Architecture 324

Structures II

Steel Beam Analysis and Design

- Steel Properties
- Steel Profiles
- Steel Codes: ASD vs. LRFD
- Analysis Method
- Design Method

Cold Form Sections

Photos by Albion Sections Ltd, West Bromwich, UK

Cold Form Sections

From:

Building Design Using Cold Formed Steel Sections: Structural Design to BS 5950-5:1998. Section Properties and Load Tables. p. 276

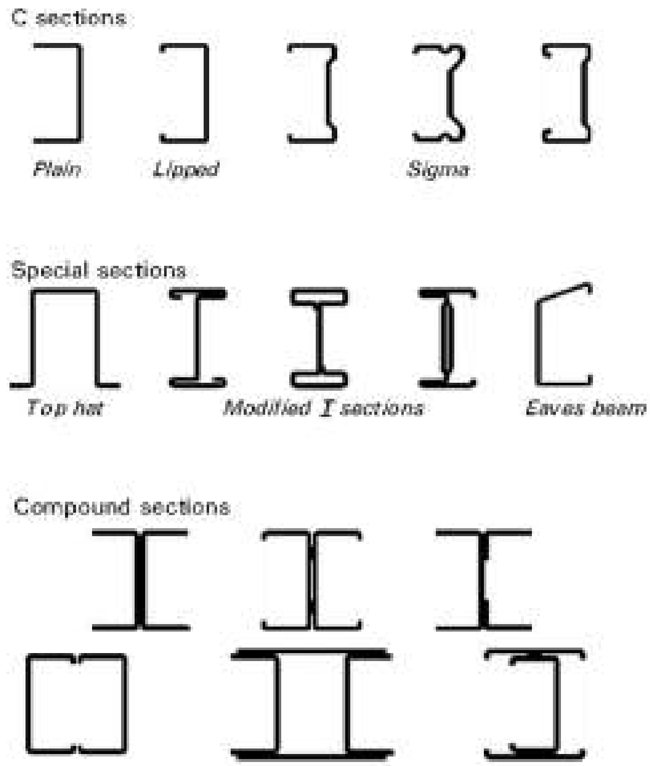
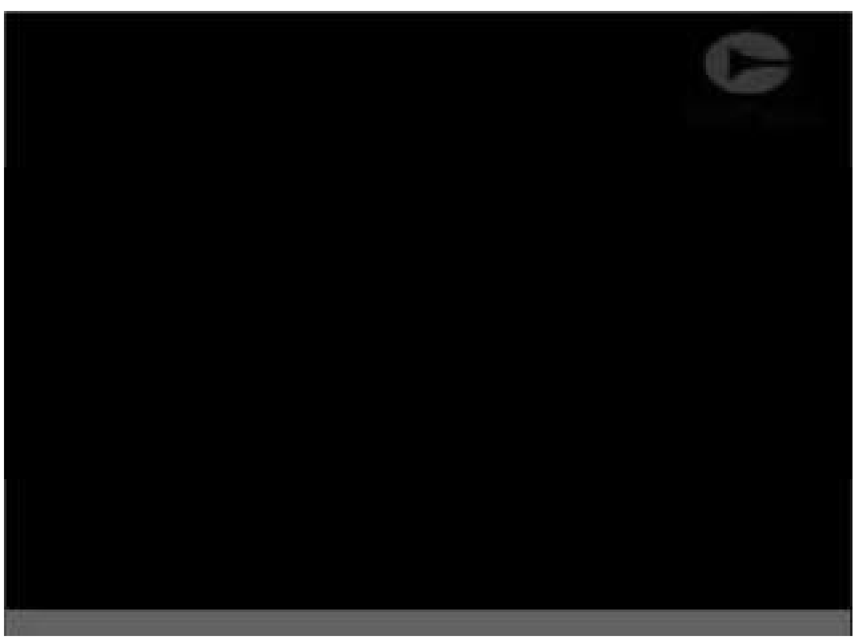
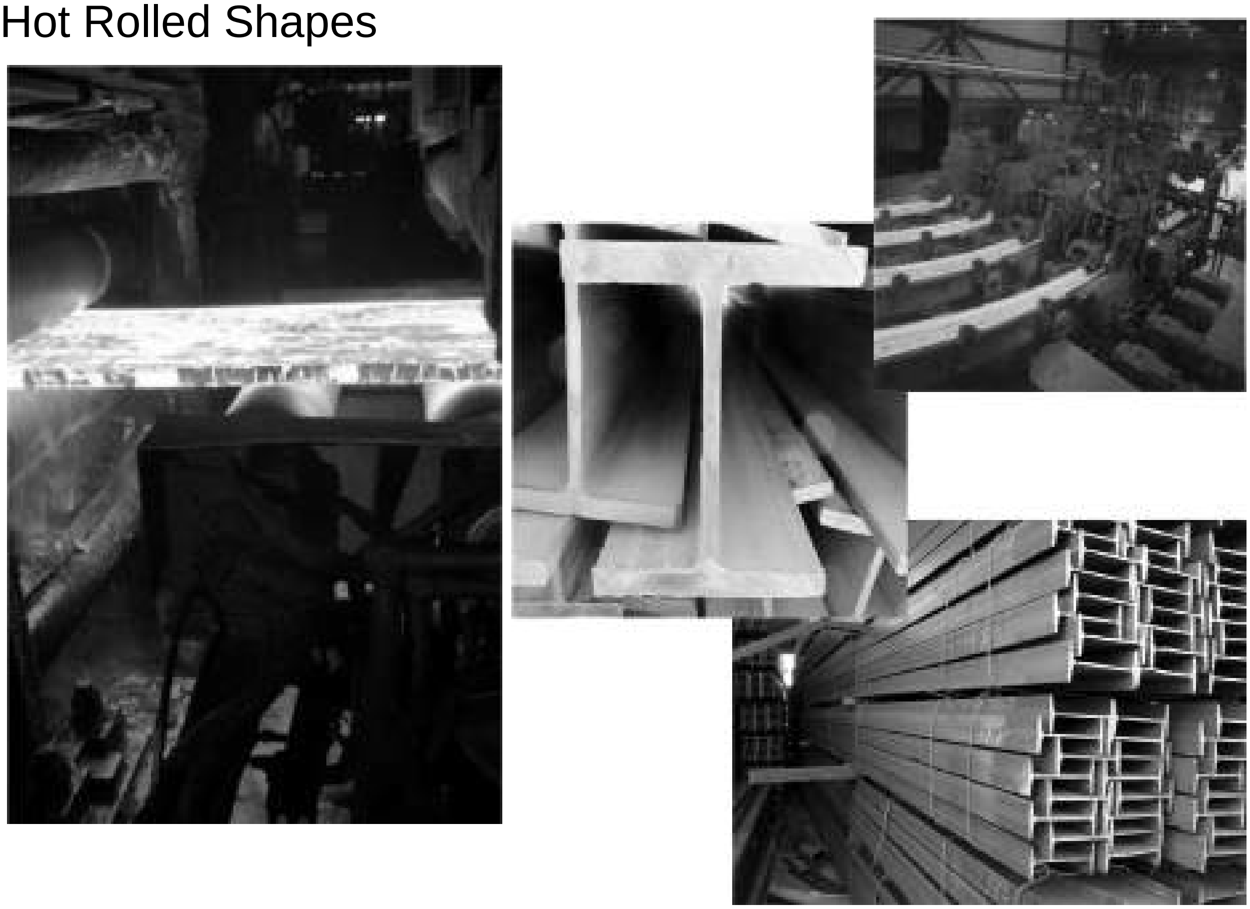


Figure 2.3 *Examples of cold formed steel sections*

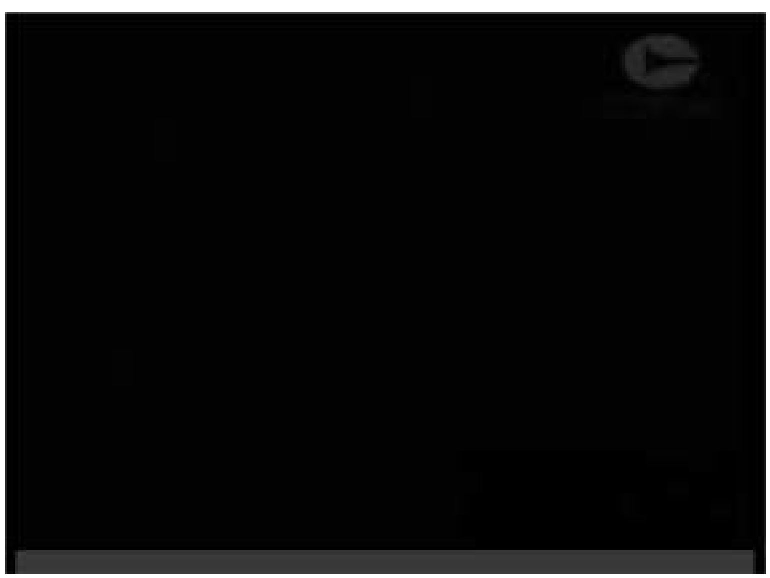
Cold Form Sections



Hot Rolled Shapes



Hot Rolled Shapes



Nomenclature of steel shapes

Standard section shapes:

W – wide flange

S – American standard beam

C – American standard channel

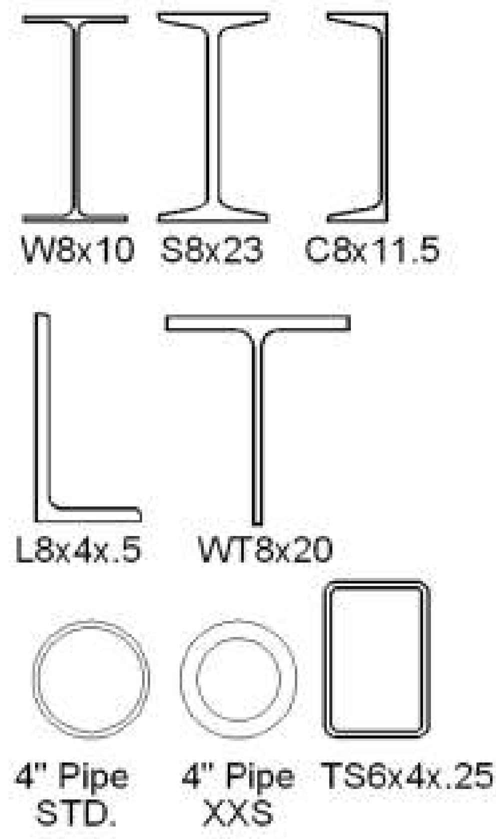
L – angle

WT or **ST** – structural T

STD, **XS** or **XXS** – Pipe

HSS – Hollow Structural Sections
Rectangular, Square, Round

LLBB , SLBB - Double Angles

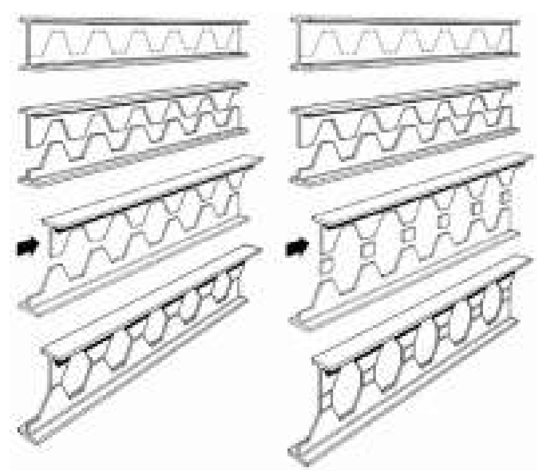


Nomenclature of steel shapes

Castellated Sections:

round

hexagonal



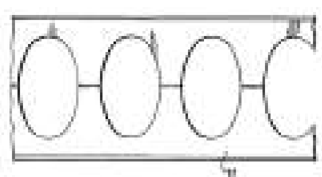
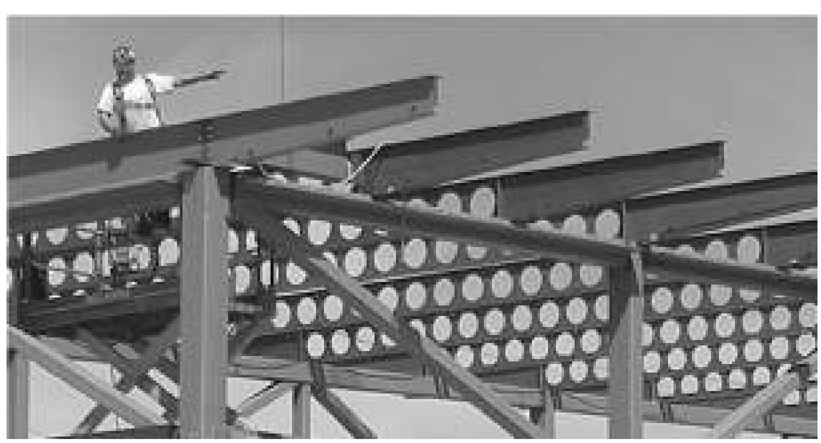


Fig.2A.

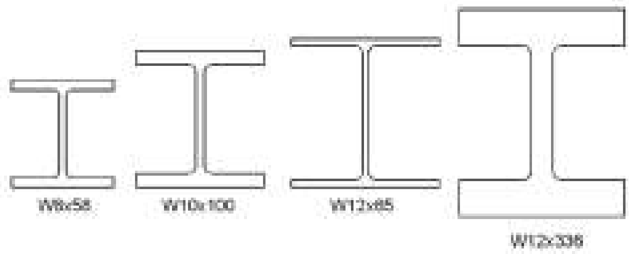
Fig.2B



Steel W-sections for beams and columns

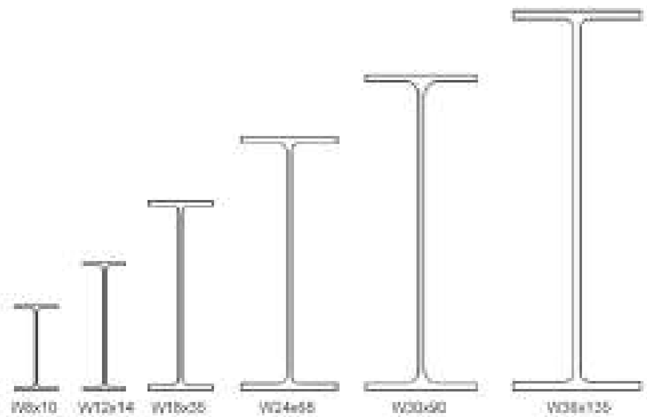
Columns:

Closer to square
Thicker web & flange



Beams:

Deeper sections
Flange thicker than web



Steel W-sections for beams and columns

Columns:

Closer to square
Thicker web & flange

Beams:

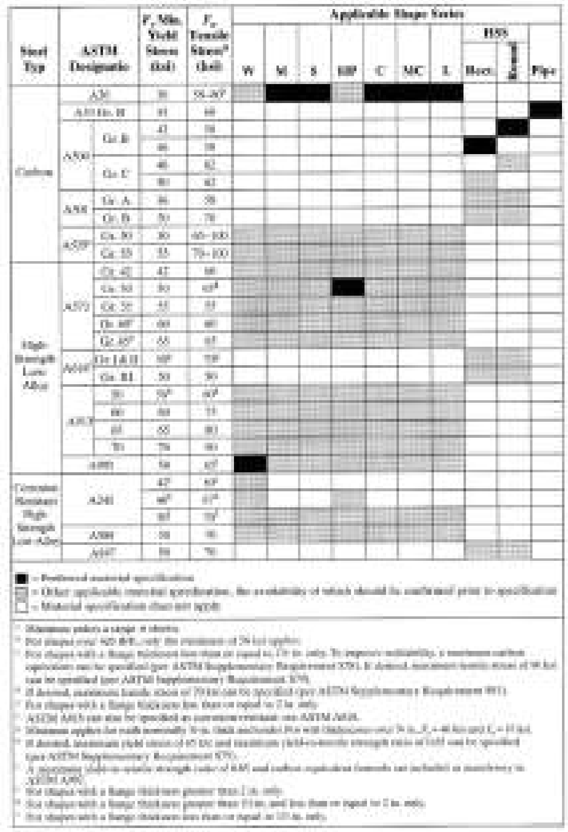
Deeper sections
Flange thicker than web

Photo by Gregor Y.

Steel Grades – Rolled Sections

Different sections are make with different grades of steel.

Most structural shapes are:
A36 Carbon Steel Fy = 36ksi
A992 High Strength Fy = 50 ksi



Steel Type	ASTM Designation		F_y Min. Yield Stress (ksi)	F_u Tensile Stress (ksi)
Carbon	A36		36	58–80
	A53 Gr. B		35	60
	A500	Gr. B	42	58
			46	58
		Gr. C	46	62
			50	62
	A501	Gr. A	36	58
		Gr. B	50	70
	A529	Gr. 50	50	65–100
		Gr. 55	55	70–100
High-Strength Low-Alloy	A572	Gr. 42	42	60
		Gr. 50	50	65
		Gr. 55	55	70
		Gr. 60	60	75
		Gr. 65	65	80
	A618	Gr. I & II	50	70
		Gr. III	50	65
	A913	50	50	60
		60	60	75
		65	65	80
		70	70	90
	A992		50	65
Corrosion Resistant High-Strength Low-Alloy	A242		42	63
			46	67
			50	70
	A588		50	70
	A847		50	70

■ = Preferred material specification.
▒ = Other applicable material specification, the availability of which should be confirmed prior to specification.
□ = Material specification does not apply.

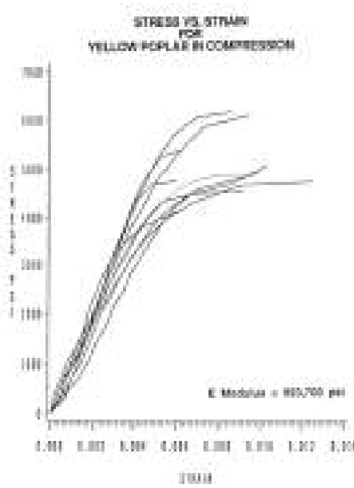
Source: AISC Manual, Table 2-4, p. 2-48, 14th ed. 2011. Copyright © American Institute of Steel Construction. Reprinted with permission. All rights reserved.

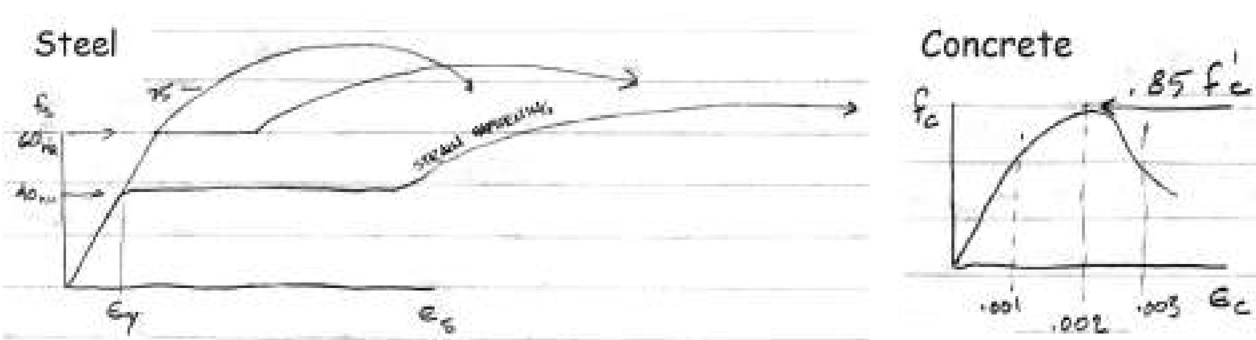
Young's Modulus

Young's Modulus or the Modulus of Elasticity, is obtained by dividing the stress by the strain present in the material. (Thomas Young, 1807)

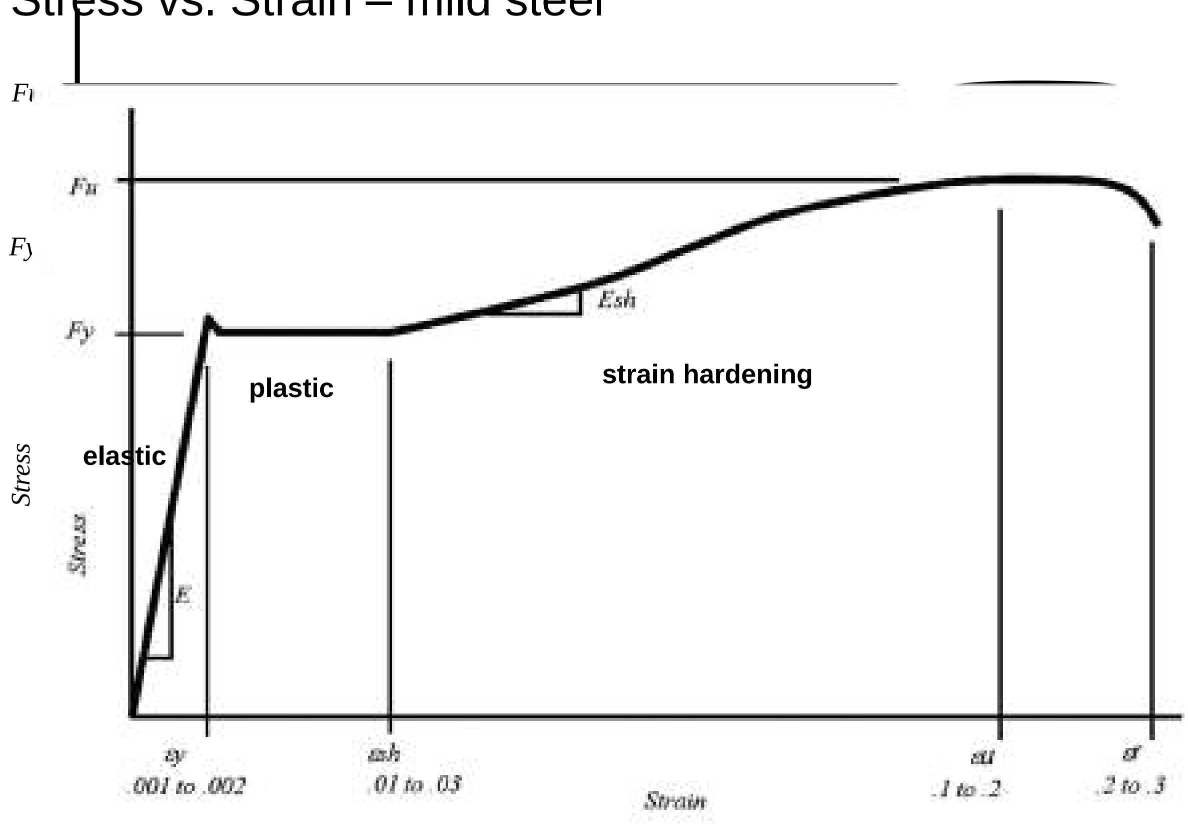
$$E = \frac{P/A}{D/L} = \frac{\sigma}{\varepsilon}$$

It thus represents a measure of the stiffness of the material.



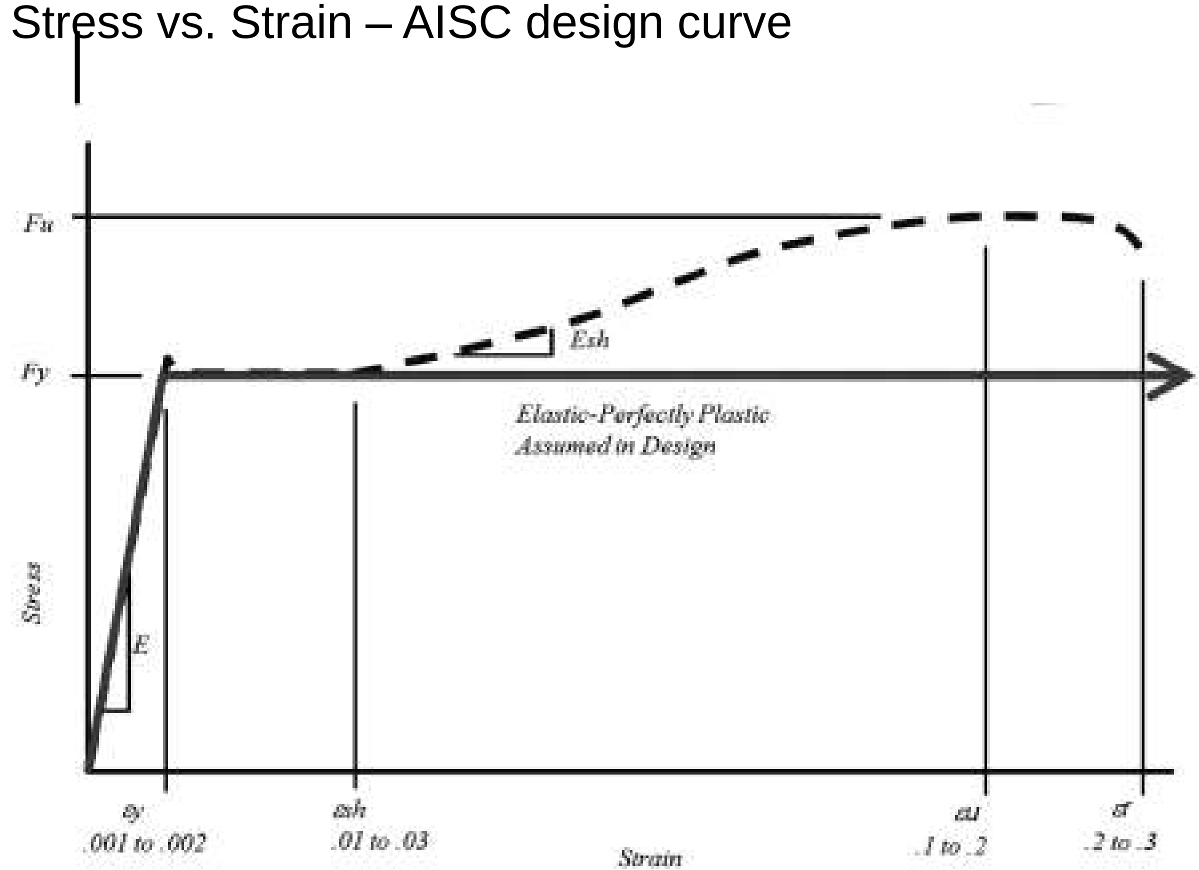


Stress vs. Strain – mild steel



Developed by Scott Civjan
University of Massachusetts, Amherst

Stress vs. Strain – AISC design curve



Stress Analysis

Allowable Stress Design (ASD)

- use design loads (no F.S. on loads)
- reduce stress by a Factor of Safety F.S.

$$f_{actual} = \frac{P}{A}$$

$$f_{actual} \le F_{allowable}$$

$$F_{allowable} = F.S. \cdot f_{yield}$$

Load & Resistance Factored Design (LRFD)

- Use loads with safety factor γ
- Use factor on ultimate strength ϕ

$$P_{load} = \gamma \cdot P_{applied\_load}$$

$$P_{load} \le P_{resisting}$$

$$P_{resisting} = \phi \cdot P_{material\_strength}$$

LRFD Analysis

Load & Resistance Factored Design (LRFD)

- Use loads with safety factor γ
- Use forces with strength factor ϕ

$$P_{load} = \gamma \cdot P_{applied}$$

$$P_{load} \le P_{resisting}$$

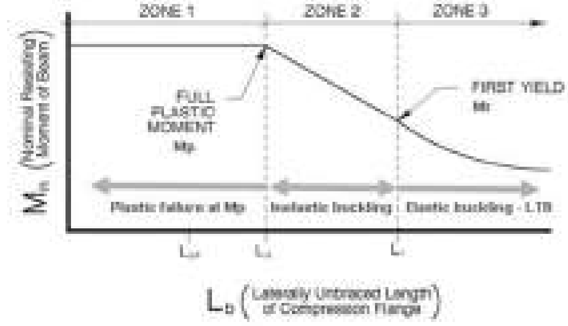
$$P_{resisting} = \phi \cdot P_{material}$$

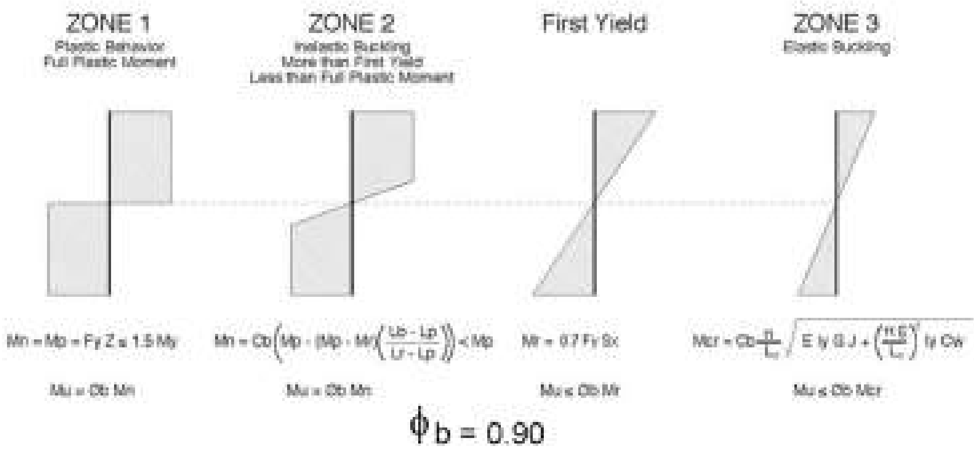
Design Strength $P_u \le \phi\ P_n$ Required (Nominal) Strength

2.3 COMBINING FACTORED LOADS USING STRENGTH DESIGN

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
3. $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$
4. $1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$
5. $1.2D + 1.0E + L + 0.2S$
6. $0.9D + 1.0W$
7. $0.9D + 1.0E$

Beam Strength vs Unbraced Length





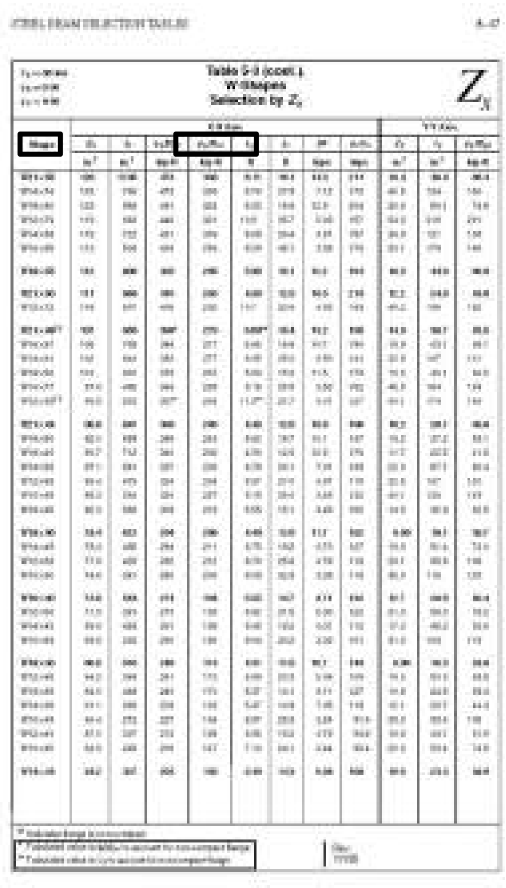
Steel Beams by LRFD

Yield Stress Values

- A36 Carbon Steel Fy = 36 ksi
- A992 High Strength Fy = 50 ksi

Elastic Analysis for Bending

- Plastic Behavior (zone 1)
  Mn = Mp = Fy Z < 1.5 My
  - Braced against LTB (Lb < Lp)
- Inelastic Buckling "Decreased" (zone 2)
  Mn = Cb(Mp-(Mp-Mr)[(Lb-Lp)/(Lr-Lp)] < Mp
  - Lp < Lb < Lr
- Elastic Buckling "Decreased Further" (zone 3)
  Mcr = Cb * π/Lb √(E*Iy*G*J + (π*E/Lb)^2 * IyCw)
  - Lb > Lr



from AISC 2003

Elastic Design for Shear

Shear stress in steel sections is approximated by averaging the stress in the web:

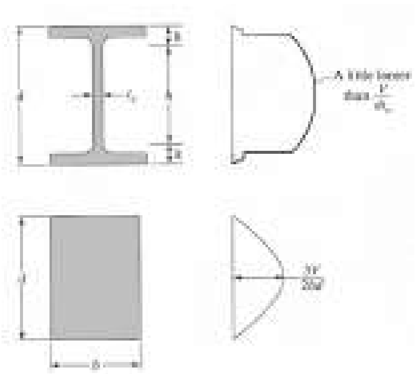
$F_v = V / A_w$

$A_w = d * t_w$

To adjust the stress a reduction factor of 0.6 is applied to F_y

$F_v = 0.6\ F_y$

so, $V_n = 0.6\ F_y\ A_w$ **(Zone 1)**



The equations for the 3 stress zones:
(ϕ in all cases = 1.0)

Zone 1:
WEB YIELDING (Most beam sections fall into this category)

if $\frac{h}{t_w} \le 2.45\sqrt{E/F_y} = 59$ (for 50 ksi steel)

then: $V_n = 0.6\ F_y\ A_w$

Zone 2:
INELASTIC WEB BUCKLING

if $2.45\sqrt{E/F_y} < \frac{h}{t_w} \le 3.07\sqrt{E/F_y} = 74$ (for 50 ksi steel)

then: $V_n = 0.6\ F_y\ A_w\ (2.45\sqrt{E/F}\ ) / \frac{h}{t_w}$

Zone 3:
ELASTIC WEB BUCKLING

if $3.07\sqrt{E/F_y} < \frac{h}{t_w} \le 260$

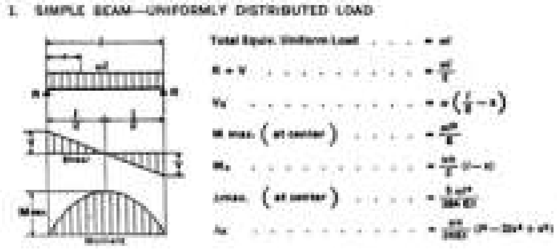
then: $V_n = A_w \left[\frac{4.26\ E}{\left(\frac{h}{t_w}\right)^2}\right]$

Design for Shear

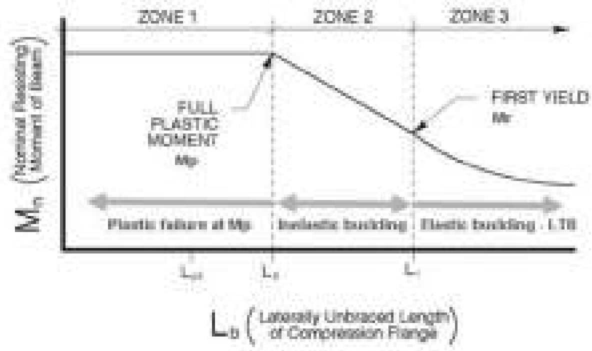
Steel

Pass/Fail Analysis of Steel Beams – $L_b < L_p$

Given: yield stress, steel section, loading
Find: pass/fail of section



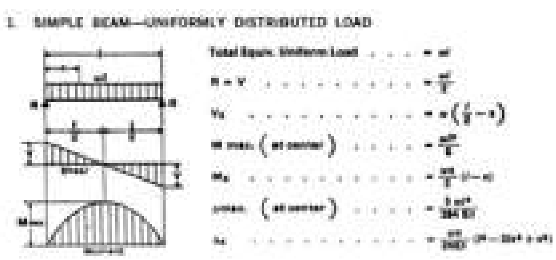
1. Calculate the factored design load wu
   $w_u = 1.2w_{DL} + 1.6w_{LL}$
2. Determine the design moment Mu.
   Mu will be the maximum beam moment using the factored loads
3. Insure that Lb < Lp (zone 1)
   $Lp = 1.76\ r_y \sqrt{E/Fy}$
4. Determine the nominal moment, Mn
   Mn = Fy Zx (look up Z for section)
5. Factor the nominal moment
   øMn = 0.90 Mn
6. Check that Mu < øMn
7. Check shear
8. Check deflection



Pass/Fail Analysis of Steel Beam – $L_b < L_p$

Example:

Given: yield stress, steel section, loading
Find: pass/fail of section



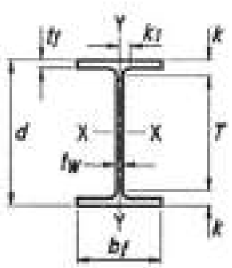
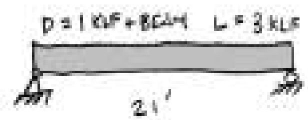


Table 1-1
W-Shapes
Dimensions



W21x44
A992 STEEL
F_y = 50 KSI

FROM TABLE 1-1 AISC $Z_x = 95.4\ in^3$

$w_u = 1.2(1 + .044) + 1.6(3) = 6.05$ KLF

$M_u = \frac{w_u \ell^2}{8} = \frac{6.05\ KLF \times 21'^2}{8} = 333.5\ K\text{-}'$

$M_n = F_y Z = 50\ ksi\ 95.4\ in^3 = 4770\ K\text{-}''$

$M_n = 4770\ ^{K\text{-}''}/12 = 397.5\ ^{K\text{-}'}$

$\phi M_n = 0.9\,(397.5) = 357.7\ ^{K\text{-}'}$

$M_u = 333.5^{K\text{-}'} < 357.7^{K\text{-}'} = \phi M_n$

∴ PASS

Analysis of Steel Beam – $L_b < L_p$

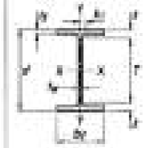


Table 1-1 (continued) W-Shapes Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		$\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	k_{det}	k_1	T	Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W21×93	27.3	21.6	21 5/8	0.580	9/16	5/16	8.42	8 3/8	0.930	15/16	1.43	1 5/8	15/16	18 3/8	5 1/2
×83	24.4	21.4	21 3/8	0.515	1/2	1/4	8.36	8 3/8	0.835	13/16	1.34	1 1/2	7/8	↓	↓
×73	21.5	21.2	21 1/4	0.455	7/16	1/4	8.30	8 1/4	0.740	3/4	1.24	1 7/16	7/8		
×68	20.0	21.1	21 1/8	0.430	7/16	1/4	8.27	8 1/4	0.685	11/16	1.19	1 3/8	7/8		
×62	18.3	21.0	21	0.400	3/8	3/16	8.24	8 1/4	0.615	5/8	1.12	1 5/16	13/16		
×55	16.2	20.8	20 3/4	0.375	3/8	3/16	8.22	8 1/4	0.522	1/2	1.02	1 3/16	13/16		
×48	14.1	20.6	20 5/8	0.350	3/8	3/16	8.14	8 1/8	0.430	7/16	0.930	1 1/8	13/16	↓	↓
W21×57	16.7	21.1	21	0.405	3/8	3/16	6.56	6 1/2	0.650	5/8	1.15	1 5/16	13/16	18 3/8	3 1/2
×50	14.7	20.8	20 7/8	0.380	3/8	3/16	6.53	6 1/2	0.535	9/16	1.04	1 1/4	13/16		
×44	13.0	20.7	20 5/8	0.350	3/8	3/16	6.50	6 1/2	0.450	7/16	0.950	1 1/8	13/16	↓	↓

Table 1-1 (continued) W-Shapes Properties W21-W18

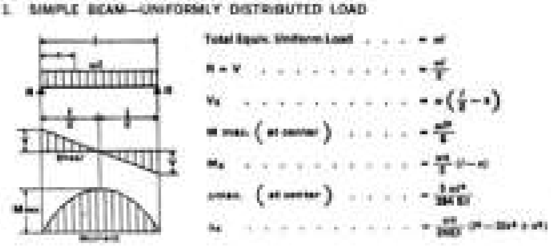
Nominal Wt.	Compact Section Criteria $\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	Axis X-X I	S	r	Z	Axis Y-Y I	S	r	Z	r_{ts}	h_o	Torsional Properties J	C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
93	4.53	32.3	2070	192	8.70	221	92.9	22.1	1.84	34.7	2.24	20.7	6.03	9940
83	5.00	36.4	1830	171	8.67	196	81.4	19.5	1.83	30.5	2.21	20.6	4.34	8630
73	5.60	41.2	1600	151	8.64	172	70.6	17.0	1.81	26.6	2.19	20.5	3.02	7410
68	6.04	43.6	1480	140	8.60	160	64.7	15.7	1.80	24.4	2.17	20.4	2.45	6760
62	6.70	46.9	1330	127	8.54	144	57.5	14.0	1.77	21.7	2.15	20.4	1.83	5960
55	7.87	50.0	1140	110	8.40	126	48.4	11.8	1.73	18.4	2.11	20.3	1.24	4980
48	9.47	53.6	959	93.0	8.24	107	38.7	9.52	1.66	14.9	2.05	20.2	0.803	3950
57	5.04	46.3	1170	111	8.36	129	30.6	9.35	1.35	14.8	1.68	20.5	1.77	3190
50	6.10	49.4	984	94.5	8.18	110	24.9	7.64	1.30	12.2	1.64	20.3	1.14	2570
44	7.22	53.6	843	81.6	8.06	95.4	20.7	6.37	1.26	10.2	1.60	20.2	0.770	2110

Pass/Fail Analysis of Steel Beam – $L_b < L_p$

Example:

Given: yield stress, steel section, loading
Find: pass/fail of section

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



CHECK SHEAR:

$V_u = \frac{w_u \ell}{2} = \frac{6.05(21)}{2} = 63.5^K$

FROM AISC TABLE 1-1

$h/t_w = 53.6 < 59$ (zone 1)

$V_n = 0.6 F_y A_w = 0.6(50)(\overset{d}{20.7} \times \overset{t_w}{0.35})$

$V_n = 217.35^K$

$\phi V_n = 1.0(217.35) = 217.35^K$

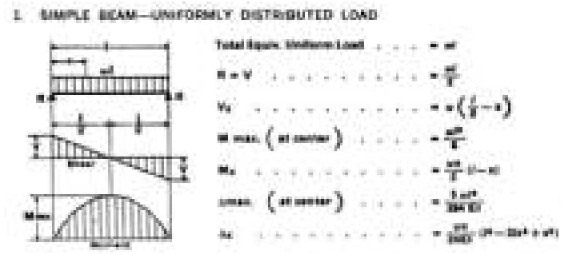
$V_u = 63.5^K < 217.3^K = \phi V_n$

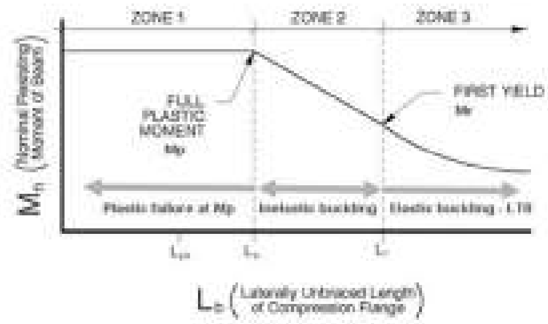
Capacity Analysis of Steel Beam

Given: yield stress, steel section

Find: moment or load capacity

1. Determine the unbraced length of the compression flange (Lb).
2. Find the Lp and Lr values from the given properties table.
3. Compare Lb to Lp and Lr and determine which equation for Mn or Mcr to be used.
4. Determine the equation for maximum moment in the beam.
5. Calculate load based on maximum moment.





Example – Capacity Analysis of Steel Beam

Find applied live load capaciy, w_{LL} in KLF

$w_u = 1.2w_{DL} + 1.6w_{LL}$

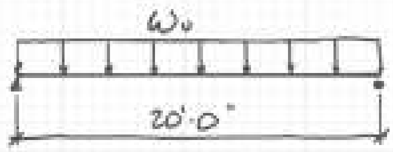
w_{DL} = beam + floor = 44plf + 1500plf

Fy = 50 ksi, Fully Braced

My = Fy * Sx = 50 ksi x 81.6 in^3 = 340 k-ft

1. Find the Plastic Modulus (Zx) and Section Modulus (Sx) for the given section from the AISC tables.
2. Determine 1.5*My
3. Determine Mn : Mn = Fy*Zx
4. Compare Mn and 1.5*My, choose lesser of the two.
5. Calculate Mu: Mu = ϕ_b * Mn
   ϕ_b = 0.90

GIVEN: F_y = 50 ksi
W21x44
FULLY BRACED



FOR A W21x44 FROM TABLE

$Z_x = 95.4 \text{ in}^3$ $S = 81.6$

$1.5M_y = 1.5(F_y \cdot S) = 6{,}120$ k-in

$M_n = F_y Z = 50 \text{ ksi} \cdot 95.4 = 4{,}770$ k-in

$M_n < 1.5 M_y$ OK

$M_u = \phi_b \cdot M_n = 0.9 \cdot 4{,}770$ k-in

$M_u = 4{,}293$ k-in $= 357.75$ k-ft

Example – Load Analysis cont.

W21x44

6. Using the maximum moment equation, solve for the factored distributed loading, w_u

$$M_u = \frac{w_u \cdot l^2}{8} \Rightarrow w_u = \frac{8 M_u}{l^2}$$

$$w_u = \frac{8 \cdot 357.75 \text{ k-ft}}{20 \text{ ft}^2}$$

$$w_u = 7.155 \text{ k/ft}$$

7. The applied (unfactored) load
   $w = w_u / (\gamma \text{factors})$
   $w_u = 1.2w_{DL} + 1.6w_{LL}$

$$w_u = 7.155 \text{ KLF} = 1.2(\overbrace{0.044 + 1.5}^{w_{DL}}) + 1.6(w_{LL})$$

$$w_u = 1.853 + 1.6\,w_{LL} = 7.155 \text{ KLF}$$

$$w_{LL} = 3.31 \text{ KLF}$$

Braced Beam Design with Plastic Modulus Table

- Calculate Required Moment
- Determine Mn

$Mu = \phi b * Mn$
$Mn = Mu / \phi b$

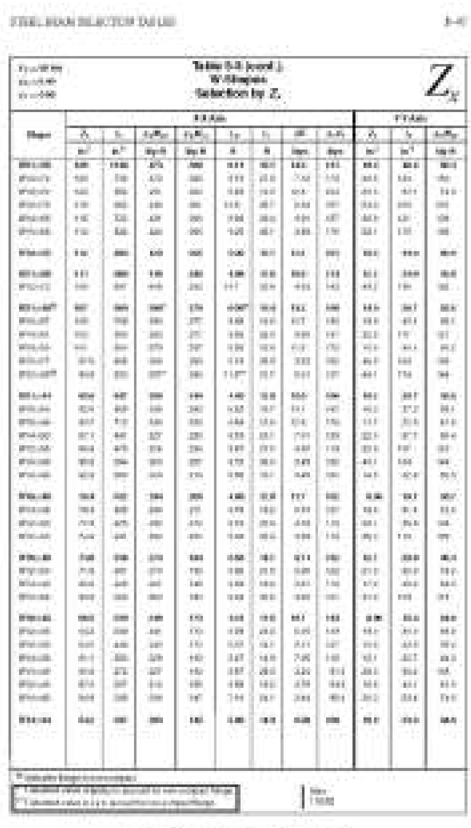
- Determine Minimum Zx required

$Mn = Fy * Zx$
$Zx > Mn / Fy$

- Choose a section based on Z from the AISC table.
  Bold faced sections are lighter
- Check Shear

If $h/tw < 59$
$Vn = 0.6 * Fy * Aw$
$Vu < \phi v * Vn$

$\phi v = 1.0$



from AISC 2003

Design of Steel Beam

Example - Bending

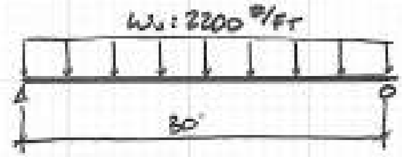
1. Use the maximum moment equation, and solve for the ultimate moment, Mu.

2. Solve for Mn

3. Determine Zx required

4. Select the lightest beam with a Zx greater than the Zx required from AISC table

GIVEN: $F_y = 50$ ksi
FULLY BRACED



$$M_u = \frac{w_u l^2}{8} = \frac{2200 \text{ PLF} \cdot 30 \text{ FT}^2}{8}$$

$$M_u = 247{,}500 \text{ \#-FT} = 247.5 \text{ KFT}$$

$$M_n = M_u / \phi_b = \frac{247.5 \text{ KFT}}{0.90} = 275 \text{ KFT}$$

$$Z_{REQ'D} = \frac{M_n}{F_y} = \frac{275 \text{ KFT} \left(\frac{12''}{FT}\right)}{50 \text{ KSI}}$$

$$Z_{x\,REQ'D} = 66 \text{ IN}^3$$

SELECT W18x35

Design of Steel Beam

Example - Shear

5. Determine if $h/t_w < 59$ (case 1, most common)
6. Determine Aw: Aw = d * tw
7. Calculate Vn: Vn = 0.6*Fy*Aw
8. Calculate Vu for the given loading
9. Check Vu < ϕv Vn
   ϕv = 1.0

FIND h/t_w FROM TABLES FOR A W18x35

$$h/t_w = 53.5 < 59$$

$$V_n = 0.6 \cdot F_y \cdot A_w$$
$$= 0.6 \cdot 50 \text{ KSI} \cdot (17.7'' \times .3'')$$
$$= 159.3 \text{ K}$$

$$V_u = \frac{2200 \text{ \#/FT} \cdot 30'}{2} = 33{,}000 \text{ \#}$$

$$V_u \le \phi_v V_n$$

$$33^k < (1.0)\, 159.3^k = 159.3^k$$ OK

Steel Beam – Deflection

Serviceability limits:

Deflection limits by application

IBC Table 1604.3

Secondary roof structural members

formed metal roofing – LL L/150

For steel structural members, the DL can be taken as zero (note g)

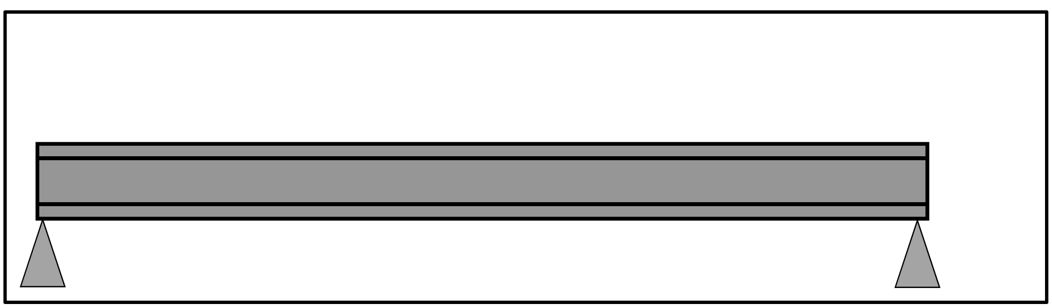
There are more stringent cases:

Machine tolerance – e.g. L/1000

DL deflection can be compensated for by beam camber

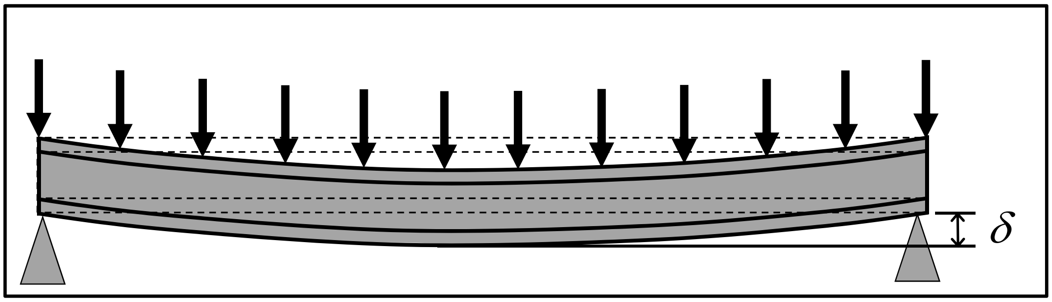
TABLE 1604.3
DEFLECTION LIMITS[a, b, c, h, i]

CONSTRUCTION	L	S or W[f]	D + L[d,g]
Roof members:[e]			
Supporting plaster ceiling	l/360	l/360	l/240
Supporting nonplaster ceiling	l/240	l/240	l/180
Not supporting ceiling	l/180	l/180	l/120
Floor members	l/360	—	l/240
Exterior walls and interior partitions:			
With brittle finishes	—	l/240	—
With flexible finishes	—	l/120	—
Farm buildings	—	—	l/180
Greenhouses	—	—	l/120



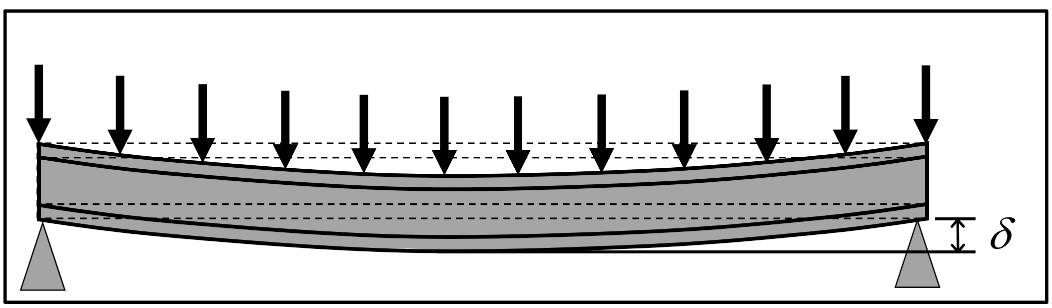
Beam without Camber

Developed by Scott Civjan
University of Massachusetts, Amherst
For AISC

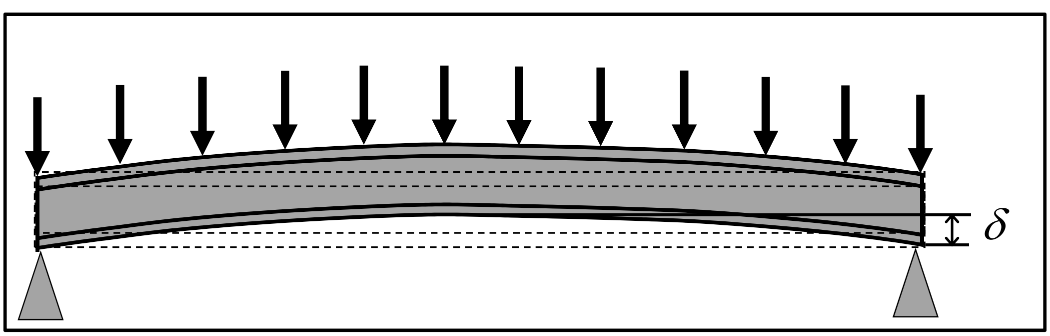


Results in deflection in floor under Dead Load.
This can affect thickness of slab and fit of non-structural components.

Developed by Scott Civjan
University of Massachusetts, Amherst
For AISC



Results in deflection in floor under Dead Load.
This can affect thickness of slab and fit of non-structural components.



Beam with Camber

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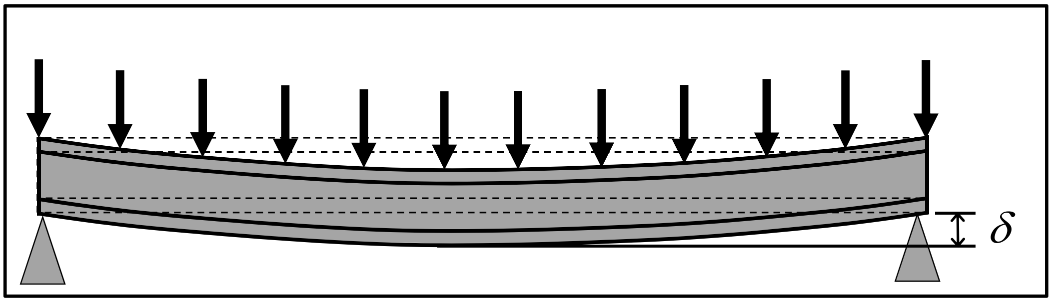
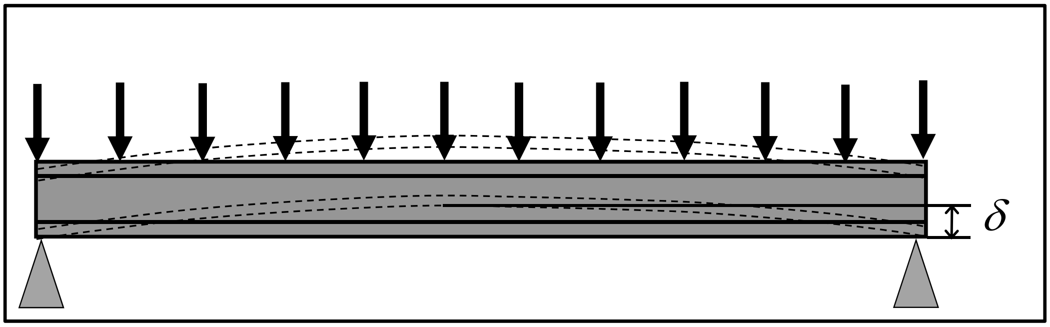


Results in deflection in floor under Dead Load.
This can affect thickness of slab and fit of non-structural components.



Cambered beam counteracts service dead load deflection.

Developed by Scott Civjan
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For AISC