Advanced Topics in hydrology

* **Instantaneous unit hydrograph (IUH):**

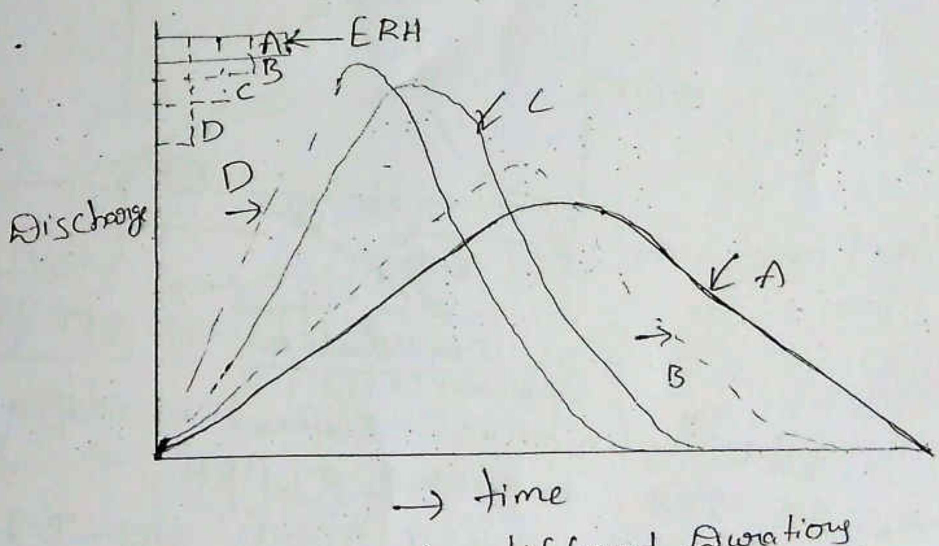
→ for a given catchment a no. of unit hydrographs of different durations are possible.

→ The shape of these different unit hydrographs depend upon the value of D.

→ the below fig shows a typical variation of the shape of unit hydrographs for different values of D.

→ If D is reduced, the intensity of rainfall excess being equal to $\frac{1}{D}$ increases & the UH becomes more skewed.

→ A finite unit hydrograph is indicated as the duration D → 0. the limiting case of a unit hydrograph of zero duration is known as "instantaneous unit hydrograph" (IUH).



UH of different Durations.

→ Thus IUH is a fictitious, (Imaginary (or) not real) ~~contemplative~~ conceptual UH which represents the surface runoff from the catchment due to an instantaneous ppt of the rainfall excess volume of 1 cm.

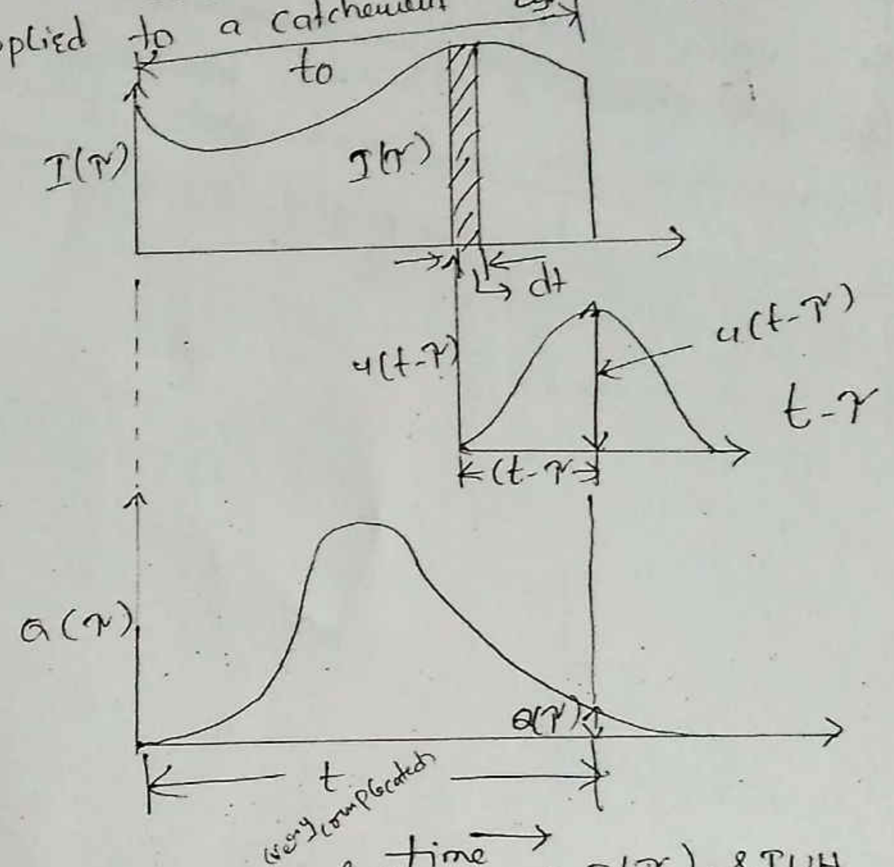
IUH It denotes $u(t)$ (or) $u(0,t)$.

it is single-peaked hydrograph with a unit Daye width

→ its important properties can be given below:

1. $0 \le u(t) \le$ a positive value , for $t > 0$
2. $u(t) = 0$, for $t \le 0$
3. $u(t) \to = 0$, for $t \to \infty$
4. $\int_0^{\infty} u(t)\,dt$ = unit depth over the catchment.
5. time to the peak time to the centroid of the curve.

Consider an Effective rainfall $I(\tau)$ of duration t_0 applied to a catchment as in fig.



Convolution of $I(\tau)$ & IUH

ERH

→ Each (very small) infinitesimal Element of this ERH will operate on the IUH to produce a DRH at discharge at given

$$Q(t) = \int_0^{t'} u(t-\tau)\, I(\tau)\, d\tau \longrightarrow (1)$$

where $t' = t$ when $t < t_0$
$t' = t_0$ when $t \ge t_0$ is called "convolution integral (or) Duhamel integral".

→ IUH can be derived in many ways i) harmonic Analysis ii) Laplace transform iii) conceptual models. & two Simple models Clark's model & Nash's model.

Derivation of IUH:

Consider an S-curve, designated as S_1, in this intensity of rainfall excess $i = 1/D$ cm/h.

Let S_2 another S-curve, its intensity i cm/h. Let S_2 is ordinates subtracted, S_2 separated from S_1 by a time dt & ordinates subtracted S_2 separated from S_1 by a time dt is

DRH due to rainfall excess of duration dt & magnitude $i\,dt = dt/D$ h

$$\text{IUH} = dt/_{\text{DRH of } i\,dt} = \frac{(S_2 - S_1)}{i\,dt} \quad \therefore ds = S_2 - S_1$$

where dt is smaller $dt \to 0$,

$$\text{IUH} \quad u(t) = \lim_{dt \to 0} \left(\frac{S_2 - S_1}{i\,dt} \right) = \frac{1}{i} \frac{ds}{dt} \rightarrow (2)$$

if $i = 1$, then $\boxed{u(t) = \frac{ds'}{dt}} \rightarrow (3)$

Derivation of D-Hour UH from IUH:

from Eq^n (1) derived for complex shaped UH of different duration D-hour UH .. adopted Eq^n (3) from

$$ds' = u(t)\, dt$$

Integrating b/w two points (1) & (2)

$$S_2' - S_1' = \int_{t_1}^{t_2} u(t)\, dt$$

$u(t)$ is essentially linear range 1-2, for small value of $\Delta t = (t_2 - t_1)$ by taking

$$u(t) = \bar{u}(t) = \frac{1}{2}[u(t_1) + u(t_2)]$$

$$S_2' - S_1' = \frac{1}{2}[u(t_1) + u(t_2)][t_2 - t_1]$$

but $(S_2' - S_1')/(t_2 - t_1)$ = ordinate unit ~~dur~~ hydrograph of duration $D_1 = (t_2 - t_1)$. ordinates D_1-UHh are obtained Eq^n

for small values of D_1, ordinates

$$\boxed{(D_1\text{- hr UH})_t = \frac{1}{2}[(\text{IUH})_t + (\text{IUH})_{t-D_1}]}$$

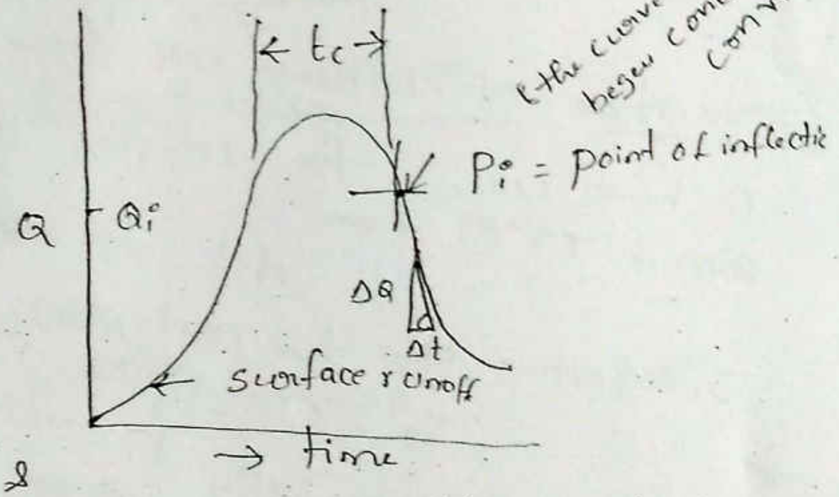
* Clarks Method for IUH. also known as Time-... histogram method aims an developing IUH.

→ the traylation is achieved by a travel time-area histogram curve by routing the results of linear reservoir at the catchment outlet.

Time-Area curve:

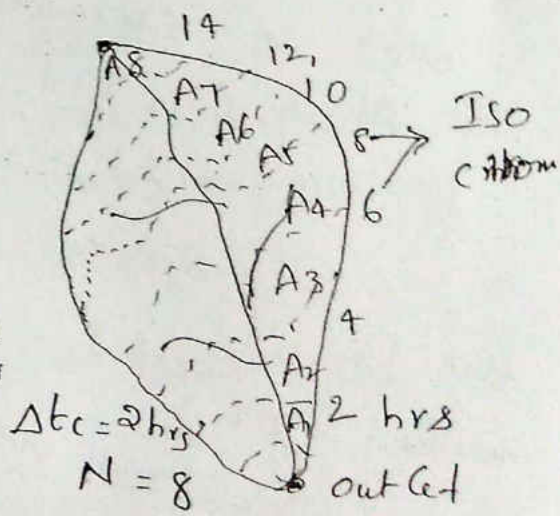
→ the time of concentration t_c is the time required for a unit volume of water from the fartest point of catchment to reach the outlet.

→ In gauged areas the time interval b/w the end of the rainfall excess & the point of inflection of the resulting surface runoff provide good estimating t_c from known data.



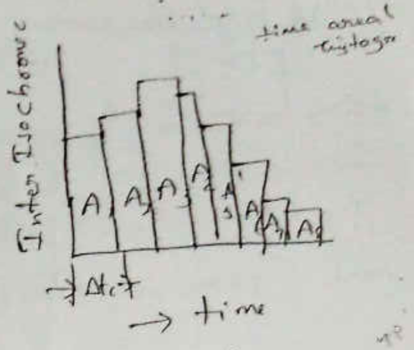
→ If the area considered is located on a map of the catchment, a line joining them is called an "Isochrone".

→ In fig a catchment divided N=8 sub areas & draw Isochrones the distance is devided into N parts of Elevations of subparts measured on the profile trasferred to the contour map of catchment.



Isochrones in a Catchment

→ In inter-Isochrone areas $A_1, A_2 \ldots A_n$ are used to construct a travel time-area histogram.



→ the time-area histogram represents the sequence in which the volume of rainfall will be moved out of the catchment & outlet.

moving time $\Delta t_c = t_c/N$ hrs.

→ In fig properly accounting for the sequence of arrival of flows, do not provide for storage properties of the catchment.

→ clark assumed a linear reservoir to be hypothetically available at the outlet to provide the requisite attenuation

Nash's models:

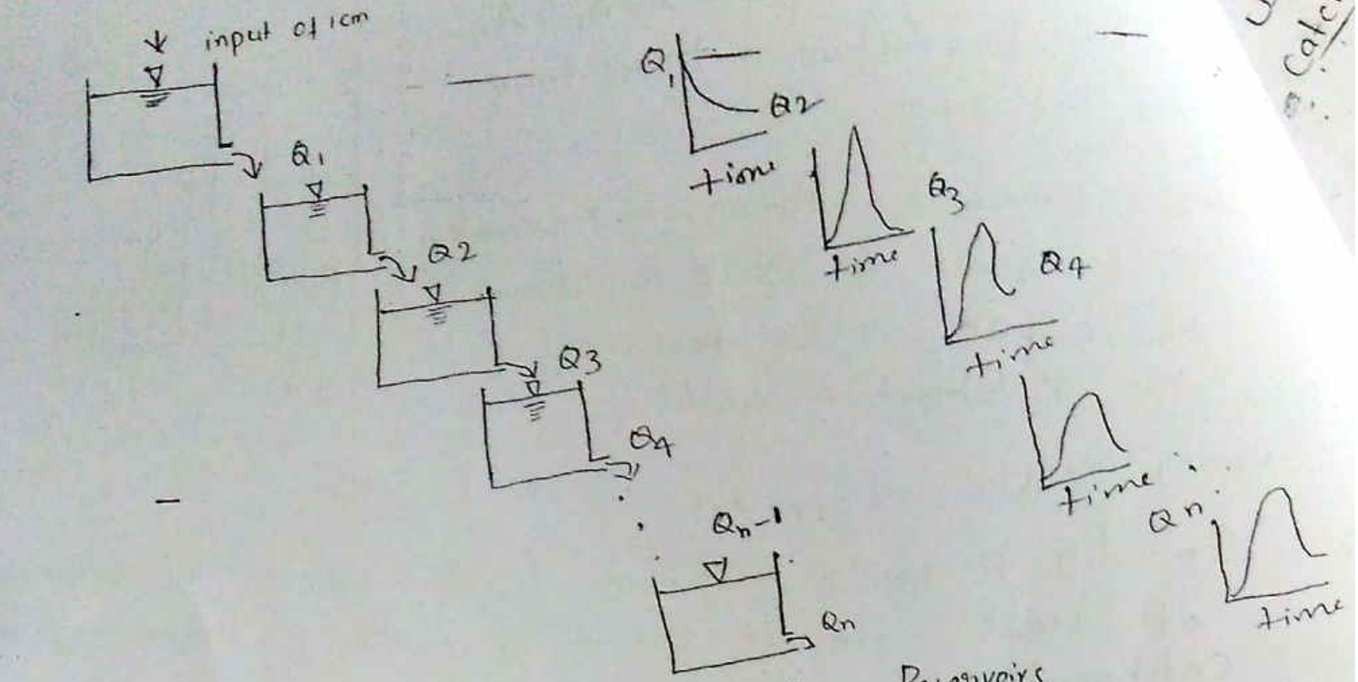
→ Nash (1957) proposed the following conceptual model of a catchment to develop an eqn for IUH.

→ The catchment is assumed to be made up of a series of n identical linear reservoirs each having the same storage constant K.

→ The first reservoir receives a unit volume = 1 cm eff rain fr the catchment.

→ This inflow is routed through the first reservoir to get the outflow hydrograph. the outflow from the first reservoir is considered as the input to second; the outflow from the second reservoir is the input to the third & so on for all the n reservoirs.

→ the shape of the outflow hydrographs from each reservoir of the cascade is shown in fig.



Cascade of Linear Reservoirs

from the eq^n of Continuity $I - Q = \frac{ds}{dt} \rightarrow ①$

for linear reservoir $S = KQ$ & $\frac{ds}{dt} = K\frac{dQ}{dt} \rightarrow ②$

Eq^n ② sub in ①

$$K\frac{dQ}{dt} + Q = I \rightarrow ③$$

differential eq^n, where Q & I are functions of time t,

$$Q = \frac{1}{K} e^{-t/K} \int e^{t/K} I\,dt \rightarrow ④$$

hence for $t>0$, $I = 0$, at $t = 0$, $\int I\,dt$ = Instaneous volume inflow = 1 cm of eff rain.

hence the first reservoir

$$Q_1 = \frac{1}{K} e^{-t/K} \rightarrow ⑤$$

for Second reservoir $Q_2 = \frac{1}{K} e^{-t/K} \int e^{t/K} I\,dt$

I = input = Q_1 given from eq^n ⑤

$$Q_2 = \frac{1}{K} e^{-t/K} \int e^{t/K} \frac{1}{K} e^{-t/K}\,dt$$
$$= \frac{1}{K^2} t e^{-t/K} \rightarrow ⑥$$

$Q_3 = \frac{1}{K} e^{-t/K} \int \frac{1}{K^2} e^{t/K} t$

$= \frac{1}{K^3} e^{-t/K} \int t$

$= \frac{1}{2}\frac{1}{K^3} e^{-t}$

for third reservoir

$I = Q_2$ & Q_3 is obtained $Q_3 = \frac{1}{2}\frac{1}{K^3} t^2 e^{-t/K}$

$||^{ly}$ hydrograph of outflow from the n^{th} reservoir Q_n

$$Q_n = \frac{1}{(n-1)!\,K^n} t^{n-1} e^{-t/K} \rightarrow ⑦$$

Using notation ..., To represent the ordinate of IUH, of catchment is

$$u(t) = \frac{1}{(n-1)!\,k^n}\, t^{n-1} e^{-t/k}$$

k & n are constants & $(n-1)!$ to integer fractional values of n, the gamma function $\Gamma(n)$ is used replace $(n-1)$

$$u(t) = \frac{1}{K\Gamma(n)} (t/k)^{n-1} e^{-t/k}$$

when n is integer, $\Gamma(n) = (n-1)!$; n, $\Gamma(n)$ is from obtained from gamma tables.

* Unsteady flow in a confined Aquifer: (or) Chow-model:

→ when a well in a confined aquifer starts discharging, the water from the aquifer is released resulting in the formation of a cone of depression of the piezometric surface.

→ This cone gradually expands with time till an equilibrium is attained.

→ the flow configuration from the start of pumping till the attainment of equilibrium is in unsteady regime & is described by

$$\nabla^2 h = \frac{S}{T}\frac{\partial h}{\partial t} \rightarrow ①$$

In polar co-ordinates, to represent the radial flow into a well,

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \rightarrow ②$$

→ Making the same assumptions as used in the derivation of the equilibrium formula (it is also known as thiem's Eq^n)

$$Q = \frac{2\pi KB(h_2-h_1)}{\log_e \frac{r_2}{r_1}}$$

$2.303 \times 2 = 4$

$T = KB$

In (1935) obtained the sol^n of Eq^n

$$S = (H-h) = \frac{Q}{4\pi T}\int_u^{\infty} \frac{e^{-u}}{u}\,du \rightarrow ③$$

where $S = (H-h)$ = drawdown at a pt, u = a parameter $= \frac{r^2}{4T}$

H = Initial constant piezometric head

where t = time from start of pumping

→ the integral on the right hand side is called the Mean time

using theis method $W(u) = \int_u^{\infty} \frac{e^{-u}}{u}\,du = -0.577216 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!}$

→ the estimation of the aquifer constants S and T from the drawdown Vs time data of a pumping well, which involve trial - and - error procedure, can be done either by a digital computer (or) by semigraphical methods such as the use of type curve (or) by chow's method.

→ for small values of u ($u \le 0.01$) from eqn ② & ⑤ Jacob showed calculation of first two terms series of $w(u)$.

$$s = \frac{Q}{4\pi T}\left[-0.5772 - \ln \frac{r^2 S}{4Tt}\right]$$

log (a-b) = log ab

$$= \frac{Q}{4\pi T} \ln\left[\frac{2.2Tt}{r^2 S}\right] \rightarrow ⑥$$

If s_1 & s_2 are drawdowns at times t_1 & t_2

$$(s_2 - s_1) = \frac{Q}{4\pi T} \ln \frac{t_2}{t_1}$$

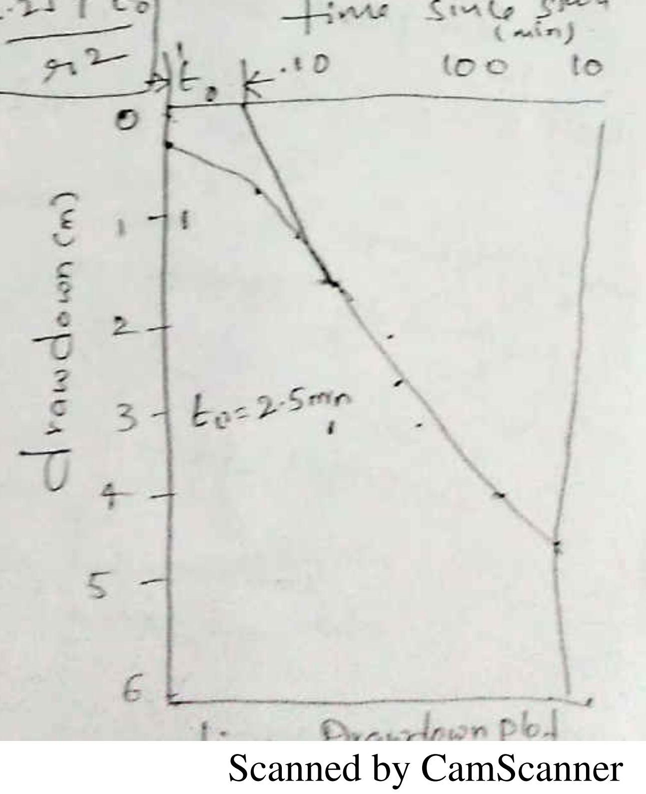
→ If the drawdown is plotted against time t on a semi-log paper, the plot will be a straight line for large values of time. the slope of this line enables the storage co-efficient S to be determined. from eqn ⑤, when s = 0,

$$\frac{2.25Tt_0}{r^2 S} = 1$$

$$\boxed{S = \frac{2.25Tt_0}{r^2}}$$

t_0 = time corresponding to "zero" drawdown obtained straight-line

→ semi-log curve of s Vs t



Drawdown plot

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Rainfall - runoff - modelling : The hydrologic water budget equation for the determination of runoff for a given period is

$$R = R_s + G_o = P - E_{et} - \Delta S$$

ppt

soil moisture storage

actual Evapotranspiration

Net ground water outflow

Surface runoff

- R_s & G_o is total runoff R.

→ the above Eqn use to calculate R by knowing values of P and functional dependence of E_{et}, ΔS & infiltration rates with catchment & climatic conditions.

→ for accurate results the functional dependence of various parameters governing the runoff in the catchment & value P at short time intervals are needed.

→ Calculation of runoff by using manually and digital computers the use of water budgeting of runoff.

→ this techinque of predicting the runoff, which is the catchment reponse to a given rainfall input is called "deterministic watershed simulation".

→ In this the mathematical relationships describing the interdependence of various parameters in the system are first prepared & this is called the "model"

→ the model is then calibrated, i.e the numerical values of various co-eff determined by simulating the known rainfall - runoff records.

→ the accuracy of the model is further checked by reproducing, the results of another string of rainfall data for which runoff values are known. this phase is known as "validation (or) verification of the model". After this model is ready for use.

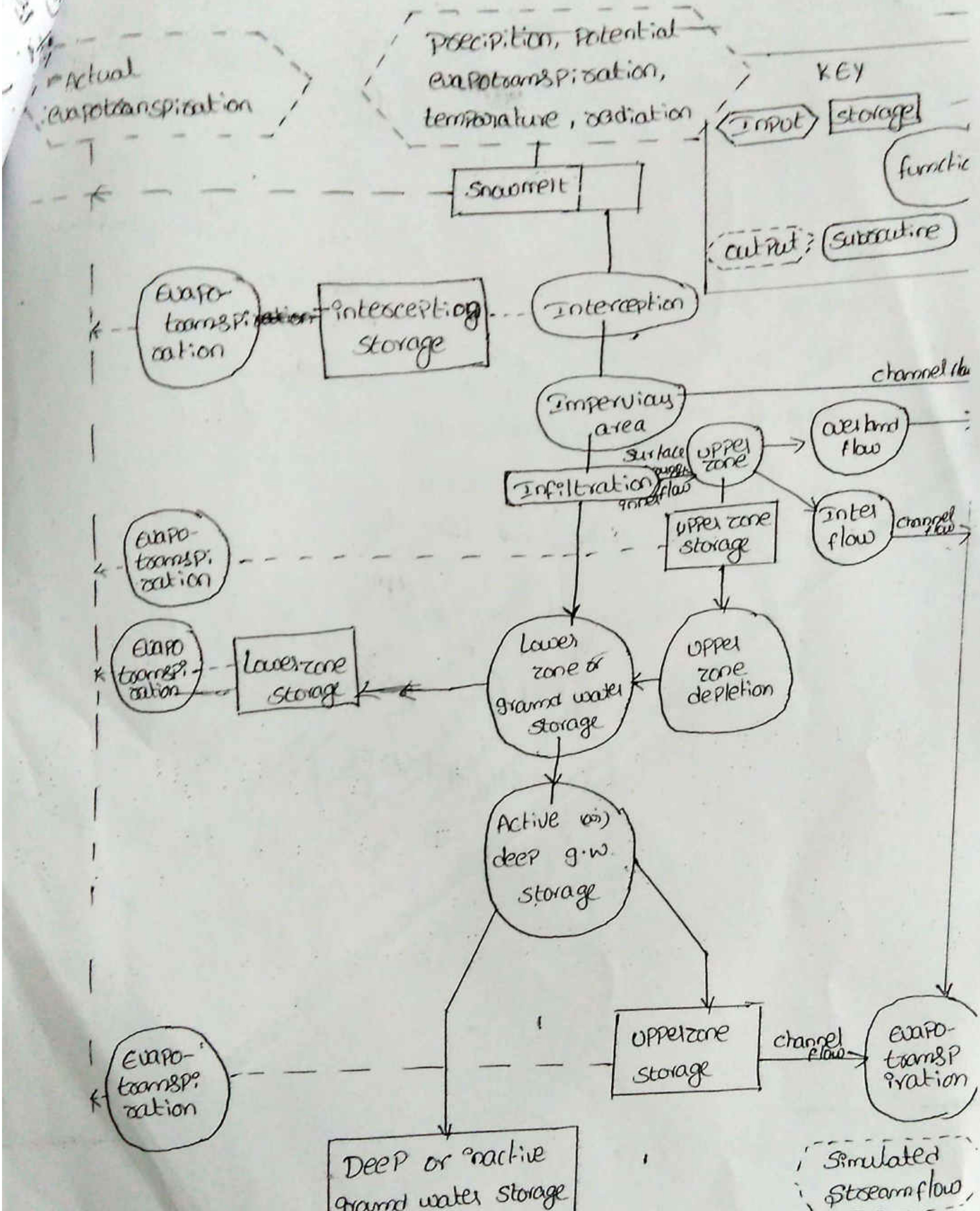
→ Crawford & Linsley (1959) pioneered this techque by proposing a watershed simulation model known as stanford watershed model (SWM).

→ this underwent successive refinements & the Stanford watershed Model-IV (SWM-IV) suitable for use on a wide variety of conditions was proposed in 1966.

→ the model consider the soil in 3 zones with distinct properties to simulate evapotranspiration, infilt overland flow, channel flow, interflow & baseflow of the runoff phenomenon.

→ for calibration about 5 years of data are needed

→ using an additional length of rainfall-runoff of about 5 years duration, the model is verified for its ability to give proper response.



flow chart of SWM-IV

Determination of n & k of Nash's Model: $u(t) = (n-1)!\, k^n$

→ first moment of the IUH about the origin $t = 0$

given $M_1 = nK$

→ Second moment of the IUH about the origin $t = 0$

$$M_2 = n(n+1)K^2$$

→ using these properties the values of n & k for a catchment can be determined if the ERH & corresponding DRH are

for 1st is $M_{Q_1} - M_{I_1} = nK$

where M_{Q_1} : first moment of DRH about time origin divided by the total direct runoff

M_{I_1} = " " ERH " " " " Effective rainfall

for 2nd $M_{Q_2} - M_{I_2} = n(n+1)K^2$

Anti ln $-0.5772 = 0.5164$ $\qquad e^{-0.5772} = 0.516$

$$S = \frac{Q}{4\pi T}\left[\cancel{\log}\, 0.5614 + \ln\frac{4Tt}{r^2 S}\right]$$

$$= \frac{Q}{4\pi T}\ln\left[\frac{2.24Tt}{r^2 S}\right]$$