Module
3
Limit State of Collapse - Flexure (Theories and Examples)

Lesson
6
Numerical Problems on Singly Reinforced Rectangular Beams

Instructional Objectives:

At the end of this lesson, the student should be able to:

- identify the main two types of problems of singly reinforced rectangular sections,
- name the inputs and outputs of the two types of problems,
- state the specific guidelines of assuming the breadth, depths, area of steel reinforcement, diameter of the bars, grade of concrete and grade of steel,
- determine the depth of the neutral axis for specific dimensions of beam (breadth and depth) and amount of reinforcement,
- identify the beam with known dimensions and area of steel if it is under-reinforced or over-reinforced,
- apply the principles to design a beam.

3.6.1 Types of Problems

Two types of problems are possible: (i) design type and (ii) analysis type. In the design type of problems, the designer has to determine the dimensions b, d, D, A_{st} (Fig. 3.6.1) and other detailing of reinforcement, grades of concrete and steel from the given design moment of the beam. In the analysis type of the problems, all the above data will be known and the designer has to find out the moment of resistance of the beam. Both the types of problems are taken up for illustration in the following two lessons.

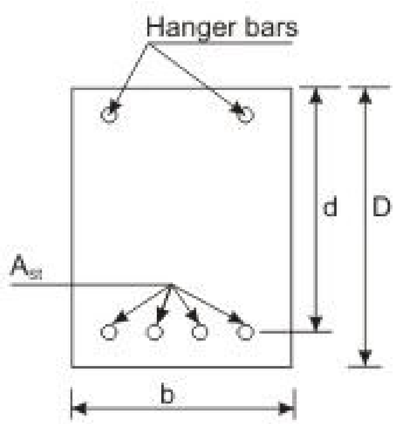

Fig. 3.6.1: Typical section of a beam

3.6.2 Design Type of Problems

The designer has to make preliminary plan lay out including location of the beam, its span and spacing, estimate the imposed and other loads from the given functional requirement of the structure. The dead loads of the beam are estimated assuming the dimensions *b* and *d* initially. The bending moment, shear force and axial thrust are determined after estimating the different loads. In this illustrative problem, let us assume that the imposed and other loads are given. Therefore, the problem is such that the designer has to start with some initial dimensions and subsequently revise them, if needed. The following guidelines are helpful to assume the design parameters initially.

3.6.2.1 Selection of breadth of the beam *b*

Normally, the breadth of the beam *b* is governed by: (i) proper housing of reinforcing bars and (ii) architectural considerations. It is desirable that the width of the beam should be less than or equal to the width of its supporting structure like column width, or width of the wall etc. Practical aspects should also be kept in mind. It has been found that most of the requirements are satisfied with *b* as 150, 200, 230, 250 and 300 mm. Again, width to overall depth ratio is normally kept between 0.5 and 0.67.

3.6.2.2 Selection of depths of the beam *d* and *D*

The effective depth has the major role to play in satisfying (i) the strength requirements of bending moment and shear force, and (ii) deflection of the beam. The initial effective depth of the beam, however, is assumed to satisfy the deflection requirement depending on the span and type of the reinforcement. IS 456 stipulates the basic ratios of span to effective depth of beams for span up to 10 m as (Clause 23.2.1)

Cantilever	7
Simply supported	20
Continuous	26

For spans above 10 m, the above values may be multiplied with 10/span in metres, except for cantilevers where the deflection calculations should be made. Further, these ratios are to be multiplied with the modification factor depending on reinforcement percentage and type. Figures 4 and 5 of IS 456 give the different values of modification factors. The total depth *D* can be determined by adding 40 to 80 mm to the effective depth.

3.6.2.3 Selection of the amount of steel reinforcement A_{st}

The amount of steel reinforcement should provide the required tensile force *T* to resist the factored moment M_u of the beam. Further, it should satisfy the minimum and maximum percentages of reinforcement requirements also. The minimum reinforcement A_s is provided for creep, shrinkage, thermal and other environmental requirements irrespective of the strength requirement. The minimum reinforcement A_s to be provided in a beam depends on the f_y of steel and it follows the relation: (cl. 26.5.1.1a of IS 456)

$$\frac{A_s}{b\,d} = \frac{0.85}{f_y}$$

(3.26)

The maximum tension reinforcement should not exceed 0.04 bD (cl. 26.5.1.1b of IS 456), where D is the total depth.

Besides satisfying the minimum and maximum reinforcement, the amount of reinforcement of the singly reinforced beam should normally be 75 to 80% of $p_{t,\,lim}$. This will ensure that strain in steel will be more than $(\frac{0.87\,f_y}{E_s} + 0.002)$ as the design stress in steel will be 0.87 f_y. Moreover, in many cases, the depth required for deflection becomes more than the limiting depth required to resist $M_{u,\,lim}$. Thus, it is almost obligatory to provide more depth. Providing more depth also helps in the amount of the steel which is less than that required for $M_{u,\,lim}$. This helps to ensure ductile failure. Such beams are designated as under-reinforced beams.

3.6.2.4 Selection of diameters of bar of tension reinforcement

Reinforcement bars are available in different diameters such as 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 36 and 40 mm. Some of these bars are less available. The selection of the diameter of bars depends on its availability, minimum stiffness to resist while persons walk over them during construction, bond requirement etc. Normally, the diameters of main tensile bars are chosen from 12, 16, 20, 22, 25 and 32 mm.

3.6.2.5 Selection of grade of concrete

Besides strength and deflection, durability is a major factor to decide on the grade of concrete. Table 5 of IS 456 recommends M 20 as the minimum grade under mild environmental exposure and other grades of concrete under different environmental exposures also.

3.6.2.6 Selection of grade of steel

Normally, Fe 250, 415 and 500 are in used in reinforced concrete work. Mild steel (Fe 250) is more ductile and is preferred for structures in earthquake zones or where there are possibilities of vibration, impact, blast etc.

3.6.3Design Problem 3.1

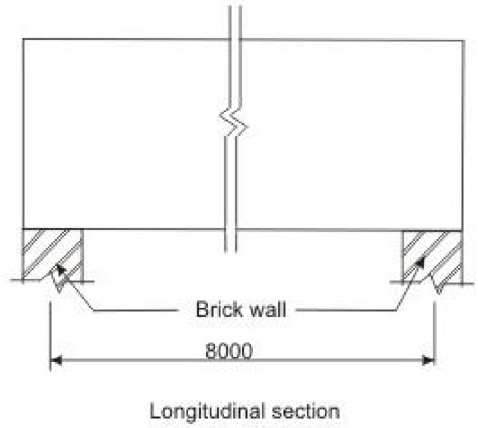

Fig. 3.6.2: Design Problem 3.1

Design a simply supported reinforced concrete rectangular beam (Fig. 3.6.2) whose centre to centre distance between supports is 8 m and supported on brick walls of 300 mm thickness. The beam is subjected to imposed loads of 7.0 kN/m.

3.6.4 Solution by Direct Computation Method

The unknowns are *b*, *d*, *D*, A_{st}, grade of steel and grade of concrete. It is worth mentioning that these parameters have to satisfy different requirements and they also are interrelated. Accordingly, some of them are to be assumed which subsequently may need revision.

3.6.4.1 Grades of steel and concrete

Let us assume Fe 415 and M 20 are the grades of steel and concrete respectively. As per clause 6.1.2 and Table 5 of IS 456, minimum grade of concrete is M 20 for reinforced concrete under mild exposure (durability requirement).

3.6.4.2 Effective span L_{eff}

Clause 22.2(a) of IS 456 recommends that the effective span is the lower of (i) clear span plus effective depth and (ii) centre to centre distance between two supports. Here, the clear span is 7700 mm. Thus

(i) Clear span + d = 7700 + 400 (assuming d = 400 from the specified ratio of span to effective depth as 20 and mentioned in the next section)

(ii) Centre to centre distance between two supports = 8000 mm.

Hence, L_{eff} = 8000 mm

3.6.4.3 Percentage of steel reinforcement p_t

The percentage of steel reinforcement to be provided is needed to determine the modification factor which is required to calculate d. As mentioned earlier in sec. 3.6.2.3, it is normally kept at 75 to 80 per cent of $p_{t,\,lim}$. Here, $p_{t,\,lim}$ = 0.96 (vide Table 3.1 of Lesson 5). So, percentage of steel to be provided is assumed = 0.75 (0.96) = 0.72.

3.6.4.4 Effective depth d

As per clause 23.2.1 of IS 456, the basic value of span to effective depth ratio here is 20. Further, Fig. 4 of IS 456 presents the modification factor which will be multiplied with the basic span to effective depth ratio. This modification factor is determined on the value of f_s where

$$f_s = 0.58 \; f_y \; \frac{\text{Area of cross-section of steel required}}{\text{Area of cross-section of steel provided}}$$

= 0.58 f_y (assuming that the A_{st} provided is the same as A_{st} required)

= 0.58 (415) = 240.7 N/mm^2.

From Fig. 4 of IS 456, the required modification factor is found to be 1.1 for f_s = 240.7 N/mm^2 and percentage of steel = 0.72. So, the span to effective depth ratio = 22 as obtained by multiplying 20 with 1.1. Accordingly, the effective depth = 8000/22 = 363.63 mm, say 365 mm. Since this value of d is different from the d assumed at the beginning, let us check the effective span as lower of (i) 7700 + 365 and (ii) 8000 mm. Thus, the effective span remains at 8000 mm. Adding 50 mm with the effective depth of 365 mm (assuming 50 mm for cover etc.), the total depth is assumed to be 365 + 50 = 415 mm.

3.6.4.5 Breadth of the beam b

Let us assume b = 250 mm to get b/D = 250/415 = 0.6024, which is acceptable as the ratio of b/D is in between 0.5 and 0.67.

3.6.4.6 Dead loads, total design loads F_d and bending moment

With the unit weight of reinforced concrete as 25 kN/m^3 (cl. 19.2.1 of IS 456):

Dead load of the beam = 0.25 (0.415) (25) kN/m = 2.59 kN/m

Imposed loads = 7.00 kN/m

Thus, total load = 9.59 kN/m, which gives factored load F_d as 9.59 (1.5) (partial safety factor for dead load and imposed load as 1.5) = 14.385 kN/m. We have, therefore, M_u = Factored bending moment = 14.385 (8) = 115.08 kNm.

3.6.4.7 Checking of effective depth *d*

It is desirable to design the beam as under-reinforced so that the ductility is ensured with steel stress reaching the design value. Let us now determine the limiting effective depth when $x_u = x_{u,\,max}$ and the factored moment $M_u = M_{u,\,lim}$ = 115.08 kNm from Eq. 3.24 of Lesson 5.

$$M_{u,\,\mathrm{lim}} = 0.36 \; \frac{x_{u,\,\max}}{d} \left\{ 1 - 0.42 \, \frac{x_{u,\,\max}}{d} \right\} b\, d^2 \; f_{ck}$$

(3.24)

Table 3.2 of Lesson 5 gives $\frac{x_{u,\,\max}}{d}$ = 0.479 for f_y = 415 N/mm^2. Thus:

(115.08) 10^6 Nmm = 0.36(0.479) [1 - 0.42(0.479)] $b\, d^2$ (20)

which gives d = 408.76 mm

So, let us revise d = 410 mm from the earlier value of 365 mm to have the total depth = 410 + 50 = 460 mm.

3.6.4.8 Area of Steel A_{st}

The effective depth of the beam has been revised to 408.76 mm from the limiting moment carrying capacity of the beam. Increasing that depth to 410 also has raised the $M_{u,\,lim}$ of the beam from the design factored moment of 115.08 kNm. Therefore, the area of steel is to be calculated from the moment equation (Eq. 3.23 of Lesson 5), when steel is ensured to reach the design stress f_d = 0.87 (415) = 361.05 N/mm^2.

$$M_u = 0.87 \; f_y \;\; A_{st} \;\; d \left\{ 1 \; - \; \frac{A_{st} \; f_y}{f_{ck} \; b \; d} \right\}$$

(3.23)

Here, all but A_{st} are known. However, this will give a quadratic equation of A_{st} and one of the values, the lower one, will be provided in the beam. The above equation gives:

$$115.08\,(10^6)\text{ Nmm} = 0.87(415)\,A_{st}\,(410)\left\{1 - \frac{A_{st}\,(415)}{20(250)(410)}\right\}\text{ Nmm}$$

$$= 148030.5\,A_{st} - 29.96715\,A_{st}^2$$

or $$A_{st}^2 - 4939.759\,A_{st} + 3840205 = 0$$

which gives

$$A_{st} = 966.5168\ \text{mm}^2 \text{ or } 3973.2422\ \text{mm}^2$$

The values of x_u determined from Eq. 3.16 of Lesson 5 are 193.87 mm and 796.97 mm respectively, when A_{st} = 966.5168 mm^2 and 3973.2422 mm^2. It is seen that the value of x_u with lower value of A_{st} is less than $x_{u,max}$ (= 216 mm). However, the value of x_u with higher value of A_{st} (= 3973.2422 mm^2) is more than the value of $x_{u,max}$ (= 0.48 d = 216 mm), which is not permissible as it exceeds the total depth of the beam (= 460 mm). In some problems, the value of x_u may be less than the total depth of the beam, but it shall always be more than $x_{u,max.}$ The beam becomes over-reinforced. Therefore, the lower value of the area of steel is to be accepted as the tensile reinforcement out of the two values obtained from the solution of the quadratic equation involving $A_{st.}$

Accepting the lower value of A_{st} = 966.5168 mm^2, the percentage of steel becomes

$$\frac{966.5168\,(100)}{250\,(410)} = 0.9429 \quad \text{per cent}$$

This percentage is higher than the initially assumed percentage as 0.72. By providing higher effective depth, this can be maintained as shown below.

3.6.4.9 Increase of effective depth and new A_{st}

Increasing the effective depth to 450 mm from 410 mm, we have from Eq. 3.23 of Lesson 5,

$$115.08\,(10^6) = 0.87(415)\,A_{st}\,(450)\left\{1 - \frac{A_{st}\,(415)}{20(250)\,(450)}\right\}$$

$$= 162472.5\ A_{st} - 29.967148\ A_{st}^2$$

or $\quad A_{st}^2 - 5421.6871\ A_{st} + 3840205.2 = 0$

or $\quad A_{st} = 0.5\ \{5421.6871 \pm 3746.1808\}$

The lower value of A_{st} now becomes 837.75315 which gives the percentage of A_{st} as

$\dfrac{837.75315\ (100)}{250\ (450)} = 0.7446$, which is close to earlier assumed percentage of 0.72.

Therefore, let us have d = 450 mm, D = 500 m, b = 250 mm and A_{st} = 837.75315 mm^2 for this beam.

For any design problem, this increase of depth is obligatory to satisfy the deflection and other requirements. Moreover, obtaining A_{st} with increased depth employing moment equation (Eq. 3.23 of Lesson 5) as illustrated above, results in under-reinforced beam ensuring ductility.

3.6.4.10 Further change of A_{st} due to increased dead load

However, increasing the total depth of the beam to 500 mm from earlier value of 415 mm has increased the dead load and hence, the design moment M_u. This can be checked as follows:

The revised dead load = 0.25 (0.5) (25) = 3.125 kN/m

Imposed loads = 7.00 kN/m

Total factored load F_d = 1.5(10.125) = 15.1875 kN/m

M_u = 15.1875 (8) = 121.5 kNm

The limiting moment that this beam can carry is obtained from using $M_{u,\,lim}/bd^2$ factor as 2.76 from Table 3.3 of of Lesson 5. Thus,

$M_{u,\,lim}$ = (2.76) bd^2 = (2.76) (250) (450)^2 Nmm

$= \; 139.72 \text{ kNm} > (M_u = 121.5 \text{ kNm})$

Hence, it is under-reinforced beam.

Equation 3.23 of Lesson 5 is now used to determine the A_{st} for M_u = 121.5 kNm

$$M_u \quad = \quad 0.87\, f_y\, A_{st}\, d \left\{ 1 - \frac{A_{st}\, f_y}{f_{ck}\, b\, d} \right\}$$

(3.23)

or $$121.5\,(10^6) \quad = \quad 0.87\,(415)\, A_{st}\,(450) \left\{ 1 - \frac{A_{st}\,(415)}{20\,(250)\,(450)} \right\}$$

$$= \; 162472.5 \; A_{st} \; - \; 29.96715 \; A_{st}^2$$

or A_{st} = 0.5 {5421.6867 $\pm$ 3630.0038} = 895.84145 mm^2

The steel reinforcement is $\frac{895.84\,(100)}{250\,(450)}$ = 0.7963 per cent which is 83 per cent of $p_{t,lim}$.

So, we have the final parameters as b = 250 m, d = 450 mm, D = 500 mm, A_{st} = 895.84 mm^2. A selection of 2-20 T bars and 2-14 T bars gives the A_{st} = 935 mm^2 (Fig. 3.6.3). Though not designed, Fig. 3.6.3 shows the holder bars and stirrups also.

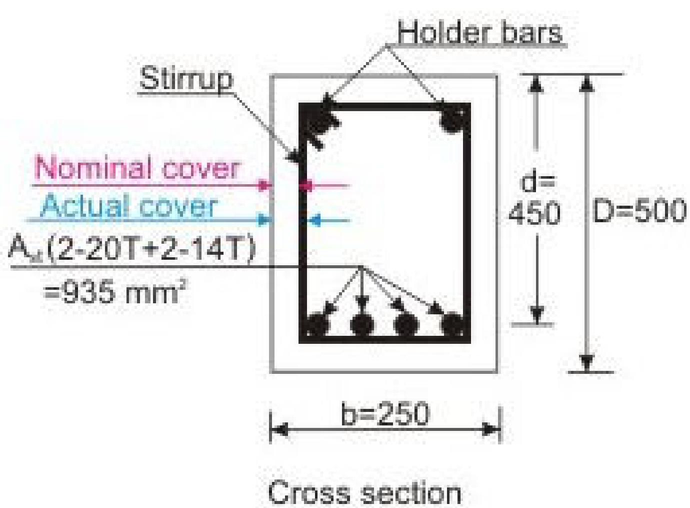

Fig. 3.6.3: Design Problem 3.1

3.6.4.11 Summary of steps

Table 3.4 presents the complete solution of the problem in eleven steps. Six columns of the table indicate (i) parameters assumed/determined, (ii) if they need revision, (iii) final parameters, (iv) major requirements of the parameter, (v) reference section numbers, and (vi) reference source material.

Table 3.4 Steps of the illustrative problem

Step	Assumed/ determined parameter(s) (i)	If need(s) revision (ii)	Final parameter(s) (iii)	Major requirement of the parameter(s) (iv)	Reference section number (v)	Reference source Material(s) (vi)
1	f_{ck}, f_y	No	f_{ck}, f_y	Durability for f_{ck} and ductility for f_y	3.6.4.1	cl.6.1.2, cl. 8 and Table 5 of IS 456
2	d	Yes	No	$d = \frac{\text{c/c span}}{20}$	3.6.4.2	cl. 23.2 of IS 456
3	L_{eff}	Yes	No	Boundary conditions	3.6.4.2	cl.22.2 of IS 456
4	$p = A_{st}/bd$	No	Yes	Ductility (p = 75 to 80% of $p_{t,\ lim}$)	3.6.4.3	Table 3.1 of Lesson 5 for $p_{t,\ lim}$
5	d	Yes	No	Control of deflection	3.6.4.4	cl.23.2 of IS 456
6	D, b	Yes for D	b	Economy	3.6.4.5	$D = d$ + (40 to 80 mm) b = (0.5 to 0.67)D
7	F_d, M_u	Yes	No	Strength	3.6.4.6	Strength of material books
8	d	Yes	No	Limiting depth considering $M_u = M_{u,lim}$	3.6.4.7	Eq. 3.24 of Lesson 5
9	A_{st}	Yes	No	Strength	3.6.4.8	Eq. 3.23 of Lesson 5
10	d, D, A_{st}	No	d, D, L_{eff}	Under-reinforced	3.6.4.9	$D = d + 50$ Eq. 3.23 of Lesson 5
11	A_{st}	No	A_{st}	Strength	3.6.4.10	Eq. 3.23 of Lesson 5

3.6.5 Use of Design Aids

From the solution of the illustrative numerical problems, it is clear that *b*, *d, D* and A_{st} are having individual requirements and they are mutually related. Thus, any design problem has several possible sets of these four parameters. After getting one set of values, obtaining the second set, however, involves the same steps as those of the first one. The steps are simple but time consuming and hence, the designer may not have interest to compare between several sets of these parameters. The client, contractor or the architect may request for alternatives also. Thus, there is a need to get several sets of these four parameters as quickly as possible. One way is to write a computer program which also may restrict average designer not having a computer. Bureau of Indian Standard (BIS), New Delhi published SP-16, Design Aids for Reinforced Concrete to IS: 456, Special Publication No. 16, which is very convenient to get several sets of these values quickly.

SP-16 provides both charts (graphs) and tables explaining their use with illustrative examples. On top left or right corner of these charts and tables, the governing parameters are provided for which that chart/table is to be used.

3.6.6 Solution by using Design Aids Charts (SP-16)

The initial dimension of effective depth *d* of Design Problem 3.1 is modified from 400 mm to 410 mm first to satisfy the deflection and other requirements and then to 450 mm as the final dimension. While using only the charts or tables of SP-16, the final results as obtained for this problem by direct calculation method will not be available. So, we will assume the percentage of steel as 0.75 (0.96) = 0.72 initially.

3.6.6.1 Effective depth *d*

Chart 22 of SP-16 for f_y = 415 N/mm^2 and f_{ck} = 20 N/mm^2 gives maximum ratio of span to effective depth as 21.5 when the percentage of steel assumed = 0.75 (0.96) = 0.72. Thus, we get effective depth *d* = 8000/21.5 = 372.09 mm with *d* = 372.09 mm and effective span L_{eff} = 8000 mm. Total depth *D* = 372.09 + 50 = 422.09 = 425 mm (say).

3.6.6.2 Breadth *b* and factored moment M_u

Here also *b* = 250 mm is assumed and accordingly,

Dead load = 0.25 (0.425) (25) = 2.66 kN/m

Imposed loads = 7.00 kN/m

Total factored load, F_d = 1.5 (9.66) = 14.50 kN/m

Factored bending moment = (14.5) (8) = 116.00 kN/m

3.6.6.3 Checking of effective depth *d* and area of steel A_{st}

Chart 14 of SP-16 is for f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2 and d varying from 300 to 550 mm. For this problem, M_u per metre width of the beam = 464 kNm/m. For the percentage of reinforcement = 0.72, chart 14 gives d = 460 mm and then D = 510 mm. Area of steel reinforcement 0.72 (25) (460)/100 = 828 mm^2.

As in the earlier problem, the increased dead load due to the increased D to 510 mm is checked below:

Revised dead load = 0.25 (0.51) (25) = 3.188 kN/m

Imposed loads = 7.000 kN/m

Total factored load F_d = 1.5 (10.188) = 15.282 kN/m

Factored moment M_u = 15.282 (8) = 122.256 kN/m

M_u per metre width of the beam = 122.256/0.25 = 489.02 kNm/m.

Chart 14 of Sp-16 gives the effective depth of the beam d = 472 mm and D = 475 + 50 = 525 mm assuming d = 475 mm.

A_{st} required = (0.72/100) (250) (475) = 855.0 mm^2

Thus, we have b = 250 mm, d = 475 mm, D = 525 mm and A_{st} = 855 mm^2

3.6.7 Solution by using Design Aids Tables (SP-16)

3.6.7.1 Effective depth *d*

Tables 1 to 4 of SP-16 present p_t for different values of M_u/bd^2 covering a wide range of f_y and f_{ck}. Table 2 is needed for this problem.

To have more confidence while employing this method, we are starting with the effective depth d as 400 mm as in the direct computational method. The total depth D is (400 + 50) mm = 450 mm. The breadth b of the beam is taken as 250 mm.

3.6.7.2 Factored load and bending moment

Dead load = 0.25 (0.45) (25) = 2.8125 kN/m

Imposed loads = 7.00 kN/m

Factored load F_d = 1.5 (2.8125 + 7.00) = 14.71875 kN/m

Factored bending moment M_u = 14.71875 (8) = 117.75 kNm

$$\frac{M_u}{b\,d^2} = \frac{117.75\,(10^6)}{250\,(400)\,(400)} = 2.94375$$

3.6.7.3 Use of Tables of SP-16

Table 2 of SP-16 shows that $\frac{M_u}{b\,d^2}$ is restricted up to 2.76 when p_t = 0.955, i.e. the limiting condition. So, increasing the effective depth by another 50 mm to have D = 500 m, the total factored moment as calculated in sec. 3.6.4.10 is 121.5 kNm,

Now, $$\frac{M_u}{b\,d^2} = \frac{121.5\,(10^6)}{250\,(450)\,(450)} = 2.4$$

From Table 2 of SP-16, the corresponding p_t becomes 0.798.

Therefore, A_{st} = 0.01 (0.798) (250) (450) = 897.75 mm^2

3.6.8 Comparison of Results of Three Methods

Results of this problem by three methods: (i) direct computation method, (ii) use of charts of SP-16 and (iii) use of tables of SP-16 are summarised for the purpose of comparison. The tabular summary includes the last two values of d and A_{st}. Other parameters (b, f_{ck} and f_y) are remaining constants in all the three methods.

Table 3.5 Comparison of d and A_{st} by three methods

Cycle	Direct computation method		Use of charts of SP-16		Use of tables of SP-16	
	d (mm)	A_{st} (mm^2)	d (mm)	A_{st} (mm^2)	d (mm)	A_{st} (mm^2)
1	410	966.5168	460	828	400	Not possible
2	450	895.84145 (2-20+2-14 = 935 mm^2)	475	855 (2-20 + 2-12 = 854 mm^2)	450	897.75 (2-20+2-14 = 935 mm^2)

3.6.9 Other Alternatives using Charts and Tables of SP-16

Any alternative solution of d will involve computations of factored loads F_d and bending moment M_u. Thereafter, Eq. 3.23 of Lesson 5 has to be solved to get the value of A_{st} by direct computation method. On the other hand, it is very simple to get the A_{st} with the help of either charts or tables of SP-16 from the value of factored bending moment. Some alternatives are given below in Table 3.6 by the use of tables of SP-16.

In sec. 3.6.7.3, it is observed that an effective depth of 400 mm is not acceptable. Hence, the effective depth is increased up to 450 mm at intervals of 10 mm and the corresponding A_{st} values are presented in Table 3.6. The width b is kept as 250 mm and M 20 and Fe 415 are used for all the alternatives.

Table 3.6 Alternative values of d, D, F_d, M_u, $\frac{M_u}{b\,d^2}$, p_t and A_{st}

Sl. No.	d (mm)	D (mm)	F_d (kN/m)	M_u (kNm)	$\frac{M_u}{b\,d^2}$ (N/mm^2)	p_t (%)	A_{st} (mm^2)
1	410	460	14.825	118.5	2.8197	Not acceptable	Not acceptable
2	420	470	14.9062	119.25	2.7041	0.93	976.5
3	430	480	15.0	120.00	2.596	0.88	946.0
4	440	490	15.09	120.75	2.4948	0.839	922.9
5	450	500	15.1875	121.5	2.4	0.798	897.75

3.6.10 Advantages of using SP-16

The following are the advantages:

(i) Alternative sets of b, d and A_{st} are obtained very quickly.
(ii) The results automatically exclude those possibilities where the steel reinforcement is inadmissible.

It has been mentioned that the reinforcement should be within 75 to 80 per cent of limiting reinforcement to ensure ductile failure. The values of charts and tables are given up to the limiting reinforcement. Hence, the designer should be careful to avoid the reinforcement up to the limiting amount. Moreover, these charts and tables can be used for the design of slabs also. Therefore, the values are also taking care of the minimum reinforcement of slabs. The minimum reinforcement of beams are higher than that of slabs. Accordingly, the designer should also satisfy the requirement of minimum reinforcement for beams while using SP-16.

It is further suggested to use the tables than the charts as the values of the charts may have personal error while reading from the charts. Tabular values have the advantage of numerical, which avoid personal error. Moreover, intermediate values can also be evaluated by linear interpolation.

3.6.11 Practice Questions and Problems with Answers

Q.1: Mention the necessary input data and unknowns to be determined for the two types of problems of singly reinforce beams.

A.1: (i) The input data for the design type of problems are layout plan, imposed loads, grades of steel and concrete. The unknowns to be determined are b, d, D, A_{st} and L_{eff}.

(ii) The input data for the analysis type of problem are b, d, D, A_{st}, L_{eff}, grades of concrete and steel. The unknowns to be determined are M_u and service imposed loads.

Q.2: State specific guidelines to select the initial dimensions/amount/grade of the following parameters before designing the reinforced concrete beams:

(i) b, (ii) d, (iii) D, (iv) A_{st}, (v) diameter of reinforcing bars, (vi) grade of concrete and (vii) grade of steel.

A.2: Sections 3.6.2.1 to 6 cover the answers.

Q.3: Name the three methods of solution of the design of reinforced concrete beam problems.

A.3: The three methods are: (i) Direct computation method, (ii) Use of charts of SP-16 and (iii) Use of tables of SP-16

Q.4: Determine the imposed loads and the tensile steel $A_{st,lim}$ of the singly reinforced rectangular beam shown in Figs. 3.6.2 and 4 of L = 8.0 m simply supported, thickness of brick wall = 300 mm, width b = 300 mm, effective depth d = 550 mm, total depth D = 600 mm, grade of concrete = M 20 and grade of steel = Fe 500. Use (i) direct computation method, (ii) design chart of SP-16 and (iii) design table of SP-16.

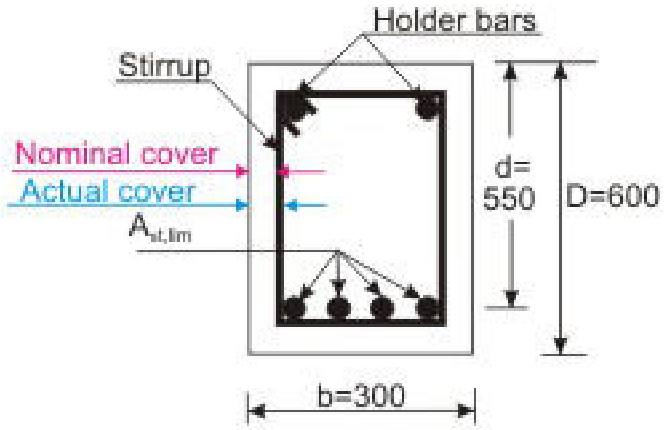

Fig. 3.6.4: Problem of Q 4

A.4: (i) Direct computation method:

The limiting moment of resistance $M_{u,lim}$ is obtained from Eq. 3.24 as follows

$$M_{u,\lim} \quad = \quad 0.36 \quad \frac{x_{u,\lim}}{d} \left\{1 - 0.42 \frac{x_{u,\lim}}{d}\right\} b\, d^2 \; f_{ck}$$

Here, $x_{u,lim}/d$ = 0.46 (cl. 38.1, Note of IS 456:2000)

Hence, $M_{u,lim}$ = 0.36 (0.46) {1 - 0.42 (0.46)} (300) (550) (500) (20) Nmm

= 22,04,50,00,000 Nmm

Tensile steel $A_{st,lim}$ is obtained from Eq. 3.23 as follows:

$$M_{u,lim} \quad = \quad 0.87\, f_y \; A_{st,\lim} \; d \left\{1 - \frac{A_{st,\lim} \; f_y}{f_{ck} \; b \; d}\right\}$$
(3.23)

Denoting the unknown $A_{st,lim}$ as A, we get:

A^2 - 6600 A + 6081379.31 = 0

Solving the above equation, the lower value of A is the $A_{st,lim}$ which is equal to 1107.14 mm^2

(ii) Use of chart of SP-16:

Using chart 17 of SP-16 for $M_{u,lim}/b$ = 220.45/0.3 = 734.833 kNm/m, we get the reinforcement percentage 100($A_{st,lim}$)/bd = 0.67.

So, $A_{st,lim}$ = 0.67 (300) (550)/100 = 1105 mm^2

(iii) Use of table of SP-16:

Table 2 of SP-16 for $M_{u,lim}/bd^2$ = 220.45/300 (0.55) (0.55) = 2.4292 N/mm^2, we get the reinforcement percentage by linear interpolation as:

0.669 + (0.007) (0.0092)/(0.02) = 0.67222.

Hence, $A_{st,lim}$ = 0.67222 (300) (550)/(100) = 1109.16 mm^2

Comparison of results:

Method	$A_{st,lim}$ (mm^2)
(i)	1107.14
(ii)	1105.00
(iii)	1109.16

Imposed loads:

The total load W per metre can be obtained from

$W = 8\,(M_{u,lim})/L_{eff}^2$

Where, L_{eff} is the lower of (i) 7700 + 550 or (ii) 8000 mm (cl. 22.2a of IS 456:2000)

Using L_{eff} = 8000 mm and $M_{u,lim}$ = 220.45 kNm

We get the total load W = 220.45/8 = 27.556 kN/m

The dead load of the beam = 0.3 (0.6) (25) = 4.5 kN/m

Hence, the imposed loads = 27.556 - 4.5 = 23.056 kN/m

3.6.12 References

1. Reinforced Concrete Limit State Design, 6th Edition, by Ashok K. Jain, Nem Chand & Bros, Roorkee, 2002.
2. Limit State Design of Reinforced Concrete, 2nd Edition, by P.C.Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2002.
3. Advanced Reinforced Concrete Design, by P.C.Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2001.
4. Reinforced Concrete Design, 2nd Edition, by S.Unnikrishna Pillai and Devdas Menon, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2003.
5. Limit State Design of Reinforced Concrete Structures, by P.Dayaratnam, Oxford & I.B.H. Publishing Company Pvt. Ltd., New Delhi, 2004.
6. Reinforced Concrete Design, 1st Revised Edition, by S.N.Sinha, Tata McGraw-Hill Publishing Company. New Delhi, 1990.
7. Reinforced Concrete, 6th Edition, by S.K.Mallick and A.P.Gupta, Oxford & IBH Publishing Co. Pvt. Ltd. New Delhi, 1996.
8. Behaviour, Analysis & Design of Reinforced Concrete Structural Elements, by I.C.Syal and R.K.Ummat, A.H.Wheeler & Co. Ltd., Allahabad, 1989.
9. Reinforced Concrete Structures, 3rd Edition, by I.C.Syal and A.K.Goel, A.H.Wheeler & Co. Ltd., Allahabad, 1992.
10. Textbook of R.C.C, by G.S.Birdie and J.S.Birdie, Wiley Eastern Limited, New Delhi, 1993.
11. Design of Concrete Structures, 13th Edition, by Arthur H. Nilson, David Darwin and Charles W. Dolan, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2004.
12. Concrete Technology, by A.M.Neville and J.J.Brooks, ELBS with Longman, 1994.
13. Properties of Concrete, 4th Edition, 1st Indian reprint, by A.M.Neville, Longman, 2000.
14. Reinforced Concrete Designer's Handbook, 10th Edition, by C.E.Reynolds and J.C.Steedman, E & FN SPON, London, 1997.
15. Indian Standard Plain and Reinforced Concrete – Code of Practice (4th Revision), IS 456: 2000, BIS, New Delhi.
16. Design Aids for Reinforced Concrete to IS: 456 – 1978, BIS, New Delhi.

3.6.13 Test 6 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1: State specific guidelines to select the initial dimensions/amount/grade of the following parameters before designing the reinforced concrete beams:

(i) b, (ii) d, (iii) D, (iv) A_{st}, (v) diameter of reinforcing bars, (vi) grade of concrete and (vii) grade of steel. (6 x 5 = 30 marks)

A.TQ.1: See secs. 3.6.2.1 to 6.

TQ.2: State the advantages of using SP-16 than employing direct computation method in the design of a beam. (15 marks)

A.TQ.2: See sec. 3.6.10 (except the last para).

TQ.3: Why the use of tables of SP-16 is better than the use of chart ? (5 marks)

A.TQ.3: See sec. 3.6.10 (last para only).

3.6.14 Summary of this Lesson

Explaining the two types of problems and giving the necessary guidelines of the preliminary selection of the parameters, this lesson illustrates step by step method of solving design type of problems employing (i) direct computation method, (ii) use of charts of SP-16 and (iii) use of tables of SP-16. The results of a specific problem are compared. The advantages of using SP-16 in general and the superiority of using the tables of SP-16 to the charts are also discussed.

<u>Design of Doubly Reinforced Rectangular Beam – Limit State Method</u>

If the applied bending moment is greater than the moment resisting capacity or ultimate moment carrying capacity of the section (i.e. over reinforced section), then there can be **three** alternatives:

1) If possible, increase the dimensions of the section, preferable ‘depth’.
2) Use higher grade of concrete
3) Steel reinforcement may be added in compression zone to increase the moment resisting capacity of the section (depth is remain as it is). This is known as a **<u>‘Doubly Reinforced Beam’</u>.**

The Double Reinforced Section is normally required under the following circumstances:

1) sectional dimensions are restricted due to requirements of head room, appearance etc. and the strength of given singly reinforced section is inadequate.
2) the beam which acts as a flanged beam at mid span becomes a rectangular beam at supports of a continuous beam. At support tension occurs at top making the flange ineffective and there for the section becomes inadequate to resist large peak value of support moment.
4) basement with lower plinth level and combined with ventilator couples and to design a double reinforced beam.
5) Compression steel is provided sometimes to reduce the deflection i.e. to increase the stiffness and also to increase the rotation capacity.
6) Compression steel is always used in structures in earthquake regions to increase their ductility.
7) Compression reinforcement will also aid significantly in reducing the long term deflection of beams.

The failure theory evolved for singly reinforced section holds good for doubly reinforced beam also. The main **assumptions** are:

1) plane sections remain plain even after bending
2) R.C. sections in bending fail when the compressive strain in the concrete reaches the value of 0.0035
3) The stress at any point in steel and concrete can be taken as equal to the stress corresponding to the strain at that point of the stress v/s strain graph for the material (steel or concrete)
4) ‘Xu lim / d’ ratio need not be strictly adhered to in double reinforced beams
5) Even though shrinkage, creep and other properties of concrete wil affect the actual stat of stress of steel and concrete, these are not taken into consideration in estimating the collapse or ultimate strength

A doubly reinforced section, may be looked upon as made up of two sections 1 & 2 given below:

Section 1: a singly reinforced section with concrete resisting compressive force C_{U1} balanced by tensile force T_{U1} provided by tension steel A_{st1}. This section is assumed to resist part moment M_{U1} (= Mu, lim) out of total moment M_U.

Section 2: an imaginary section (shown dotted) consisting of compression steel providing additional compression force C_{U2}, which is balanced by tensile force T_{U2} given by tension steel Ast_2. This section is assumed to resist balancing moment $M_{U2} = M_U - M_{U1}$.

Where,

C_{U1} = compression provided by concrete in section 1

C_{U2} = compression provided by compression steel in section 2 making due allowance for loss of compression due to replacement of concrete area by steel area

T_{U1} = tension provided by tension steel A_{st1} in section 1 to balance C_{U1}

T_{U2} = tension provided by tension steel A_{st2} in section 2 to balance C_{U2}

A_{st2} = area of additional tensile reinforcement in mm^2

A_{SC} = area of compression reinforcement in mm^2

fsc = stress in compression reinforcement in MPa

fcc = compressive stress in concrete at the level of compression steel in MPa

According to IS:456-2000, Annex G 1.2 p-96,

A double reinforced beam subjected to a moment M_U can be expressed as a rectangular section with tension reinforcement $A_{st,lim}$ reinforced for balanced condition giving M.R., M_{Ulim} plus an auxiliary section reinforced with compression reinforcement A_{SC} and tensile reinforcement Ast_2 giving a M.R. M_{U2} such that, **$M_U = M_{U\,lim} + M_{U2}$**

For the moment Mu,lim the tension steel Ast,lim is found out as explained for singly reinforced beams.

Considering Tension Steel ⇨ $M_{U2} = T_{U2}$ x lever arm = $Ast_2 * 0.87 * fy * (d – d')$

Considering Compression Steel ⇨ $M_{U2} = C_{U2}$ x lever arm = $Asc * (fsc – fcc) * (d – d')$

Now, additional tensile force = additional compressive force
So, $0.87 * fy * (d – d') = Asc * (fsc – fcc)$

So, $Ast_2 = \dfrac{Asc*(fsc \quad fcc)}{0.87*fy}$

The value of fcc = 0.45 * fck is much less compare to the value of fsc, so we are ignoring the value of fcc. So, $Ast_2 = \dfrac{Asc*fsc}{0.87*fy}$

So, Total Tensile Reinforcement = Ast = Ast lim + Ast_2

For finding the value of Xu, take Total Compression = Total Tension
So, $C_{U1} + C_{U2} = T_{U1} + T_{U2}$
$0.36 * fck * b * Xu + Asc * fsc = 0.87 * fy * Ast$

As per IS:456-2000, p-69, find Xu,max, compare the Xu & Xu max & state the type of beam.

Fe	$\frac{Xu,max}{d}$
250	0.53
415	0.48
500	0.46

Note : If the section is over reinforced, take Xu = Xu,max, do not increase the type of beam.

Now, $M_U = M_{U\,lim} + M_{U2}$
So, **$M_U = 0.36 * fck * b * X_U * (d – 0.42 * X_U) + Asc * fsc* (d – d')$**
Ast lim = Mu lim / 0.87 * fy * (d – 0.42 * Xu)
Asc = M_{U2} / fsc (d - d')
Ast_2 = Asc * fsc / 0.87 * fy
Ast = Ast lim + Ast_2

Sr. No.	Types of Problems	Data Given	Data Determine
1	Calculate the ultimate moment of resistance	Grade of Concrete & Steel, Size of beam & Reinforcement provided	If Xu = Xu max $\Rightarrow$ Balanced If Xu < Xu max $\Rightarrow$ Under Reinforced If Xu > Xu max $\Rightarrow$ Over Reinforced Equating the, Total Compressive Force = Total Tensile Force **0.36 * fck * b * Xu + Asc * fsc = 0.87 * fy * Ast** From this equation find Xu. As per IS:456-2000, p-69 Fe $\frac{Xu, max}{d}$ 250 0.53 415 0.48 500 0.46 **Mu = 0.36 * fck * b * Xu * (d – 0.42 * Xu) + Asc * fsc * (d – d')** This equation is valid for Balanced & Under Reinforced Section. If the section is Over Reinforced, then take Xu = Xu max, in above equation & use the same equation for finding Mu. Do not increase the depth of the beam, for over reinforced section.

Sr. No.	Types of Problems	Data Given	Data Determine
2	Design the beam. Find out the Reinforcement Ast	Grade of Concrete & Steel, Size of beam, ultimate moment	Mu is given to us. For a <u>balanced design</u>, Xu = Xu,max **Mu lim = 0.36 * fck * b * Xu max * (d – 0.42 * Xu max)** So, Mu_2 = Mu – Mu lim Astlim=Mulim / 0.87*fy*(d – 0.42*Xumax) Asc = M_{U2} / fsc (d - d') Ast_2 = Asc * fsc / 0.87 * fy Ast = Ast lim + Ast_2

Where,
d = effective depth of beam in mm = distance between the extreme compression fibre to the centroid of the tensile reinforcement
d' = distance between the extreme compression fibre to the cenroid of the compression reinforcement in mm
b = width of beam in mm
Xu = depth of actual neutral axis in mm from extreme compression fibre
Xu,max = depth of critical neutral axis in mm from extreme compression fibre
Ast = total area of tensile reinforcement in mm^2
fck = characteristic compressive strength of concrete in Mpa
fy = characteristic strength of steel in MPa
Mu,lim = limiting M.R. of a section without compression reinforcement in kN.m
Mu = ultimate M.R. of a section in kN.m
Asc = area of compression reinforcement in mm^2