

UNIT-VI

EM Waves characteristics

Plane waves: Refraction & reflection

Normal and oblique incidences: for perfect conductors and perfect dielectrics

Brewster angle, critical angle and total internal reflection

Surface impedance, Poynting vector & Poynting theorem

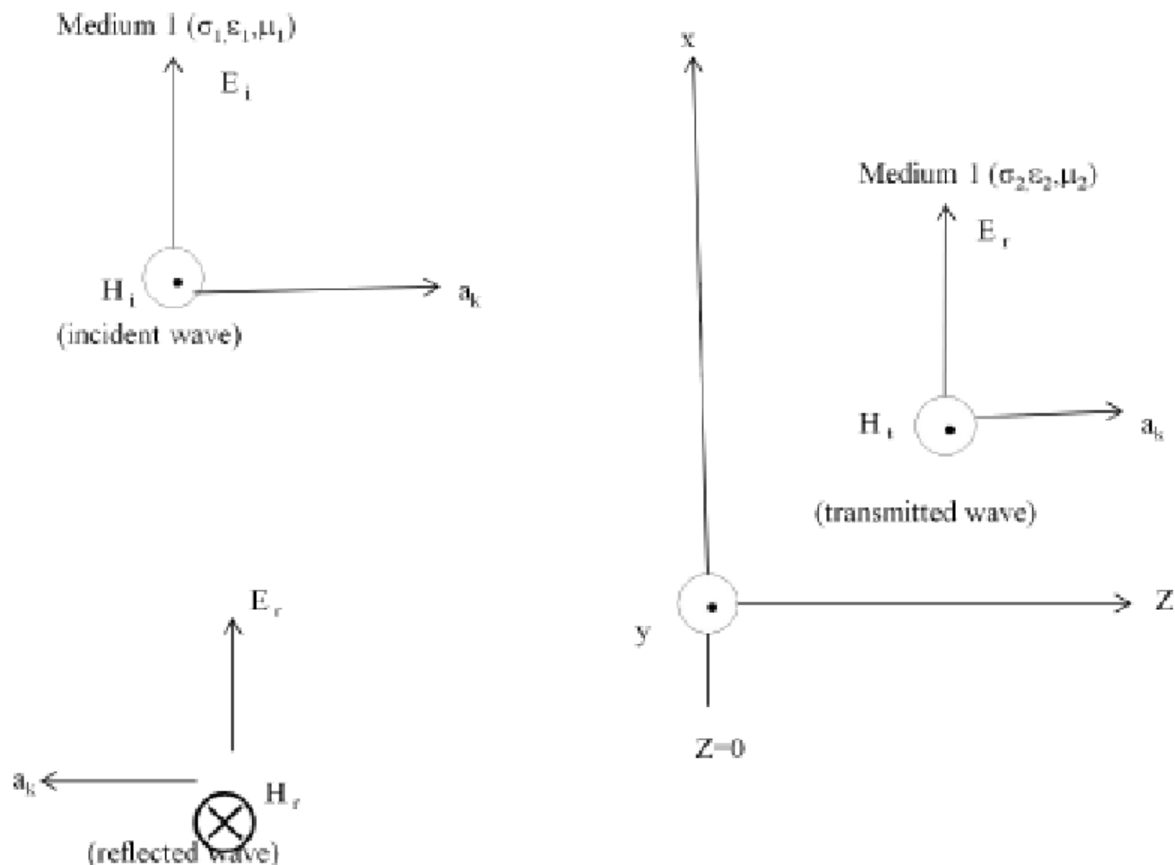
Power loss in plane conductors, problems

5.1 REFLECTION AND REFRACTION OF PLANE WAVES:

At Normal Incidence:

When a plane wave from one medium meets a different medium, it is partly reflected and partly transmitted. The proportion of the wave reflected or transmitted depends on (ϵ, μ, σ) of the two media involved.

Suppose a plane wave propagating along the $+z$ direction is incident normally on the boundary $z=0$ between medium 1 ($z < 0$) with parameters $\epsilon_1, \sigma_1, \mu_1$ and medium 2 ($z > 0$) with parameters $\epsilon_2, \sigma_2, \mu_2$. The subscripts i, r and t represent the incident, reflected and transmitted waves.



Incident wave:

(E, H) is travelling along $+z$ in medium. The time factor $e^{j\omega t}$ is suppressed and we have

$$E_z(z) = E_{i0} e^{-\gamma_1 z} \quad \text{for } z < 0$$

then

$$H_y(z) = H_{i0} e^{-\gamma_1 z} = \frac{E_{i0}}{\eta_1} e^{-\gamma_1 z}$$

Reflected Wave:

(E_r, H_r) is travelling along $-z$ in medium

1. If

$$E_{rs}(z) = E_{rs} e^{-\eta_1 z} \mathbf{ax} = \frac{E_{io}}{\eta_1} e^{-\eta_1 z} \mathbf{ay}$$

Then

$$H_{rs}(z) = H_{rs} e^{-\eta_1 z} (-\mathbf{ay}) = -\frac{E_{io}}{\eta_1} e^{-\eta_1 z} \mathbf{ay}$$

Where E_{rs} has been assumed to be along \mathbf{ax} .

Transmitted wave:

(E_o, H_o) is travelling along $+\mathbf{az}$ in medium

2. If

$$E_{to}(z) = E_{to} e^{-\eta_2 z} \mathbf{ax}$$

Then

$$H_{to}(z) = H_{to} e^{-\eta_2 z} \mathbf{ay} = \frac{E_{to}}{\eta_2} e^{-\eta_2 z} \mathbf{ay}$$

The total fields in medium 1 and medium 2 can be written as follows:

$$\overline{E}_1 = \overline{E}_i + \overline{E}_r, \overline{H}_1 = \overline{H}_i + \overline{H}_r$$

$$\overline{E}_2 = \overline{E}_t, \overline{H}_2 = \overline{H}_t$$

At the interface $z=0$, the boundary conditions require that the tangential components of E and H must be continuous.

At $z=0$

$$\overline{E}_i(0) + \overline{E}_r(0) = \overline{E}_t(0) \rightarrow \overline{E}_{io} + \overline{E}_{ro} = \overline{E}_{to}$$

$$\overline{H}_i(0) + \overline{H}_r(0) = \overline{H}_t(0) \rightarrow \frac{1}{\eta_1} (\overline{E}_{io} - \overline{E}_{ro}) = \frac{\overline{E}_{to}}{\eta_2}$$

From the above two eqns, we obtain

$$\overline{E}_{ro} = \frac{n_2 - n_1}{n_2 + n_1} \overline{E}_{io} \rightarrow (1)$$

and

$$\overline{E}_{to} = \frac{2n_2}{n_2 + n_1} \overline{E}_{io} \rightarrow (2)$$

We know define the reflection coefficient Γ and the transmission coefficient τ from (1) and

(2).

$$\Gamma = E_{ro} = \frac{n_2 - n_1}{n_2 + n_1}$$

$$\text{and } \tau = \frac{E_{to}}{E_{io}} = \frac{2n_2}{n_2 + n_1}$$

We have

1. $1 + \Gamma = \tau$
2. Both Γ and τ are dimensionless and may be complex.
3. $0 \leq |\Gamma| \leq 1$

The above case considered is a general one. When medium 1 is perfect dielectric (loss less, $\sigma_1=0$) and medium 2 is a perfect conductor ($\sigma_2 \cong \infty$) the wave is totally reflected and $N_2 = 0$, $\Gamma = -1$ and $\tau = 0$. Standing waves are formed in the medium 1 because of combination of incident and reflected waves. The standing wave in medium 1 as

$$\bar{E}_{1z} = \bar{E}_i + \bar{E}_r = (\bar{E}_{10}e^{-\alpha_1 z} + \bar{E}_r e^{+\alpha_1 z})\bar{a}_z$$

But

$$\Gamma = \frac{\bar{E}_r}{\bar{E}_{10}} = -1, \sigma_1 = 0, \alpha_1 = 0, \eta_1 = j\beta_1$$

$$\text{Hence, } \bar{E}_{1z} = -\bar{E}_{10} (e^{j\beta_1 z} - e^{-j\beta_1 z})\bar{a}_z$$

$$\text{or } \bar{E}_{1z} = -2j\bar{E}_{10} \sin \beta_1 z \bar{a}_z$$

$$\text{Thus, } \bar{E}_1 = \text{Re}(E_{10} e^{j\omega t})$$

$$\text{Or } \bar{E}_1 = 2E_{10} \sin \beta_1 z \sin \omega t \bar{a}_z \rightarrow (3)$$

By taking similar steps, the magnetic field component of the wave is given by

$$\bar{H}_1 = \frac{2E_{10}}{\eta_1} \cos \beta_1 z \cos \omega t \bar{a}_y \rightarrow (4)$$

The standing wave in equation (3) is presented in figure below for $t=0, \frac{T}{8}, \frac{T}{4}, \frac{3T}{8}, \frac{T}{2}$, and so

on, where $T = \frac{2\pi}{\omega}$.

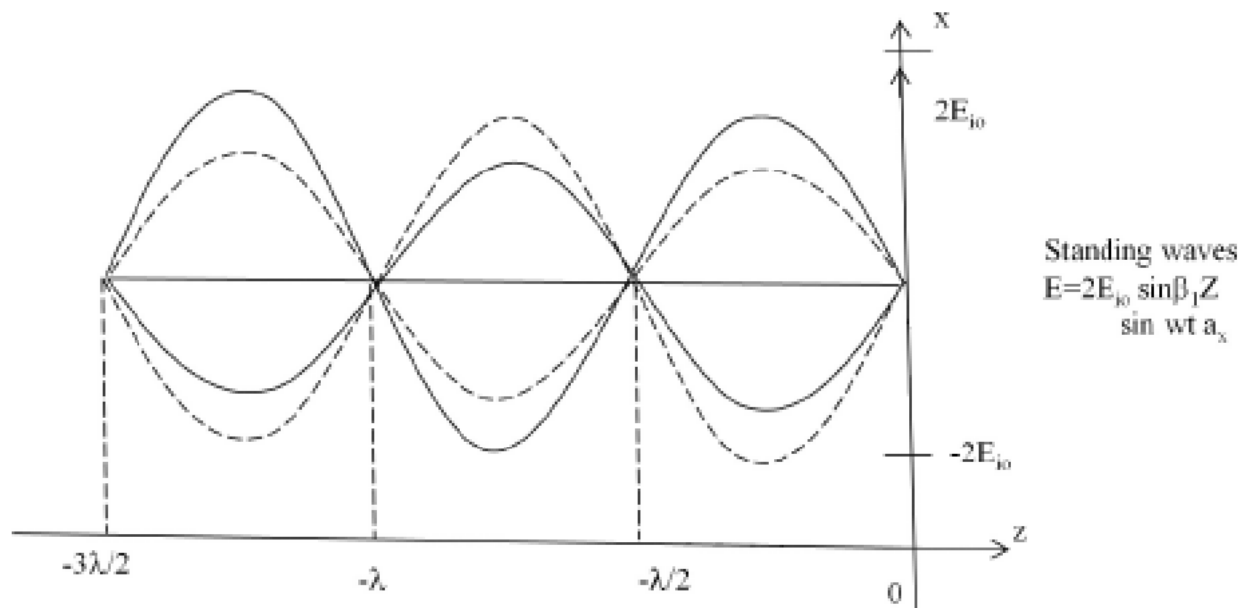
When both the media are loss less i.e $\sigma_1 = \sigma_2 = 0$. In this case, n_1 and n_2 are real and so are Γ and τ .

Case (i):

If $n_2 > n_1$, $\Gamma < 0$. There is a standing wave in medium 1 and a transmitted wave in medium 2. The amplitudes of incident and reflected waves are not equal in magnitude. The maximum value of $|C_1|$ occurs at

$$-\beta_1 Z_{\max} = n\pi$$

$$\text{Or } Z_{\max} = \frac{n\pi}{\beta_1} = \frac{-n\lambda_1}{2}, n = 0, 1, 2, \dots \rightarrow (5)$$



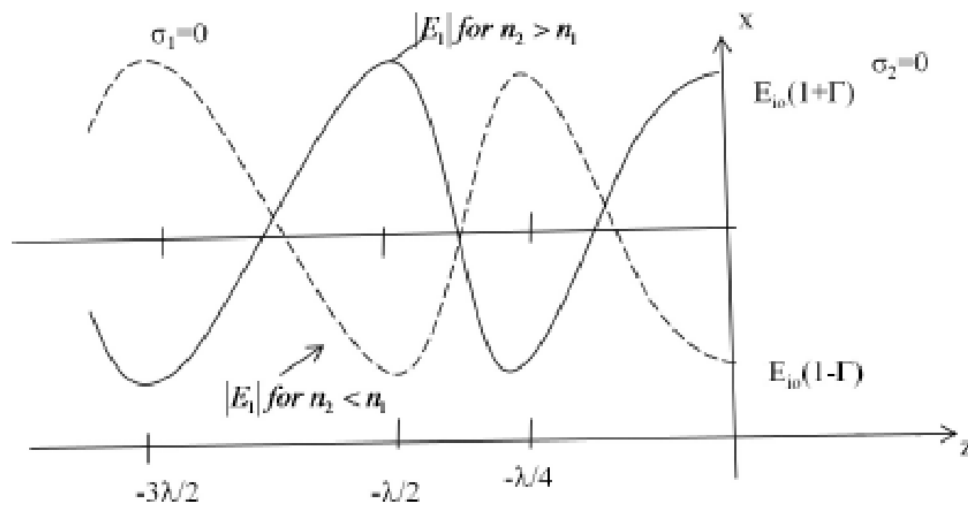
The minimum values of $|E_1|$ occur at

$$-\beta_1 Z_{\min} = (2n+1)\pi/2$$

$$Z_{\min} = -\frac{(2n+1)\pi}{2\beta_1} = \frac{-(2n+1)\lambda_1}{4}, n = 0, 1, 2, \dots \rightarrow (6)$$

Case (ii):

If $n_2 < n_1$, $\Gamma < 0$, the locations of $|E_1|$ minimum are given by eqn (5) and the maximum locations are given by eqn (6). All these are illustrated in graph below:



1. $|H_1|$ is minimum occurs whenever there is $|E_1|$ maximum and vice versa.
2. The transmitted wave in medium 2 is a pure travelling wave and there are no maxima or minima in this region.

$$S = \frac{|E_1|_{\max}}{|E_1|_{\min}} = \frac{|H_1|_{\max}}{|H_1|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\text{or } |\Gamma| = \frac{S-1}{S+1}$$

OBLIQUE INCEDANCE

Consider a uniform plane wave in the general form.

$$E(r,t) = E_0 \cos(\bar{K}\bar{r} - \omega t)$$

$$= \text{Re}[E_0 e^{j(\bar{K}\bar{r} - \omega t)}]$$

Where $\bar{r} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$ is the radius or position vector and $\bar{K} = K_x\bar{a}_x + K_y\bar{a}_y + K_z\bar{a}_z$

is the propagation vector. \bar{K} is always in the direction of wave propagation, we have

$$K^2 = K_x^2 + K_y^2 + K_z^2 = \omega^2 \mu \epsilon$$

Thus, for lossless media, k is the same as β in the previous, derivations.

Maxwell's eqns are

$$\bar{K} * \bar{E} = \omega \mu \bar{H}$$

$$\bar{K} * \bar{H} = -\omega \epsilon \bar{E}$$

$$\bar{K} \cdot \bar{H} = 0$$

$$\bar{K} \cdot \bar{E} = 0$$

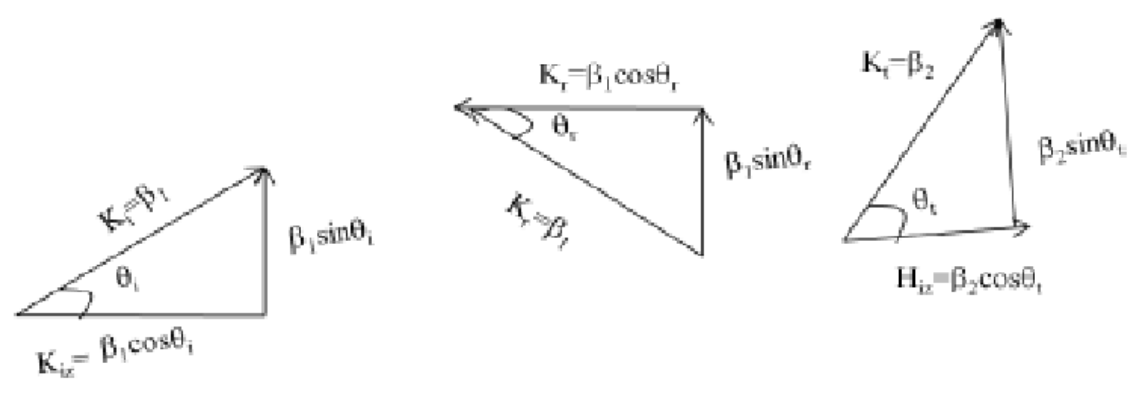
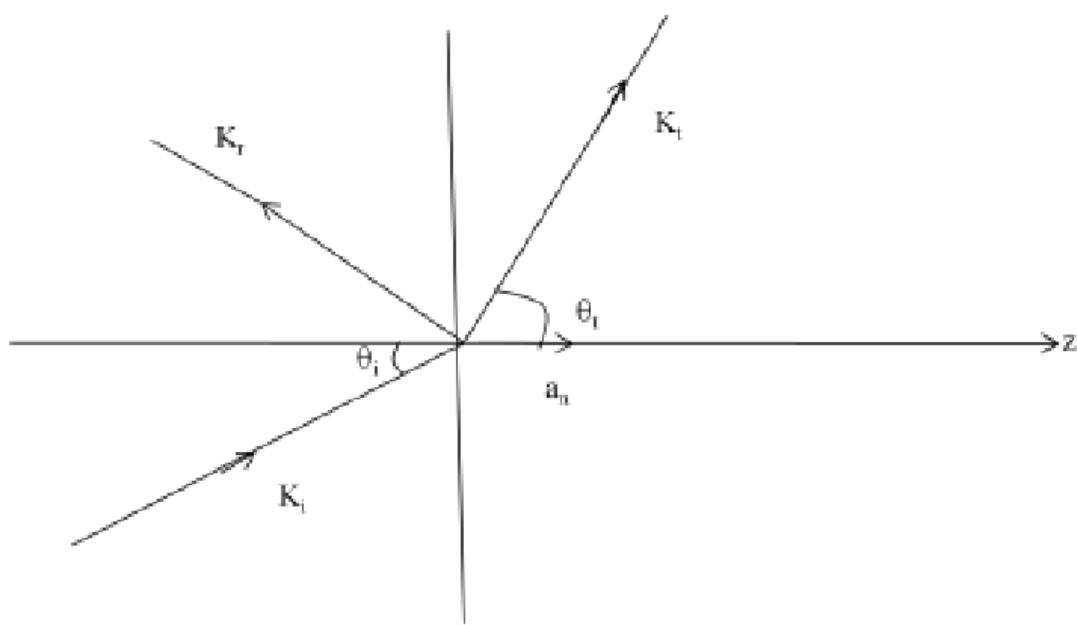
Showing that \bar{E} , \bar{H} and \bar{K} are mutually orthogonal.

$$\bar{k}\bar{r} = k_x x + k_y y + k_z z = \text{constant}$$

The corresponding H field

$$\bar{H} = \frac{1}{\omega \mu} \bar{K} * \bar{E} = \frac{\bar{a}_k * \bar{E}}{\mu}$$

The plane defined by the propagation vector and a unit normal vector \bar{a}_n to the boundary is called the plane of incidence. The angle θ_i between \bar{K} and \bar{a}_n is the angle of incidence.



$K_i \sin \theta_i = k_r \sin \theta_r$ ----- (i)
 And $K_i \sin \theta_i = k_t \sin \theta_t$ ----- (ii)
 For loss less media

$$k_t = k_r = \beta_1 = w\sqrt{\mu_1 \epsilon_1} \text{ -----(iii)}$$

$$k_t = \beta_2 = w\sqrt{\mu_2 \epsilon_2} \text{ -----(iv)}$$

From equations (i) and (iii) we have

$$\theta_r = \theta_i$$

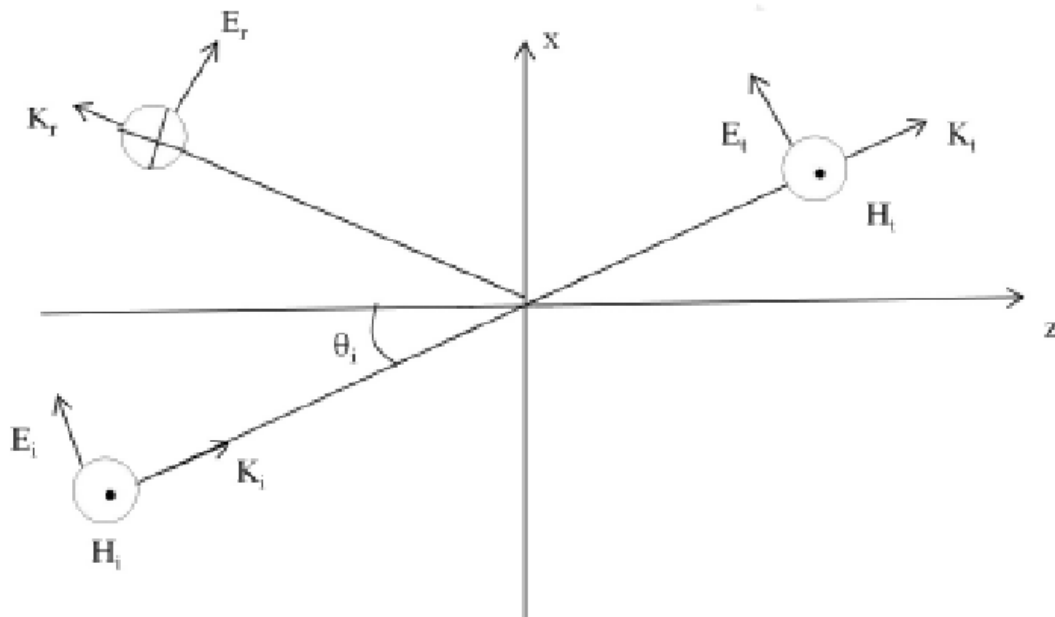
$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{K_t}{K_i} = \frac{\mu_2}{\mu_1} = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}} \text{ -----(v)}$$

The Snell's law can be written as

$$n_1 \sin \theta_i = n_2 \sin \theta_t \text{ -----(vi)}$$

a. parallel Polarization:

The E field lies in the xz-plane, the plane of incidence. This can be understood from the figure below:



$$E_{is} = E_{i0} (\cos \theta_1 a_x - \sin \theta_1 a_z) - j\beta_1 (x \sin \theta_1 + z \cos \theta_1) a_y$$

$$H_{is} = -\frac{E_{i0}}{\eta_1} e^{-j\beta_1 (x \sin \theta_1 + z \cos \theta_1)} a_y$$

$$\text{Where } \beta_1 = w\sqrt{\mu_1 \epsilon_1}$$

We define E_s such that

$$\nabla \cdot \bar{E}_s = 0 \text{ or } \bar{K} \cdot \bar{E}_s = 0 \text{ and then } \bar{H}_s \text{ is obtained from}$$

$$\bar{H}_s = \frac{\bar{K}}{w\mu} * \bar{E}_s = \bar{a}_1 * \frac{\bar{E}}{\eta}$$

The transmitted fields exist in medium 2 and are given by

$$E_{ts} = E_{t0} (\cos \theta_2 a_x - \sin \theta_2 a_z) - j\beta_2 (x \sin \theta_2 + z \cos \theta_2) a_y$$

$$\sin^2 \theta \beta_{11} = \frac{1}{1 + \frac{\epsilon_1}{\epsilon_2}}$$

$$\Rightarrow \sin \theta \beta_{11} = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}$$

$$\text{or } \tan \theta \beta_{11} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{n_2}{n_1}$$

b. perpendicular polarization:

The E field is perpendicular to the plane of incidence over here. The incident and reflected fields in medium 1 are given by

$$\bar{E}_{i,z} = E_{i0} e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)} \bar{a}_y$$

$$\bar{H}_{i,z} = \frac{E_{i0}}{n_1} (-\cos \theta_i \bar{a}_x + \sin \theta_i \bar{a}_z) e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)}$$

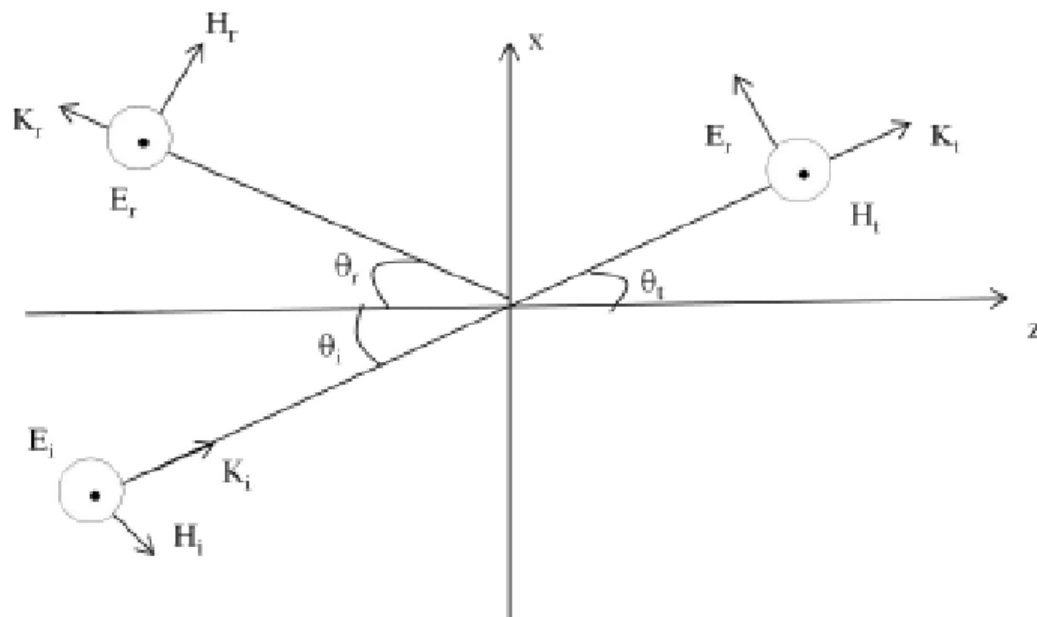
$$\bar{E}_{r,z} = E_{r0} e^{-j\beta_1(x \sin \theta_r + z \cos \theta_r)} \bar{a}_y$$

$$\bar{H}_{r,z} = \frac{E_{r0}}{n_1} (-\cos \theta_r \bar{a}_x + \sin \theta_r \bar{a}_z) e^{-j\beta_1(x \sin \theta_r + z \cos \theta_r)}$$

The transmitted fields in medium 2 are given by

$$\bar{E}_{t,z} = E_{t0} e^{-j\beta_2(x \sin \theta_t + z \cos \theta_t)} \bar{a}_y$$

$$\bar{H}_{t,z} = \frac{E_{t0}}{n_2} (-\cos \theta_t \bar{a}_x + \sin \theta_t \bar{a}_z) e^{-j\beta_2(x \sin \theta_t + z \cos \theta_t)}$$



As the tangential components of \vec{E} and \vec{H} are continuous at $z = 0$ and setting θ_r equal to θ_i , we get

$$E_{io} + E_{ro} = E_{to}, \frac{1}{n_1}(E_{io} - E_{ro})\cos\theta_i = \frac{1}{n_2}E_{to}\cos\theta_i$$

Expressing E_{ro} and E_{to} in terms of E_{io} leads to

$$\Gamma_{\perp} = \frac{E_{ro}}{E_{io}} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$

$$\text{and } \Gamma_{\parallel} = \frac{E_{ro}}{E_{io}} = \frac{2n_2 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$

So we have

$$1 + \Gamma_{\perp} = \tau_{\perp} =$$

For no reflection, $\Gamma_{\perp} = 0$ (or $E_r = 0$)

This is the same case of total transmission ($\tau_{\perp} = 1$). By replacing θ_t with Brewster angle θ_{β_1}

$$n_2 \cos\theta_{\beta_1} = n_1 \cos\theta_i$$

$$n_2^2(1 - \sin^2\theta_{\beta_1}) = n_1^2(1 - \sin^2\theta_i)$$

$$\Rightarrow \sin^2\theta_{\beta_1} = \frac{1 - \frac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1}}{1 - \left(\frac{\mu_2}{\mu_1}\right)^2}$$

For non magnetic media $\mu_1 = \mu_2 = \mu_0$

$\sin^2\theta_{\beta_1} \rightarrow \infty$ if $\delta\mu_1 \neq \mu_2$ and $\epsilon_1 = \epsilon_2$ then

$$\Rightarrow \sin\theta_{\beta_1} = \sqrt{\frac{\mu_2}{\mu_2 + \mu_1}}$$

$$\tan\theta_{\beta_1} = \sqrt{\frac{\mu_2}{\mu_1}}$$

Total Internal Reflection:

The reflection coefficient of parallel and perpendicular polarizations becomes complex when

$$\sin\theta_i > \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

When this happens total internal reflection takes place.

QUESTIONS:

Surface impedance:

The surface impedance is defined as,

$$Z_s = \frac{E_{\text{tan}}}{J_x}$$

Where, E_{tan} is the electric field component parallel to the surface of the conductor.

J_x is the linear current density due to E_{tan} .

From the definition of current density, it is the conduction current per meter width flowing in the thin sheet. If it is assumed that the conductor is flat with its surface at $y = 0$, the current distribution in this direction is given by

$$J = J_0 e^{-\eta y}$$

Where J_0 is the current density at the surface.

It is assumed that the conductor thickness is very much greater than the depth of Penetration. Then there is no reflection from the back surface of the conductor

$$J_x = \int_0^{\infty} J \cdot dy = J_0 \int_0^{\infty} e^{-\eta y} dy = \frac{J_0}{\eta} [e^{-\eta y}]_0^{\infty} = \frac{J_0}{\eta}$$

we know that $J_0 = \sigma E_{\text{tan}}$

$$\therefore Z_s = \frac{E_{\text{tan}}}{J_x} \cdot \frac{\eta}{\sigma}, \because \eta = \sqrt{j\omega\mu\sigma}, Z_s = \sqrt{\frac{j\omega\mu}{\sigma}} = n$$

For a good conductor

$$Z_s = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

Pointing vector and pointing theorem

Poynting theorem:

Statement: It states that the vector product of Electric field intensity vector E and magnetic field intensity vector H at any point is a measure of the rate of energy flow per unit area at that point. The direction of power flow is perpendicular to E and H in the direction of the vector $E \times H$.

Symbolically, the theorem is expressed as

$$P = E \times H \text{ V A.m}^2 \text{ or Watts/m}^2$$

Where P = Poynting's radiation vector

Proof: start with Maxwell's eqn:

$$\begin{aligned}\nabla \times H &= J + \frac{\partial D}{\partial t} \\ J &= \nabla \times H - \frac{\partial D}{\partial t} \rightarrow (1)\end{aligned}$$

This is a relation having dimension of current density and if multiplied by E the resultant would be having dimensions of power per unit volume so by taking dot product of equation (1).

$$E \cdot J = E \cdot (\nabla \times H) = E \cdot \frac{\partial D}{\partial t}$$

But by the vector identity equation we have

$$\begin{aligned}\nabla \cdot (E \times H) &= H(\nabla \cdot E) - E(\nabla \cdot H) \\ \Rightarrow E \cdot (\nabla \times H) &= H(\nabla \cdot E) - \nabla \cdot (E \times H) \rightarrow (2)\end{aligned}$$

Putting equation (2) in (1)

$$\begin{aligned}E \cdot J &= H(\nabla \cdot E) - \nabla \cdot (E \times H) - E \cdot \frac{\partial D}{\partial t} \\ E \cdot J &= H \left[-\frac{\partial B}{\partial t} \right] - \nabla \cdot (E \times H) - E \cdot \frac{\partial E}{\partial t} \\ E \cdot J &= \mu H \cdot \frac{\partial H}{\partial t} - \epsilon E \cdot \frac{\partial E}{\partial t} - \nabla \cdot (E \times H)\end{aligned}$$

$$\mu H \frac{\partial H}{\partial t} = \frac{\mu}{2} \frac{\partial H^2}{\partial t} = \frac{\partial}{\partial t} \left[\frac{\mu H^2}{2} \right]$$

$$\epsilon E \frac{\partial E}{\partial t} = \frac{\epsilon}{2} \frac{\partial E^2}{\partial t} = \frac{\partial}{\partial t} \left[\frac{\epsilon E^2}{2} \right]$$

$$E \cdot J = -\frac{\partial}{\partial t} \left[\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2} \right] - \nabla \cdot (E \times H)$$

Integrating throughout the volume we have

$$\int_V E \cdot J d\vartheta = -\frac{\partial}{\partial t} \int_V \left[\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2} \right] d\vartheta - \int_V \nabla \cdot (E \times H) d\vartheta$$

Applying now Divergence Theorem to the last term to change it from a volume integral to surface integral, we get

$$-\int_S (E \times H) \cdot ds = \int_V E \cdot J d\vartheta + \frac{\partial}{\partial t} \int_V \left[\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2} \right] d\vartheta$$

Alternatively

$$-\int_S (E \times H) \cdot ds = \int_V E \cdot J d\vartheta + \frac{\partial}{\partial t} \int_V \left[\frac{B \cdot H}{2} + \frac{E \cdot D}{2} \right] d\vartheta$$

<p>↓</p> <p>Ingoing power flux over surface S</p>	<p>↓</p> <p>Total dissipated or generated power within volume V at any instant</p>	<p>↓</p> <p>Time rate of increase of total electric magnetic energy within the volume V enclosed by the surfaces at any instant.</p>
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Pointing vector:

From pointing theorem, the vector product of electric field intensity E and magnetic field intensity H is another vector which is denoted by P. This vector P is known as pointing vector. It measures the rate of flow of energy and its direction is the direction of power flow and is perpendicular to the plane containing the \vec{E} and H vectors.

Complex pointing vector:

Complex Pointing Vector:

Maxwell's curl equations in phasor form are:

$$\nabla \times H = (\sigma + j\omega\epsilon)E + J$$

$$\text{and } \nabla \times E = -j\omega\mu H$$

These equations are used to prove the theorem. Consider the identity

$$\nabla \cdot (E \times H^*) = H^* (\nabla \times E) - E (\nabla \times H)$$

The asterisk indicates the complex conjugate.

So we have

$$\nabla \cdot (E \times H^*) = H^* (-j\omega\mu H) - E [(\sigma - j\omega\epsilon)E^* + J^*]$$

$$\nabla \cdot (E \times H^*) = -j\omega\mu H \cdot H^* - (\sigma - j\omega\epsilon) E \cdot E^* - E \cdot J^*$$

Integrating above eqn over the volume V, surrounded by surface S and by using divergence theorem, we get

$$\oint_S (E \times H^*) \cdot ds = -j\omega \int_V (\mu H \cdot H^* - \epsilon E \cdot E^*) d\mathcal{V} - \int_V \sigma E \cdot E^* d\mathcal{V} - \int_V E \cdot J^* d\mathcal{V} \rightarrow (1)$$

$$P_{av} = \frac{1}{2} R_e (E \times E^*) = \int_V \frac{\sigma E^2}{2} d\mathcal{V} / m^2$$

$$R_e (J^* \cdot E) = R_e (\sigma E \cdot E^*) = \omega E^2$$

From(1)

$$\frac{1}{2} \int_S I_m (E \times H^*) \cdot ds = -2\omega \int_V \left[\frac{\mu H \cdot H^*}{4} - \epsilon \frac{E \cdot E^*}{4} \right] d\mathcal{V}$$

$$-I_m \int_S (E \times H^*) \cdot ds = -4\omega \int_V (u_m - u_e) d\mathcal{V}$$

$$u_e = \frac{1}{4} \epsilon H \cdot H^* = \text{Average stored energy in electric field}$$

$$u_e = \frac{1}{4} \mu H \cdot H^* = \text{Average stored energy in magnetic field}$$

The imaginary part of pointing flow through the surface is the reactive power flowing back and forth to supply the instantaneous changes in net stored energy in the volume.

$$v_e = \frac{1}{2} \epsilon E \cdot E^*$$

$$\begin{aligned}
&= \frac{1}{2} \in \operatorname{Re} \left[E_0 e^{j\omega t} e^{j\theta} E_0^* e^{-j\omega t} e^{-j\theta} \right] \\
&= \frac{1}{2} \in E_0 \cos(\omega + t\theta) E_0 \cos(\omega + t\theta) \\
&= \frac{1}{2} \in |E_0|^2 \cos^2(\omega + t\theta) \\
&= \frac{1}{4} \in |E_0|^2 + \frac{1}{4} \in |E_0|^2 \cos 2(\omega + t\theta) \\
&= \frac{1}{4} \in |E_0|^2 + 0 = \frac{1}{4} \in E \cdot E^*
\end{aligned}$$

$$v_n = \frac{1}{4} \mu H \cdot H^*$$

We have

$$\begin{aligned}
&\frac{1}{2} \oint_{\gamma} E^* H^* ds \\
&= \frac{1}{2} \operatorname{Re} \oint_{\gamma} E^* H^* ds + j \frac{1}{2} I_n \oint_{\gamma} E^* H^* ds \\
&\frac{1}{2} \int_{\gamma} E \cdot J^* d\mathcal{G} = \frac{1}{2} \operatorname{Re} \int_{\gamma} E \cdot J^* d\mathcal{G} + j \frac{1}{2} I_n \int_{\gamma} E \cdot J^* d\mathcal{G} \\
&= -j\omega \int_{\gamma} \left(\frac{\mu H \cdot H^*}{2} - \frac{\epsilon E \cdot E^*}{2} \right) d\mathcal{G} \\
&- \frac{1}{2} \int_{\gamma} E \cdot E^* d\mathcal{G} \\
&- \frac{1}{2} \operatorname{Re} \int_{\gamma} E \cdot J^* d\mathcal{G} - j \frac{1}{2} I_n \int_{\gamma} E \cdot J^* d\mathcal{G}
\end{aligned}$$

Equating Real and Imaginary

$$P_{\text{avg}} = \frac{1}{2} R_e \oint_{\gamma} E^* H^* ds = -\frac{1}{2} \int_{\gamma} \sigma E \cdot E^* d\mathcal{G} - \frac{1}{2} R_e \int_{\gamma} (E \cdot J^*) d\mathcal{G}$$

Applications of Poynting Theorem:

1. Pointing vector specifies a coordinate free way of specifying the direction of propagation
2. It is also used to determine the direction of fields if the direction of propagation is unknown.
3. It is applicable to the power flow which is associated with a wire carrying direct current and to a plane wave which is in a loss or conductive region.

Power loss in a plane conductor:

The power flow per unit area through the surface and power loss in conductor can be found out by using pointing vector.

E and H's product gives the power.

$$\text{Surface resistance, } R_s = \sqrt{\frac{w\mu}{2\sigma}}$$

$$P_{avg} = \frac{1}{2} R_s (E_{tan} * H_{tan})$$

For a good conductor E_{tan} leads H_{tan} by 45° in time phase

$$P_{avg} = \frac{1}{2} |E_{tan}| |H_{tan}| \cos 45^\circ$$

$$= \frac{1}{2} |n_m| |H_{tan}|^2$$

$$= \frac{1}{2\sqrt{2}} \frac{|H_{tan}|^2}{|n_m|}$$

$$n_m = Z_s \cdot 2\sqrt{\frac{w\mu}{\sigma}}$$

$$Z_s = R_s + jX_s$$

The linear current density J_s is equal in magnitude to the tangential magnetic field strength at the surface, so

$$P_{avg} = \frac{1}{2\sqrt{2}} |Z_s| |H_{tan}|^2$$

$$= \frac{1}{2\sqrt{2}} |Z_s| |J_s|^2$$

$$\text{Then } P = \frac{1}{\sqrt{2}} |Z_s| (J_s^{eff})^2$$

$$= R_s J_s^2 \text{ eff Watt / Sq-m}$$

Questions:

- (a) Derive the expression for attenuation and phase constants of uniform plane wave. [8]
- (b) If $\epsilon_r = 9, \mu = \mu_0$ for the medium in which a wave with frequency $f = 0.3$ GHz is propagating, determine propagation constant and intrinsic impedance of the medium when [8]
- $\sigma = 0$ and
 - $\sigma = 10$ mho/m.
- (a) Starting from Maxwell's equations, derive the wave equations for an e.m wave in free space. [8]
- (b) A uniform plane wave is incident normally on a plane surface separating two loss less dielectric media. Discuss quantitatively the phenomena that takes place. [8]
- (a) Define surface impedance and explain how it exists? [5]
- (b) Derive expression for reflection and transmission coefficients of an EM wave when it is incident normally on a dielectric. [5]
- (c) The reflected magnetic field $H_r = -\sqrt{2}mA/m$, and the incident electric field in medium 1 (free space) is 1.0 mv/m. The medium2 has $\epsilon_{r,2} = 18.0$ and $\sigma_2 = 0$. Determine the permeability of medium2. [6]

Write short notes on [16]

- Surface impedance
 - Brewster angle
 - Uniform plane wave characteristics
 - Total internal reflection.
- (a) State and Prove Poynting Theorem. [10]
- (b) A Plane wave traveling in a free space has an average poynting vector of 5 watts/ m^2 . Find the average energy density. [6]
- (a) Define complex pointing vector and explain how to obtain an average power. [8]
- (a) Define reflection and transmission coefficients of a plane wave? [8]
- (b) Obtain an expression for reflection coefficient when a wave is incident on a dielectric with oblique incident parallel polarization. [8]
- (b) A plane wave of frequency $= 2$ MHz is incident upon a copper conductor normally. The wave has an electric field amplitude of $E = 2$ mV/m. The copper has $\mu_r = 1, \epsilon_r = 1$ and $\sigma = 5.8 \cdot 10^7$ mho/m. Find out average power density absorbed by the copper. [8]

- (a) Define plane of incidence and reflection coefficient?
 (b) Derive an expression for reflection when a wave is incident on a dielectric obliquely with parallel polarization.

Bits:

1. E . H of a Uniform plane wave is zero
2. Velocity of the wave in an ideal conductor is zero
3. If $E=2V/m$ of a wave in free space ,(H) is $\frac{1}{60} A/m$
4. The velocity of EM wave in free space is independent of f
5. If E is a vector ,then $\nabla \cdot E$ is zero
6. For a uniform plane wave propagating in z-direction $E_z=0$ and $H_z=0$
7. The velocity of propagation of EM wave is $\sqrt{\mu_0 / \epsilon_0}$
8. The Intrinsic impedance of a medium is given by $\sqrt{\mu_r / \epsilon_r}$ is 40
9. Electric field just above a conductor is always normal to the surface
10. Intrinsic impedance of free space is 120
11. Dissipation factor, $D_f = \frac{\sigma}{\omega \epsilon}$
12. Phase constant in good conductors is $\frac{\omega \sqrt{\mu \epsilon}}{2}$
13. β in good dielectric is $\beta = \frac{\omega \sqrt{\epsilon}}{2}$
14. Transmission coefficient = $\frac{\text{Transmitted wave}}{\text{Incident wave}}$
15. Reflection coefficient = $\frac{\text{Reflected wave}}{\text{Incident wave}}$
16. Snell's law is given by $\frac{\sin \theta_i}{\sin \theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$
17. Unit of μ and ϵ are the same (yes/no)
18. The solution of uniform plane wave propagating in x direction is $E = f(x-v_0t)$

19. Propagation constant is $\gamma = \sqrt{\omega^2 \mu \epsilon - j\omega \mu \sigma}$

20. Propagation constant is $\gamma = \sqrt{\omega^2 \mu \epsilon - j\omega \mu \sigma}$

21. Poynting vector $\mathbf{P} = \mathbf{E} \times \mathbf{H}$