \dfrac{Wx^2}{2}$

$= WRx - \dfrac{Wx^2}{2}$

$= WR \times R(1-\sin\theta) - \left[\dfrac{W}{2}\left(R(1-\sin\theta)\right)^2\right]$

$= WR^2(1-\sin\theta) - \dfrac{WR^2}{2}(1-\sin\theta)^2$

$= WR^2\left[1 - \sin\theta - \dfrac{(1-\sin\theta)^2}{2}\right]$

$= \dfrac{WR^2}{2}(1-\sin\theta)\left[2 - (1-\sin\theta)\right]$

$= \dfrac{WR^2}{2}\left[(1-\sin\theta)(1+\sin\theta)\right]$

$= \dfrac{WR^2}{2}\left[1 - \sin^2\theta\right] = \left(\dfrac{WR^2}{2}\right)\cos^2\theta$

Horizontal thrust is given by $\quad H = \dfrac{\int M'y\,ds}{\int y^2\,ds}$

$\therefore$ Numerator $\int M'y\,ds = 2\displaystyle\int_0^{\pi/2}\left(\dfrac{WR^2}{2}\right)\cos^2\theta \times R\cos\theta \times R\,d\theta$

$= WR^4 \displaystyle\int_0^{\pi/2} \cos^3\theta\, d\theta$

Now, consider $I = \int \cos^3\theta\,d\theta = \int (1-\sin^2\theta)\cos\theta\,d\theta$

let $\quad t = \sin\theta$

$dt = \cos\theta\,d\theta$

$\therefore\ I = \int (1-t^2)\,dt$

$= \left[t - \dfrac{t^3}{3}\right] = \sin\theta - \dfrac{\sin^3\theta}{3}$

$\therefore$ Numerator $= WR^4\left[\sin\theta - \frac{\sin^3\theta}{3}\right]_0^{\pi/2}$

$$= WR^4\left[1-\frac{1}{3}\right]$$

$$= \frac{2}{3} WR^4$$

Denominator $= 2\int_0^{\pi/2} R^2\cos^2\theta \, R\,d\theta$

$$= 2R^3\int_0^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)d\theta$$

$$= R^3\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= R^3\left[\frac{\pi}{2} + 0 - 0 - 0\right]$$

$$= \frac{\pi R^3}{2}$$

$$\therefore \quad H = \frac{2}{3} WR^4 \times \frac{2}{\pi R^3}$$

$$\boxed{\therefore H = \frac{4}{3}\left(\frac{WR}{\pi}\right)}$$

//

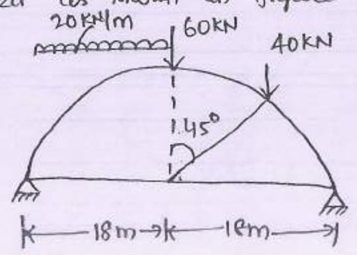
* Determine the horizontal thrust developed in a semi-circular arch subjected to a UDL on only one-half of the ar
EI is constant

④

$$\therefore 2H = \frac{4}{3}\left(\frac{WR}{\pi}\right)$$

$$\therefore H = \frac{2}{3}\left(\frac{WR}{\pi}\right) \text{ //}$$

* Determine the horizontal thrust developed in the Semi-Circu
of radius 18 loaded as shown in figure



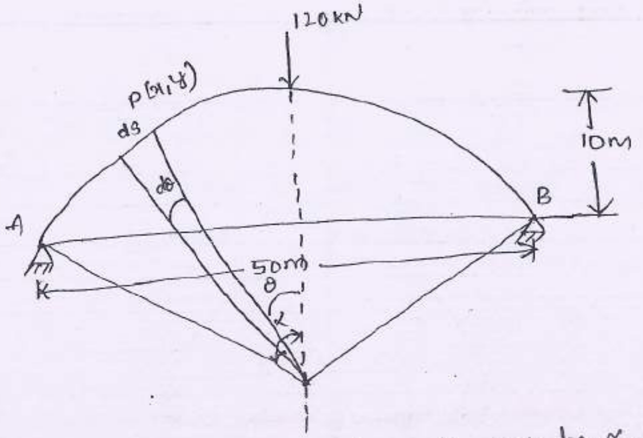
using the results obtained in standard cases, the thru developed due to each case is determined separately and superposed get the horizontal thrust in this case

$$H = \left[\frac{2}{3}\frac{WR}{\pi}\right] + \frac{8W}{\pi} + \frac{W}{\pi}\cos^2\alpha$$

$$= \left[\frac{2}{3} \times \frac{20\times 18}{\pi}\right] + \frac{60}{\pi} + \left[\frac{40}{\pi}\cos^2 45^\circ\right]$$

$$= 105.47 \text{ kN}$$

the general expression for the horizontal thrust developed
inged segmental arch subjected to a concentrated load W at the
and then evaluate its value for the arch shown in figure.
constant flexural rigidity.



let the semi-central angle of the arch be α. considering
emental length ds at an angular distance θ from the vertical axis
gh the crown and taking A as origin, we get

$$x = R(\sin\alpha - \sin\theta)$$

$$y = R(\cos\theta - \cos\alpha)$$

$$ds = R\,d\theta$$

action, $V_A = V_B = \frac{W}{2}$

$$M' = \frac{W}{2} \times x = \frac{W}{2} \times R(\sin\alpha - \sin\theta)$$

Since, EI is constant,

$$H = \frac{\int M'y \frac{ds}{EI}}{\int y^2 \frac{ds}{EI}} = \frac{\int M'y\,ds}{\int y^2\,ds}$$

∴ Now, Numerator (N) $= \int M'y\,ds$

$$= \int \left(\frac{W}{2}\right) R(\sin\alpha - \sin\theta)\, R(\cos\theta - \cos\alpha)\, R\,d\theta$$

$$= \frac{WR^3}{2}\left[2\int_0^{\alpha}(\sin\alpha - \sin\theta)(\cos\theta - \cos\alpha)\,d\theta\right]$$

$$= WR^3 \int_0^{\alpha}(\sin\alpha\cos\theta - \sin\alpha\cos\alpha - \sin\theta\cos\theta + \sin\theta\cos\alpha)\,d\theta$$

$$= WR^3 \int_0^{\alpha}\left(\sin\alpha\cos\theta - \sin\alpha\cos\alpha - \frac{\sin 2\theta}{2} + \sin\theta\cos\alpha\right)d\theta$$

$$= WR^3\left[\sin\alpha\sin\theta - \theta\sin\alpha\cos\alpha - \frac{1}{2}\left(-\frac{\cos 2\theta}{2}\right) + (-\cos\theta)\cos\alpha\right]_0^{\alpha}$$

$$= WR^3\left[\sin^2\alpha - \alpha\sin\alpha\cos\alpha + \frac{1}{4}(\cos 2\alpha - 1) - \cos\alpha(\cos\alpha - 1)\right]$$

$$= WR^3\left[\sin^2\alpha - \alpha\sin\alpha\cos\alpha + \frac{1}{4}(2\cos^2\alpha - 1 - 1) - \cos\alpha(\cos\alpha - 1)\right]$$

$$= WR^3\left[\sin^2\alpha - \alpha\sin\alpha\cos\alpha - \frac{1}{2}\sin^2\alpha - \cos^2\alpha + \cos\alpha\right]$$

$$= WR^3\left[\frac{\sin^2\alpha}{2} - \cos\alpha(\cos\alpha - 1 + \alpha\sin\alpha)\right)$$

$$N = WR^3\left[\sin^2\alpha - 2\cos\alpha(\cos\alpha - 1 + \alpha\sin\alpha)\right]$$

Denominator (D) $= \int y^2\,ds$

$$= \int R^2(\cos\theta - \cos\alpha)^2 R\,d\theta$$

$$= R^3\, 2\int_0^{\alpha}(\cos^2\theta - 2\cos\alpha\cos\theta + \cos^2\alpha)\,d\theta$$

$$= 2R^3 \int_0 \left(\frac{1+\cos 2\theta}{2} - 2\cos\alpha \cos\theta + \cos^2\alpha \right) d\theta$$

$$= 2R^3 \left[\frac{\theta}{2} + \frac{\sin 2\theta}{2\times 2} - 2\cos\alpha \sin\theta + \theta \cos^2\alpha \right]_0^\alpha$$

$$= 2R^3 \left[\frac{\alpha}{2} + \frac{\sin 2\alpha}{4} - 2\cos\alpha \sin\alpha + \alpha \cos^2\alpha \right]$$

$$= R^3 \left[\alpha + \frac{\sin 2\alpha}{2} - 4\cos\alpha \sin\alpha + 2\alpha \cos^2\alpha \right]$$

$$= R^3 \left[\alpha + \frac{\sin 2\alpha}{2} - 2\sin 2\alpha + 2\alpha \cos^2\alpha \right]$$

$$= R^3 \left[\alpha - \frac{3\sin 2\alpha}{2} + 2\alpha \cos^2\alpha \right]$$

$$D = \frac{R^3}{2} \left[4\alpha \cos^2\alpha + 2\alpha - 3\sin 2\alpha \right]$$

$$\therefore H = \frac{\int M' y \, ds}{\int y^2 ds} = \frac{W\cancel{R^3}\left[\sin^2\alpha - 2\cos\alpha \left(\cos\alpha - 1 + \alpha \sin\alpha \right) \right]}{\frac{\cancel{R^3}}{2}\left[4\alpha \cos^2\alpha + 2\alpha - 3\sin 2\alpha \right]}$$

$$\boxed{\therefore H = \frac{W}{2} \left(\frac{\sin^2\alpha - 2\cos\alpha (\cos\alpha - 1 + \alpha \sin\alpha)}{4\alpha \cos^2\alpha + 2\alpha - 3\sin 2\alpha} \right)}$$

∴ On the given problem, from geometry of circle

$\cancel{\frac{H}{2} \times K}$

$R = \cancel{\frac{L^2}{8R} + \frac{h}{2}}$

$$R = \frac{L^2}{8h} + \frac{h}{2}$$

$$= \frac{50^2}{8\times 10} + \frac{10}{2} = \frac{50\times 5\cancel{0}}{8\times 1\cancel{0}} + 5 = \frac{250}{8} + 5$$

$$= 31.25 + 5$$

$$\boxed{\therefore R = 36.25\text{ m}}$$

and $R \sin\alpha = 25$ m

$$\therefore \sin\alpha = \frac{25}{R}$$

$$\alpha = \sin^{-1}\left(\frac{25}{36.25} \right) = 43.6^\circ$$

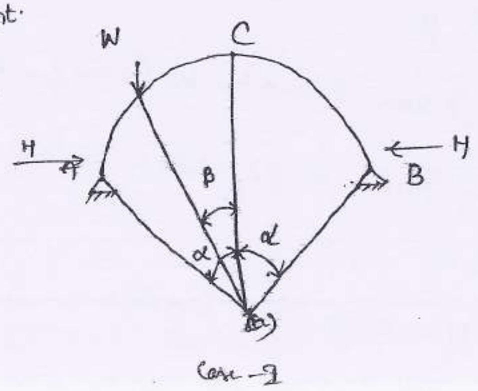
$$\therefore H = \frac{W}{2} \left[\frac{\sin^2\alpha - 2\cos\alpha (\cos\alpha - 1 + \alpha \sin\alpha)}{4\alpha \cos^2\alpha + 2\alpha - 3\sin 2\alpha} \right]$$

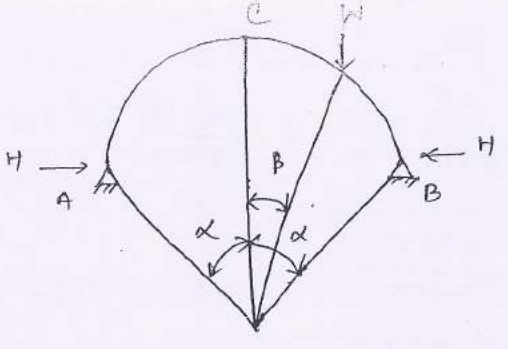
$$= \frac{120}{2} \left[\frac{0.4756 - 1.4483(0.7241 - 1 + 0.5248)}{1.5902 + 1.5220 - 2.9964} \right]$$

$H = 113.45$ KN //

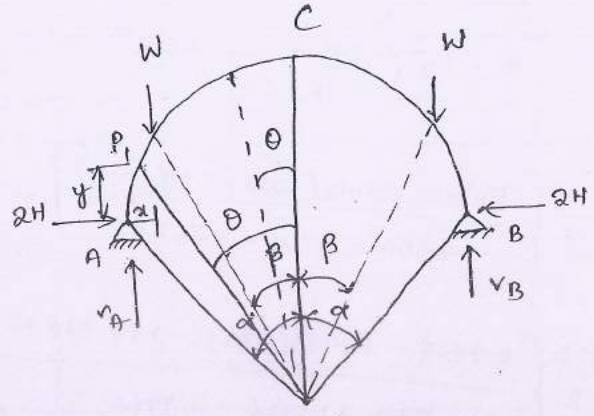
* Determine the expression for horizontal thrust in a segmental a
to a concentrated load at angular distance β as shown in figure
EI to be constant.

Sol:-





Case-II



Case III

Case III ·

$V_A = V_B = W$

For θ = 0 to β

$M' = W(R\sin\alpha - R\sin\theta) - W(R\sin\beta - R\sin\theta)$

$= WR(\sin\alpha - \sin\beta)$ which is constant w.r.t θ

For θ = β to α

$M' = WR(\sin\alpha - \sin\theta)$

$$2H = \frac{\int M' y \frac{ds}{EI}}{\int y^2 \frac{ds}{EI}} = \frac{\int M' y\, ds}{\int y^2 ds} \quad (\because EI \text{ is constant})$$

$$\text{Numerator}(N) = 2\int_0^\beta WR(\sin\alpha - \sin\beta)\,R(\cos\theta - \cos\alpha)\,R\,d\theta$$

$$+ 2\int_\beta^\alpha WR(\sin\alpha - \sin\theta)\,R(\cos\theta - \cos\alpha)\,R\,d\theta$$

$$= N_1 + N_2$$

$$\therefore N_1 = 2WR^3 \int_0^\beta (\sin\alpha - \sin\beta)(\cos\theta - \cos\alpha)\,d\theta$$

$$= 2WR^3(\sin\alpha - \sin\beta)\int_0^\beta (\cos\theta - \cos\alpha)\,d\theta$$

$$= 2WR^3(\sin\alpha - \sin\beta)\left[\sin\theta - \theta\cos\alpha\right]_0^\beta$$

$$= 2WR^3(\sin\alpha - \sin\beta)\left[\sin\beta - \beta\cos\alpha\right]$$

$$\therefore N_2 = 2WR^3 \int_\beta^\alpha (\sin\alpha - \sin\theta)(\cos\theta - \cos\alpha)\,d\theta$$

$$= 2WR^3\left[\sin\alpha\sin\theta - \theta\sin\alpha\cos\alpha + \frac{1}{4}\cos 2\theta - \cos\alpha\cos\theta\right]_\beta^\alpha$$

as in previous example

$$= 2WR^3\left[\sin\alpha\sin\theta - \theta\sin\alpha\cos\alpha + \frac{1}{4}(2\cos^2\theta - 1) - \cos\alpha\cos\theta\right]_\beta^\alpha$$

$2WR^3[\ \cancel{\sin^2\alpha} - \alpha \sin\alpha\cos\alpha$

$$= 2WR^3\Big[\sin^2\alpha - \sin\alpha\sin\beta - \alpha\sin\alpha\cos\alpha + \beta\sin\alpha\cos\alpha + \frac{1}{4}\left(2\cos^2\alpha - 1 - 2\cos^2\beta + 1\right) - \cos^2\alpha + \cos\alpha\cos\beta\Big]$$

$$N_2 = 2WR^3\Big[\sin^2\alpha - \sin\alpha\sin\beta - \alpha\sin\alpha\cos\alpha + \beta\sin\alpha\cos\alpha + \frac{\cos^2\alpha}{2} - \frac{\cos^2\beta}{2} - \cos^2\alpha + \cos\alpha\cos\beta\Big]$$

$$\therefore N_1 + N_2 = 2WR^3\left(\sin\alpha - \sin\beta\right)\left(\sin\beta - \beta\cos\alpha\right)$$

$$+ 2WR^3\Big[\sin^2\alpha - \sin\alpha\sin\beta - \alpha\sin\alpha\cos\alpha + \beta\sin\alpha\cos\alpha + \underset{\left(\frac{1-\sin^2\alpha}{2}\right)}{\frac{\cos^2\alpha}{2}} - \underset{\left(\frac{1-\sin^2\beta}{2}\right)}{\frac{\cos^2\beta}{2}} - \cos^2\alpha + \cos\alpha\cos\beta\Big]$$

$$= 2WR^3\Big[\sin\alpha\cancel{\sin}\beta - \beta\cancel{\sin\alpha\cos}\alpha - \sin^2\beta - \beta\sin\beta\cos\alpha$$

$$- \sin\alpha\cancel{\sin}\beta - \alpha\sin\alpha\cos\alpha + \beta\cancel{\sin\alpha}\cos\alpha - \cos^2\alpha$$

$$+ \cos\alpha\cos\beta + \frac{\sin^2\alpha}{2} - \frac{\sin^2\beta}{2}\Big]$$

$$= WR^3\Big[\sin^2\alpha - \sin^2\beta - 2\cos\alpha\left(\cos\alpha + \alpha\sin\alpha - \beta\sin\beta - \cos\beta\right)\Big]$$

$$= WR^3\Big[\sin^2\alpha - \sin^2\beta - 2\cos\alpha\left(\cos\alpha - \cos\beta + \alpha\sin\alpha - \beta\sin\beta\right)\Big]$$

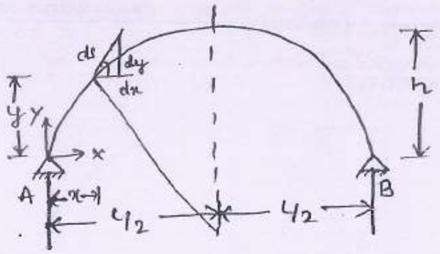
$$N = WR^3\Big[\sin^2\alpha - \sin^2\beta - 2\cos\alpha\left(\cos\alpha - \cos\beta + \alpha\sin\alpha - \beta\sin\beta\right)\Big] //$$

$\frac{}{2}[4\cos\alpha + 2\alpha$ [illegible] $]$ //

$$\therefore 2H = \frac{N}{D} = \frac{2WR^3\left[\sin^2\alpha - \sin^2\beta - 2\cos\alpha\left(\cos\alpha - \cos\beta + \alpha\sin\right.\right.}{\frac{R^3}{2}\left[4\alpha\cos^2\alpha + 2\alpha - 3\sin 2\alpha\right]}$$

$$\therefore H = W\frac{\left[\sin^2\alpha - \sin^2\beta - 2\cos\alpha\left(\cos\alpha - \cos\beta + \alpha\sin\alpha -\right.\right.}{4\alpha\cos^2\alpha + 2\alpha - 3\sin 2\alpha}$$

* Analysis of Parabolic Arches:-



Consider a typical parabolic arch shown in figu

The left springing point as the origin, the equation of the parabola is g

$$y = \frac{4hx(L-x)}{L^2}$$

Where L is the span and h is the rise. Let p(x,y) be a

the arch and ds be the elemental length. Let θ be the slope of the arch

horizontal at p. Then,

$$\frac{dx}{ds} = \tan\theta$$

& $ds = dx\sec\theta$

Horizontal thrust H is given by

If I is varying such that at any point $H = \dfrac{\int M'y\left(\frac{ds}{EI}\right)}{\int y^2\left(\frac{ds}{EI}\right)} = \dfrac{\int M'y\left(\frac{dx}{EI}\right.}{\int y^2\left(\frac{dx}{EI}\right)}$

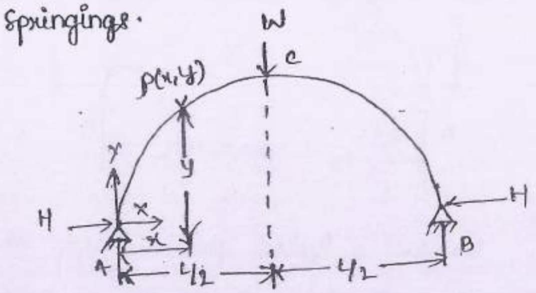
$$H = \frac{\int M' y \frac{dx}{EI_0}}{\int y^2 \frac{dx}{EI_0}} = \frac{\int M' y\, dx}{\int y^2 dx} \quad (\text{since, } EI_0 \text{ is a constant})$$

us, in a parabolic arch with variation of moment of inertia in the

$I = I_0 \sec\theta$

$$\boxed{H = \frac{\int M' y\, dx}{\int y^2 dx}}$$

hinged parabolic arch of span L and rise h carries a concentrated W at the crown. Determine the expression for horizontal thrust oped at springings.



Consider the typical arch shown in figure. Due to symmetry, ctions at A and B are the same and its value is equal to $\frac{W}{2}$.

At any point $p(x,y)$, beam moment (M') is given by

$$M' = \left(\frac{W}{2}\right)x$$

d in the parabola, $y = \frac{4hx(L-x)}{L^2}$

Now, $H = \frac{\int M' y\, dx}{\int y^2 dx}$

Considering the Numerator (N), $\int M' y\, dx = \int \left(\frac{W}{2}\right) x\, y\, dx$

$$= \left(\frac{2Wh}{L^2}\right) 2 \int_0^{L/2} (Lx^2 - x^3)\, dx$$

$$= \frac{4Wh}{L^2}\left[\frac{Lx^3}{3} - \frac{x^4}{4}\right]_0^{L/2}$$

$$= \frac{4Wh}{L^2}\left[\frac{L^4}{24} - \frac{L^4}{64}\right]$$

$$= 4WhL^2\left[\frac{8-3}{192}\right]$$

$$N = \frac{5}{48} WhL^2 \quad //$$

Denominator (D) is given by

$$D = \int y^2 dx$$

$$= \int \left[\frac{4hx(L-x)}{L^2}\right]^2 dx$$

$$= \left(\frac{16h^2}{L^4}\right) 2\left[\int_0^{L/2} x^2(L^2 + x^2 - 2Lx)\, dx\right]$$

$$= \frac{32h^2}{L^4}\left[\int_0^{L/2} L^2x^2 + x^4 - 2Lx^3\, dx\right]$$

$$= \frac{32h^2}{L^4}\left[\frac{L^2x^3}{3} + \frac{x^5}{5} - \frac{2Lx^4}{4}\right]_0^{L/2}$$

$$= \frac{32h^2}{L^4} L^5\left[\frac{1}{24} - \frac{1}{32} + \frac{1}{160}\right]$$

$$= 32h^2L\left[\frac{20-15+3}{480}\right]$$

$$D = \left(\frac{8}{15}\right) h^2 L$$

$$\therefore \quad H = \frac{N}{D} = \frac{5}{48} WhL^2 \times \frac{15}{8h^2L}$$

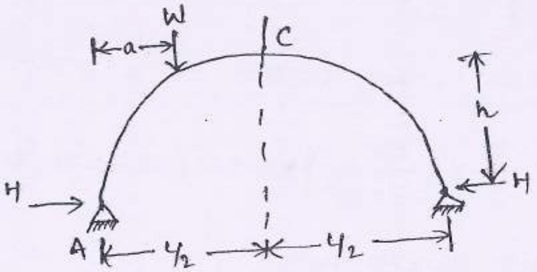
$$\boxed{\therefore \quad H = \frac{25}{128}\left(\frac{WL}{h}\right)}$$

//

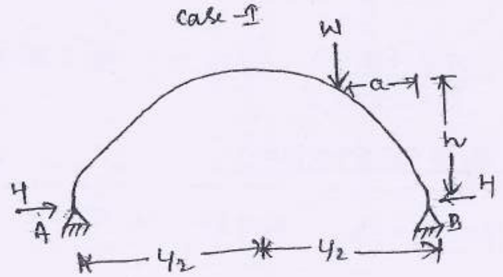
* Show that the horizontal thrust developed in a parabolic arch of span L and rise h, subjected to a concentrated load W at a distance a from a springing is given by

$$H = \left(\frac{5}{8}\right)\left(\frac{W}{hL^3}\right) a(L-a)(L^2+La-a^2)$$

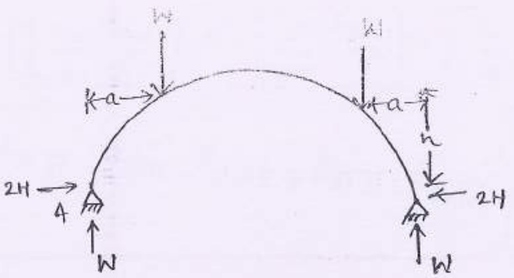
sol:-



case - I



Case - II



Case - III

In case - III, Vertical reaction at end A is W

$\therefore$ Beam moment at a point P (x, y) is

$$M' = Wx, \text{ for the portion } x = 0 \text{ to } a$$

$$M' = Wx - W(x-a)$$

$$= Wa \quad \text{for the portion of } x = a \text{ to } L/2$$

Horizontal thrust,

$$2H = \frac{\int M' y\, dx}{\int y^2\, dx}$$

Now, $\int y^2\, dx = \frac{8}{15} h^2 L$, as in previous problem

$$\int M' y\, dx = 2\left[\int_0^a Wxy\, dx + \int_a^{L/2} Way\, dx\right]$$

$$= 2\int_0^a Wx \times \frac{4hx(L-x)}{L^2}\, dx + 2\int_a^{L/2} Wa \times \frac{4hx(L-x)}{L^2}\, dx$$

$$= \left(\frac{8Wh}{L^2}\right)\int_0^a (Lx^2 - x^3)\, dx + \left(\frac{8Wh}{L^2}\right) a \int_a^{L/2} (Lx - x^2)\, dx$$

$$= \frac{8Wh}{L^2}\left[\left[\frac{Lx^3}{3} - \frac{x^4}{4}\right]_0^a + a\left[\frac{Lx^2}{2} - \frac{x^3}{3}\right]_a^{L/2}\right]$$

$$\frac{Wh}{L^2}\left[\frac{aL^3}{3}-\frac{a^4}{4}+a\left(\frac{L^3}{8}-\frac{L^3}{24}\right)-a\left[\frac{La^2}{2}-\frac{a^3}{3}\right]\right]$$

$$\left(\frac{8Wh}{L^2}\right)\frac{1}{24}\left[8La^3-6a^4+3aL^3-aL^3-12a^3L+8a^4\right]$$

$$\frac{Wha}{3L^2}\left[2L^3-4a^2L+2a^3\right]$$

$$=\frac{Wha}{3L^2}\times 2\left[L^3-2a^2L+a^3\right]$$

$$=\left(\frac{2}{3}\right)\left(\frac{Wha}{L^2}\right)\left[L^3-a^2L-a^2L+a^3\right]$$

$$=\frac{2}{3}\left(\frac{Wha}{L^2}\right)\left[L(L^2-a^2)-a^2(L-a)\right]$$

$$=\frac{2}{3}\left(\frac{Wha}{L^2}\right)\left[L(L-a)(L+a)-a^2(L-a)\right]$$

$$=\frac{2}{3}\left(\frac{Wha}{L^2}\right)(L-a)\left[L^2+aL-a^2\right]$$

$$\therefore\ 2H=\frac{\int M'y\,dx}{\int y^2dx}=\frac{2}{3}\left(\frac{Wha}{L^2}\right)(L-a)(L^2+aL-a^2)\times\frac{15}{8h^2L}$$

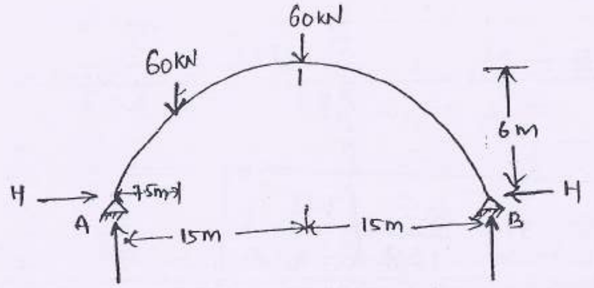
$$=\frac{5}{4}\left(\frac{W}{hL^3}\right)a(L-a)(L^2+La-a^2)$$

$$\boxed{\therefore\ H=\frac{5}{8}\left(\frac{W}{hL^3}\right)a(L-a)(L^2+La-a^2)}$$

//

[illegible] span 30m and rise 6m carries two point loads, each 60 kN, acting at 7.5 m and 15 m from the left end, respectively. The moment of inertia varies as the secant of slope. Determine the horizontal thrust and maximum positive and negative moments in the arch rib.

Sol:-



The arch is shown in figure. Let the horizontal thrust due to 60 kN load at 7.5 m from A be H_1 and that due to 60 kN load at 15 m from A be H_2. Then,

$$H=H_1+H_2$$

$$\therefore\ H_1=\frac{5}{8}\times\frac{W}{hL^3}\,a(L-a)(L^2+La-a^2)$$

where $W=60\text{ kN}$, $h=6\text{m}$, $L=30\text{m}$ and $a=7.5\text{m}$

$$H_1=\frac{5}{8}\times\left(\frac{60}{6\times30^3}\right)\times7.5\times(30-7.5)\times\left(30^2+(30\times7.5)-(7.5)^2\right)$$

$$H_1=41.748\ \text{kN}$$

III'y $a=15\text{m}$,

$$H_2=\frac{5}{8}\left(\frac{60}{6\times30^3}\right)15(30-15)(30^2+30\times15-15^2)$$

$$H_2=58.594\ \text{kN}$$

$$\therefore\ H=H_1+H_2=41.748+58.594=100.342\ \text{kN}$$

$$\boxed{\therefore\ H=100.342\ \text{kN}}$$

[illegible] +ve [illegible]

end point. Now,

$V_A + V_B = 120$

$\Sigma M_B = 0,$

$V_A \times 30 - 60 \times (30 - 7.5) - 60 \times 15 = 0$

$V_A \times 30 = 60 \times 22.5 + 60 \times 15$

$\boxed{\therefore V_A = 75\text{ kN}}$

$\therefore V_B = 120 - 75 = 45\text{ kN}$

$\boxed{\therefore V_B = 45\text{ kN}}$

At 7.5m from the left Spring point A,

$M_x = \cancel{8}\, V_A \times x - H \times y$ $\qquad y = \frac{4hx(L-x)}{L^2}$

$= 75 \times 7.5 - 100.342 \times 4.5$ $\qquad = \frac{4 \times 6 \times 7.5(30-7.5)}{30^2}$

$M_x = 110.961\text{ kNm}$ $\qquad = 4.5$

At crown,

$M_x = V_B \times 15 - H \times y$ $\qquad (\because y = h = 6)$

$= 45 \times 15 - 100.342 \times 6$

$= 72.948\text{ kNm}$

∴ Max. negative moment occurs in the right half portion, measuring x from B.

$M_x = 45x - 100.342y$

$= 45x - 100.342\left(\frac{4 \times 6 \times x(30-x)}{30^2}\right)$

$= 45x - 2.676(30x - x^2)$

$\frac{dM_x}{dx} = 0$, $= 45 - 2.676 \times (30 - 2x)$

$\therefore x = 6.59\text{m}$

∴ maximum negative moment $=$ ~~$45 \times (6.59) - 2.676 \times 30 \times$~~

$= 45x - 100.342y$

$= 45 \times 6.59 - 100.342\left[\frac{4(6)(6.59)(30-}{30^2}\right.$

$= -116.28\text{ kNm}$ ///

* Determine the horizontal reaction in the above problem carrying integrations.

Sol

$V_A = 75\text{ kN}$ & $V_B = \cancel{8}\,45\text{ kN}$

for $x = 0$ to 7.5 m from A, $M'_x = V_A \times x = 75x$

for $x = 7.5$ to 15m from A, $M'_x = V_A x - 60(x - 7.5)$

$= 75x - 60x + 450$

$= 15x + 450$

For $x = 0$ to 15 from B, $M'_x = V_B \times x = 45x$

$\therefore y = \frac{4hx(L-x)}{L^2} = \frac{4(6)x(30-x)}{30^2} = \frac{24x(30-x)}{900}$

$\therefore \int M'y\,dx = \int_0^{7.5} 75xy\,dx + \int_{7.5}^{15} (15x+450)y\,dx + \int_0^{15} 45xy\,dx$

$= \int_0^{7.5} 75x \times \frac{24}{900} \times (30x - x^2)dx + \int_{7.5}^{15} (15x+450) \times \frac{24}{900} \times \cancel{x}(30x -$

$+ \int_0^{15} 45x \times \frac{24}{900} \times (30 - x)\,dx$

$$7.5x \times \frac{24}{900} \times (30x-x^2)\,dx + \int_{7.5}^{15} (15x+450) \times \frac{24}{900} \times (30x-x^2)\,dx$$

$$+ \int^{15} 45x \times \frac{24}{900} \times (30x-x^2)\,dx$$

5

$$2(30x^2-x^3)\,dx + \int_{7.5}^{15} \frac{24}{900} \times (15x+450)(30x-x^2)\,dx$$

$$+ \int^{15} 1.2\,(30x^2-x^3)\,dx$$

$$\int_0^{7.5} (30x^2-x3)\,dx + \frac{24}{900} \int_{7.5}^{15} (15x+450)(30x-x^2)\,dx$$

$$+ 1.2 \int_0^{15} (30x^2-x^3)\,dx$$

$$\left[30 \times \frac{x^3}{3} - \frac{x^4}{4}\right)_0^{7.5} + 1.2\left[30 \times \frac{x^3}{3} - \frac{x^4}{4}\right]_0^{15}$$

$$+ \frac{24}{900} \int_{7.5}^{15} (\cancel{450x^2} - 15x^3 + 13500x - \cancel{450x^2})\,dx$$

$$2\left[30 \times \frac{7.5^3}{3} - \frac{7.5^4}{4}\right] + 1.2\left(\frac{30 \times 15^3}{3} - \frac{15^4}{4}\right)$$

$$+ \frac{24}{900}\left[-15\left(\frac{x^4}{4}\right) + 13500\left(\frac{x^2}{2}\right)\right]_{7.5}^{15}$$

$$= 32167.964 + \frac{24}{900}\left[-15\left(\frac{15^4-7.5^4}{4}\right) + 13500\left(\frac{15^2-7.5^2}{2}\right)\right]$$

$$= [illegible]167.964 + 25628.906 = 57798.87$$

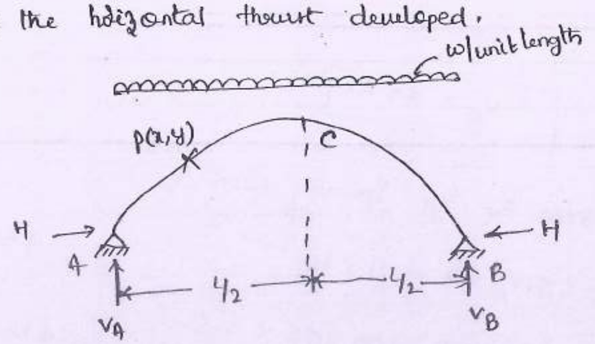
$$\int M'y\,dx = 57798.87$$

$$\int y^2 dx = \frac{8}{15} \times h^2 L = \frac{8}{15} \times 6^2 \times 30 = 576$$

$$\therefore H = \frac{\int M'y\,dx}{\int y^2 dx} = \frac{57798.87}{576} = 100.345 \text{ kN}$$

* A 2-hinged parabolic arch of span L and rise h carries UDL of w/m run over the whole span. Assuming $I = I_0 \sec\alpha$, find the expression for the horizontal thrust developed.

Sol:



$$V_A = V_B = \frac{WL}{2}$$

Beam moment at point P(x, y) is given by

$$M'_x = W'_A \times x - W \times x \times \frac{x}{2}$$

$$= \left(\frac{WL}{2}\right)x - \frac{Wx^2}{2}$$

$$= \frac{Wx}{2}(L-x)$$

Ordinate $$y = \frac{4hx(L-x)}{L^2}$$

∴ Horizontal Thrust

$$H = \frac{\int M' y \, dx}{\int y^2 dx}$$

$$= \frac{\int \frac{Wx}{2}(L-x) \times \frac{4hx(L-x)}{L^2} dx}{\int \frac{16h^2 x^2 (L-x)^2}{L^4} dx}$$

$$= \frac{2\left(\frac{Wh}{L^2}\right) \int x^2 \cancel{(L-x)^2} dx}{\frac{16h^2}{L^4} \int x^2 \cancel{(L-x)^2} dx}$$

$$= \frac{\cancel{2}W\cancel{h}}{\cancel{L^2}} \times \frac{L^{\cancel{4}\,2}}{8\,\cancel{16}\,h^{\cancel{2}}}$$

$$= \frac{WL^2}{8h}$$

$$\boxed{\therefore H = \frac{WL^2}{8h}}$$ //

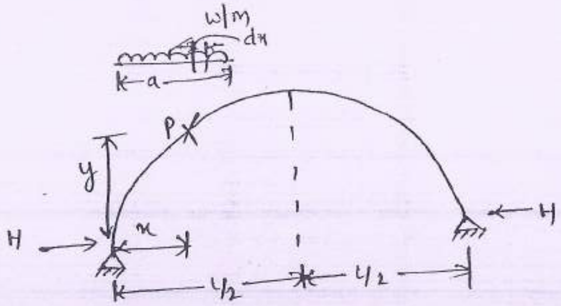
∴ If the load UDL is acted in half portion

$$\therefore H = \frac{1}{2} \times \frac{WL^2}{8h}$$

$$\boxed{\therefore H = \frac{WL^2}{16h}}$$ //

Subjected to UDL of W per unit length over a length 'a' f

left support.



Consider the typical arch shown in figure. Le

let dx be an elemental length at P(x, y).

Due to elemental load 'Wdx' acting on the element, Horizontal thrust produced is given by

$$dH = \frac{5}{8}\left(\frac{W}{hL^3}\right) a(L-a)(L^2+La-a^2)$$

$$= \frac{5}{8}\left(\frac{Wdx}{hL^3}\right)x(L-x)(L^2+Lx-x^2)$$

$$= \frac{5}{8}\left(\frac{W}{hL^3}\right)(L^3x + \cancel{L^2x^2} - x^3L - \cancel{x^2L^2} - Lx^3 + x$$

$$= \frac{5}{8}\left(\frac{W}{hL^3}\right)(L^3x - 2Lx^3 + x^4)\,dx$$

$$H = \frac{5}{8}\left(\frac{W}{hL^3}\right)\int_0^a (L^3x - 2Lx^3 + x^4)\,dx$$

$$= \frac{5}{8}\left(\frac{W}{hL^3}\right)\left[\frac{L^3x^2}{2} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]_0^a$$

$$= \frac{5}{8}\left(\frac{W}{hL^3}\right)\left[\frac{L^3a^2}{2} - \frac{2La^4}{4} + \frac{a^5}{5}\right)$$

$$= \frac{2}{8}\left(\frac{w}{hL^3}\right)\left(\frac{1}{\frac{10}{2}}\right)\left(5L^3a^2 - 5La^4 + 2a^5\right)$$

$$= \left(\frac{1}{16}\right)\left(\frac{w}{hL^3}\right) a^2 \left(5L^3 - 5La^2 + 2a^3\right) //$$

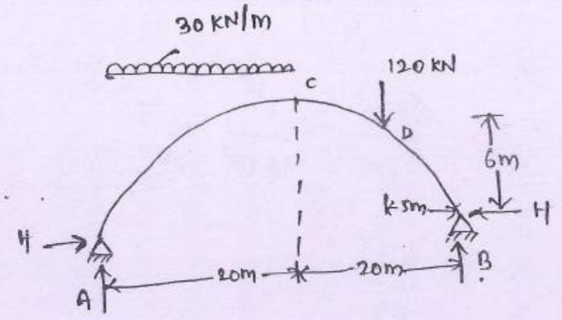
* A 2- hinged parabolic arch is loaded as shown in figure. Determine the

i) horizontal thrust,

ii) Max. +ve & -ve moments,

iii) Shear force & normal thrust at 10 m from the left support

Assume $I = I_0 \sec\theta$, where I_0 is the moment of inertia at the crown and θ is the slope at the section under consideration.



Let H_1 be the horizontal thrust due to given UDL and H_2 be due to given concentrated load

Then

$$H_1 = \frac{WL^2}{16h} = \frac{30 \times 40^2}{16 \times 6} = 500 \text{ kN}$$

$$H_2 = \frac{5}{8}\left(\frac{Wdx}{hL^3}\right) a(L-a)(L^2+La-a^2)$$

$$= \frac{5}{8}\left(\frac{120}{6 \times 40^3}\right) 5(40-5)(40^2+40\times5-5^2)$$

$$H_2 = 60.669 \text{ kN}$$

$\therefore H = H_1 + H_2$

$= (500 + 60.669)$ kN

$$\boxed{H = 560.669 \text{ kN}}$$

$V=0$, $\therefore V_A + V_B = 30\times20 + 120$ —①

$\therefore \Sigma M_B = 0$, $V_A \times 40 - 30\times20\left(\frac{20}{2}+20\right) - 120\times5 = 0$

$V_A = \frac{30\times20\times30 + 120\times5}{40}$

$$\boxed{V_A = 465 \text{ kN}}$$

$\therefore$ from①

$V_B = 30\times20 + 120 - 465$

$$\boxed{\therefore V_B = 255 \text{ kN}}$$

For the portion AC,

$M_x = V_A \times x - 30\times x\times\frac{x}{2} - H\times y$

$= 465\times x - 15x^2 - 560.669 \times \frac{(4)(6)x(40-x)}{40^2}$

$= 465x - 15x^2 - 8.41(40x - x^2)$

$= 128.6x - 6.59x^2$

$\therefore$ for, M_{max}, $\frac{dM_x}{dx} = 0 = 128.6 - 659(2x)$

$x = \frac{128.6}{2\times6.59} = 9.757$ m

$\therefore M_{max} = 1286x - 6.59x^2$

$= 128.6\times9.757 - 6.59\times9.757^2$

$= 627.388$ kNm

In portion CD, measuring x from right support,

$M_x = V_B x - 120(x-5) - HY$

$= 255x - 120x + 600 - 8.41(40x - x^2)$

$= 600 - 201.4x + 8.41x^2$

$\therefore \frac{dH_x}{dx} = 0 = -201.4 + 8.41(2x)$

$\therefore x = \frac{201.4}{8.41\times2} = 11.974$ m

$\therefore M_{min} = 600 - 201.4\times11.974 + 8.41\times11.974^2$

$= -605.766$ KNm

In portion BD, $M_x = V_B \times x - HY$

$= 255x - 8.41(40x - x^2)$

$= -8.41x + 8.41x^2$

$\therefore \frac{dM_x}{dx} = 0 = -81.4 + 8.41(2x)$

$\therefore x = \frac{81.4}{8.41\times2} = 4.839$ m

$\therefore M = -8.41(4.839) + 8.41(4.839)^2$

$= -196.967$ kNm

ot and max. -ve moment ≃ -605.766 kNm at 11.974 m from
ht support

om from the right support,

$$\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left[\frac{4hx(L-x)}{L^2}\right]$$

$$= \frac{4h}{L^2}(L-2x)$$

$$= \frac{4(6)}{40^2}(40-2(10))$$

$$= 0.3$$

$$\theta = 16.694^\circ$$

$$V = V_A - 120 = 255-120 = 135$$

$$N = V\sin\theta + H\cos\theta$$

$$= 135\sin 16.694^\circ + 560.669\cos 16.694^\circ$$

$$= 575.815 \text{ kN}$$

$$Q = V\cos\theta - H\sin\theta$$

$$= 135\cos 16.694^\circ - 560.669\sin 16^\circ.694^\circ$$

$$Q = -31.800 \text{ kN}$$

Determine H by carrying out suitable integrations for the above problm.

for the portion 0 to 20m from end A

$$M^1 = \left(V_A \times x - 30\frac{x^2}{2}\right) = 465x - \frac{30x^2}{2}$$

$$M^1 = V_B \times x = 255x$$

and from 5m to 20m from end B

$$M^1 = V_B \times x - 120(x-5)$$

$$= 255x - 120x + 600$$

$$= 135x + 600$$

$$\therefore \int M^1 y\, dx = \int_0^{20} (465x - 15x^2) \times \frac{(4)(6)\times x(40-x)}{40^2}\, dx$$

$$+ \int_0^5 255x \times \frac{(4)(6)\times x(40-x)}{40^2}\, dx$$

$$+ \int_0^{20} (135x+600)\frac{(4)(6)\times x(40-x)}{40^2}\, dx$$

$$\cancel{= \frac{24}{1800}\int_0^{20} (465x^2 - 15x^3)(40}$$

$$= \frac{24}{1600}\int_0^{20} (465x - 15x^2)(40x - x^2)\, dx$$

$$+ 255\times\frac{24}{1600}\int_0^5 (40x^2 - x^3)\, dx$$

$$+ \frac{24}{1600}\int_0^{20} (135x+600)(40x-x^2)\, dx$$

$$= \frac{24}{1600}\int_0^{20} 18600x^2 - 1065x^3 + 15x^4\, dx + \frac{(24)(255)}{1600}\int_0^5 (40x^2 - x^3)\, dx$$

$$+$$

$$\frac{+24}{1600} \int_0 (42000x + 4800x^2 - 600x^3)\,dx$$

$= 249000 + 5777.344 + 175816.406$

$= 430593.75$

$$\int y^2 dx = \frac{8}{15} \times h^2 L = \frac{8}{15} \times 6^2 \times 40 = 768$$

$$\therefore H = \frac{\int M' y\, dy}{\int y^2 dx} = \frac{430593.75}{768}$$

$$\boxed{H = 560.669\ N}$$

* EFFECT OF yielding of supports:-

$$H = \frac{\int M' y \left(\frac{ds}{EI}\right)}{\int y^2 \frac{ds}{EI} + K}$$

$K = 0.0001$

K - yielding of support per unit horizontal

* Effecting of shortening of RIB :-

$$H = \frac{\int M' y \frac{ds}{EI}}{\int y^2 \left(\frac{ds}{EI}\right) + \frac{L}{EAm} + K}$$

Effect of temperature change

$$H = \frac{\int M' y \left(\frac{ds}{EI}\right) + L\alpha t}{\int y^2 \left(\frac{ds}{EI}\right) + \left(\frac{L}{EAm}\right) + K}$$

* A two hinged parabolic arch of span 50m and rise 5 Subjected to a Central Concentrated load of 60 kN. It has an el

Suppot which yields by 0.0001 mm/kN.

Taking $E = 200\ kN/mm^2$, $I = 5 \times 10^9\ mm^4$,

Average area $Am = 10000\ mm^2$,

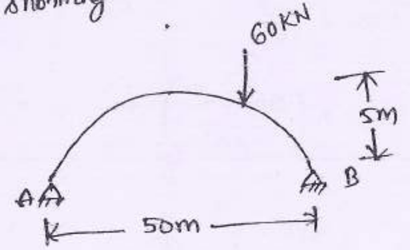
$\alpha = 12 \times 10^{-6}\ /°C$

and assuming secant variation, Calculate the horizontal developed when the temperature rises by 20°C

(i) neglecting rib shortening.

(ii) Considering rib shortning

Ⓐ



Taking left springing A as the origin,

$$y = \frac{4hx(L-x)}{L^2} = \frac{4 \times 5 \times x(50-x)}{50^2}$$

$$y = \frac{x(50-x)}{125}$$

$$y = \frac{50x - x^2}{125}$$

$$y^2 dx = \int_0^{50} \frac{(50x - x^2)^2}{125^2}$$

$$= \frac{1}{125^2} \int_0^{50} (50^2x^2 + x^4 - 100x^3)\, dx$$

$$= \frac{1}{125^2} \left[2500\frac{x^3}{3} - 100\frac{x^4}{4} + \frac{x^5}{5} \right)_0^{50}$$

$$= 666.67$$

$\therefore V_A + V_B = 60$

$V_A = 30 = V_B$

Beam moment at a p(x,y) at x from A

$$M' = V_A \times x$$

$$= 30x$$

$$N \therefore \int M' y \frac{dx}{EI} = 2\int_0^{25} 30x\, y \frac{dx}{EI}$$

$$= 2\int_0^{25} 30x \left(\frac{50x - x^2}{125}\right) \frac{dx}{EI}$$

$$= 60 \int_0^{25} \frac{50x^2 - x^3}{125} \frac{dx}{EI}$$

$$= \frac{60}{125(EI)} \left[50\frac{x^3}{3} - \frac{x^4}{4} \right)_0^{25}$$

$$= \frac{78125}{EI}$$

Now, $E = 200$ kN/mm^2 and $I = 5 \times 10^9$ mm^4

and $A_m = 1000$ mm^2

Converting E, I, Am to the metric unit

$$EI = 200 \times 10^6 \times 5 \times 10^9 \times 10^{-12}$$

$$= 1000000 \text{ KNm}^2$$

$$EA_m = 200 \times 10^6 \times 1000 \times 10^{-6}$$

$$= 2000000$$

$$\therefore \int M' y \frac{dx}{EI} = 0.078125$$

Now, $L\alpha t = 50 \times 12 \times 10^{-6} \times 20 = 0.012$

Yielding support per unit thrust

$k = 0.0001$

$\therefore$ neglecting rib shortening;

$$H = \frac{\int M' y \left(\frac{dx}{EI}\right) + L\alpha t}{\int y^2 \left(\frac{ds}{EI}\right) + 0 + k} \qquad \left(\because \frac{L}{E_{Am}} : \text{rib shortening}\right)$$

$$= \frac{0.078125 + 0.012}{\frac{(666.67)}{1000000} + 0.0001}$$

$$= \frac{0.090125}{7.667 \times 10^{-4}}$$

$$= 117.554 \text{ KN}$$

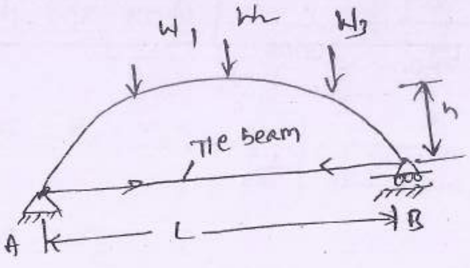
Rib shortening considered,

$$H = \frac{\int My \frac{ds}{EI} + L\alpha t}{\int y^2 \frac{ds}{EI} + \frac{L}{EAM} + K}$$

$$= \frac{0.078125 + 0.012}{7.667 \times 10^{-4} + \frac{50}{2000000}}$$

$$= 113.842 \text{ kN}$$

TIED Arch :-



A_T - Area of tie beam

E_T = Modulus of elasticity

$$\therefore K = \frac{L}{A_T E_T}$$

$$\therefore H = \frac{\int M'y \frac{ds}{EI} + L\alpha T}{\int y^2 \frac{ds}{EI} + \frac{L}{EAM} + \frac{L}{A_T E_T}}$$

A tied arch of [illegible]

shown in figure. Determine the axial force develo

in the tie beam and moment at the Crown in

arch. Consider rib shortening. Assume secant

for moment of inertia of the arch rib.

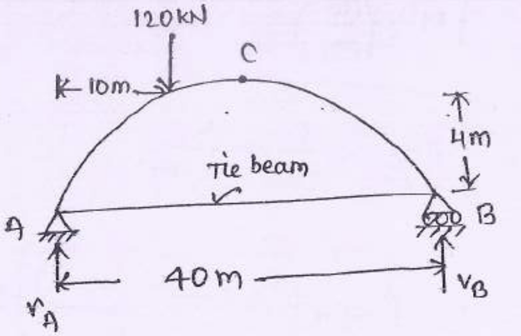
Curved surface area of rib = 1800 mm² (A

E_r of rib = 200 kN/mm². (E_r

EI of rib = 150000 kNm².

Curved surface of tie = 4000 mm² (Average ar

E of tie = 2000 kN/mm²



Sol:-

$\Sigma F_V = 0,$

$V_A + V_B = 120$

$\Sigma M_A = 0,$

$V_B \times 40 = 120 \times 10$

$$\boxed{V_B = 30 \text{ kN}}$$

$$\boxed{V_A = 90 \text{ kN}}$$

[illegible]

aing x from A

$1' = V_A x = 90x$, when $0 < x < 10$ m

$= V_B(L-x) = 30(40-x)$, when $10 < x < 40$

he I varies in secant form

$$\int \frac{ds}{EI} = \int \frac{dx}{EI}$$

idering rib shortening of the tied arch

$$H = \frac{\int M'y\left(\frac{ds}{EI_0}\right)}{\int y^2 \frac{ds}{EI} + \frac{L}{EA_m} + \frac{L}{A_T E_T}}$$

$$\int M'y \frac{ds}{EI_0} = \frac{1}{EI_0}\int_0^{10} 90x\left[\frac{4\times 4\times x(40-x)}{40^2}\right]dx$$

$$+ \frac{1}{EI_0}\int_{10}^{30} 30(40-x)\left[\frac{4(4)x(40-x)}{40^2}\right]dx$$

$$= \frac{0.9}{EI_0}\int_0^{10}(40x^2-x^3)dx + \frac{0.3}{EI_0}\int_0^{30} x(40-x)^2 dx$$

$$= \frac{0.9}{EI_0}\left[\frac{40x^3}{3} - \frac{x^4}{4}\right)_0 + \frac{0.3}{EI_0}\left[\int_0 x(1600+x^2-80x)dx\right.$$

$$= \frac{0.9}{EI_0}\left[\frac{40000}{3} - \frac{40000}{4}\right] + \frac{0.3}{EI_0}\left[\frac{1600x^2}{2} + \frac{x^4}{4} - \frac{80x^3}{3}\right)_0^{30}$$

$$= \frac{57000}{EI_0} = \frac{57000}{150000} = 0.38$$

$$\therefore D = \int_0^{40} y^2 \frac{dx}{EI_0} = 2\int_0^{20}\left(\frac{4(4)x(40-x)}{40}\right)^2 \frac{dx}{EI_0}$$

$$= \frac{2}{EI_0} \times \frac{1}{10000}\int_0^{20}(40x-x^2)^2 dx$$

$$= \frac{2}{10000EI_0}\left[\int_0^{20} 40^2x^2 + x^4 - 80x^3\, dx\right]$$

$$= \frac{2}{10000EI_0}\left[1600\frac{x^3}{3} + \frac{x^5}{5} - \frac{80x^4}{4}\right)_0^{20}$$

$$\frac{40\times 40}{1600}$$

$$= \frac{341.333}{EI_0} = 2.27555\times10^{-3}$$

$$\therefore H = \frac{0.38}{2.27555\times10^{-3} + \left(\frac{40}{200\times10^6\times18000\times10^{-6}}\right) + \left(\frac{40}{200\times10^6\times4000\times10^{-6}}\right)}$$

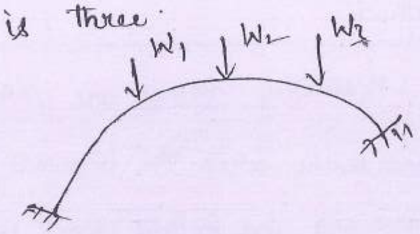
$= 162.625$ kN//

$\therefore M_{crown} = V_B\times20 - Hh$

$= 30\times20 - 162.625\times4 = -50.5$ kNm//

FIXED Arch :

Fixed arches are also called hingeless arches. At each end there are 3 reaction components, thus giving a total of six reaction components. Only three independent static equilibrium equations are available. Hence, degree of indeterminacy is three.



To analyse such arches

Two methods are available

1. Consistent Deformation method
2. Elastic Centre method.

✓ UNIT II

LATERAL LOAD ANALYSIS USING APPROXIMATE METHODS :-

* Approximate methods in Building Frames

1. portal method
2. Cantilever method

Multistorey frames are subjected to horizontal forces due to wind and seismic forces. The horizontal forces are assumed to act at joints and any one method may be used to their analysis.

1. portal method :-

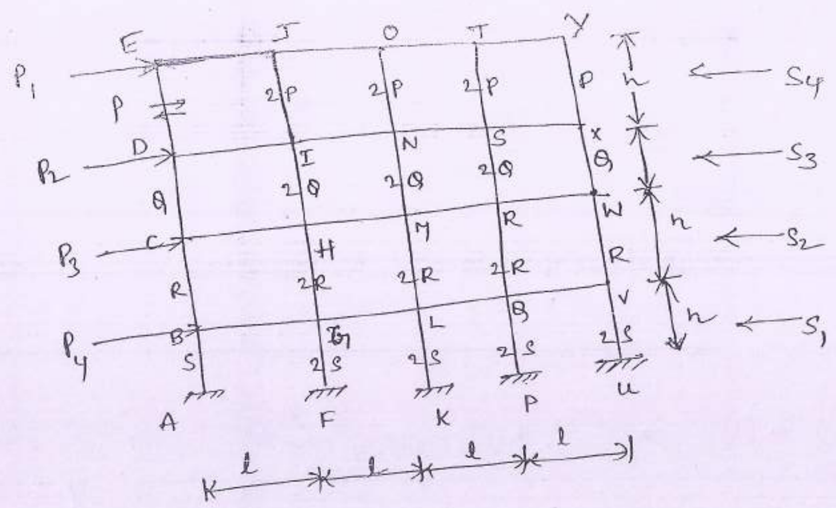
In this method, the following two assumptions are made

i. point of contraflexure occurs at the middle of all members of the frame

ii, Horizontal shear taken by each interior column is double of that taken by external columns.

● with the above two assumptions, the frame becomes a determinate and hence analysis is easy.

This method was presented by Albert Smith in the journal of 'Western Society of Engineers' in 1915.



1) Column Shear

consider column S_4:

$$P+2P+2P+2P+P = P_1$$

$$\therefore 8P = P_1$$

$$\therefore P = \frac{P_1}{8}$$

$\therefore$ Column shear in outer Column $= \frac{P_1}{8}$

" in interior " $= 2P = \frac{2P_1}{8} = \frac{P_1}{4}$

column S_3:-

$$8Q = P_1+P_2$$

$$Q = \frac{P_1+P_2}{8}$$

S.F in outer, $Q = \frac{P_1+P_2}{8}$

S.F in inner, $2Q = \frac{P_1+P_2}{4}$

$$8R = P_1+P_2+P_3 \qquad \therefore R = \frac{P_1+P_2+P_3}{8}$$

S.F in outer column, $R = \frac{P_1+P_2+P_3}{8}$

" in inner, " $2R = \frac{P_1+P_2+P_3}{4}$

Column S_1 :-

$$8S = P_1+P_2+P_3+P_4$$

$$S = \frac{P_1+P_2+P_3+P_4}{8}$$

S.F in outer column $S = \frac{P_1+P_2+P_3+P}{8}$

" " inner " $2S = \frac{P_1+P_2+P_3+}{4}$

2) Column moments :-

Column Moment = S.F × half (distance) (hieght of T

3) Beam Moment :-

At point E,

$M_{ED} = M_{EJ}$

At J :-

$M_{JE} + M_{JO} = M_{JI}$

$\therefore M_{JO} = M_{JI} - M_{JE}$

At point O :-

$M_{OJ} + M_{OT} =$

$\therefore M_{OT} = M_{ON}$

At point T :-

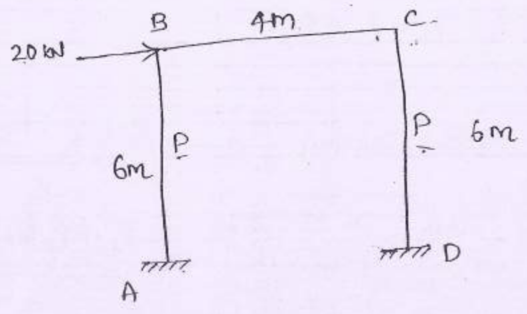
$M_{OT} + M_{TY}$

$\therefore M_{TY} = M_T$

$$B.M = S.F \times \frac{l}{2}$$

$$\therefore S.F = B.M \times \frac{2}{l}$$

se the portal frame shown in the figure by method



let 'p' be the shear force in the Column

Column shear :-

$\therefore$ p + p = 20

2p = 20

P = 10 kN

$\therefore$ S.F in Column AB = 10 kN

" " CD = 10 kN

) Column Moment :-

Column moment = $S.F \times \frac{h}{2}$

$\therefore M_{AB} = 10 \times \frac{6}{2} = 30$ kN-m

$\therefore M_{CD} = 10 \times \frac{6}{2} = 30$ kN-m

$\therefore$

iii) Beam moment :-

At point B :

$M_{BC} = M_{AB} = 30$ kN-m

iv) Beam shear :-

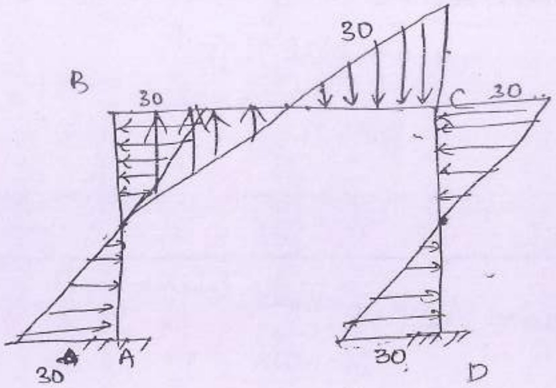
$B.M = S.F.B \times \frac{L}{2}$

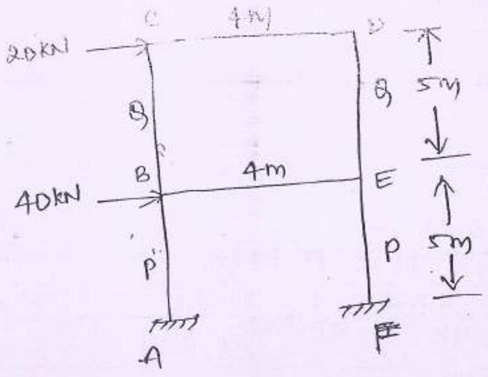
$\because$ S.F.B = B.M ,

$M_{BC} = S.f \text{ in Beam} \times \frac{4}{2}$

30 = S.f.B × 2

S.f.B = 15 kN





Column shear:

let 'Q' be the S.F in Column CB & DE

$Q + Q = 20$

$2Q = 20$

$\underline{Q = 10kN}$

$\therefore$ S.F in Column CB & DE = 10 KN //

let 'P' be the S.F in Column AB & FE

$P + P = 20 + 40$

$\therefore 2P = 60$

$\underline{P = 30 kN}$

$\therefore$ S.F in Column AB & FE = 30kN //

ii) Column moments:-

Column moment $= S.F.C \times \frac{h}{2}$

$\therefore M_{CB} = M_{DE} = Q \times \frac{5}{2} = \frac{10 \times 5}{2} = 25$ kN-m

$\therefore M_{AB} = M_{EF} = P \times \frac{5}{2} = \frac{30 \times 5}{2} = 75$ KN-M

iii) Beam moment :-

At point C :-

$M_{CB} = M_{CD} = \underline{\underline{25}}$ kN-m

At joint B :-

$M_{CB} + M_{BA} = M_{BE}$

$\therefore M_{BE} = 25 + 75 = 100$ kN-M

iv) Beam shear :-

$M_{CD} = S.F \times \frac{L}{2}$

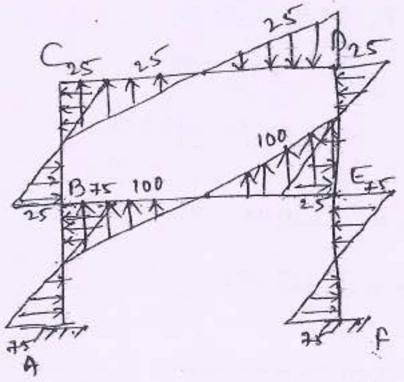
$25 = $ S.F of beam CD $\times \frac{4}{2}$

$\therefore$ S.F on beam CD $= 25 \times \frac{2}{4} = 12.5$ kN //

$M_{BE} = $ S.F on Beam BE $\times \frac{L}{2}$

$100 = S.F \cdot BE \times \frac{4}{2}$

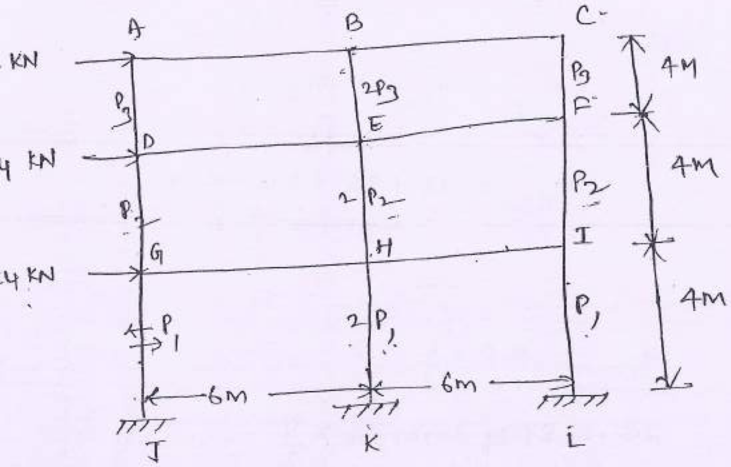
$\therefore$ S.F of beam BE = 50 kN //



given frame, wind loads transferred to joints A, D & G

2KN, 24 KN and 24 KN respectively. Analyse the frame by

method.



n shears:-

let the horizontal shears taken by each column by

to exterior columns P_1, P_2, P_3 and for

for interior columns be $2P_1$, $2P_2$, $2P_3$.

∴ let P_3 be the S.F in column AD & CF and $2P_3$ be the BE

∴ $P_3 + 2P_3 + P_3 = 12$

∴ $4P_3 = 12$

∴ $P_3 = 3$ KN

∴ S.F in column AD & CF = 3 KN//

& S.F in Column BE = 6 KN//

III'y

$P_2 + 2P_2 + P_2 = 12 + 24$ ∴ S.F in Column DG & FI = 9KN//

$4P_2 = 36$ 〃 EH = 18 KN//

$P_2 = 9$ KN

III'y

$P_1 + 2P_1 + P_1 = 12 + 24 + 24$

$4P_1 = 60$

$P_1 = 15$ KN

∴ S.F in column GJ & IL = 15 KN//

〃 HK = 30 KN//

ii) Column moment :- $= \text{S.F in Column} \times \frac{h}{2}$

$M_{AD} = M_{CF} = 3 \times \frac{4}{2} = 6$ KN-m//

$M_{BE} = 6 \times \frac{4}{2} = 12$ KN-m//

$M_{DG} = M_{FI} = 9 \times \frac{4}{2} = 18$ KN-m//

$M_{EH} = 18 \times \frac{4}{2} = 36$ KN-m//

$M_{GJ} = M_{IL} = 15 \times \frac{4}{2} = 30$ KN-m//

$M_{HK} = 30 \times \frac{4}{2} = 60$ KN-m//

iii) Beam moment:-

At point A:-

$M_{AD} = M_{AB} = 6$ KN-m//

At point B:-

$M_{AB} + M_{BC} = M_{BE}$

$\therefore M_{BC} = M_{BE} - M_{AB}$

$= 12 - 6 = 6$ KN-m//

At point D :

$$M_{AD} + M_{DG} = M_{DE}$$

$$6 + 18 = M_{DE}$$

$$\therefore M_{DE} = 24 \text{ kN-m} //$$

At point F :-

$$\therefore M_{EF} = M_{CF} + M_{FI}$$

$$= 6 + 18 = 24 \text{ kN-m} //$$

$$\therefore M_{EF} = 24 \text{ kN-m} //$$

At point G :-

$$\therefore M_{GH} = M_{DG} + M_{GJ} = 18 + 30 = 48 \text{ kN-m} //$$

At point I :-

$$\therefore M_{HI} = M_{FI} + M_{IL} = 18+30 = 48 \text{ kN-M} //$$

iv) Beam Shear :-

S.F on AB :

$$\therefore M_{AB} = S.F \times \frac{L}{2}$$

$$6 = S.F on AB \times \frac{6}{2}$$

$$\therefore SF on AB = 2 \text{ KN} = S.F on BC$$

S.F on DE & EF :-

$$\therefore M_{DE} = S.F on DE \times \frac{L}{2}$$

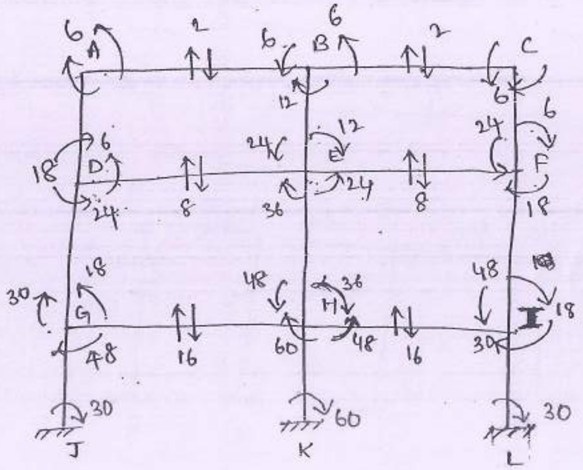
$$24 = S.F \times DE \times \frac{6}{2}$$

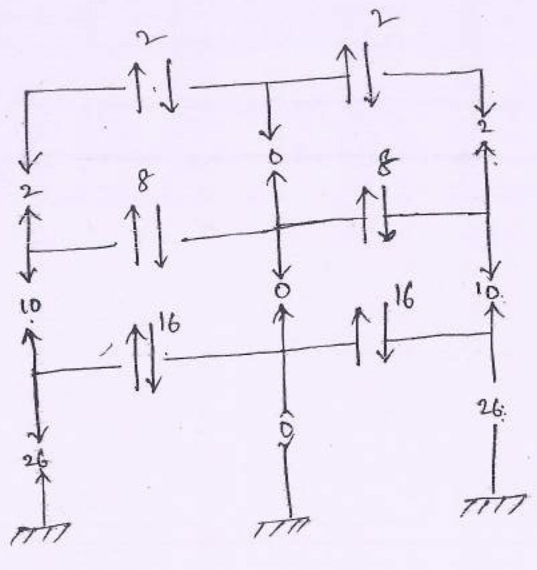
$$\therefore S.F on DE = 8 \text{ KN} = S.F on EF$$

$$\therefore M_{GH} = S.F on G.H \times \frac{L}{2}$$

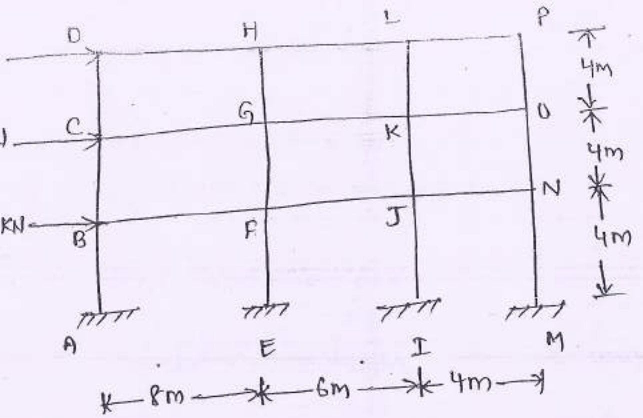
$$48 = S.F \times \frac{6}{2}$$

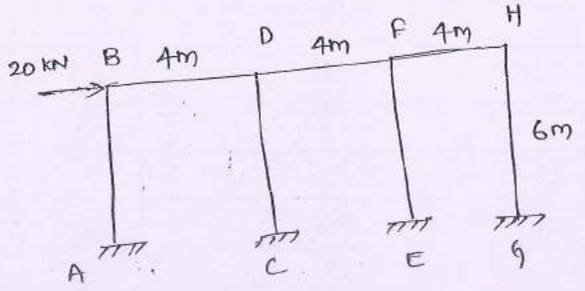
$$\therefore S.F on GH = 16 = S.F on HI //$$





vertical loads





2. CANTILEVER METHOD :

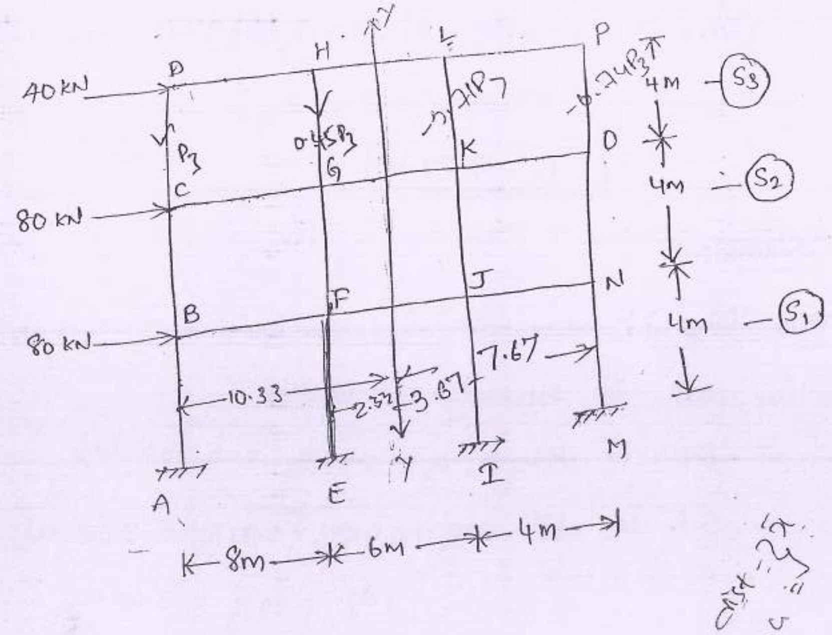
The following assumptions are made in this method

- There is a point of contraflexure at the center of each member
- The intensity of axial stress in each column of a storey is ∝ proportional to the horizontal distance of that column from the centre of gravity of all the columns of the storey under consideration

(The analysis is carried out storey by storey)

- The area of each intermediate column is twice that of area of exterior & end column.

Analyse the frame shown below by cantilever method



i) Consider above assumptions

Let the area of each end column = A

and " " intermidate " = 2A

∴ Total area of column = 6A

Let the distance of C.G of the columns be $\bar{x}$ from the axis of column ABCD

$$\therefore \bar{x} = \frac{A_1x_1 + A_2x_2 + \dots + A_nx_n}{A_1 + A_2 + \dots + A_n}$$

$$\bar{x} = \frac{(A\times 0)+(2A\times 8)+(2A\times 14)+(A\times 18)}{6A}$$

$$\bar{x} = \frac{16A + 28A + 18A}{6A} = \frac{62A}{6A}$$

$$\boxed{\bar{x} = 10.33\ m}$$

Let the axial force in the column AB = P_1

$$\text{Stress in the column AB} = \frac{\text{distance}}{\bar{x}} \times \frac{P_1}{A} = \frac{10.33}{10.33} \times \frac{P_1}{A} =$$

$$\text{Stress in the column EF} = \frac{2.33}{10.33} \times \frac{P_1}{A} = 0.225\frac{P_1}{A}$$

$$\text{Stress in the column IJ} = -\frac{3.67}{10.33} \times \frac{P_1}{A} = 0.355\frac{P_1}{A}$$

$$\text{Stress in the column MN} = -\frac{7.67}{10.33} \times \frac{P_1}{A} = -0.74\frac{P_1}{A}$$

∴ Axial force in column AB = stress in column × Area

$$= \frac{P_1}{A} \times A = P_1 \text{ (T)}$$ (+ve

" $EF = 0.225 \times \frac{P_1}{A} \times 2A = 0.45 P$

" $IJ = 0.355 \times \frac{P_1}{A} \times 2A = 0.71 P_1$

" $MN = 0.74 \times \frac{P_1}{A} \times A = 0.74 P_1$

Now taking moments about the point of contraflexu of column AB

$$40\times(4+4+2) + 80(4+2) + 80\times 2 + P_1\times 0 + 0.45 P$$
$$- 0.71P_1\times 14 - 0.74P_1\times 18 = 0$$

$$400 + 480 + 160 + 3.6P_1 - 9.8P_1 - 13.32P_1 = 0$$

$$19.52 P_1 = 1040$$

y②:-

let P_2 be the axial force in the Column BC

al force in the column BC = P_2 (T)

" " FG $= \frac{2.33}{10.33} \times \frac{P_2}{A} \times 2A = 0.45 P_2$ (T)

" " JK $= 0.71 P_2$ (C)

" " NO $= 0.74 P_2$ (C)

ow taking moments about point of Contraflexure of BC.

$$40 \times (4+2) + 80(2) + P_2 \times 0 + (0.45 P_2 \times 8)$$
$$- (0.71 P_2 \times 14) - (0.74 P_2 \times 18) = 0$$

$$400 = 19.52 P_2$$

$$\boxed{\therefore P_2 = 20.49 \text{ kN}}$$

torey③:- let 'P_3' be the axial force in the Column 'CD'

ly axial force in Column CD $= P_3$ (T)

" " GH $= 0.45 P_3$ (T)

" " KL $= 0.71 P_3$ (C)

" " OP $= 0.74 P_3$ (C)

Now taking moments about point of contraflexure of D

$$(40 \times 2) + (0.45 P_3 \times 8) - (0.71 P_3 \times 14) - (0.74 P_3 \times 18) = 0$$

$$80 = 19.52 P_3$$

$$\boxed{\therefore P_3 = 4.09 \text{ kN}}$$

BEAM SHEAR:-

From Storey ③:-

S.F in DH $= P_3 = 4.10$ kN

S.F in HL $= P_3 + 0.45 P_3 = 5.945$ kN

S.F in LP $= P_3 + 0.45 P_3 - 0.71 P_3 = 3.07$ kN

(or) $0.74 P_3 = 3.03$ kN

From Storey ②:-

S.F in CG $= P_2 - P_3 = 16.36$ kN

S.F in GK $= P_2 + 0.45 P_2 - P_3 - 0.45 P_3 = 23.76$ kN

S.F in KO $= P_2 + 0.45 P_2 - 0.71 P_2 - P_3 - 0.45 P_3 + 0.7 P_3$
$= 12.29$ kN

Storey 1:-

S.F in BF $= P_1 - P_2 = 32.79$ kN

S.F in FJ $= P_1 + 0.45 P_1 - P_2 - 0.45 P_2 = 47.54$ kN

S.F in JN $= P_1 + 0.45 P_1 - 0.7 P_1 - P_2 - 0.45 P_2 + 0.7 P_2 = 24.59$ kN

Beam Moment:-

Beam moment = Shear force ~~on~~ beam x half Span

$M_{DH} = 4.1 \times 4 = 16.4$ kN-m

$M_{HL} = 5.945 \times 3 = 17.835$ kN-m

$M_{LP} = 3.07 \times 2 = 6.14$ kN-m

$M_{CG} = 16.36 \times 4 = 65.44$ kN-m

$M_{GK} = 23.26 \times 3 = 69.78$ kN-m

$M_{KO} = 12.29 \times 2 = 24.58$ kN-m

$M_{BF} = 32.7 \times 4 = 130.8$ kN-m

$M_{FJ} = 47.54 \times 3 = 142.62$ kN-m

$M_{JN} = 24.59 \times 2 = 49.18$ kN-m

column moment :-

@ point D

$M_{DC} = M_{DH} = 16.4$ kN-m //

@ point H

$M_{HG} = M_{DH} + M_{HL} = 16.4 + 17.835 = 34.235$ kNm //

@ point L

$M_{LK} = M_{HL} + M_{LP} = 17.835 + 6.14 = 23.97$ kNm //

@ point P

$M_{PO} = M_{LP} = 6.14$ kN-m //

@ point C

$M_{CG} = M_{DC} + M_{CB}$

$\therefore M_{CB} = M_{CG} - M_{DC} = 65.44 - 16.4 = 49.04$ kN-m //

@ point G

$M_{GC} + M_{GK} = M_{GH} + M_{GF}$

$M_{GF} = M_{GC} + M_{GK} - M_{GH}$

$= 65.44 + 69.78 - 34.235$

$= 100.98$ KNm //

@ point K

$M_{KG} + M_{KO} = M_{KL} + M_{KJ}$

$69.78 + 24.58 = 23.97 - M_{KJ}$

$\therefore M_{KJ} = 70.39$ kNm //

@ point O

$M_{PO} + M_{ON} = M_{OK} \Rightarrow M_{ON} = M_{OK} - M_{PO}$

$M_{ON} = 18.44$ kNm //

@ point B

$M_{BA} + M_{BC} = M_{BF}$

$\therefore M_{BA} = 81.76$ kN-m //

@ point F

$M_{FB} + M_{FJ} = M_{FG} + M_{FE}$

$M_{FE} = 172.44$ kNm //

@ point J

$M_{JF} + M_{JN} = M_{JK} + M_{JI}$

$M_{JI} = 121.41$ kNm //

@ point N

$M_{NO} + M_{NM} = M_{JN}$

$M_{NM} = 30.74$ kNm //

Shear [illegible] ([illegible])

Column $\overset{(M)}{\text{moment}}$ = S.F × half distance

S.F DC $= M_{DC} \times \frac{2}{h}$

$= 16.4 \times \frac{2}{4}$

$= 16.4/2 = 8.2$ KN//

S.F HG $= M_{HG}/2 = 34.235/2 = 17.71$ kN//

S.F LK $= \frac{23.97}{2} = 11.98$ kN//

S.F PO $= \frac{6.14}{2} = 3.07$ kN//

S.F CB $= \frac{49.04}{2} = 24.52$ KN//

S.F GF $= \frac{100.98}{2} = 50.49$ kN//

S.F KJ $= \frac{70.39}{2} = 35.19$ KN//

S.F ON $= \frac{18.44}{2} = 9.22$ KN//

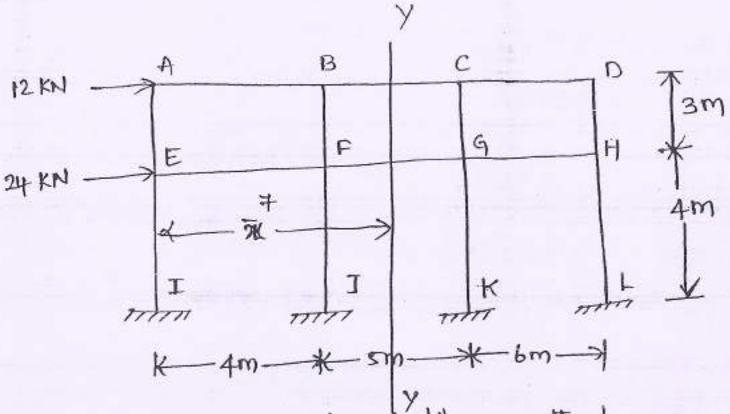
S.F BA $= \frac{81.76}{2} = 40.88$ KN//

S.F FE $= \frac{172.44}{2} = 86.22$ KN//

S.F JI $= \frac{121.41}{2} = 60.7$ kN//

S.F NM $= \frac{30.74}{2} = 15.37$ kN//

* Analyse the frame shown in figure by Cantilever method. Take cross-sectional areas of all columns as the Same.



Sol: Considering the assumptions in Cantilever method

let the area of each column be 'A' given is same

Total area = 4A

$$\therefore \bar{x} = \frac{(A\times0)+(A\times4)+(A\times9)+(A\times15)}{4A}$$

$$\bar{x} = \frac{28A}{4A} = 7$$

From Storey ① let the axial force in the column be P_1

Axial force in column EI = stress in the column EI × Area

$= \frac{\text{distance}}{\bar{x}} \times \frac{P_1}{A} \times A$

$= \frac{7}{7} \times \frac{P_1}{A} \times A = P_1$ (T)

" in FJ $= \frac{3}{7} \times \frac{P_1}{A} \times A = 0.428\,P_1$ (T)

" in GK $= \frac{2}{7} \times \frac{P_1}{A} \times A = 0.285\,P_1$ (C)

" in HL $= \frac{8}{7} \times \frac{P_1}{A} \times A = 1.142\,P_1$ (C)

~~On storey ② let P_2 be the Axial force in column AE~~ =

Now taking moments about the point of contraflexure of column EI

$$12 \times (3+2) + 24 \times 2 + P_1 \times 0 + (0.428 P_1 \times 4)\ (0.285 P_1 \times 9)$$
$$- (1.142 P_1 \times 15) = 0$$

$$108 = -1.712 P_1 + 2.565 P_1 + 17.13 P_1$$

$$108 = 17.983 P_1$$

~~∴ P_1 = [illegible] KN~~ // $\boxed{P_1 = 6.0\ KN}$

From storey ②

Let 'P_2' be the axial force in AE

IIIly

Axial force in AE = P_2 (T)

" BF = $0.428 P_2$ (T)

" CG = $0.285 P_2$ (C)

" DH = $1.142 P_2$ (C)

Now taking moments about the point of contraflexure of column AE

$$12 \times 1.5 + P_2 \times 0 + (0.428 P_2 \times 4) - (0.285 P_2 \times 9)$$
$$- (1.142 P_2 \times 15) = 0$$

$18 = \overset{17.983}{\cancel{[illegible]}} P_2$

~~P_2 = 0.84 KN~~ $\boxed{P_2 = 1.0\ KN}$

BEAM SHEAR :-

From storey ②

S.F in A.B = P_2 = 1 KN//

S.F in BC = $P_2 + 0.428 P_2$ = 1+0.428 =

S.F in CD = $P_2 + 0.428 P_2 - 0.285 P_2$

= 1.428 − 0.285 (or) $1.142 P_2$

= 1.143 KN//

From storey ①

S.F in EF = $P_1 - P_2$ = 6−1 = 5 KN//

S.F in FG = $P_1 + 0.428 P_1 - P_2 - 0.428 P_2$

= 6 + (0.428 × 6) − 1 − 0.428

= 7.14 KN//

S.F in GH = $P_1 + 0.285 P_1 - P_2 - 0.428 P_2$ + 0

= 6 + 0.285×6 − 1 − 0.428×1 + 0.2

= 6 + 1.71 − 1 − 0.428 + 0.285

= 6.567 KN//

BEAM MOMENT :-

Beam moment = S.F × half span

$M_{AB} = 1 \times \frac{4}{2} = 2$ KN-m//

$M_{BC} = 1.428 \times \frac{5}{2} = 3.57$ KN-m//

$M_{CD} = 1.143 \times \frac{6}{2} = 3.429$ KN-m//

$M_{EF} = 5 \times \frac{4}{2} = 10$ KN-m//

$M_{FG} = 7.14 \times \frac{5}{2} = 17.85$ KN-m//

$M_{GH} = 6.567 \times \frac{6}{2} = 19.70$ KN-m//

ment:-

@ point A

$M_{AE} = M_{AB} = 2$ KN-m//

@ point B

$M_{BF} = M_{AB} + M_{BC}$

$= 2 + 3.57 = 5.57$ KN-m//

@ point C

$M_{CG} = M_{BC} + M_{CD}$

$= 3.57 + 3.429 = 6.99$ KN-m//

@ point D

$M_{DH} = M_{CD} = 3.429$ KN-m//

@ point E

$M_{EI} + M_{AE} = M_{EF}$

$\therefore M_{EI} = M_{EF} - M_{AE} = 10 - 2 = 8$ KN-m//

@ point F

$M_{EF} + M_{FG} = M_{BF} + M_{FJ}$

$M_{FJ} = M_{EF} + M_{FG} - M_{BF}$

$= 10 + 17.85 - 5.57 = 22.28$ KN-m

$M_{FJ} = 22.28$ KN-m//

@ point G

$M_{FG} + M_{GH} = M_{CG} + M_{GK}$

$M_{GK} = M_{FG} + M_{GH} - M_{CG}$

85 + 19.70 − 6.99 = 30.56

M_{GK} = [illegible] //

@ Point H

$M_{GH} = M_{DH} + M_{HL}$

$\therefore M_{HL} = M_{GH} - M_{DH}$

$= 19.70 - 3.429$

$M_{HL} = 16.271$ KN-m//

Column shear :-

column moment = S.F × $\frac{\text{height of column}}{2}$

∴ S.F ~~in~~ ~~AE~~ = Column Moment M × $\frac{2}{h}$

∴ S.F in AE $= 2 \times \frac{2}{3} = 1.33$ KN//

S.F in BF $= 5.57 \times \frac{2}{3} = 3.71$ KN//

S.F in CG $= 6.99 \times \frac{2}{3} = 4.66$ KN//

S.F in DH $= 3.429 \times \frac{2}{3} = 2.28$ KN//

S.F in EI $= 8 \times \frac{2}{4} = 4$ KN//

S.F in FJ $= \frac{22.28}{2} = 11.14$ KN//

S.F in GK $= \frac{30.56}{2} = 15.28$ KN//

S.F in HL $= \frac{16.271}{2} = 8.13$ KN//