**UNIT-3**
**UNIT HYDROGRAPH ANALYSIS**

Sherman (1932) first proposed the unit hydrograph concept. The Unit Hydrograph (UH) of a watershed is defined as the direct runoff hydrograph resulting from a unit volume of excess rainfall of constant intensity and uniformly distributed over the drainage area. The duration of the unit volume of excess or effective rainfall, sometimes referred to as the *effective duration*, defines and labels the particular unit hydrograph. The unit volume is usually considered to be associated with 1 cm (1 inch) of effective rainfall distributed uniformly over the basin area.

The fundamental assumptions implicit in the use of unit hydrographs for modeling hydrologic systems are:

a) Watersheds respond as linear systems. On the one hand, this implies that the proportionality principle applies so that effective rainfall intensities (volumes) of different magnitude produce watershed responses that are scaled accordingly. On the other hand, it implies that the superposition principle applies so that responses of several different storms can be superimposed to obtain the composite response of the catchment.

b) The effective rainfall intensity is uniformly distributed over the entire river basin.

c) The rainfall excess is of constant intensity throughout the rainfall duration.

d) The duration of the direct runoff hydrograph, that is, its time base, is independent of the effective rainfall intensity and depends only on the effective rainfall duration

When the effective rainfall is given as a hyetograph, that is, as a sequence of *M* rainfall pulses of the same duration $\Delta t$, the corresponding direct runoff hydrograph can be expressed as the discrete convolution of the rainfall hyetograph and the Unit Hydrograph as,

$$Q_n = \sum_{m=1}^{m^*} P_m U_{n-m+1} \quad , \quad m^* = min\ (n, M) \tag{1a}$$

$$Q_n = Q(n\Delta t), \qquad n = 1, \cdots, N \tag{1b}$$

$$P_m = \int_{(m-1)\Delta t}^{m t} I_e(\tau)\,d\tau, \qquad M = 1,\ \cdots, M \tag{1c}$$

Where $P_m$ is the volume of the $m^{th}$ effective rainfall pulse, $Q_n$ is the direct runoff, and $U_{n-m+1}$ are the Unit Hydrograph ordinates.

Although the above assumptions lead to acceptable results, watersheds are indeed nonlinear systems. For example, unit hydrographs derived from different rainfall-runoff events, under the assumption of linearity, are usually different, thereby invalidating the linearity assumption.

The determination of unit hydrographs for particular basins can be carried out either using the theoretical developments of linear system theory; or using empirical techniques. For either case, simultaneous observations of both precipitation and stream flow must be available. These two approaches are presented in more detail in later sections.

### Hydrograph Components

Total stream flow during a precipitation event includes the base flow existing in the basin prior to the storm and the runoff due to the given storm precipitation. Total stream flow hydrographs are usually conceptualized as being composed of:

a) Direct Runoff, which is composed of contributions from *surface runoff* and *quick interflow*. Unit hydrograph analysis refers only to direct runoff.

b) Base flow, which is composed of contributions from *delayed interflow* and *Groundwater runoff*.

*Surface runoff* includes all overland flow as well as all precipitation falling directly onto stream channels. Surface runoff is the main contributor to the peak discharge.

*Interflow* is the portion of the stream flow contributed by infiltrated water that moves laterally in the subsurface until it reaches a channel. Interflow is a slower process

than surface runoff. Components of interflow are *quick interflow*, which contributes to direct runoff, and *delayed interflow*, which contributes to baseflow (*e.g.*, Chow, 1964.)

*Groundwater runoff* is the flow component contributed to the channel by groundwater. This process is extremely slow as compared to surface runoff.

Schematically in Figure 11.1, the stream flow hydrograph is subdivided into a) *Rising Limb:* rising portion of the hydrograph, composed mostly of surface runoff. b) *Crest*: zone of the hydrograph around peak discharge. c) *Falling (or Recession) Limb*: Portion of the hydrograph after the peak discharge, composed mostly of water released from storage in the basin. The lower part of this recession corresponds to groundwater flow contributions.

The main factors affecting hydrograph shape are:

1) *Drainage characteristics*: basin area, basin shape, basin slope, soil type and land use, drainage density, and drainage network topology. Most changes in land use tend to increase the amount of runoff for a given storm (*e.g.*, Chow et al., 1988; Singh, 1989; Bras, .1990).

2) *Rainfall characteristics*: rainfall intensity, duration, and their spatial and temporal distribution; and storm motion, as storms moving in the general downstream direction tend to produce larger peak flows than storms moving upstream (*e.g.*, Chow et al., 1988; Singh, 1989; Bras, .1990).

Hydrographs are also described in terms of the following time characteristics (see Figure 11.1):

*Time to Peak*, $t_p$: Time from the beginning of the rising limb to the occurrence of the peak discharge.

The time to peak is largely determined by drainage characteristics such as drainage density, slope, channel roughness, and soil infiltration characteristics. Rainfall distribution in space also affects the time to peak.

*Time of Concentration*, $t_c$: Time required for water to travel from the most hydraulically remote point in the basin to the basin outlet. For rainfall events of very long duration, the time of concentration is associated with the time required for the system to achieve the maximum or equilibrium discharge. Kibler (1982) and Chow et al. (1988) summarize several of many empirical and physically-based equations for $t_c$ that have been developed.

The drainage characteristics of length and slope, together with the hydraulic characteristics of the flow paths, determine the time of concentration.

*Lag Time*, $t_l$: Time between the center of mass of the effective rainfall hyetograph and the center of mass of the direct runoff hydrograph.

The basin lag is an important concept in linear modeling of basin response. The lag time is a parameter that appears often in theoretical and conceptual models of basin behavior. However, it is sometimes difficult to measure in real world situations. Many empirical equations have been proposed in the literature. The simplest of these equations computes the basin lag as a power function of the basin area.

*Time Base*, $t_b$: Duration of the direct runoff hydrograph.

### Base flow separation.

As the Unit Hydrograph concept applies only to direct runoff, the direct runoff must be separated from the base flow. *Base flow separation* or hydrograph analysis is the process of separating the direct runoff (surface runoff and quick interflow) from the base flow. This separation is somewhat arbitrary, but corresponds to theoretical concepts of basin response.

*Subjective methods.* Several subjective methods are shown in Figure 11.1. The simplest one consists in arbitrarily selecting the discharge marking the beginning of the rising limb as the value of the base flow and assuming that this base flow discharge remains constant throughout the storm duration. A second method consists in arbitrarily selecting the beginning of the groundwater recession on the falling limb of the hydrograph (usually assumed to occur at a theoretical inflection point) and connecting this point by a straight line to the beginning of the rising limb. A third example of subjective methods consists in extending the recession prior to the storm by a line from the beginning of the rising limb to a point directly beneath the peak discharge and then connecting this point to the beginning of the groundwater recession on the falling limb.

*Area method.* The area method of base flow separation consists in determining the beginning of the base flow on the falling limb with the following empirical equation,

$$N = bA^{0.2} \tag{2}$$

relating the time in days from the peak discharge, *N*, to the basin area, *A*. When *A* is in square miles, *b* equals 1. When *A* is in square kilometers, *b* equals 0.8. This equation is unsuitable for smaller watersheds and should be checked for a number of hydrographs before using.

*The master recession curve method.* This method consists in modeling the response of the groundwater aquifer as a linear reservoir of parameter *k*. This assumption leads to the following equation for the groundwater recession hydrograph,

$$Q(t) = Q(t_o)e^{-(t-t_o)/k} \tag{3}$$

where $Q(t)$ is the baseflow at time *t*; $Q(t_o)$ is a reference baseflow discharge at time $t_o$, and *k* is the recession constant for baseflow. This method is based on a linear reservoir model of unforced basin response (that is, response from storage) and it can be used to separate

the contributions to the recession flow from surface storage, subsurface storage, and groundwater aquifer storage. It involves the determination of several recession constants.
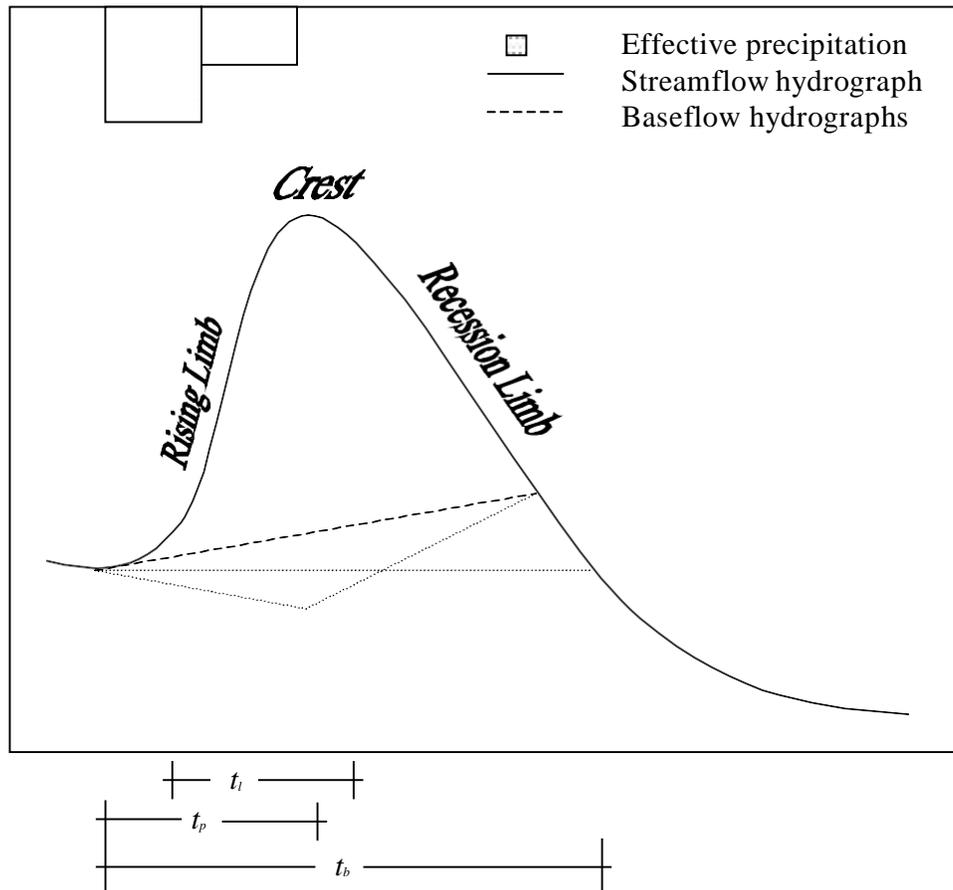


Figure 11.1: Schematic Description of Hydrograph

**UNIT HYDROGRAPHS: EMPIRICAL DERIVATION**

The following are essential steps in deriving a unit hydrograph from a single storm:

1) Separate the baseflow and obtain the direct runoff hydrograph (DRH).

2)  Compute the total volume of direct runoff and convert this volume into equivalent depth of effective rainfall (in centimeters or in inches) over the entire basin.

3)  Normalize the direct runoff hydrograph by dividing each ordinate by the equivalent volume (in or cm) of direct runoff (or effective rainfall).

4) Determine the effective duration of excess rainfall. To do this, obtain the effective rainfall hyetograph (*e.g.*, use the ϕ-index, the Horton, Green and Ampt, or Philip equations, or some other method to determine infiltration losses) and its associated duration. This duration is the duration associated with the unit hydrograph.

Unit hydrographs are fundamentally linked to the duration of the effective rainfall event producing them. They can only be used to predict direct runoff from storms of the same duration as that associated with the UH, or from storms which can be described as a sequence of pulses, each of the same duration as that associated with the UH (see Equation 1).

### Unit Hydrographs for Different Effective Duration

A unit hydrograph for a particular watershed is developed for a specific duration of effective rainfall. When dealing with a rainfall of different duration a new unit hydrograph must be derived for the new duration. The linearity property implicit in the UH analysis can be used to generate UH's associated with larger or smaller effective rainfall pulse duration. This procedure is sometimes referred to as the S-curve Hydrograph method.

### S-Curve Hydrograph Method

An S-hydrograph represents the response of the basin to an effective rainfall event of infinite duration. Assume that an UH of duration D is known and that an UH for the same basin but of duration D' is desired. The first step is to determine the S-curve hydrograph by adding a series of (known) UH's of duration D, each lagged by a time interval D. The resulting superposition represents the runoff resulting from a continuous rainfall excess of intensity 1/D.

Lagging the S-curve in time by an amount D' and subtracting its ordinates from the original unmodified S-curve yields a hydrograph corresponding to a rainfall event of intensity 1/D and of duration D'. Consequently, to convert this hydrograph whose volume is D'/D into a unit hydrograph of duration D', its ordinates must be normalized by multiplying them by D/D'. The resulting ordinates represent a unit hydrograph associated with an effective rainfall of duration D'.

### UNIT HYDROGRAPHS: LINEAR SYSTEM THEORY

A hydrologic system (a basin) is said to be a linear system if the relationship between storage, inflow, and outflow is such that it leads to a linear differential equation. The hydrologic response of such systems can be expressed in terms of an impulse response function (IRF) through a so-called Convolution Equation. Linear systems possess the properties of additivity and proportionality, which are implicit in the convolution equation. Linear reservoirs, for example, are special cases of a general hydrologic system model in which the storage is linearly related to the output by a constant *k*.

### Impulse Response Function

The IRF of a linear system represents the response of the system to an instantaneous impulse (excitation) of unit volume applied at the origin in time (*t=0*). The response of continuous linear systems can be expressed, in the time domain, in terms of the *impulse* response function via the convolution integral as follows,

$$Q(t) = \int_0^t I_e(\tau)u(t-\tau)d\tau \qquad (4)$$

where *u(t)* is the impulse response function of the system.

In hydrology, it has been customary to assume that watersheds behave as linear systems. When dealing with hydrologic systems, *u(t)* represents the instantaneous unit hydrograph (IUH), and *Q(t)* and *I_e(t)* represent direct runoff and excess or effective precipitation, respectively. Thus, an Instantaneous Unit Hydrograph represents the response of a watershed (discharge at its outlet as a function of time) to a unit volume of precipitation uniformly distributed over the basin and occurring instantaneously at time *t = 0*.

### Unit Step Response Function

The unit step response function (SRF) is the theoretical counterpart to the S-curve hydrograph concept presented earlier in the empirical UH analysis section. It represents the runoff hydrograph from a continuous effective rainfall of unit intensity. As can be seen from its definition, it is the convolution of *1* and *u(t)*, and obtained as,

$$g(t) = \int_0^t u(t)dt \qquad (5)$$

### Unit Pulse Response Function

The unit pulse response function (PRF) is the theoretical counterpart to the UH concept presented earlier. It represents the runoff hydrograph from a constant effective rainfall of intensity *1/Δt* and of duration Δ*t*.

$$h(t) = \frac{1}{\Delta t}[g(t) - g(t-\Delta t)] = \frac{1}{\Delta t}\int_{t-\Delta t}^t u(\tau)d\tau \qquad (6)$$

From its definition, the PRF can be seen as the normalized difference between two lagged SRF's (S-curve hydrographs), lagged by an amount Δ*t*. This is analogous to the procedure presented earlier in the section on S-hydrograph analysis.

### Discrete Convolution Equation

When the effective rainfall is given as a hyetograph, that is, as a sequence of rainfall pulses of the same duration Δ*t*, the corresponding direct runoff hydrograph can be expressed as the discrete convolution of the rainfall hyetograph and a Unit Hydrograph,

$$Q_n = \sum_{m=1}^{m^*} P_m U_{n-m+1} \quad , \quad m^* = min(n, M) \tag{1a}$$

$$Q_n = Q(n\Delta t), \qquad n = 1, \cdots, N \tag{1b}$$

$$P_m = \int_{(m-1)\Delta t}^{m t} I_e(\tau)d\tau, \qquad m = 1, \cdots, M \tag{1c}$$

where $P_m$ is the volume of the $m^{th}$ effective rainfall pulse and the Unit Hydrograph ordinates are given by,

$$U_{n-m+1} = h[(n - m + 1)\Delta t] = \frac{1}{\Delta t} \int_{(n-m)\Delta t}^{(n-m+1)\Delta t} u(\tau)d\tau \tag{7}$$

The UH ordinates correspond to the area under the IUH between two consecutive time intervals.

The discrete convolution can be expressed alternatively as a matrix equation as,

$$[P][U] = [Q] \tag{8a}$$

$$\begin{bmatrix} P_1 & & & \\ P_2 P_1 & & & \\ & P_2 \cdots & & \\ & & & \\ P_M & \cdots & P_1 & \\ & P_M & P_2 & \\ & \cdots & & \\ & & P_M \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_{N-M+1} \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \\ \\ Q_N \end{bmatrix} \tag{8b}$$

where *M* is the number of effective rainfall pulses; *N-M+1* is the number of UH ordinates; and *N* is the number of direct runoff ordinates. Matrix $\boldsymbol{P}$ is of dimensions (*N x (N-M+1)*). Recasting the discrete convolution equation in this manner allows for an objective (and optimal) mathematical derivation of the UH ordinates. The estimation of the Unit Hydrograph from simultaneous observations of effective precipitation ($P_m$) and direct runoff ($Q_n$) can be seen as a linear static estimation problem for which a method like least-squares, linear programming, or others can be used.

The least-squares approach tries to obtain a set of UH ordinates that minimizes the sum of squares of the errors, and leads to the following solution for the UH,

$$[U] = [[P]^T [P]]^{-1} [P]^T [Q] \qquad (9)$$

9

where $[[P]^T[P]]^{-1}$ is the inverse of matrix $[[P]^T[P]]$, and the superscript $T$ stands for transpose. This approach is adequate when the data (precipitation and discharge) are relatively error free or the errors can be expected to be small. When the data can be expected to contain large errors a different approach may be more adequate. Such approach would minimize the sum of the absolute value of the errors. Linear programming would then be the appropriate solution procedure.

## CONCEPTUAL (SYNTHETIC) UNIT HYDROGRAPHS

As indicated earlier, sets of concurrent observations of effective rainfall and direct runoff are required for the derivation of unit hydrographs. Thus, the resultant UH is specific to the particular watershed defined by the point on the stream where the direct runoff observations were made. When no direct observations are available, or when UH's for other locations on the stream in the same watershed or for nearby watersheds of similar characteristics are required, Synthetic Unit Hydrograph procedures must be used.

Synthetic Unit Hydrograph procedures can be categorized as (*e.g.*, Chow et al., 1988): 1) those based on models of watershed storage (*e.g.*, Nash, 1957, 1958, 1959; Dooge, 1959; etc.); 2) those relating hydrograph characteristics (time to peak, peak flow, etc.) to watershed characteristics (*e.g.*, Snyder, 1938; Geomorphologic Instantaneous Unit Hydrograph); and 3) those based on a dimensionless unit hydrograph (*e.g.*, Soil Conservation Service, 1972).

### Conceptual UH's Based on Models of Watershed Storage

*The Nash and Dooge Models.*

*Linear Reservoirs* - A linear reservoir is characterized by a linear relationship between the storage and the output as,

$$S(t) = kQ(t) \qquad (10)$$

The impulse response function of such linear reservoir is,

$$u(t) = \frac{1}{k} e^{-t/k} \qquad (11)$$

Nash (1957, 1958, 1959, 1960) proposed a cascade of $n$ equal linear reservoirs as a model on which to base the derivation of IUH's for natural watersheds. The Nash model is one of the most widely used models in applied hydrology. Using the convolution equation (eq. 4) and the impulse response function for a single linear reservoir (eq. 11), the IUH corresponding to the Nash Model can be easily obtained as follows,

$$u_n(t) = \frac{1}{k\Gamma(n)} \left(\frac{t}{k}\right)^{n-1} e^{-t/k} \qquad (12)$$

10

This equation is a two parameter gamma distribution function, where *n* is the shape parameter, and *k* is the scale parameter. When rainfall and runoff data are available, the shape parameter can be estimated as the inverse of the second non-dimensional moment of the IUH about the centroid (*i.e.*, the inverse of the square of the coefficient of variation); and the scale parameter can be estimated as the ratio of the first order moment of the IUH about the origin to *n*. A general form for the peak discharge, $Q_{max}$, due to a finite pulse of rainfall excess with duration $t_r$ and uniform intensity $I_e$ was suggested by Adom et al. (1989) in the form of a Weibull cumulative distribution function (*CDF*), *i.e.*

$$Q_{max} = AI_e\left[1 - e^{-\alpha\left(\frac{t_r}{t_l}\right)^{\beta}}\right] \qquad (13)$$

where *A* is the contributing area, and $t_l$ is the basin lag-time, which is given by $t_l = nk$. The values of *a* and *b*, which are parameters depending on *n*, *i.e.* the number of reservoirs in the cascade, are given in Table 1 (Ramírez, et al., 1994).

TABLE 1. Parameter values for the peak discharge equation of the linear reservoir cascade model

| *n* | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| *a* | 1 | 1.017 | 1.175 | 1.348 |
| *b* | 1 | 1.220 | 1.313 | 1.370 |

*Linear Channels* - A linear channel is characterized by continuity and momentum equations given by,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad (14a)$$

$$A = C(x)Q \qquad (14b)$$

For conditions of no lateral inflow, the unit impulse response function of such linear channel is,

$$u(t) = \delta(t - T) \qquad (15)$$

which implies that the downstream outflow is equal to the upstream inflow delayed by a lag-time *T* representing the travel time through the system.

The cascade of linear reservoirs model can be modified in order to account for

translation by using combinations of linear channels and linear reservoirs. Dooge (1959) developed a general unit hydrograph theory under the assumption that basin response can

be represented by a cascade of linear channels and linear reservoirs in series. For this model the drainage area of the watershed is divided into sub areas using the isochrones. A linear channel in series with a linear reservoir represents each sub area. If this is done assuming a system of *n* equal linear reservoirs of parameter *k* and *n* equal linear channels of lag-time *T*, the IUH or impulse response function is,

$$u_n(t) = \frac{1}{k\Gamma(n)} \left(\frac{t-nT}{k}\right)^{n-1} e^{-(t-nT)/k} \qquad (16)$$

As a general rule, linear channels and linear reservoirs can be combined in series and/or in parallel to model complex hydrologic systems.

### Conceptual UH's Based on Relationships Between Hydrologic Response and Watershed Characteristics

Flow hydrographs travel through a stream network after runoff is generated on the hillslopes. The drainage network develops so that a certain load of water and sediment (as a result of erosive processes) can be evacuated in an optimal manner (least energy expenditure (*e.g.*, Molnár and Ramírez, 1998a, 1998b)). Therefore, the characteristics of the outflow hydrograph and the characteristics of the hillslopes and drainage network must be intimately linked as they shape and modify each other.

*Geomorphologic Unit Hydrographs*. Geomorphologic parameters have been widely used to characterize and model hydrologic response. In particular, geomorphologic parameters have been used in synthesizing unit hydrographs for gauged and ungauged watersheds; as well as for estimating flood potential indices and flood frequency distributions. Recent efforts have concentrated on establishing a theoretical link between hydrologic response on the one hand, and climatic and geomorphologic characteristics on the other. Even though climate and geologic processes are continually changing and affecting hydrologic response, at the time scales of interest, climate and geomorphology may be considered constant boundary conditions, which uniquely define watershed behavior. The theoretical link was developed in the late seventies and early eighties and was based on the ideas of the topologically random model for drainage networks. The fundamental postulates of the random topological model can be summarized as follows (e.g., Shreve, 1967; Gupta and Waymire, 1983; Bras, 1990):

a) in the absence of natural controls, natural drainage networks are topologically random; and,

b) interior and exterior link lengths, as well as their associated areas, are independent random variables.

*The GIUH*. Rodríguez-Iturbe and Valdés (1979) linked the hydrologic response of watersheds in terms of the IUH to the their geomorphologic parameters, and to a dynamic parameter. The IUH is interpreted as the probability density function of the travel times to the basin outlet of water droplets randomly and uniformly distributed over the watershed. The resulting IUH or impulse response function for the basin is a function of the

probability density function of the travel times of droplets in streams of a given order and of the transition probabilities of water droplets going from a given state (stream segment) to another of higher order. In this model, streams of a given order define the states of the system. The travel times in the streams are assumed to be exponentially distributed and independent of one another. The initial probabilities of a drop landing anywhere on the basin, as well as the transition probabilities which are required in order to define the probability function of a particular path that a drop may take to the outlet, can be defined as functions of only geomorphologic and geometric parameters. The initial probability of a drop falling in an area of order *i* is equal to the percent contributing area for the given order. The initial probabilities are thus related to the average ratio of the average area of sub-basins of a given order to the average area of sub-basins of the next higher order (*i.e.*, the area ratio, $R_A$). Similarly, the transition probabilities from state *i* (stream order) to state *j*, where *j* represents a stream of higher order, are functions of the number of streams of order *i* draining into streams of order *j*, divided by the total number of streams of order *i*. Thus, the transition probabilities are related to the average ratio of the number of streams of a given order to the number of streams of the next order (*i.e.*, bifurcation ratio, $R_B$). These ratios are the parameters of the so-called Horton Laws summarized below.

The Horton law of stream numbers states that there exists a geometric relationship between the number of streams of a given order $N_\omega$ and the corresponding order, $\omega$. The parameter of this geometric relationship is the Bifurcation Ratio, $R_B$. This ratio has been found in nature to be between 3 and 5.

$$N_\omega = R_B^{\Omega - \omega} \tag{17}$$

The Horton law of stream lengths states that there exists a geometric relationship between the average length of streams of a given order and the corresponding order, $\omega$. The parameter of this relationship is the so-called Length Ratio, $R_L$.

$$\bar{L}_\omega = \bar{L}_1 R_L^{\omega - 1} \tag{18}$$

The Horton law of stream areas states that there exists a geometric relationship between the average area drained by streams of a given order and the corresponding order $\omega$. The parameter of this relationship is the so-called Area Ratio, $R_A$. This ratio has been observed in nature to vary within a narrow range of 3 to 5.

$$\bar{A}_\omega = \bar{A}_1 R_A^{\omega - 1} \tag{19}$$

In the equations above, $\Omega$ is the order of the basin, and the over-bar indicates the average value of the corresponding variable.

Rodríguez-Iturbe and Valdés (1979) suggested that the probability distribution function of travel time in streams of a given order is an exponential distribution with parameter $\lambda$ representing the inverse of the mean travel time. They suggest estimating $\lambda$ as the ratio of a characteristic velocity, *V*, and a characteristic length scale given by the mean length of streams of the given order. Valdés et al. (1979) suggest using the peak velocity

for the characteristic velocity. Rodríguez-Iturbe and Valdés (1979) obtain expressions for the time to peak, $t_p$, and peak discharge, $q_p$, of the IUH which are functions of geomorphology, and of the dynamic parameter represented by the peak velocity. Their results, obtained through numerical integration of the full GIUH, and multiple regression analysis, are simple, elegant, and easy to use. They are,

$$q_P = \frac{1.31}{L_\Omega} R_L^{0.43} V \tag{20}$$

and,

$$t_p = \frac{0.44 L_\Omega}{V} \left(\frac{R_B}{R_A}\right)^{0.55} R_L^{-0.38} \tag{21}$$

where $L_\Omega$, is the length of the highest order stream in kilometers, $V$ is in meters per second, and the units of $q_p$ and $t_p$ are the customary units of inverse time in hours, and time in hours, respectively.

The dependence of the GIUH on the dynamic parameter $V$ can be used to address non-linear watershed response issues. Non-linear effects in the basin response manifest themselves in the characteristic discharge velocity. The theoretical framework of the GIUH above was verified through numerical experiments with a detailed physically-based watershed model on several basins in Venezuela and Puerto Rico with excellent results (Valdés et al., 1979). They indicate that the nonlinear characteristics of the response function of a basin can be modeled with a linear scheme such as the GIUH but with a characteristic velocity that is representative of the discharge velocity for the given event. Rodríguez-Iturbe et al. (1979) show that the dynamic parameter of the GIUH can be taken as the space-time average flow velocity for a given rainfall-runoff event in a basin. This is based on the hypothesis of spatial uniformity of the flow velocity distribution throughout the river network (Pilgrim, 1976 and 1977). However, Agnese et al. (1988) showed that the estimation of the dynamic parameter of the GIUH could also be performed for those basins where flow velocity distribution depends on stream order.

The parameters of the Nash model and the geomorphologic descriptors of drainage networks can be related by relating approximate measures of volume of the IUH for a cascade of linear reservoirs and the volume of the GIUH (Rosso, 1984). Rosso equated the product of the time to peak and peak flow of the GIUH with the product of the time to peak and peak flow of the Nash model IUH. It is simple to show that for the GIUH this product equals,

$$(t_p q_p)_{GIUH} = 0.58 \left(\frac{R_A}{R_B}\right)^{0.55} R_L^{0.05} \tag{22}$$

and for the Nash model IUH, this product equals,

$$(t_p q_{Nash})_h = (\alpha - 1)^\alpha e^{(1-\alpha)} / \Gamma(\alpha) \tag{23}$$

These equations allow estimation of the shape parameter. Using numerical algorithms to solve the resulting equation Rosso obtained,

$$\alpha = 3.29 (\frac{R_B}{R_A})^{0.78} R_L^{0.07} \tag{24}$$

$$k = 0.70 [R_A / (R_B R_L)]^{0.48} V^{-1} L_\Omega \tag{25}$$

where $\alpha$ and $k$ are the shape parameter and the scale parameter of the Nash model IUH, respectively. When the parameters of the Nash model are estimated using these equations, and results of their application are compared to other empirical parameter estimation procedures, the adequacy of the geomorphologic connection is clearly demonstrated. These results improve the prediction capability of the Nash model for regions where no hydrologic records exist. The shape parameter of the IUH is shown to be a function only of Horton ratios; whereas the scale parameter is shown to depend both on geomorphology and streamflow velocity.

Geomorphologic IUH's have also been proposed that are based on characterizing drainage networks as a function of network links as opposed to streams (Gupta et al., 1986; Mesa and Mifflin, 1986; Troutman and Karlinger, 1984, 1985, 1986; and Gupta and Mesa, 1988.) In that case, the GIUH is expressed in terms of the width function as,

$$u(t) = \int_0^\infty g(x, t) N^*(x, t) dx \tag{26}$$

where $g(x, t)$ is the hydrologic response function of a single channel a distance $x$ from the basin outlet; and $N^*(x)$ is the normalized width function which represents the probability density function of the number of links at a distance $x$ from the outlet. $N^*(x, t)$ is given by,

$$N^*(x, t) = N(x, t) / \int_0^\infty N(x, t) dx \tag{27}$$

where $N(x, t)$ is the width function, which measures the number of links at a distance x from the basin outlet. The link-based GIUH above has been approximated by its conditional expected value, conditional on a topological parameter vector (Karlinger and Troutman, 1985). When the topological parameter is the magnitude $M$ of the network, Karlinger and Troutman have shown that, asymptotically for large $M$ and for $g(x, t)$ representing a simple translation, the expected conditional value of $u(t)$ is,

$$E[u(t)/M] = \frac{t}{2M(\ell_i/V)^2} \exp(\frac{-V^2 t^2}{4M \ell_i^2}), \quad t > 0 \tag{28}$$

where *M* is the magnitude of the basin and equal to the number of first order streams, $\bar{\ell_i}$ is the mean length of the interior links, and *V* is the velocity of translation. Troutman and Karlinger show that this result is always true regardless of the form of *g(x, t)*.

Finally, Rinaldo et al. (1991) studied hydrologic response by decomposing the process of river runoff into two distinct contributions, one accounting for the travel time within individual reaches (hydrodynamic dispersion), and the other accounting for river network composition (geomorphologic dispersion). Because the analysis showed the latter one to play the major role in determining basin response, models based on the accurate specification of the geometry and the topology of the network and simplified dynamics are theoretically validated irrespective of the choice of the travel time probability density function.

*Channel Losses in the GIUH*. The GIUH has undergone several modifications and enhancements in order to make it more physically-based. One of those modifications consists in incorporating channel infiltration losses. Díaz-Granados et al. (1983a), using previous results by Kirshen and Bras (1983), derived expressions for the probability distribution function of travel times in streams of any given order taking into account channel infiltration losses. The channel response to an instantaneous input anywhere along the channel is interpreted as the conditional probability distribution function of the travel time of a drop traveling a given distance along the channel. This response is obtained as the solution of the linearized equations of motion for unsteady flow in a wide rectangular channel in which the infiltration losses are assumed to be proportional to the instantaneous discharge at any point along the channel. The proportionality coefficient is known as the infiltration parameter. Results indicate that the linear reservoir assumption, implicit in the GIUH (exponential distribution of stream travel times), is in fact adequate.

Studies on the geomorphologic response of river basins have increasingly stressed the importance of providing an accurate description of the quantitative properties of river network systems. Within this context, for example, La Barbera and Rosso (1987, 1989) first indicated the fractal nature of river networks and its relation to quantitative geomorphology as initiated by Horton's studies on river network composition. In addition, general theories of the evolution of drainage systems have been developed. For example, energy dissipation concepts have been used to link the local and global properties of a river network to the hydrologic conditions of its watershed (*e.g.*, Howard, 1990; Rodríguez-Iturbe et al., 1992; Rigon et al., 1993; Molnár and Ramírez, 1998a, 1998b). These general theories allow for a fundamental description of global network properties and thus should be incorporated into improved geomorphologic models of basin response.

*The Climatic GIUH - The GcIUH*. The instantaneous unit hydrograph is derived under the assumption that it is a random function of climatic and geomorphologic characteristics and that it varies with the characteristics of the rainfall excess. Accordingly, Rodríguez-Iturbe et al. (1982a) obtain probability density functions of the peak discharge and the time to peak as functions of the rainfall intensity, *i*, and duration, $t_r$, as well as geomorphology. Rainfall is characterized by a constant intensity over the storm duration,

or a rectangular pulse process. These two characteristics, together with geomorphologic parameters for a first order basin, define the dynamic parameter of the GIUH.

Using the kinematic wave approximation for flow routing along streams of first order, and a derived distribution approach, Rodríguez-Iturbe et al. (1982a) obtained analytical expressions for the probability density functions of the time to peak, $t_p$, and the peak discharge, $q_p$, of the GIUH that depend on the mean rainfall intensity; hence the name GcIUH. These probability distribution functions assume that the time of concentration of first order streams is much smaller than the duration of the rainfall. They are,

$$f(q_p) = 3.534 \Pi q_p^{1.5} \exp(-1.412 \Pi q_p^{2.5})$$

$$f(t_p) = \frac{0.656\Pi}{t_p^{3.5}} \exp\left(\frac{-0.262\Pi}{t_p^{2.5}}\right)$$

$$(29)$$

where,

$$\Pi = \frac{L_\Omega^{2.5}}{\bar{i} \, A_\Omega R_L \alpha_\Omega^{1.5}} \tag{30}$$

The first moments of peak discharge and time to peak of the GcIUH can be obtained as,

$$E(q_p) = \frac{0.774}{\Pi^{0.4}} \qquad\qquad E(t_p) = 0.858\Pi^{0.4}$$

$$\sigma_q = \frac{0.327}{\Pi^{0.4}} \qquad\qquad \sigma_{t_p} = 0.915\Pi^{0.4} \tag{31}$$

where $L_\Omega$, is the length of the highest order stream in kilometers, $A_\Omega$, is the basin area in square kilometers, $\bar{i}$ is the mean rainfall intensity in centimeters per hour, and $\alpha_\Omega$ is in s$^{-1}$m$^{-1/3}$. The units of $q_p$ and $t_p$ are the customary units of inverse time in hours, and time in hours, respectively.

For individual storm events, the above equations can be manipulated to obtain expressions for the peak discharge and time to peak as functions of the particular storm intensity and duration. These expressions lead to,

$$q = \frac{0.871}{\Pi^{0.4}} \tag{32a}$$

$$t_p = 0.585\Pi^{0.4} \tag{32b}$$

$$\Pi_i = \frac{L_\Omega^{2.5}}{i A_\Omega R_L \alpha_\Omega^{1.5}} \tag{32c}$$

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The GcIUH establishes a link between climate, watershed geomorphology, and the hydrologic response of the basin. Furthermore, the GcIUH allows estimation of the IUH for a given particular rainfall input, so that problems associated with nonlinear basin behavior are avoided. For example, runoff from a given rainfall event can be computed using an IUH derived from the given storm event characteristics, thereby avoiding the well-known errors incurred when using an IUH based on a different event.

Rodríguez-Iturbe et al. (1982b) compare results by the GcIUH theory with results obtained from IUH's derived from simulated rainfall-runoff events for one basin and three different climates. The theoretical distributions of the time to peak and peak discharge compare very well with the experiments. Variations in $q_p$ and $t_p$ resulting from non-linear basin behavior induce large uncertainty in the predicted peak discharge and time to peak for individual events, when using traditional UH approaches (*e.g.*, Caroni et al. 1986, Caroni and Rosso 1986, and Rosso and Caroni 1986.) These problems are avoided if the GcIUH is used, due to its functional dependence on the rainfall characteristics.

### Snyder's Synthetic Unit Hydrograph

The synthetic unit hydrograph of Snyder (1938) is based on relationships found between three characteristics of a *standard unit hydrograph* and descriptors of basin morphology. These relationships are based on a study of 20 watersheds located in the Appalachian Highlands and varying in size from 10 to 10,000 square miles. The hydrograph characteristics are the effective rainfall duration, $t_r$, the peak direct runoff rate, $q_p$, and the basin lag time, $t_l$. From these relationships, five characteristics of a *required unit hydrograph* for a given effective rainfall duration may be calculated (e.g., Chow et al., 1988; Bras, 1990): the peak discharge per unit of watershed area, $q_{pR}$, the basin lag, $t_{lR}$, the base time, $t_b$, and the widths, $W$ (in time units) of the unit hydrograph at 50 and 75 percent of the peak discharge.

*Standard unit hydrograph.* A standard unit hydrograph is associated with a specific effective rainfall duration, $t_r$, defined by the following relationship with basin lag, $t_l$,

$$t_l = 5.5t_r \tag{33}$$

For a standard unit hydrograph the basin lag, $t_l$, and the peak discharge, $q_p$, are given by,

$$t_l = C_l C_r (LL_c)^{0.3} \tag{34}$$

$$q_p = \frac{C_2 C_p A}{t_l} \tag{35}$$

The basin lag time of the standard unit hydrograph (equation 34) is in hours, $L$ is the length of the main stream in kilometers (miles) from the outlet to the upstream divide, $L_c$ is the distance in kilometers (miles) from the outlet to a point on the stream nearest the centroid of the watershed area, and $C_1 = 0.75$ (1.0 for English units). The product $LL_c$ is a

measure of watershed shape. $C_t$ is a coefficient derived from gauged watersheds in the same region, and represents variations in watershed slopes and storage characteristics. The peak discharge of the standard unit hydrograph (equation 35) is in $m^3/s$ (*cfs*), $A$ is the basin area in $km^2$ (*mi*$^2$), and $C_2 = 2.75$ (640 for English units). As $C_t$, $C_p$ is a coefficient derived from gauged watersheds in the area, and represents the effects of retention and storage.

To compute $C_t$ and $C_p$ for a gauged watershed, the values of $L$ and $L_c$ are measured. From a derived unit hydrograph of the watershed, values of its associated effective duration $t_R$ in hours, its basin lag $t_{lR}$ in hours, and its peak discharge $q_{pR}$ in $m^3/s$ are obtained. If $t_{lR} = 5.5t_R$, then the derived unit hydrograph is a standard unit hydrograph and $t_r = t_R$, $t_l = t_{lR}$, and $q_p = q_{pR}$, and $C_t$ and $C_p$ are computed by the equations for $t_l$ and $q_p$ given above, corresponding to the standard unit hydrograph.

If $t_{lR}$ is quite different from $5.5t_R$ , the standard basin lag is computed using:

$$t_l = t_{lR} + \frac{t_r - t_R}{4} \tag{36}$$

This equation must be solved simultaneously with the equation for the standard unit hydrograph lag time, $t_l = 5.5t_r$, in order to obtain $t_r$ and $t_l$. The value of $C_t$ is then obtained using the equation for $t_l$ corresponding to the standard unit hydrograph. The value of $C_p$ is obtained using the expression for $q_p$ corresponding to the standard unit hydrograph, but using $q_p = q_{pR}$ and $t_l = t_{lR}$.

When an ungauged watershed appears to be similar to a gauged watershed, the coefficients $C_t$ and $C_p$ for the gauged watershed can be used in the above equations to derive the *required synthetic unit hydrograph* for the ungauged watershed.

*Required unit hydrograph.* The peak discharges of the standard and required UH's are related as follows,

$$q_{pR} = \frac{q_p t_l}{t_{lR}} \tag{37}$$

Assuming a triangular shape for the UH, and given that the UH represents a direct runoff volume of 1 cm (1 in), the base time of the required UH may be estimated by,

$$t_b = \frac{C_3 A}{q_{pR}} \tag{38}$$

where $C_3$ is 5.56 (1290 for the English system).

As an aid in drawing an adequate UH, the U.S. Army Corps of Engineers developed relationships for the widths of the UH at values of 50% ($W_{50}$) and 75% ($W_{75}$)

of $q_{pR}$. The width in hours of the UH at a discharge equal to a certain percent of the peak discharge $q_{pR}$ is given by Chow et al. (1988) as,

$$W_\% = C_w \left( \frac{q_{lR}}{A} \right)^{-1.08} \tag{39}$$

where the constant $C_w$ is 1.22 (440 for English units) for the 75% width and equal to 2.14 (770 for English units) for the 50% width. Usually, one-third of this width is distributed before the peak time and two-thirds after the peak time, as recommended by the U.S. Army Corps of Engineers. However, several other authors have recommended different distribution ratios. For example, Hudlow and Clark (1969) recommend a partition of 4/10 and 6/10, respectively.

Figure 11.2 illustrates the form of Snyder's synthetic UH. Note that the time lag is not the same as the time to peak. Also, note that the widths of the hydrograph at 50% and 75% of the peak flow are distributed such that the longer time is to the right of the time to peak.
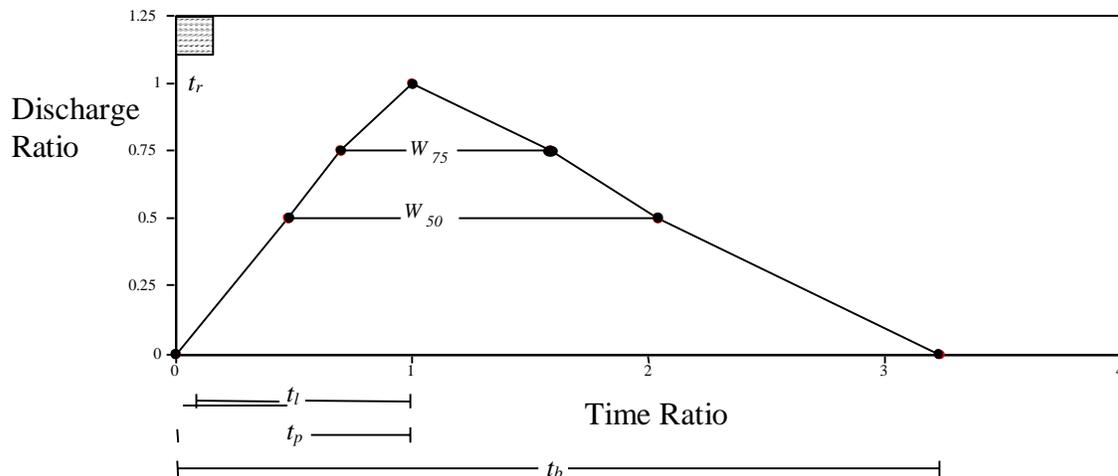


Figure 11.2: Snyder's Synthetic Unit Hydrograph

### Conceptual UH's Based on Dimensionless Hydrographs

*Soil Conservation Service Dimensionless Hydrograph.* The dimensionless unit hydrograph developed by the Soil Conservation Service has been obtained from the UH's for a great number of watersheds of different sizes and for many different locations. The SCS dimensionless hydrograph is a synthetic UH in which the discharge is expressed as a ratio of discharge, $q$, to peak discharge, $q_p$ and the time by the ratio of time, $t$, to time to peak of the UH, $t_p$. Given the peak discharge and the lag time for the duration of the excess rainfall, the UH can be estimated from the synthetic dimensionless hydrograph for the given basin.
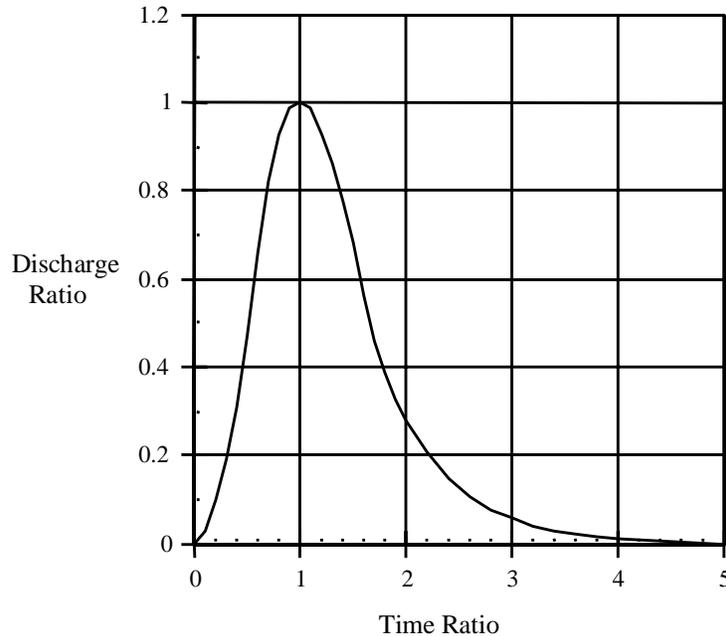
Figure 11.3: SCS Dimensionless Unit Hydrograph

The SCS suggests that the dimensionless UH can be described in terms of an equivalent triangular hydrograph. The values of $q_p$ and $t_p$ can then be estimated using this simplified triangular unit hydrograph whose height is equal to $q_p$ and whose time base, $t_{b,}$ is equal to 2.67 $t_p$. The time is usually expressed in hours (SCS), and the discharge in *m³/s/cm* (or *cfs/in*). After analysis of a great number of UH's, the SCS recommends a recession duration of 1.67 $t_p$. Because the volume of direct runoff must equal 1 cm, it can be shown that $q_p=C A/ t_p$ where $C = 2.08$ (483.4 in the English system) and *A* is the drainage area in square kilometers (square miles). From a study of many large and small rural watersheds the basin lag is $t_l = 0.6t_c$, where $t_c$ is the time of concentration of the watershed. The time to peak, $t_p$, is then equal to $t_r/2+t_l$ (*e.g.*, U.S. Soil Conservation Service, 1985).

*Flood Frequency Distributions and Climatic GIUH*. Eagleson (1972) presents a general framework for the development of flood frequency distributions, $F(Q_p)$(see also Stedinger, Chapter 12, this volume). Based on Eagleson's framework (see Figure 11.4), several authors have developed flood frequency distributions that include various assumptions about the characteristics of the rainfall process in terms of its probability density function, $f_I(i)$, and the watershed processes transforming rainfall into runoff encoded in the function $g(i,t;\theta)$. Either using numerical or analytical procedures these authors have arrived at different flood frequency distributions. One of the most recent and most interesting developments has been the use of the GIUH concept in order to quantify and analyze the effect of geomorphologic characteristics in the determination of regional flood frequency.
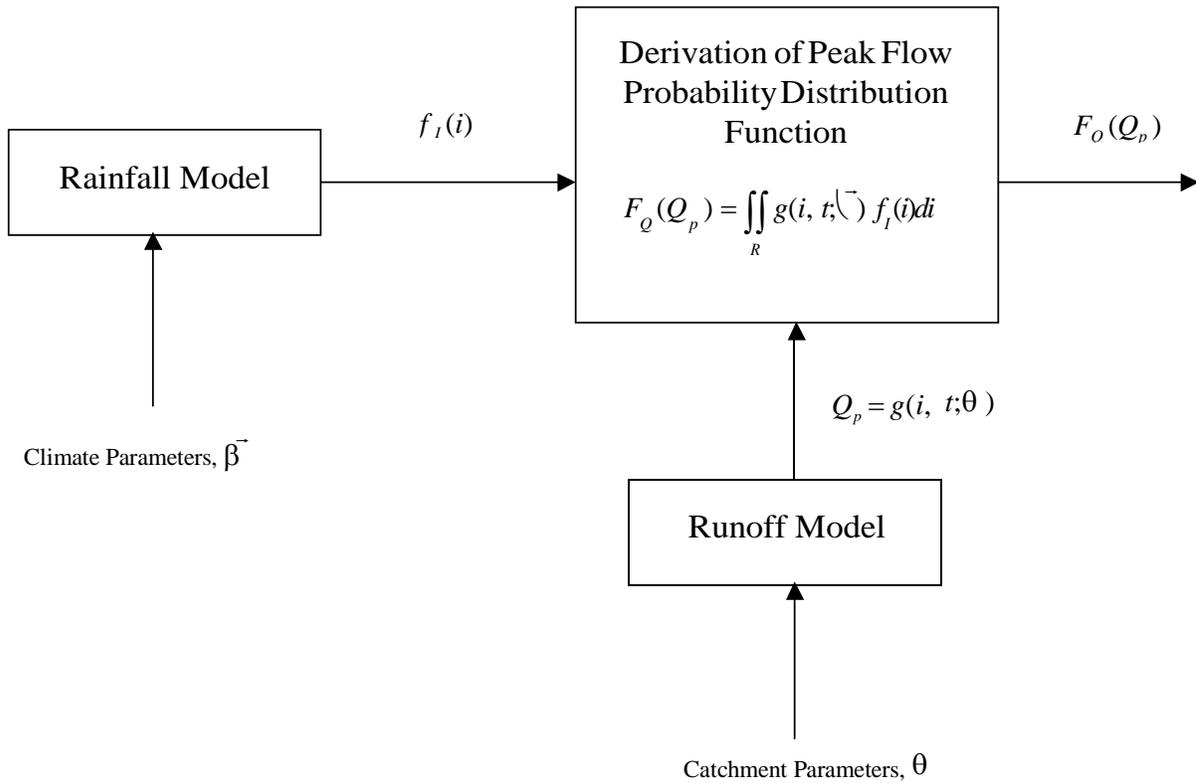
Figure 11.4. General framework for flood frequency analysis (adapted from Eagleson, 1972.)

Hebson and Wood (1982) derived a flood frequency distribution function from assumed climatic distributions for the rainfall parameters and using the GIUH as the transformation function of rainfall into runoff. These authors assume a model of rectangular pulses for storms whose intensity and duration are independent, and exponentially distributed random variables. In terms of the equivalent recurrence interval, $T_E$, Hebson and Wood obtain,

$$\ln T_E = -\ln[\lambda \int_0^{t_k} \exp(-\lambda t_{re} - \beta^*(Q_p - Q_b)/A_c \, g(t_{re})) \, dt_{re} + \exp(-\lambda t_k - \beta^*(Q_p - Q_b)/A_c)] - \ln n \tag{40}$$

where $n$ is the average number of annual direct runoff events, $Q_p$ is the flood peak under consideration, $Q_b$ is the base flow, $A_c$ is the contributing area, $g(t)$ is the area under the GIUH as a function of time, $t_{re}$ is the duration of excess rainfall, $t_k$ is the duration of the GIUH or kernel length, $\lambda$ is the inverse mean duration of rainfall events, and $\beta^*$ is a modified areal inverse mean intensity of rainfall events. They applied this concept to two third-order basins, namely the Bald Eagle Creek in Pennsylvania, and the Davidson River in North Carolina. Comparisons of their flood frequency distributions with the one

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derived by Eagleson (1972), which is based on kinematic wave concepts for overland flow, indicate that the best-fit GIUH produces better representation of the observed flood frequency distributions.

Díaz-Granados et al. (1983, 1984) derived a flood frequency distribution based on the GcIUH. Effective rainfall intensity, $i_e$, and effective rainfall duration, $t_e$, were obtained by assuming precipitation to be a process of rectangular pulses, and runoff to be generated only by the Horton infiltration excess mechanism. Runoff is then generated only when the precipitation intensity exceeds the soil infiltration capacity. Infiltration capacity was defined by using Philip's solution to the one-dimensional, concentration-dependent equation of the diffusion process in unsaturated porous media. Storm intensity and duration were assumed to be independent and exponentially distributed random variables. They used a derived distribution approach in order to link the joint probability distribution function of storm duration and storm intensity and a physically-based representation of the infiltration process in order to obtain the joint probability density function of effective storm duration and effective storm intensity. This derived distribution was used together with a triangular representation of the GcIUH in order to obtain a flood frequency distribution. The flood frequency distributions obtained by this method depend on climatic and geomorphologic parameters of the watershed as well as on soil properties governing the infiltration process. In terms of the equivalent recurrence interval, their derived flood frequency distribution is,

$$T_E^{-1} = n[1 - F_Q(Q_P)] \tag{41}$$

$$F_Q(Q_Q) = 1 - \delta \exp(\beta a - 2\sigma)\Gamma(\sigma + 1)\sigma^{-\sigma}[I + \sum_{i=1}^{4} J_i] \tag{42}$$

where $I$, and $J_i$ are integral equations that cannot be solved analytically, and which depend on the mean intensity and the mean duration of precipitation events, as well as on the infiltration sorptivity of the soil, the discharge under consideration, and geomorphologic parameters of the watershed (see Díaz-Granados et al., 1984).

Given that the above distribution depends on the elusive initial soil moisture, Díaz-Granados et al. (1984) resort to using arguments of ecological optimality for water-limited natural systems in order to define a long term average soil moisture, $s_o$. Using this approach, they verified their flood frequency distribution against observed distributions from two very different climatic regimes. Their results indicate good agreement for both wet and arid climates, although the agreement is better for the arid climate of Santa Paula Creek in California than for the wet climate of the Nashua River basin in New Hampshire.

However, obtaining the analytical form of the flood frequency distribution from climate and geomorphology often results in a cumbersome derivation, sometimes yielding implicit equations to be solved numerically. To overcome this problem, Adom et al. (1989) introduced the method of approximate moments by Taylor series to estimate

second order statistics of both flood peak and volume. For this purpose, they used the Soil Conservation Service's Curve Number method to describe basin rainfall excess, and the GIUH to model surface runoff response. After a given type of frequency distribution has been identified at the regional scale by analyzing annual flood series, this method can be used to estimate the flood frequency distribution for ungagged catchments.

## FLOOD ROUTING

The term flood routing refers to procedures to determine the outflow hydrograph at a point downstream in a river (or reservoir) as a function of the inflow hydrograph at a point upstream.

As flood waves travel downstream they are attenuated and delayed. That is, the peak flow of the hydrograph decreases and the time base of the hydrograph increases. The shape of the outflow hydrograph depends upon the channel geometry and roughness, bed slope, length of channel reach, and initial and boundary flow conditions.

The propagation of flood waves in a channel is a gradually varied unsteady flow process, which is governed by conservation of mass and momentum equations. The solution of these equations in a distributed manner is referred to as *distributed* routing of flood waves. When no spatial variability is taken into account and when the channel reach or reservoir is considered as a black box, the corresponding routing procedure is referred to as *lumped* routing.

### Lumped Flood Routing

Lumped or hydrologic routing is governed by the continuity equation and a storage-discharge functional relationship. The continuity equation is,

$$\frac{dS(t)}{dt} = I(t) - O(t) \tag{43}$$

In this equation, $S(t)$ is the storage within the system (channel reach or reservoir), $I(t)$ is the inflow hydrograph at the upstream end of the reach, and $O(t)$ is the outflow hydrograph at the downstream end of the reach. The continuity equation can be integrated over a given $\Delta t$ to obtain,

$$S(t_{i+1}) - S(t_i) = \int_{S(t_i)}^{S(t_{i+1})} dS(t) = \int_{t_i}^{t_{i+1}} I(t)\,dt - \int_{t_i}^{t_{i+1}} O(t)\,dt \tag{44}$$

where the subscripts $i$ and $i+1$ refer to the beginning of two consecutive time intervals. Assuming a linear variation of input and output fluxes during the $\Delta t$ leads to,

$$S(t_{i+1}) - S(t_i) = \frac{t}{2}[I(t_{i+1}) + I(t_i)] - \frac{t}{2}[O(t_{i+1}) + O(t_i)] \tag{45}$$

In equation (46) there are two unknown quantities, $S(t_{i+1})$ and $O(t_{i+1})$. Thus, a second equation relating $S(t)$ and $O(t)$ is required. The nature of this relationship may be linear, or non-linear, or one to one, or hysteretic. The complexity of the routing procedure depends on the nature of this relationship.

*Reservoir or level pool routing* refers to flood routing for systems whose storage and outflow are related by a function of the type $S(t) = f[O(t)]$ which is one-to-one (*i.e.*, unique, non-hysteretic). This characteristic of their *S vs. O* relationships implies that for a given set of conditions (*e.g.*, stage or storage) the outflow is unique, independent of how that storage is achieved. Such systems have a pool that is wide and deep compared to its length in the direction of flow, low flow velocities, and horizontal water surfaces or negligible backwater effects. Reservoirs or systems with horizontal water surfaces have relationships of the one-to-one type.

The solution procedure involves rearranging the continuity equation such that all unknown quantities are on the left hand side of the equation,

$$\frac{2S(t_{i+1})}{\Delta t} + O(t_{i+1}) = [I(t_{i+1}) + I(t_i)] + [\frac{2S(t_i)}{\Delta t} - O(t_i)] \tag{46}$$

For level pool systems, the storage and the outflow are one-to-one functions of elevation (see Figures 11.5 and 11.6).
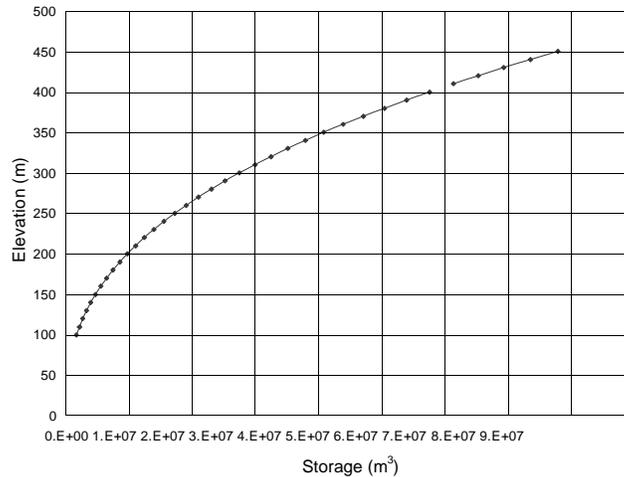


Figure 11.5: Storage *vs*. Elevation Relationship
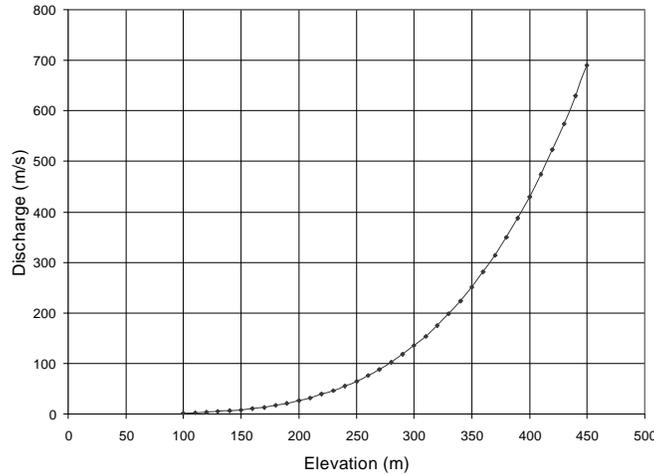
Figure 11.6: Elevation *vs.* Discharge Relationship

Thus, the left hand side of the equation above is a unique function of water surface elevation in the system, only. Usually, the storage-elevation relationship is available from topographic surveys, and the outflow-elevation relationship is available from hydraulic considerations with respect to the outlet structures (*e.g.*, spillways, etc.).

The solution involves the development of the function *2S/Δt + O = f[O(h)]* (see Figure 11.7) and then using the function sequentially to solve for *O(t$_{i+1}$)* for every time step. In this equation, *h* is water surface elevation in the reservoir.
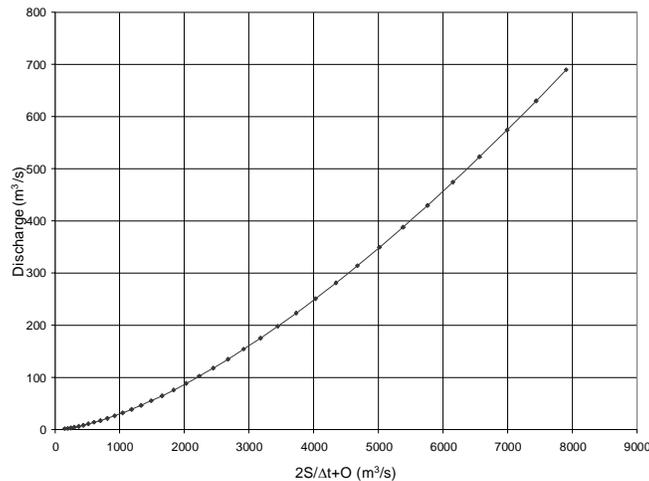


Figure 11.7: Storage-Indication Function

As observed in Figure 11.8, the effect of storage is to redistribute the flow hydrograph by shifting its centroid from the position of that of the inflow hydrograph to that of the outflow hydrograph. For these systems, the peak outflow occurs when the

outflow hydrograph intersects the inflow hydrograph, at which time the maximum storage in the system is also achieved (see Figure 11.8).
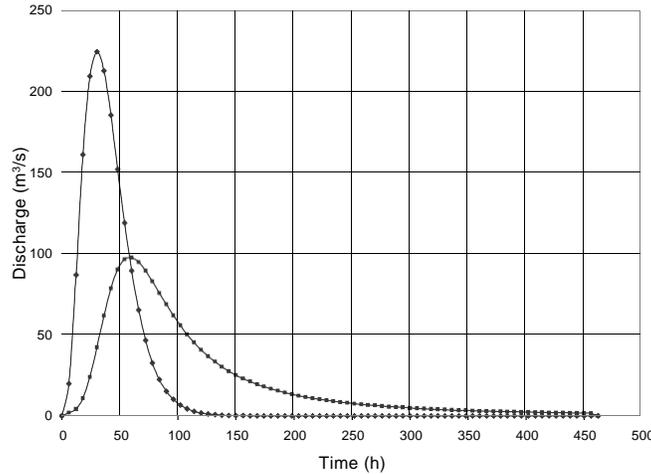


Figure 11.8: Level Pool Routing: Inflow and Outflow Hydrographs

*Channel Routing* The storage-discharge relationship in the case of channel routing is not one-to-one, but hysteretic. In other words, the outflow from a given reach is not a single valued function of the storage in the reach. In its most simple interpretation, the magnitude of the outflow discharge depends on whether the outflow is occurring during the rising limb or the recession limb of the inflow hydrograph. This behavior is a result of backwater effects, which are important in long and narrow flow routing conditions such as occur in channels.

The Muskingum method is one of the most widely used methods of lumped, channel flow routing. In addition to the lumped form of the continuity equation,

$$\frac{dS(t)}{dt} = I(t) - O(t) \tag{43}$$

this method assumes a linear storage discharge relationship as given below,

$$S(t) = k[\, xI(t) + (1 - x)O(t)] \tag{47}$$

This relationship specifies that the storage is proportional to a convex linear combination of inflow and outflow rates. The Muskingum method has two parameters, $k$ and $x$. Parameter $k$ has units of time and represents the average travel time of a kinematic wave through the reach, and it is known as the storage time constant. Parameter $x$ is dimensionless and represents the relative importance of the effects of inflow rates on defining the storage in the reach. Theoretically this parameter varies between 0 and 1.

Combining the continuity and storage equations, and integrating over a short time interval, the outflow at the end of a given time interval, $O(t_{i+1})$ can be expressed as a linear combination of the inflow rates at the beginning and end of the given interval, $I(t_i)$ and $I(t_{i+1})$, as well as the outflow rate at the beginning of the interval, $O(t_i)$ as,

$$O(t_{i+1}) = C_o I(t_{i+1}) + C_1 I(t_i) + C_2 O(t_i) \qquad (48)$$

The coefficients or weights of this linear combination can be computed as a function of $k$, $x$, and $\Delta t$ as indicated below,

$$C_o = (-kx + 0.5\Delta t)/C_3 \qquad (49a)$$

$$C_1 = (kx + 0.5\Delta t)/C_3 \qquad (49b)$$

$$C_2 = (k - kx - 0.5\Delta t)/C_3 \qquad (49c)$$

$$C_3 = k - kx + 0.5\Delta t \qquad (49d)$$

It is simple to show that the Muskingum routing solution can be expressed also as a discrete convolution equation as,

$$Q_n = \sum_{m=1}^{n} I_{n-m+1} U_m \qquad (50)$$

where $U_m$ is the UH of the Muskingum method whose ordinates are given by the following equations,

$$\begin{aligned}
U_1 &= C_o \\
U_2 &= C_o C_2 + C_1 \\
U_m &= U_{m-1} C_2 , \qquad m > 2
\end{aligned} \qquad (51)$$

The Muskingum parameters $k$ and $x$ can be easily estimated graphically, or by using the least-squares method. The graphical method consists in choosing $x$ such that the loop resulting from the plot of *S vs. (xI + (1-x)O)* becomes as close to a straight line as possible (see Figure 11.9a-d). The slope of the resulting straight line is an estimate of the parameter $k$ (see equation 47). In order to ensure positivity of the unit hydrograph ordinates, $\Delta t$ must be selected such that $x < 0.5 \Delta t/k < 1-x$.

This method results in a trial and error procedure which, when properly carried out, constitutes a graphical implementation of the least-squares method. Or in other words, the least-squares method is a numerical expression of the graphical method. Observe that routing in systems with hysteretic *S vs. O* relationships leads to additional modifications of the inflow hydrograph represented in a delay of the response of the system and in asynchronous occurrences of the maximum storage and maximum outflow, the latter occurring after the former (see Figure 11.10).
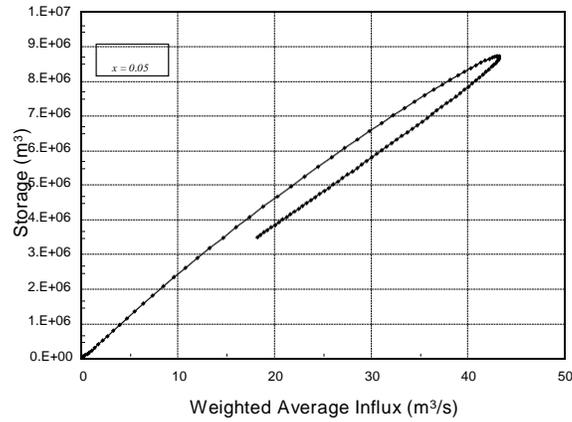
Figure 11.9a: Weighted Average Influx *vs*. Storage Function - *x* = 0.05.
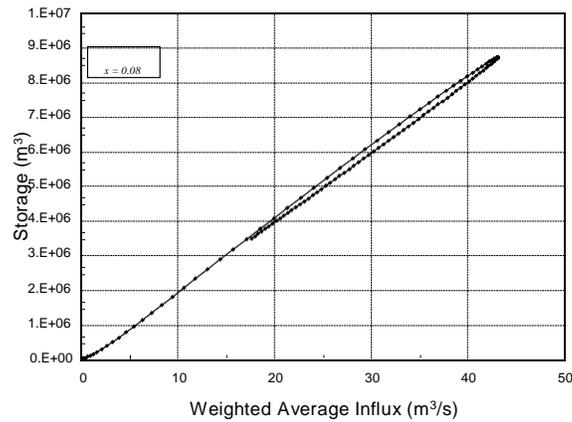


Figure 11.9b: Weighted Average Influx *vs*. Storage Function - *x* = 0.08.
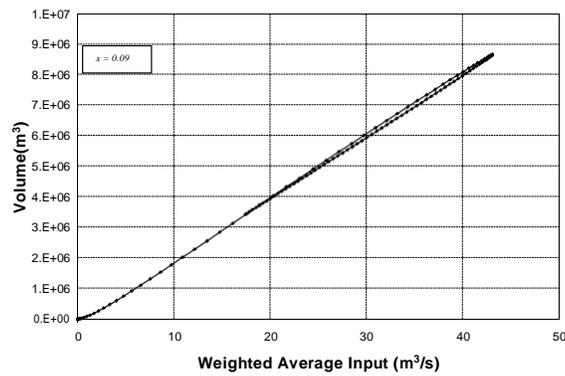


Figure 11.9c Weighted Average Influx *vs*. Storage Function - *x* = 0.09.
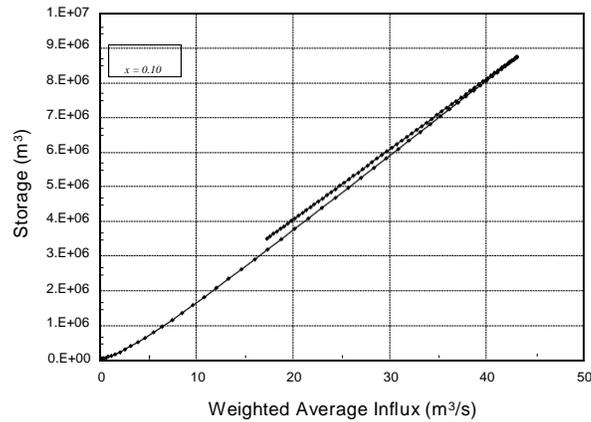
Figure 11.9d Weighted Average Influx *vs*. Storage Function - *x* = 0.10
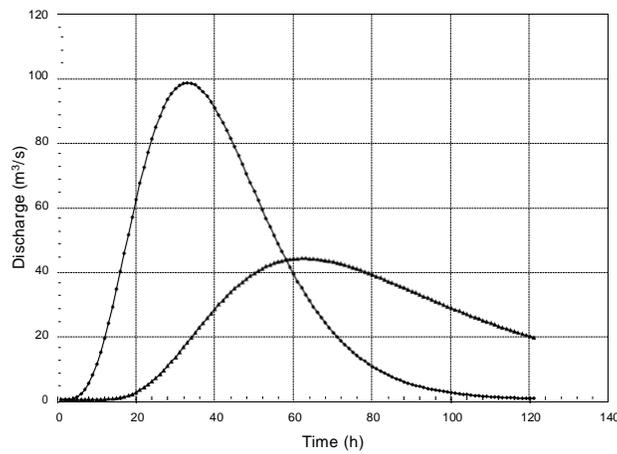


Figure 11.10: Muskingum Routing: Inflow and Outflow Hydrographs

Using a hydraulic analogy and applying the Muskingum equations over a short Δ*x*, the Muskingum routing equation constitutes an approximate solution to the kinematic wave equations (see Distributed Flood Routing section below) (Cunge, 1969). Expressing the solution in a space-time domain, the Muskingum routing equations can be rewritten as,

$$Q^{j+1}_{i+1} = C_o Q^{j+1}_i + C_1 Q^j_i + C_2 Q^j_{i+1} \tag{52}$$

where the flow discharge $Q^j_i$ refers to position *i* in space and *j* in time. In this solution, the parameters *k*, and *x*, required in order to obtain $C_o$, $C_1$, $C_2$, and $C_3$ are estimated as,

$$k = \frac{x}{c_k} \tag{53a}$$

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$$x = \frac{1}{2}(1 - \frac{Q}{c_k B S_o \Delta x}) \quad (53b)$$

In addition, Cunge also demonstrated that this solution constitutes an approximate solution of a modified diffusion equation if the parameters $k$ and $x$ are estimated as expressed above. In those equations, $c_k$ is the celerity of the kinematic wave corresponding to $Q$ and $B$, where $B$ is the width of the water surface, $Q$ is the discharge, $A$ is the cross sectional area of flow, $S_o$ is the bottom slope, and $\Delta x$ is the increment in space. Based on this analysis, it can be shown that $x$ should be between 0 and 0.5.

### Distributed Flood Routing

The characteristics of hydrologic systems are extremely variable in space and time. The excitation of the system (*i.e.,* precipitation), the initial conditions, and the boundary conditions are, in general, functions of space and time. Therefore, the response of the system, that is, the flow of water through the soil and in the channels, is a distributed process in which the characteristics of the flow change both in time and space. Flow characteristics such as flow discharge, flow depth, and flow velocity can be obtained as a function of space and time using a dynamic, distributed flow routing procedure.

The governing equations for one-dimensional, unsteady flow in an open channel are,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \quad (54)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial(\beta Q^2 / A)}{\partial x} + gA(\frac{\partial y}{\partial x} - S_o + S_f) - \beta q v_x = 0 \quad (55)$$

where $Q$ is discharge, $A$ is the cross sectional flow area, $y$ is flow depth, $S_o$ is the channel bottom slope, $S_f$ is the friction slope, $\beta$ is the momentum correction coefficient for non-uniform flow velocity distribution, $q$ is lateral inflow per unit length of flow, and $v_x$ is the magnitude of the lateral inflow velocity component in the direction of flow.

These two equations are known as the de Saint-Venant equations. The main assumptions encoded in these equations are: 1) the flow is one-dimensional; 2) the vertical accelerations are negligible so that the vertical pressure distribution is hydrostatic (gradually varied flow); 3) the slope of the channel bottom is small and the channel bottom is fixed; 4) resistance coefficients for steady uniform turbulent flow are applicable; and 5) the fluid is incompressible and of constant density.

The momentum equation includes terms for changes in momentum due to: a) changes in velocity over time (local acceleration); b) changes in velocity in space along the channel (convective acceleration); c) water level differences along the channel and consequent imbalance of pressure forces; d) gravitational acceleration; e) frictional forces; and f) lateral inflow. Local and convective accelerations represent the effect of

inertial forces in the flow. Depending on the flow conditions, some of these terms may be negligible as compared to others in the equation giving rise to either *kinematic*, or *diffusive*, or *full dynamic* wave situations. This sometimes leads to simplifications of the flood routing problem (*e.g.*, Chow et al, 1988; Singh, 1989; Bras, 1990).

*Kinematic Waves.*  Kinematic waves govern the flow when inertial and pressure forces are not important. In natural flood wave conditions both kinematic and dynamic waves are present. However, for situations where the Froude number is less than 2, dynamic waves tend to attenuate more rapidly than kinematic waves (*e.g.*, Eagleson, 1970). Although there is no single set of criteria to define when a kinematic wave solution is acceptable, the magnitude of the so-called kinematic parameter is useful. The kinematic parameter is defined as,

$$K = \frac{S_o L}{y_o F^2} \tag{56}$$

where $S_o$ is the bottom slope, $L$ is the length of (overland or channel) flow, $y_o$ is the normal flow depth at the downstream end of the flow length, and $F$ is Froude number for normal flow at the downstream end of the flow length. Woolhiser and Liggett (1967) have shown that for $K$ values greater than 10, the flow dynamics tend to be dominated by kinematic waves. In overland flow situations the value of $K$ usually exceeds this threshold, ($K \sim O(10^2 - 10^3)$), warranting a kinematic wave approach for such flow routing problems.

For a kinematic wave, the momentum equation reduces to,

$$S_o = S_f \tag{57}$$

implying that the energy grade line is parallel to the channel bottom, and that the flow is steady and uniform. The above momentum equation can be shown to be equivalent to the following stage-discharge relationship,

$$Q = \alpha A^\beta \tag{58}$$

which, for example, can be satisfied by Manning equation,

$$Q = \frac{\sqrt{S_f}}{n} R^{2/3} A \tag{59}$$

Together with the continuity equation,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{54}$$

these equations represent the routing equations for the kinematic wave flow routing approach.

If an observer moves with the kinematic wave at a speed equal to the kinematic wave celerity, the observer would see the flow rate increase at a rate equal to the lateral inflow rate, $q$, as shown below,

$$\frac{dQ}{dx} = \frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial t}\frac{dt}{dx} = \frac{\partial Q}{\partial x} + \frac{1}{c_k}\frac{\partial Q}{\partial t} = q \tag{60}$$

It can be seen that for conditions of no lateral inflow, that is, for $q = 0$, $dQ/dt = 0$. Thus, kinematic waves do not attenuate; they simply translate downstream without dissipation. Given that at any cross section $Q$ and $A$ are functionally related as $Q = \alpha A^\beta$, the continuity equation can be rewritten as,

$$\frac{\partial Q}{\partial x} + \frac{dA}{dQ}\frac{\partial Q}{\partial t} = q \tag{61}$$

and,

$$\frac{\partial A}{\partial t} = \frac{dA}{dQ}\frac{\partial Q}{\partial t} = \frac{1}{\alpha\beta A^{\beta-1}}\Big(\frac{\partial Q}{\partial t}\Big) \tag{62}$$

$$\frac{\partial Q}{\partial x} + \frac{1}{\alpha\beta A^{\beta-1}}\Big(\frac{\partial Q}{\partial t}\Big) = \frac{\partial Q}{\partial x} + \frac{Q^{1/\beta-1}}{\alpha^{1/\beta}\beta}\frac{\partial Q}{\partial t} = q \tag{63}$$

From equations 60 and 63, it is easy to obtain the following expression for the kinematic wave celerity,

$$c_k = \frac{dQ}{dA} = \alpha\beta A^{\beta-1} \tag{64}$$

Assuming that overland flow situations can be described as flow over planes, the kinematic wave approximation has been widely applied to construct models of the rainfall-runoff process. A basin is described as a collection of such overland flow planes representing the hillslope processes feeding into a network of collector channels which feed into a main channel. In this application, the lateral inflow is taken to be equal to the rainfall excess, and the channel inflow is taken to be flow per unit width of plane. The kinematic wave model offers the advantage over the unit hydrograph method that it is a one-dimensional, spatially-distributed solution of the physical equations governing the surface flow. Furthermore, for overland flow waves in the case of homogeneous planes, the kinematic wave equations have explicit analytical solutions. However, more complex representations of the flow process must sometimes be used -- two-dimensional and fully dynamic.

In general, the Saint-Venant equations do not have explicit analytical solutions, except in a few particular cases (*e.g.*, kinematic waves for homogeneous planes). Thus, numerical solution procedures must be used, like *finite-differences methods*, or the

*method of characteristics.* Finite difference methods solve the partial differential equations on a grid placed over the *x-t* plane. For the kinematic wave problem presented above, these solutions can be linear or non-linear.

A simple implementation of the linear and non-linear solutions suggested by Chow et al., (1988) is adapted below. For the linear solution, the equation is linearized by substituting an average of known solutions for the coefficient of the nonlinear term. This leads to the following solution,

$$\frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} + \frac{1}{\alpha^{1/\beta}\beta}\left(\frac{Q_{i+1}^j + Q_i^{j+1}}{2}\right)^{1/\beta - 1}\left(\frac{Q_{i+1}^{j+1} - Q_{i+1}^j}{\Delta t}\right) = \left(\frac{q_{i+1}^{j+1} + q_{i+1}^j}{2}\right) \tag{65}$$

$$Q_{i+1}^{j+1} = \frac{\left[\frac{\Delta t}{\Delta x}Q_i^{j+1} + \frac{1}{\alpha^{1/\beta}\beta}Q_{i+1}^j\left(\frac{Q_{i+1}^j + Q_i^{j+1}}{2}\right)^{1/\beta - 1} + \Delta t\left(\frac{q_{i+1}^{j+1} + q_{i+1}^j}{2}\right)\right]}{\left[\frac{\Delta t}{\Delta x} + \frac{1}{\alpha^{1/\beta}\beta}\left(\frac{Q_{i+1}^j + Q_i^{j+1}}{2}\right)^{1/\beta - 1}\right]} \tag{66}$$

In this linear solution, the subscript refers to the space coordinate and the superscript refers to the time coordinate. The solution advances on a time line from upstream to downstream.

For the nonlinear kinematic wave solution, the partial differential equations are recast in finite difference form as shown below. The same subscript and superscript convention applies as for the linearized solution.

$$\frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} + \frac{A_{i+1}^{j+1} - A_{i+1}^j}{\Delta t} = \frac{q_{i+1}^{j+1} + q_{i+1}^j}{2} \tag{67}$$

$$\frac{\Delta t}{\Delta x}Q_{i+1}^{j+1} + \alpha^{-1/\beta}(Q_{i+1}^{j+1})^{1/\beta} = \frac{\Delta t}{\Delta x}Q_i^{j+1} + \alpha^{-1/\beta}(Q_{i+1}^j)^{1/\beta} + \Delta t\left(\frac{q_{i+1}^{j+1} + q_{i+1}^j}{2}\right) \tag{68}$$

$$C = \frac{\Delta t}{\Delta x}Q_i^{j+1} + \alpha^{-1/\beta}(Q_{i+1}^j)^{1/\beta} + \Delta t\left(\frac{q_{i+1}^{j+1} + q_{i+1}^j}{2}\right) \tag{69}$$

An iterative solution based on Newton's method is shown below.

$$f(Q_{i+1}^{j+1}) = \frac{t}{\Delta x}Q_{i+1}^{j+1} + \alpha^{-1/\beta}(Q_{i+1}^{j+1})^{1/\beta} - C \tag{71}$$

$$f'(Q^{j+1}) = \frac{t}{ } + \frac{1}{ }(Q^{j+1})^{1/\beta - 1}$$

$$^{i+1} \qquad \Delta x \quad \alpha^{1/\beta}\beta \qquad ^{i+1} \tag{72}$$

$$(Q^{j+1})_{i+1 \ k+1} = (Q^{j+1})_{i+1 \ k} - \cfrac{\dfrac{-t}{\Delta x}(Q^{j+1})_{i+1 \ k} + \alpha^{-1/\beta}[(Q^{j+1})_{i+1 \ k}]^{1/\beta} - C}{\dfrac{\Delta t}{\Delta x} \ \dfrac{1}{\alpha^{1/\beta}\beta} + [(Q^{j+1})_{i+1 \ k}]^{1/\beta -1}} \tag{73}$$

Newton's method is an iterative numerical solution procedure, which progressively determines improvements to a trial solution as a function of the gradient of the function at the trial solution. The method converges relatively quickly. A convergence criterion can be defined in advance, so that a solution is reached when this criterion is met (*e.g.*, the magnitude of the improvement is less than a threshold).

*Dynamic Waves.* Dynamic waves govern the flow whenever inertial and pressure forces are significant when compared to the gravitational and frictional forces. This condition occurs for unsteady, non-uniform flows for which backwater or tidal effects are important as, for example, during the movement of a flood wave in a system of channels of shallow slopes, and may be induced by downstream reservoirs or other downstream controls, by tributaries, by tides, by storm surges (*e.g.*, caused by hurricanes), or by large discontinuities in the flow characteristics as a result of dam breaks or large reservoir releases. As indicated above for the case of hydraulic river routing, backwater effects induce rating curves that are hysteretic, that is, rating curves for which the discharge is not a single valued function of stage (*i.e.*, storage per unit length of channel) in the section.

Full dynamic wave flood routing models can be both steady-state and unsteady-state models. The Hydrologic Engineering Center's River Analysis System (HEC-RAS) model is an example of a steady-state flow model used for the computation of water surface profiles associated with one dimensional, gradually varied, steady flows. HEC-RAS is designed to perform one-dimensional hydraulic simulations for a network of channels, and computes surface water profiles associated with subcritical, supercritical, and mixed flow regimes. Surface water profiles are solved sequentially by solving the energy equation using an iterative algorithm called the standard step method. The energy equation is,

$$Y_2 + Z_2 + \frac{\alpha_2 v_2^2}{2g} = Y_1 + Z_1 + \frac{\alpha_1 v_1^1}{2g} + h_e \tag{74}$$

where $h_e$ is the energy head loss, $Y_i$ is the surface water depth at section $i$, $Z_i$ is the elevation of the channel bottom at section $i$, $v_i$ is the average velocity at cross section $i$, and $g$ is the acceleration of gravity. The energy head loss between two cross sections accounts for friction losses and for losses induced by contractions or expansions and is evaluated as,

$$h_e = LS_f + C \left| \frac{\alpha_1 v_1^2}{2g} - \frac{\alpha_2 v_2^2}{2g} \right| \tag{75}$$

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where $L$ is the reach length weighted by discharge, $S_f$ is a representative friction slope between the two sections, and $C$ is a coefficient for expansion/contraction losses. For situations where flow regimes change abruptly from supercritical to subcritical or vice versa, the HEC-RAS model uses the steady state momentum equation instead, as the energy equation is not applicable under those conditions. The momentum equation is written in this case as,

$$P_2 - P_1 + W_x - F_f = \rho Q \Delta v_x \qquad (76)$$

where $P_i$ is the hydrostatic pressure force at section $i$, $W_x$ is the component of the weight of the water in the section in the direction of flow, $F_f$ is the force due to friction in the reach between sections, $Q$ is the discharge, $\rho$ is the density of water, and $\Delta v_x$ is the change in average velocity within the reach. In addition, the model allows for the specification of variable cross sectional characteristics, both at a section and along the channel. Common applications of the HEC-RAS model include flood plain delineation and analysis for flood protection and insurance purposes, modeling of single and/or multiple culverts and bridges, modeling of gated spillways and weirs, modeling of floodway encroachment, and of ice-covered rivers.

In the context of fully dynamic, unsteady flows, the models of the National Weather Service developed by Fread and collaborators are well documented and commonly used (*e.g.*, Fread, 1974, 1976, 1978, 1988; Fread and Lewis, 1988). Among these models are the DWOPER model (Dynamic Wave Operational Model) (Fread, 1978), the DAMBRK model for dam failure studies (Fread, 1988), and the FLDWAV model (Fread and Lewis, 1988) which synthesizes DWOPER and DAMBRK. These and similar models solve the fully unsteady dynamic wave equations using finite differences. However, the treatment and presentation of the details of the solution is beyond the scope of this chapter. For those details the reader is kindly referred to the above references and those given therein.

**DISTRIBUTED WATERSHED MODELS**

Distributed and semi-distributed watershed models are generally categorized into *single-event* simulation models and *continuous* simulation models. The former models are concerned with simulating a single flood as a result of the occurrence of an individual precipitation or flood-inducing event. The latter models are concerned with the simulation of watershed processes during a period of time that encompasses more than one precipitation or flood-causing event. Thus, continuous models are also concerned with evaporation and sub-surface soil moisture redistribution during the inter-storm periods. Most of these models include elements or components that are described using the concepts of previous sections in this chapter. However, it is implied that watershed models deal with the transformation of precipitation into runoff, whereas some of the approaches presented earlier in this chapter are concerned simply with the translation of a flood wave, not necessarily with the complete precipitation-runoff transformation. These models are then referred to as *flood hydraulics* models (*e.g.*, HEC-RAS.)

### Single-Event Simulation Models

There exists a great variety of single event watershed models. Singh (1989) provides a summary table listing fifteen such models and reviews some of them. The following sections are based on the review presented in Ramírez et al. (1994). Reference is made to two models developed by the U.S. Geological Survey and one developed by the Hydrologic Engineering Center of the U.S. Corps of Engineers. However, many other alternative models are available such as the TR-20 (U.S. Soil Conservation Service, 1973), SWMM (Metcalf & Eddy Inc., 1971), IHM (Morris, 1980), and FLEA (Ranzi and Rosso, 1991a).

The USGS rainfall-runoff model (Dawdy and O'Donnell, 1965) was one of the first single event models developed to predict flood volume and peak rates of runoff for small drainage areas. The model has been utilized by the U.S. Geological Survey in over twenty states to develop peak flows and hydrographs for state highway agencies. The process is triggered by the precipitation input $P$, which augments surface storage $R$, which in turn is depleted by infiltration $F$ and evaporation $E_R$. The channel outflow $Q$ begins when the threshold value $R^*$ is exceeded. The infiltration is evaluated by a Horton type equation. The infiltration $F$ and capillary rise $C$ augment the soil storage $M$, which is depleted by transpiration $E_M$. Deep percolation $D$ occurs when the soil moisture threshold $M^*$ is exceeded, augmenting groundwater storage $G$, which in turn is depleted by capillary rise $C$ and base flow $B$. If $G$ exceeds the threshold value $G^*$, $M$ is absorbed into $G$, then $C$ and $D$ disappear and $E_M$ and $F$ operate on $G$. The channel storage $S$ and groundwater storage $G$ are assumed to be linear reservoirs with storage constants $K_S$ and $K_G$, respectively. The channel flow $Q$ is routed through a linear reservoir $S$ yielding the flow $Q_S$, which added to the base flow $B$ gives the total flow $Q$.

An alternative USGS model is the distributed routing rainfall-runoff model developed by Dawdy et al. (1972). This model uses a modified version of the Philip's infiltration equation and the infiltration capacity concept of Crawford and Linsley (1966) to determine the excess rainfall. A kinematic wave model component is used for routing. The model was specifically designed for routing urban flood discharges through a branched system of pipes or natural channels using rainfall as input. The drainage basin is presented as a set of segments (overland flow, channel reservoir and nodal) which jointly describe all sub-basins of the entire basin.

A single event watershed model which has been widely used for estimation of surface runoff and river/reservoir flow in a basin *during floods* is the HEC-1 Flood Hydrograph Package. It was originally developed in 1967 by the staff of the Hydrologic Engineering Center at Sacramento, California (now at Davis, California). The current version of the HEC-1 package includes computational capabilities of dam breaks, project optimization and kinematic wave programs (HEC, 1981).

The hydrologic simulation capabilities of HEC-1 include several techniques to input and distribute the precipitation, treat the precipitation as rainfall or snowfall,

compute rainfall and snowmelt losses and excess, and determine sub-basin outflow hydrographs by various hydrologic routing techniques. The model may be used to simulate a simple single-basin watershed or a very complex basin with practically unlimited number of sub-basins and river reaches. The HEC-1 model can account for temporal and spatial variability of the precipitation runoff process in a semi-distributed sense. That is, within a sub-basin, HEC-1 uses spatially and temporally lumped parameters to simulate the precipitation-runoff process. The precipitation hyetograph (rain and/or snow) is input over the sub basin, and the losses are computed, leaving an excess precipitation hyetograph which in turn is transformed into surface runoff hydrograph through a specified unit hydrograph. The subsurface runoff hydrograph is computed separately and added to the surface runoff hydrograph to yield the total subbasin runoff hydrograph. In addition to being capable of hydrologic simulation, the HEC-1 package also has a provision for evaluating reservoir and channel development plans for flood control purposes by performing the economic analyses of flood damages for existing and post-development conditions. An additional application of the calibrated model is for impact assessment studies of basin modifications and channel improvements. This can be accomplished by modifying the physical parameters of the sub-basin or routing reach in which the *change* is expected or proposed and by re-analyzing the response of the watershed.

### Continuous Simulation Models

Three continuous watershed models developed in the United States of America are summarized in this section. The review is based on that of Ramírez et al. (1994). The models are: the Stanford Watershed Model (SWM), the National Weather Service Model (NWS) and the Precipitation Runoff Modeling System (PRMS) of the US Geological Survey. However, many alternative models exist such as the SSARR (U.S. Army Corps of Engineers, 1972) for daily streamflow simulations and forecasting for large watersheds, the SHE model developed in Europe (Abbott, 1986a, 1986b), or the LBRM developed by the U.S. Great Lakes Environmental Research Laboratory (Croley, 1985) for simulation of daily runoff in the Great Lakes basins.

The Stanford Watershed Model (SWM) was originally developed by Crawford and Linsley (1966) for the continuous simulation of the hydrologic processes in a watershed at hourly time scales. Since then, the original model has undergone a number of revisions and expansions to satisfy specific needs and requirements (*e.g.*, Claborn and Moore, 1970; Liou, 1970; Anderson, 1971; Ricca, 1972). Several models such as the Agricultural Runoff Management Model, ARM, (Donigian and Crawford, 1979) and the Hydrologic Simulation Program, HSPF, of EPA (Johanson et al., 1980) employ the basic structure of the original SWM for runoff simulation but in addition simulate water quality and sediment erosion.

The SWM model partitions the watershed into land segments whose boundaries are established according to hydrologic characteristics and user's needs. The segmentation of land is usually based on meteorologic, soil, topographic, and drainage considerations. Thus, each land segment is assumed to be homogeneous with respect to hydrologic

characteristics, and is assumed to produce a homogeneous hydrologic response. Streamflow may be computed at several locations in the stream network.

Two storage zones control overland flow, infiltration, interflow and inflow to the groundwater storage. The upper zone storage may be physically interpreted as consisting primarily of depression storage whereas the lower zone represents the soil moisture storage above the capillary zone. The role of the upper zone is to simulate the initial response to rainfall, and therefore it is of importance for small storms, and for the first few hours of major storms. The lower zone plays a major role during large storms in controlling temporal variation of infiltration rates. Infiltrated water moves to the lower zone and groundwater storage. Additional infiltration may occur from depression storage or from runoff, giving rise to delayed infiltration. A fraction of total infiltrated water reaches the groundwater storage, which then contributes to baseflow. Moisture may be lost from the system as evapotranspiration from interception, upper zone, lower zone, and groundwater storage, and stream surfaces. Direct inflows from the impervious areas, overland flow, interflow, and groundwater flow are combined to become total inflow into the channel network which contributes to the streamflow hydrograph.

The U.S. National Weather Service (NWS) developed a computerized system of operational river forecast procedures including data acquisition and processing, computation of mean basin precipitation, snow accumulation and ablation, explicit soil moisture accounting, channel routing, parameter optimization and verification, and operational forecasting. This set of models, known as the National Weather Service River Forecast System, (NWSRFS), has been extensively described (*e.g.*, Monro and Anderson, 1974; Peck, 1976). The soil moisture accounting model of the NWSRFS has been developed by the NWS Sacramento, California River Forecast Center (*e.g.*, Burnash et al., 1973). The NWS model can be partitioned into two major components, namely: the Sacramento soil moisture accounting model, and the flow routing model. The Sacramento soil moisture accounting model is a continuous, conceptual, lumped input and lumped parameter model which transforms the rainfall input in the land phase of the hydrologic cycle into channel inflows. The flow routing model is responsible for routing the channel inflows to the basin or subbasin outlet using unit hydrograph and Muskingum routing methods.

The Sacramento soil moisture accounting (SMA) model assumes that a catchment consists of two reservoirs, the upper zone and the lower zone. This conceptualization is intended to provide a simple but effective representation of the hydrologic process of the catchment starting with precipitation, the subsequent vertical and horizontal movement of water through and over the soil, and finally the production of runoff. The upper zone represents the upper soil layer and interception storage, whereas the lower zone represents the bulk of the soil moisture and groundwater storage. Each zone stores water in two forms, tension water and free water. Tension water is only depleted by evapotranspiration. Depletion of free water occurs vertically as percolation and horizontally as evapotranspiration and interflow. In the lower zone, there are two types of free water storage: primary storage, which is slow draining, and provides baseflow over long periods of time; and supplementary storage, which is fast draining, and supplements

baseflow after a period of relatively recent rainfall. Movement of water from upper zone to lower zone is achieved through percolation, which is a nonlinear function of upper zone free available water and lower zone moisture deficiency.

Two levels of watershed partitioning are required in the model. The first level partitions the watershed into soil moisture accounting areas where each area is a homogeneous unit in terms of the SMA model parameters. Rainfall and evapotranspiration input data are assumed uniform over each SMA area. The second level of partitioning divides an SMA area into smaller homogeneous units representing individual flow planes (overland and channel). Each flow plane is assumed to have homogeneous unit graph-Muskingum model parameters. Although there are two levels of watershed partitioning, one can assume a flow plane to coincide with the SMA area. Streamflow is computed at the outlet or flow points of each flow plane.

The model is set up to simulate basin hydrology on hourly or longer time intervals. Rainfall input data can only be given on an hourly basis whereas evapotranspiration and streamflow input data can be given on a longer time interval as an integer multiple of the time interval of rainfall. In such cases, the model uniformly distributes the evapotranspiration data and computes streamflow over the time basis of rainfall data and over the time interval specified for streamflows.

Finally, the model generates five components of flow: 1) direct runoff from permanent and temporary impervious areas, 2) surface runoff which occurs when the upper zone free water storage is full and precipitation intensity exceeds the rate of percolation and interflow, 3) interflow resulting from lateral drainage of the upper zone free water storage, 4) supplemental baseflow, and 5) primary baseflow. The first three runoff components represent the total channel inflow whereas the latter two are the total baseflow. In the NWS model, the total channel inflow constitutes the surface runoff contribution to the streamflow hydrograph routed via unit hydrograph/Muskingum routing methods, and a portion of the total baseflow is the subsurface runoff contribution to streamflow. This subsurface flow contribution is added to the routed streamflow at the basin or subbasin outlet using a linear decay weighting function which is similar to the hydrograph routing method.

The U.S. Geological Survey (USGS) developed a Precipitation-Runoff Modeling System (PRMS) to simulate surface water runoff, sediment yields and general basin hydrology under various combinations of precipitation, climate and land use (Leavesley et al., 1983). The system is physically based such that each component of the hydrologic cycle is expressed as functional relationships of known physical laws or empirical knowledge of the hydrologic process. The PRMS model can be either a lumped or distributed parameter model, capable of simulating mean daily flows and storm flow hydrographs.

The watershed is partitioned into hydrologic response units (HRU's) which may or may not be associated with individual sub watersheds. HRU's are defined based on characteristics such as slope, orientation of hillslope or aspect, soil type, vegetation type,

elevation and precipitation distribution (Leavesley et al., 1983), which imply similar hydrologic response behavior. Water and energy balances are determined daily for each HRU. The inputs are daily precipitation, maximum and minimum daily air temperature, daily solar radiation and daily streamflow (for calibration). Precipitation is reduced by interception. The energy inputs of air temperature and solar radiation drive the processes of evaporation, transpiration, sublimation and snowmelt.

The model is represented by a series of storage (reservoirs). Each HRU has a soil-zone reservoir. This represents the soil mantle that can lose water by evaporation and transpiration. Average root depth of the predominant vegetation on the HRU defines the depth of the soil-zone reservoir. Water storage in this zone is increased by infiltration and is decreased by evapotranspiration. The soil zone is represented by two layers. The upper layer (recharge layer) loses water by evaporation and transpiration, whereas the lower layer loses water only by transpiration.

When the soil zone reservoir reaches field capacity, excess water percolates to the subsurface and groundwater reservoirs. The subsurface reservoir responds faster than the groundwater reservoir. The subsurface reservoir can be linear or nonlinear, whereas the groundwater reservoir responds linearly. Subsurface and groundwater reservoirs can be defined for each HRU or can be defined for larger areas comprising several HRU's. The snow component of PRMS simulates the accumulation and depletion of a snowpack on each HRU.

The USGS model can simulate basin hydrology on a daily and on a shorter time interval for flood simulation. The daily mode simulates hydrologic components on daily averages or total values. Shorter time simulation, with time steps in the order of minutes, computes water discharge and sediment yield from selected rainfall events. Normally the model operates on a daily mode until it reaches dates of storm events. In these cases, the model shifts to the shorter time interval mode until the storm period terminates. The model then returns to the daily mode. Leavesley and Stannard (1990) used ARC/INFO with PRMS to simulate hydrographs in the East Fork Carson River, and Luellwitz (1991) used ARC/INFO with PRMS to simulate hydrographs in a sub alpine watershed in Colorado. This model has been incorporated into the Modular Modeling System of the U.S.G.S., which is briefly described in the next section.

### Current Developments

Flood hydrology has developed very rapidly after the works of Sherman and Horton in the 1930's and 40's. This has been the result of improvements in our understanding of the fundamental laws governing hydrologic process behavior and of improvements in computational power and observational capabilities. Parallel to the development of increasingly more powerful computer hardware has been the development of computer programming languages, data-base systems, and graphics and image processing tools which have made easier the use of large amounts of input data and have enhanced the analysis and interpretation of the output.

The development of digital terrain models (DTM) and data base systems for spatial data, so-called Geographic Information Systems (GIS), has had a tremendous impact on our ability to describe and understand the effects of highly heterogeneous boundary conditions on hydrologic response. GIS makes it possible to integrate efficiently not only the topography, but the geomorphology, soil type, vegetation and land use characteristics of the basin, with physically-based hydrologic models of watersheds. These geomorphoclimatic and hydraulic characteristics, whose extreme variability in space and time has profound nonlinear effects on hydrologic response, have become increasingly available (*e.g.*, NEXRAD radar precipitation fields) with the advent and rapid development of remote sensing. GRASS of the U.S. Corps of Engineers and ARC/INFO of Environmental Systems Research Institute of California, USA, are two examples of GIS packages widely used in conjunction with DTM's, remotely sensed data (*i.e.*, precipitation), and hydrologic modeling.

The Modular Modeling System (MMS) (Leavesley et al, 1996) is an integrated system of computer software that has been developed with financial support from the U.S.G.S. to provide the research and operational framework needed to support development, testing, and evaluation of physical-process algorithms and to facilitate integration of user-selected sets of algorithms into operational physical-process models. As such, the MMS integrates all of the above referred to improvements in computer software and hardware within a modeling framework aimed at facilitating and improving hydrologic modeling. MMS uses a module library that contains modules for simulating a variety of water, energy, and biogeochemical processes (*e.g.*, it includes the PRMS model). A model is created by selectively coupling the most appropriate modules from the library to create a *suitable* model for the desired application. Where existing modules do not provide appropriate process algorithms, new modules can be developed. The MMS modeling system also provides installation capabilities for new module development, and model development and application using the MMS graphical user interface. The MMS enhances the ability of the modeler to address the specific conditions under consideration, as it allows building of different model configurations, either from a bank of hydrologic simulation modules, or by an explicit coding of new modules. As an example, Epstein and Ramírez (1993, 1994) used the GRASS GIS package with PRMS in an earlier version of the MMS to simulate streamflow hydrographs in the Rio Grande basin. Their work addresses issues of climatic variability and its impact on hydrologic response.

During the last decade, recognition of the significant impact of spatial variability on hydrologic response has prompted renewed interest in the development of spatially distributed hydrologic models for flood routing and for flood risk analyses. In addition to describing the complex interactions of local hillslope processes, variable soil and vegetation characteristics, and channel network, spatially distributed modeling has the advantage of providing predictions at any location throughout the river network, not only at the outlet. This is an important advantage in planning and design problems where predictions are required at a specific river cross section, at which perhaps no historical observations are available.

Mancini and Rosso (1989) used a GIS to investigate the spatial variability of SCS Curve Number within the basin, thus assessing the effects of such variability on basin scale estimates. Brath et al. (1989*a* and 1989*b*) analyzed scale effects in distributed rainfall-runoff modeling, Pilotti and Rosso (1990) presented SHELL (*i.e.*, a general framework for modeling the distributed response of drainage basins to storm rainfall), and Ranzi and Rosso (1991*b*) reported a physically based approach to modeling distributed snowmelt in alpine catchments.

Burlando et al. (1994) developed a methodology for the evaluation of flood risk in a distributed modeling framework. Their flood risk analyzer (FLORA) combines a geomorphologic model of flood frequency with a distributed description of the basin given in terms of soil types, land use, and topography. Runoff potential and infiltration characteristics are described as a function of the SCS Curve Number model. The geomorphologic model of flood frequency is based on the ideas of Eagleson (1972) introduced earlier in this chapter. In their application, Burlando et al. use the approach of Bacchi and Rosso (1988) to derive the second order properties of the frequency distribution of flood peaks for any point of the basin as a function of the stochastic properties of precipitation, and of the soil type, land use, and geomorphologic characteristics of the section in question. Assuming an Independent Poisson Marks model for precipitation (Eagleson, 1972), and the Soil Conservation Service Curve Number model for infiltration losses, they show that the frequency distribution of the annual maxima is an Extreme Value Type II distribution, and obtain expressions for its second order moments as a function of climate and geomorphology. They applied this methodology successfully to the Sansobbia river basin in Northern Italy.

Physically based distributed modeling of watershed processes has become increasingly feasible in recent years. In addition to the development of improved computational capabilities, DTM's and GIS tools, this has been brought about by advances in our understanding of the fundamental physical processes underlying the hydrologic cycle and of the solution of the mathematical equations representing those processes. Generally, the various hydrologic processes governing the transformation of rainfall into runoff are extremely complex, non-linear, and spatially and temporally variable, giving rise to complex sets of strongly coupled non-linear differential equations. Analytical solutions for these equations have been found for the simplest cases only, and one dimensional and two dimensional numerical solution schemes have been developed and applied. For example, Saghafian (1992), Ogden (1992), Ogden and Julien (1993), Ogden et al. (1995), and Saghafian et al. (1995) developed two dimensional numerical schemes for routing the excess rainfall through the overland phase of a watershed. The CASC2D model (*e.g.*, Saghafian et al., 1995), is a distributed-parameter, two-dimensional, unsteady diffusion wave model that routes overland flow using a diffusion wave approximation for the flow dynamics. The equations of motion are solved using a finite difference scheme. The model includes continuous soil-moisture accounting, rainfall interception, infiltration, surface and channel runoff routing, soil erosion and sediment transport. Over each pixel of the domain, flow is allowed to take place only one-dimensionally and in the direction of the steepest slope of either of 2 orthogonal directions in an orthogonal cascade of planes. Input to each pixel is in the form of

precipitation and upstream inflow from all directions. Infiltration losses are diagnosed at each pixel using the Green-Ampt infiltration equations. CASC2D is one of the surface-water hydrologic models support by the Watershed Modeling System (WMS) under development at Brigham Young University. More recently, Fiedler and Ramírez (1998) developed a fully unsteady, fully dynamic, 2-dimensional hydrodynamic wave model for the simulation of the rainfall-runoff process that allows for fully interactive infiltration. In addition to the explicit consideration of fully interactive infiltration, Fiedler and Ramírez developed a numerical scheme capable of describing the flow dynamics as affected by microtopographic conditions and other rapidly varying soil and vegetation characteristics. As such, this capability allows for consideration of issues of scale and takes into account the impact of microchannel formation (*e.g.*, rills, etc.) on runoff production and flood dynamics.