Chapter 5

Moving Loads and Influence Lines

Instructional Objectives:

The objectives of this lesson are as follows:
• Understand the moving load effect in simpler term
• Study various definitions of influence line
• Introduce to simple procedures for construction of influence lines

37.1 Introduction

In earlier lessons, you were introduced to statically determinate and statically indeterminate structural analysis under non-moving load (dead load or fixed loads). In this lecture, you will be introduced to determination of maximum internal actions at cross-sections of members of statically determinate structured under the effects of moving loads (live loads).

Common sense tells us that when a load moves over a structure, the deflected shape of the structural will vary. In the process, we can arrive at simple conclusion that due to moving load position on the structure, reactions value at the support also will vary.

From the designer's point of view, it is essential to have safe structure, which doesn't exceed the limits of deformations and also the limits of load carrying capacity of the structure.

37.2 Definitions of influence line

In the literature, researchers have defined influence line in many ways. Some of the definitions of influence line are given below.

• An influence line is a diagram whose ordinates, which are plotted as a function of distance along the span, give the value of an internal force, a reaction, or a displacement at a particular point in a structure as a unit load move across the structure.
• An influence line is a curve the ordinate to which at any point equals the value of some particular function due to unit load acting at that point.
• An influence line represents the variation of either the reaction, shear,

moment, or deflection at a specific point in a member as a unit concentrated force moves over the member.

37.3 Construction of Influence Lines

In this section, we will discuss about the construction of influence lines. Using any one of the two approaches (Figure 37.1), one can construct the influence line at a specific point P in a member for any parameter (Reaction, Shear or

Moment). In the present approaches it is assumed that the moving load is having dimensionless magnitude of unity. Classification of the approaches for construction of influence lines is given in Figure 37.1.
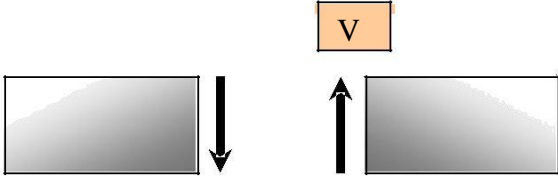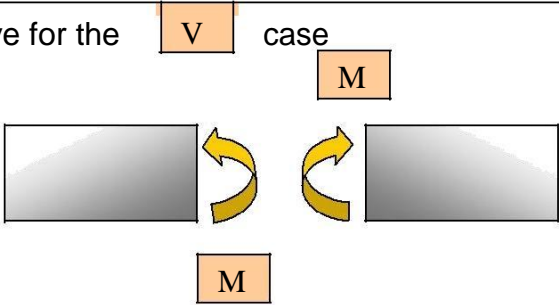
Construction of Influence Lines

Tabulate Values

for

Influence Line-Equation
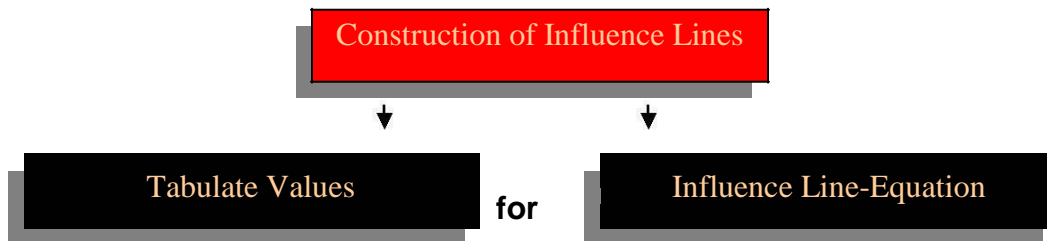
37.3.1 Tabulate Values

Apply a unit load at different locations along the member, say at **x**. And these locations, apply statics to compute the value of parameter (reaction, shear, or moment) at the specified point. The best way to use this approach is to prepare a table, listing unit load at x versus the corresponding value of the parameter calculated at the specific point (i.e. Reaction R, Shear V or moment M) and plot the tabulated values so that influence line segments can be constructed.

37.3.2 Sign Conventions

Sign convention followed for shear and moment is given below.

Parameter	Sign for influence line
Reaction R	Positive at the point when it acts upward on the beam.
Shear V	Positive for the following case V
Moment M	Positive for the V case M M

37.3.3 Influence Line Equations

Influence line can be constructed by deriving a general mathematical equation to compute parameters (e.g. reaction, shear or moment) at a specific point under the effect of moving load at a variable position *x*.

The above discussed both approaches are demonstrated with the help of simple numerical examples in the following paragraphs.

37.4 Numerical Examples

Example 1:

Construct the influence line for the reaction at support B for the beam of span 10 m. The beam structure is shown in Figure 37.2.
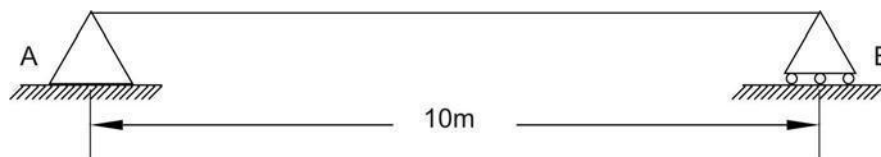
Figure 37.2: The beam structure

Solution:

As discussed earlier, there are two ways this problem can be solved. Both the approaches will be demonstrated here.

Tabulate values:

As shown in the figure, a unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A. Let us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows (Figure 37.3).

$$\Sigma \ M_A = 0 : R_B \ x \ 10 - 1 \ x \ 2.5 = 0 \Rightarrow R_B = 0.25$$
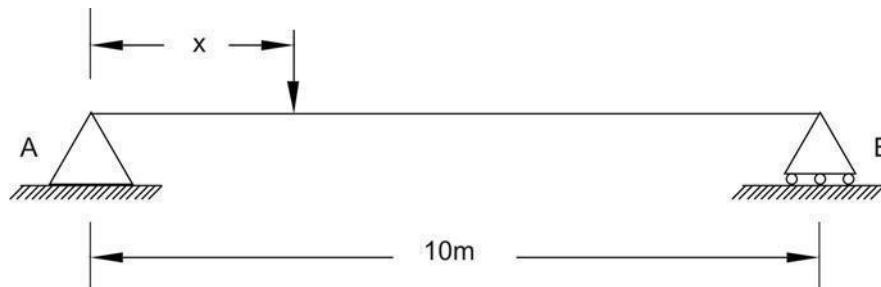

Figure 37.3: The beam structure with unit load

Similarly, the load can be placed at 5.0, 7.5 and 10 m. away from support A and reaction R_B can be computed and tabulated as given below.

x	RB
0	0.0
2.5	0.25
5.0	0.5
7.5	0.75
10	1

Graphical representation of influence line for R_B is shown in Figure 37.4.
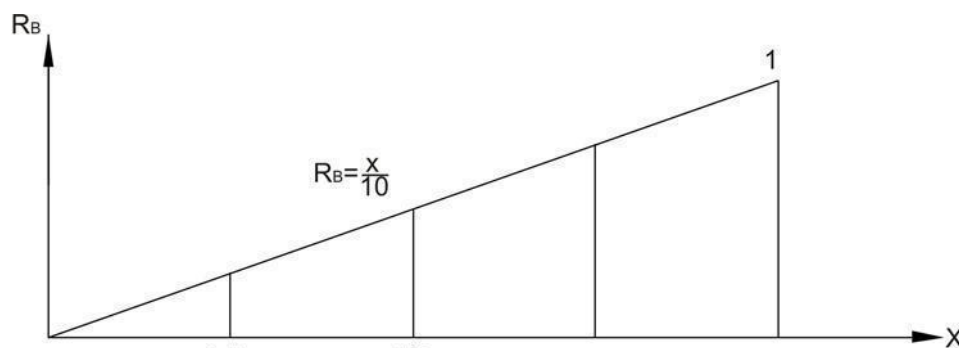

Figure 37.4: Influence line for reaction R_B.

Influence Line Equation:

When the unit load is placed at any location between two supports from support A at distance x then the equation for reaction R_B can be written as

$$\Sigma\ M_A = 0 : R_B \times 10 - x = 0 \Rightarrow R_B = x/10$$

The influence line using this equation is shown in Figure 37.4.

Example 2:

Construct the influence line for support reaction at B for the given beam as shown in Fig 37.5.
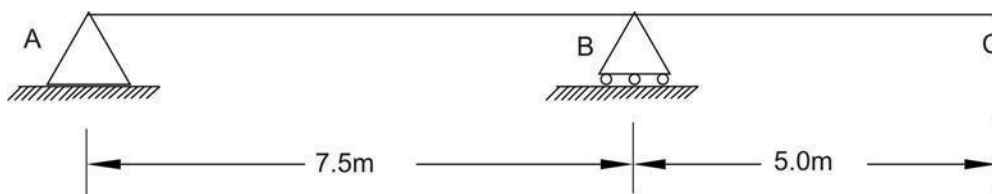

Figure 37.5: The overhang beam structure

Solution:

As explained earlier in example 1, here we will use tabulated values and influence line equation approach.

Tabulate Values:

As shown in the figure, a unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A. Let

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us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows.

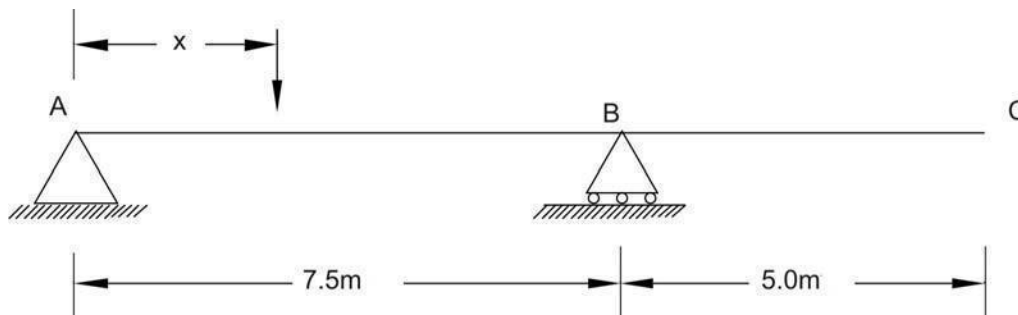$$\Sigma\ M_A\ = 0 : R_B \times 7.5 - 1 \times 2.5 = 0 \Rightarrow R_B\ = 0.33$$

Figure 37.6: The beam structure with unit load

Similarly one can place a unit load at distances 5.0 m and 7.5 m from support A and compute reaction at B. When the load is placed at 10.0 m from support A, then reaction at B can be computed using following equation.

$$\Sigma\ M_A\ = 0 : R_B \times 7.5 - 1 \times 10.0 = 0 \Rightarrow R_B\ = 1.33$$

Similarly a unit load can be placed at 12.5 and the reaction at B can be computed. The values of reaction at B are tabulated as follows.

x	R_B
0	0.0
2.5	0.33
5.0	0.67
7.5	1.00
10	1.33
12.5	1.67

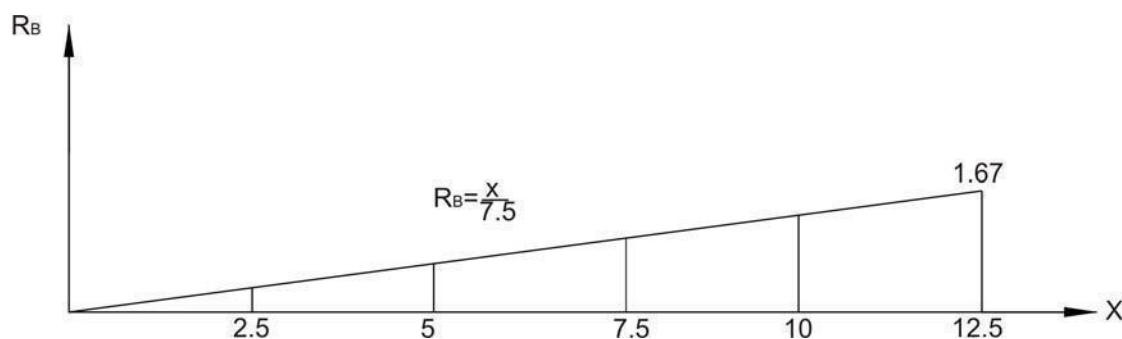Graphical representation of influence line for R_B is shown in Figure 37.7.

Figure 37.7: Influence for reaction R_B.

Influence line Equation:

Applying the moment equation at A (Figure 37.6),

$$\Sigma\, M_A = 0 : R_B \times 7.5 - 1 \times x = 0 \Rightarrow R_B = x/7.5$$

The influence line using this equation is shown in Figure 37.7.

Example 3:

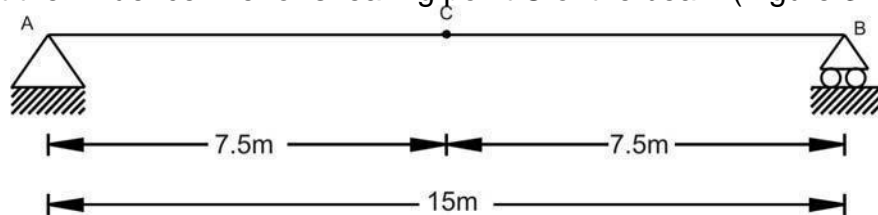Construct the influence line for shearing point C of the beam (Figure 37.8)

Figure 37.8: Beam Structure

Solution:
Tabulated Values:

As discussed earlier, place a unit load at different location at distance x from support A and find the reactions at A and finally computer shear force taking section at C. The shear force at C should be carefully computed when unit load is placed before point C (Figure 37.9) and after point C (Figure 37.10). The resultant values of shear force at C are tabulated as follows.
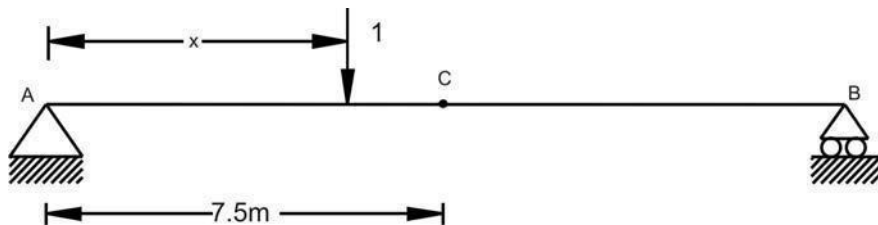

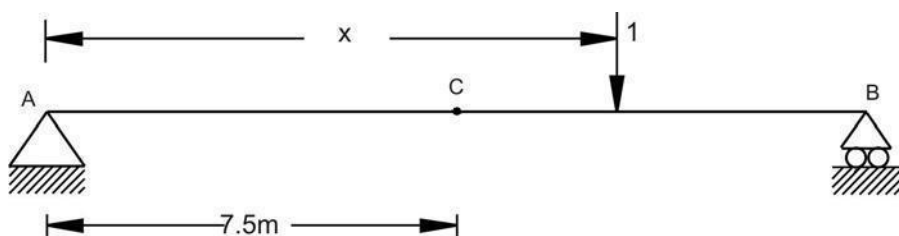**Figure 37.9: The beam structure – a unit load before section**

Figure 37.10: The beam structure - a unit load before section

X	V_C
0	0.0
2.5	-0.16
5.0	-0.33
7.5(-)	-0.5
7.5(+)	0.5
10	0.33
12.5	0.16
15.0	0

Graphical representation of influence line for V_C is shown in Figure 37.11.
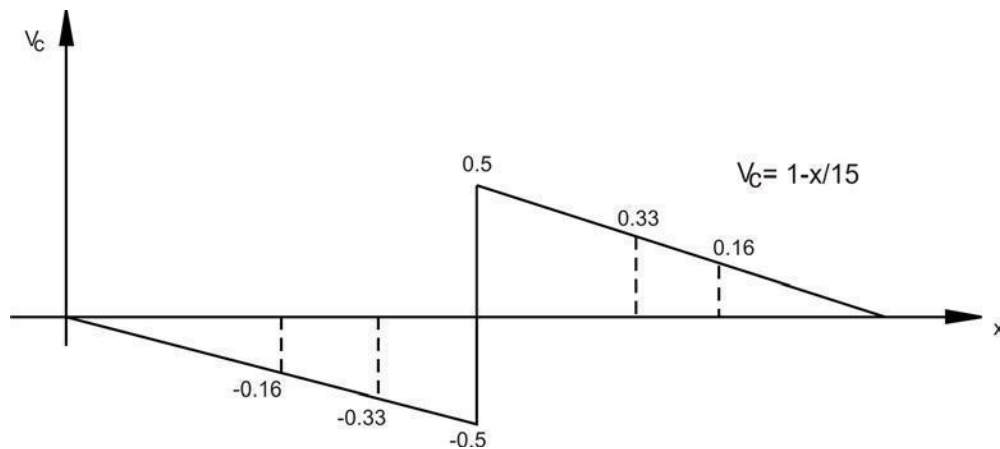

Figure 37.11: Influence line for shear point C

Influence line equation:

In this case, we need to determine two equations as the unit load position before point C (Figure 37.12) and after point C (Figure 37.13) will show different shear force sign due to discontinuity. The equations are plotted in Figure 37.11.
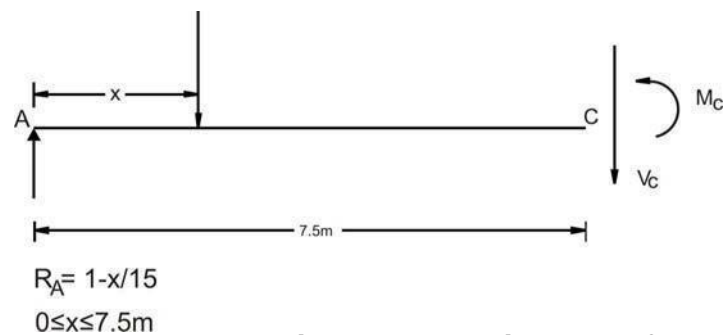

$R_A = 1 - x/15$

$0 \leq x \leq 7.5m$

Figure 37.12: Free body diagram – a unit load before section
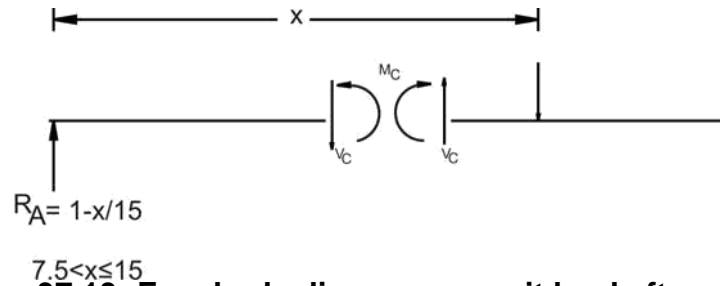
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$R_A = 1-x/15$

$7.5 < x \leq 15$

Figure 37.13: Free body diagram – a unit load after section

Influence Line for Moment:

Like shear force, we can also construct influence line for moment.

Example 4:

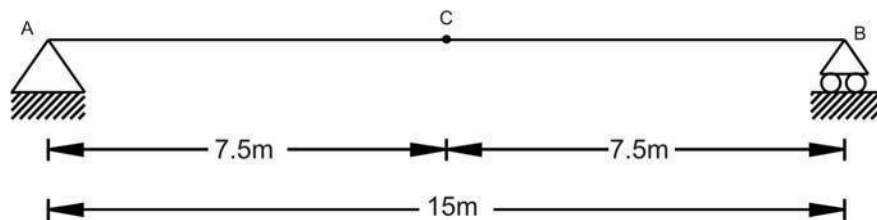Construct the influence line for the moment at point C of the beam shown in Figure 37.14

Figure 37.14: Beam structure

Solution:
Tabulated values:

Place a unit load at different location between two supports and find the support reactions. Once the support reactions are computed, take a section at C and compute the moment. For example, we place the unit load at x=2.5 m from support A (Figure 37.15), then the support reaction at A will be 0.833 and support reaction B will be 0.167. Taking section at C and computation of moment at C can be given by

$\Sigma\ M_C = 0 : -M_C + R_B \times 7.5 - = 0 \Rightarrow -M_C + 0.167 \times 7.5 - = 0 \Rightarrow M_C = 1.25$
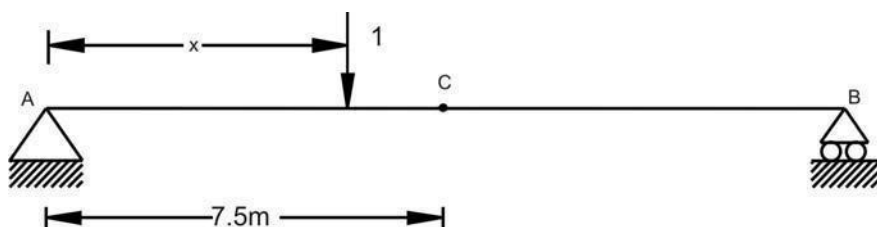

Figure 37.15: A unit load before section

Similarly, compute the moment M_C for difference unit load position in the span.

The values of Mc are tabulated as follows.

X	M_C
0	0.0
2.5	1.25
5.0	2.5
7.5	3.75
10	2.5
12.5	1.25
15.0	0

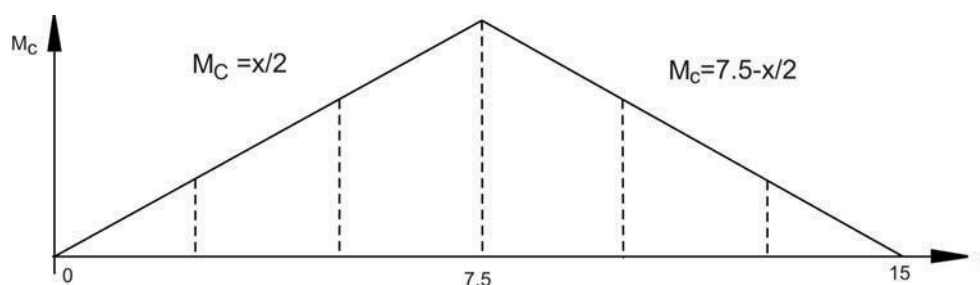Graphical representation of influence line for M_C is shown in Figure 37.16.

Figure 37.16: Influence line for moment at section C

Influence Line Equations:

There will be two influence line equations for the section before point C and after point C.

When the unit load is placed before point C then the moment equation for given Figure 37.17 can be given by

$\Sigma M_C = 0 : M_C + 1(7.5 - x) - (1 - x/15)x7.5 = 0 \Rightarrow M_C = x/2$, where $0 \le x \le 7.5$
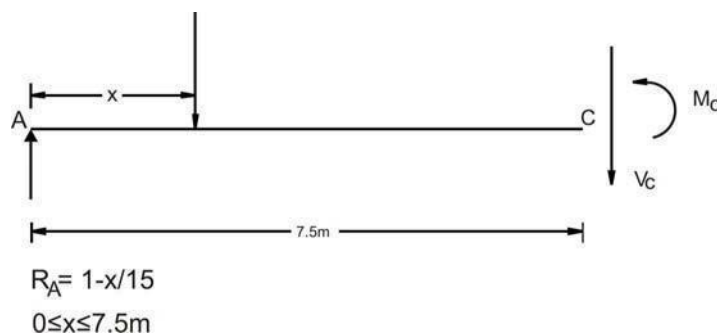

$R_A = 1 - x/15$

$0 \le x \le 7.5m$

Figure 37.17: Free body diagram - a unit load before section

When the unit load is placed after point C then the moment equation for given Figure 37.18 can be given by

$\Sigma M_C = 0 : M_C - (1 - x/15) \times 7.5 = 0 \Rightarrow M_C = 7.5 - x/2$, where $7.5 < x \le 15.0$
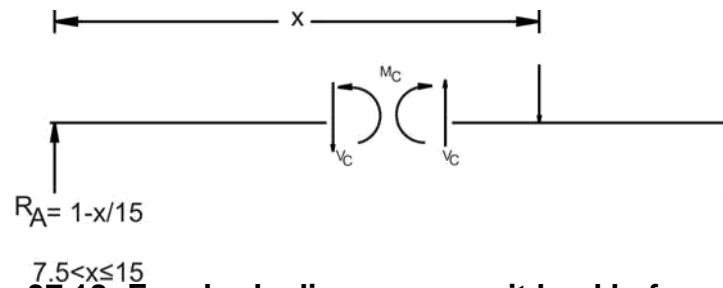
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Figure 37.18: Free body diagram - a unit load before section

The equations are plotted in Figure 37.16.

Example 5:

Construct the influence line for the moment at point C of the beam shown in Figure 37.19.
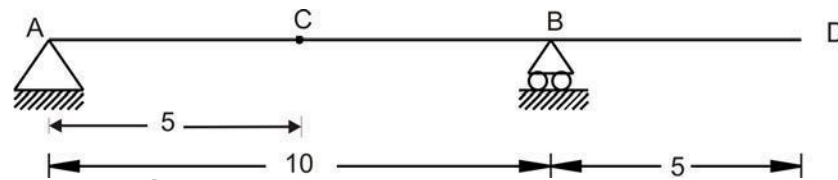

Figure 37.19: Overhang beam structure

Solution:
Tabulated values:

Place a unit load at different location between two supports and find the support reactions. Once the support reactions are computed, take a section at C and compute the moment. For example as shown in Figure 37.20, we place a unit load at 2.5 m from support A, then the support reaction at A will be 0.75 and support reaction B will be 0 .25.

Figure 37.20: A unit load before section C

Taking section at C and computation of moment at C can be given by

$$\Sigma\ M_C\ = 0 : - M_C + R_B \times 5.0 \text{-} = 0 \Rightarrow - M_C + 0.25 \times 5.0\ = 0 \Rightarrow M_C = 1.25$$

Similarly, compute the moment M_C for difference unit load position in the span. The values of Mc are tabulated as follows.

x	M_C
0	0
2.5	1.25
5.0	2.5
7.5	1.25
10	0
12.5	-1.25
15.0	-2.5

Graphical representation of influence line for M_C is shown in Figure 37.21.

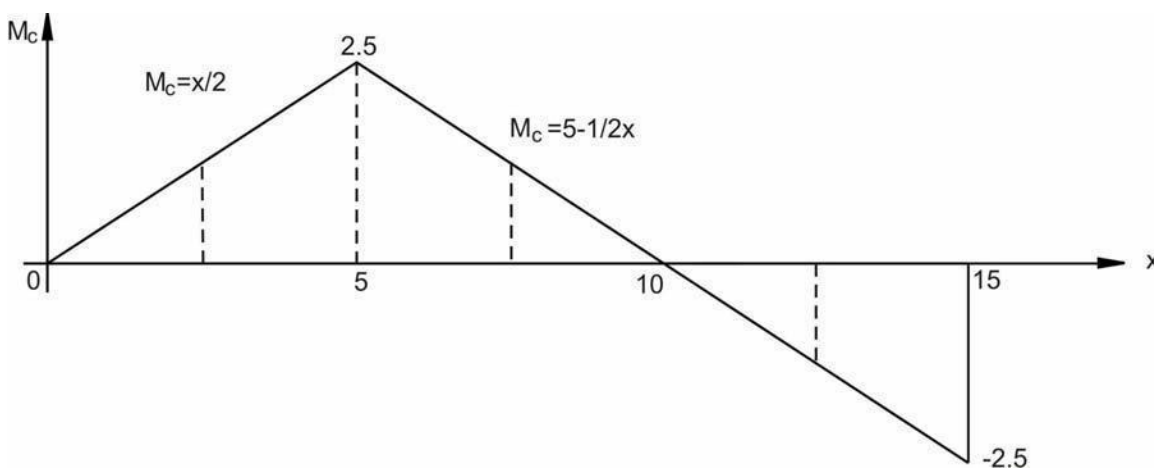**Figure 37.21: Influence line of moment at section C**

Influence Line Equations:

There will be two influence line equations for the section before point C and after point C.

When a unit load is placed before point C then the moment equation for given Figure 37.22 can be given by

$\Sigma M_C = 0 : M_C + 1(5.0 - x) - (1 - x/10) \times 5.0 = 0 \Rightarrow M_C = x/2$, where $0 \le x \le 5.0$
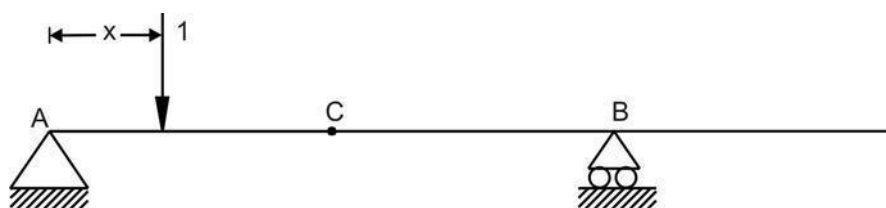

Figure 37.22: A unit load before section C

When a unit load is placed after point C then the moment equation for given Figure 37.23 can be given by

$\Sigma M_C = 0 : M_C - (1 - x/10) \times 5.0 = 0 \Rightarrow M_C = 5 - x/2$, where $5 < x \le 15$
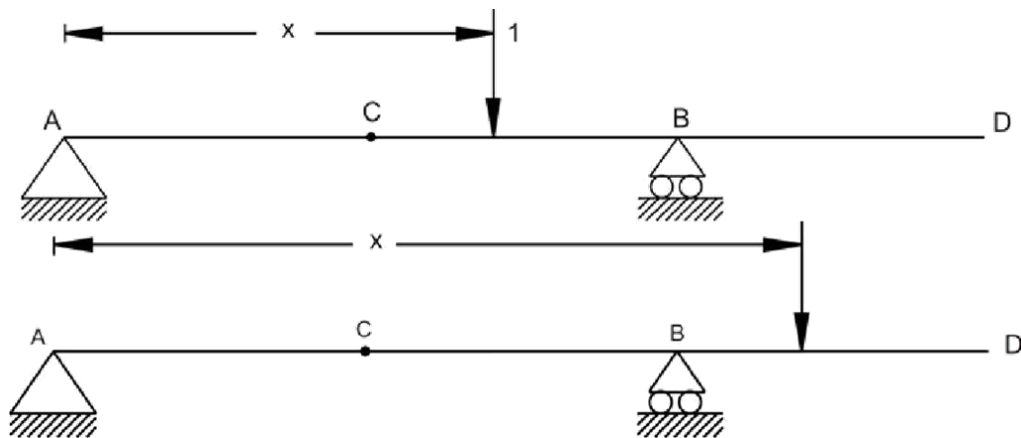
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Figure 37.23: A unit load after section C

The equations are plotted in Figure 37.21.

37.5 Influence line for beam having point load and uniformly distributed load acting at the same time

Generally in beams/girders are main load carrying components in structural systems. Hence it is necessary to construct the influence line for the reaction, shear or moment at any specified point in beam to check for criticality. Let us assume that there are two kinds of load acting on the beam. They are concentrated load and uniformly distributed load (UDL).

37.5.1 Concentrated load

As shown in the Figure 37.24, let us say, point load P is moving on beam from A to B. Looking at the position, we need to find out what will be the influence line for reaction B for this load. Hence, to generalize our approach, like earlier examples, let us assume that unit load is moving from A to B and influence line for reaction A can be plotted as shown in Figure 37.25. Now we want to know, if load P is at the center of span then what will be the value of reaction A? From Figure 37.24, we can find that for the load position of P, influence line of unit load gives value of 0.5. Hence, reaction A will be 0.5xP. Similarly, for various load positions and load value, reactions A can be computed.
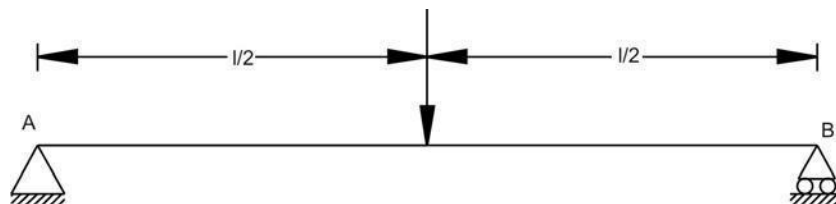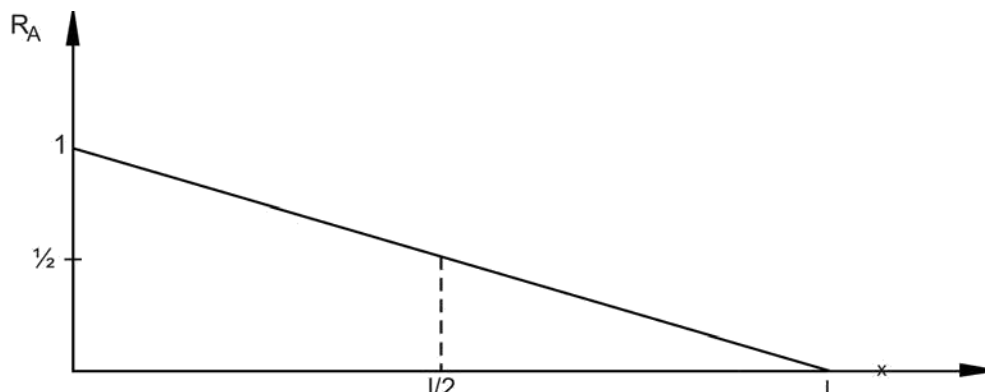

Figure 37.24: Beam structure

Figure 37.25: Influence line for support reaction at A

37.5.2 Uniformly Distributed Load

Beam is loaded with uniformly distributed load (UDL) and our objective is to find influence line for reaction A so that we can generalize the approach. For UDL of w on span, considering for segment of dx (Figure 37.26), the concentrated load dP can be given by w.dx acting at x. Let us assume that beam's influence line ordinate for some function (reaction, shear, moment) is y as shown in Figure 37.27. In that case, the value of function is given by (dP)(y) = (w.dx).y. For computation of the effect of all these concentrated loads, we have to integrate over the entire length of the beam. Hence, we can say that it will be ∫ w.y.dx = w ∫ y.dx. The term ∫ y.dx is equivalent to area under the influence line.
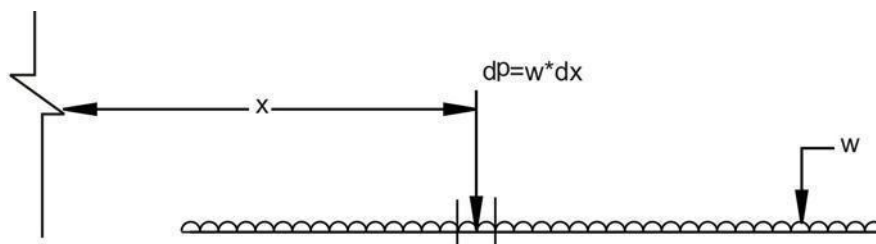

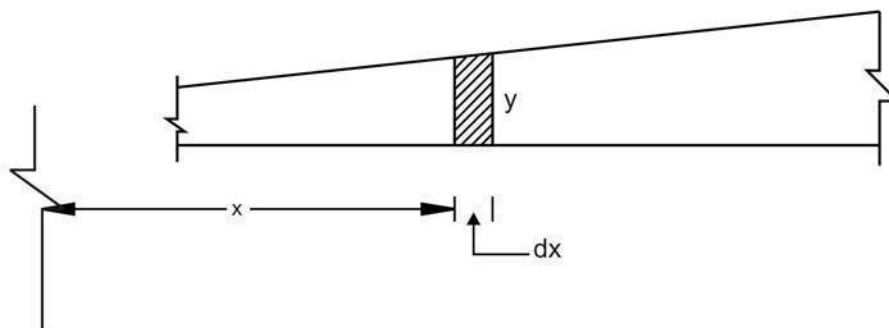**Figure 37.26: Uniformly distributed load on beam**

Figure 37.27: Segment of influence line diagram

For a given example of UDL on beam as shown in Figure 37.28, the influence line (Figure 37.29) for reaction A can be given by area covered by the influence line for unit load into UDL value. i.e. [0.5x (1)xl] w = 0.5 w.l.
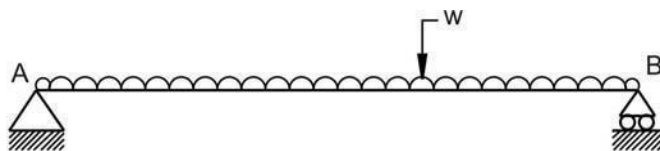

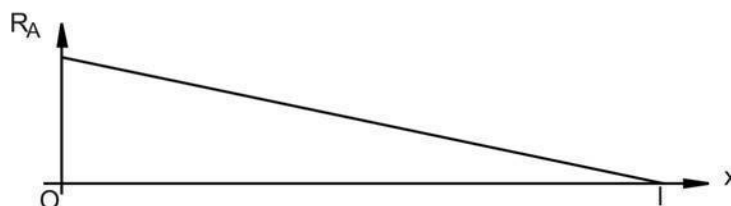**Figure 37.28: UDL on simply supported beam**

Figure 37.29: Influence line for support reaction at A.

37.6 Numerical Example

Find the maximum positive live shear at point C when the beam (Figure 37.30) is loaded with a concentrated moving load of 10 kN and UDL of 5 kN/m.
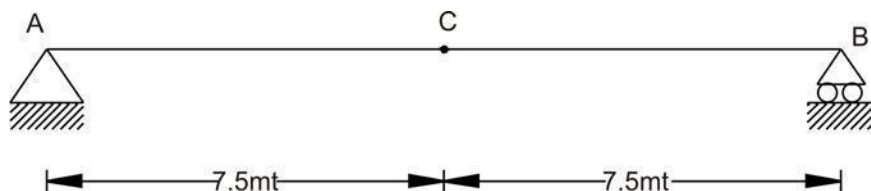

Figure 37.30: Simply supported beam

Solution:

As discussed earlier for unit load moving on beam from A to B, the influence line for the shear at C can be given by following Figure 37.31.
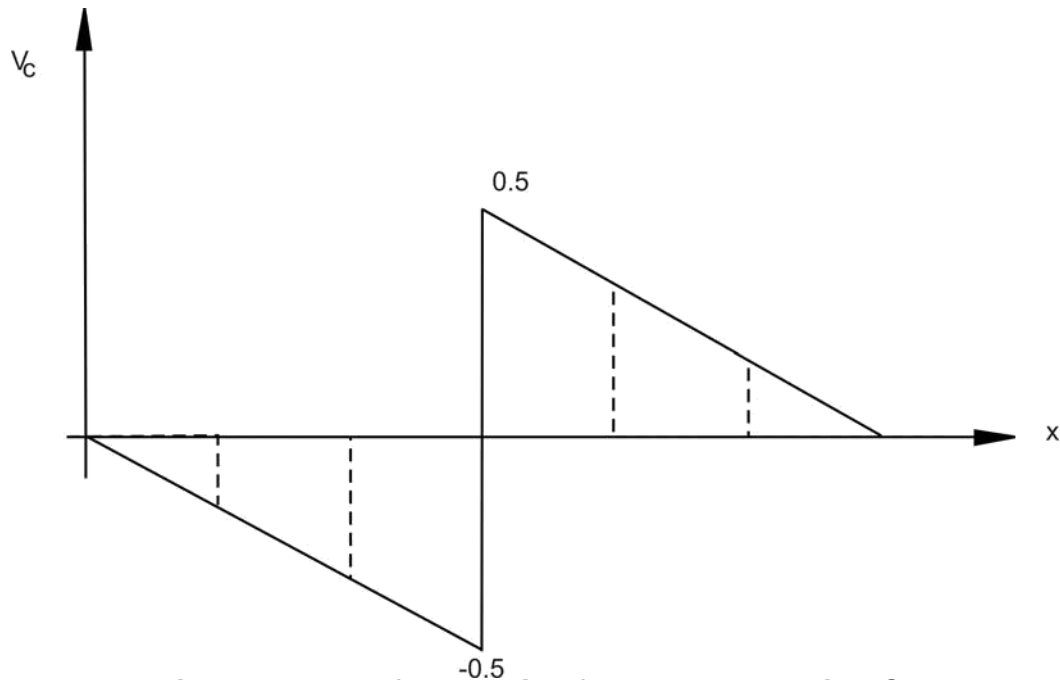
Figure 37.31: Influence line for shear at section C.

Concentrated load: As shown in Figure 37.31, the maximum live shear force at C will be when the concentrated load 10 kN is located just before C or just after C. Our aim is to find positive live shear and hence, we will put 10 kN just after C. In that case,

$$V_C = 0.5 \text{ x } 10 = 5 \text{ kN.}$$

UDL: As shown in Figure 37.31, the maximum positive live shear force at C will be when the UDL 5 kN/m is acting between x = 7.5 and x = 15.

$$V_C = [\; 0.5 \text{ x } (15 - 7.5) \; (0.5)] \text{ x } 5 = 9.375$$

Total maximum Shear at C:

$$(V_C) \text{ max} = 5 + 9.375 = 14.375.$$

Finally the loading positions for maximum shear at C will be as shown in Figure 37.32. For this beam one can easily compute shear at C using statics.
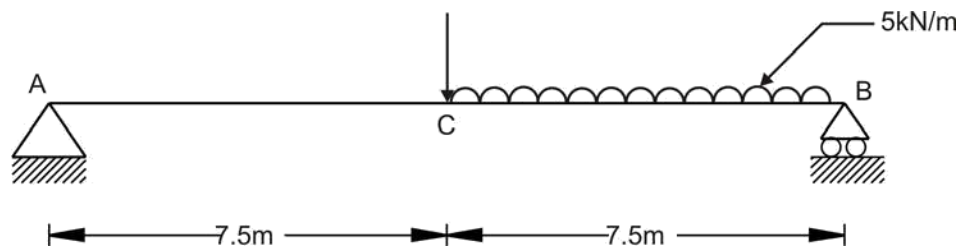
Figure 37.32: Simply supported beam

Suggested Text Books for Further Reading

- Armenakas, A. E. (1988). *Classical Structural Analysis – A Modern Approach*, McGraw-Hill Book Company, NY, ISBN 0-07-100120-4

- Hibbeler, R. C. (2002). *Structural Analysis*, Pearson Education (Singapore) Pte. Ltd., Delhi, ISBN 81-7808-750-2

- Junarkar, S. B. and Shah, H. J. (1999). *Mechanics of Structures – Vol. II*, Charotar Publishing House, Anand.

- Leet, K. M. and Uang, C-M. (2003). *Fundamentals of Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-058208-4

- Negi, L. S. and Jangid, R.S. (2003). *Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0 -07-462304-4

- Norris, C. H., Wilbur, J. B. and Utku, S. (1991). *Elementary Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-058116-9