

Plate Girders

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1. Plate Girder Behaviour and Design I

SUMMARY

Modern plate girders are introduced by explaining typical usage, types and the reasons for their inherent slender proportions. Their behaviour is described with particular emphasis on the different forms of buckling that can occur. Post-buckling and tension field action are introduced and the roles of the main components in a plate girder identified.

1. INTRODUCTION

Modern plate girders are normally fabricated by welding together two flanges and a web plate, as shown in Figure 1. Such girders are capable of carrying greater loads over longer spans than is generally possible using standard rolled sections or compound girders. Plate girders are typically used as long-

span floor girders in buildings, as bridge girders, and as crane girders in industrial structures.

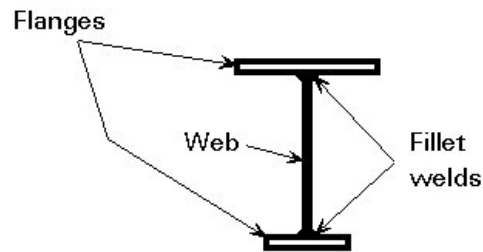
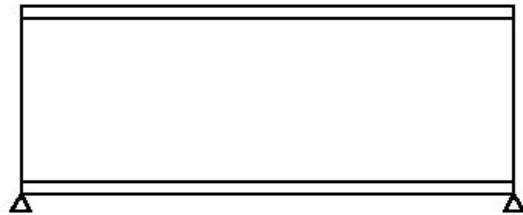


Figure 1 Plate girder composed of three plates

Plate girders are at their most impressive in modern bridge construction where main spans of well over 200m are feasible, with corresponding cross-section depths, haunched over the supports, in the range of 5-10m. Because plate girders are fabricated separately, each may be designed individually to resist the applied actions using proportions that ensure low self-weight and high load resistance.

For efficient design it is usual to choose a relatively deep girder, thus minimising the required area of flanges for a given

applied moment, M_{sd} . This obviously entails a deep web whose area will be minimised by reducing its thickness to the minimum required to carry the applied shear, V_{sd} . Such a web may be quite slender (i.e. a high d/t_w ratio) and may be prone to local buckling and shear buckling (see below). Such buckling problems have to be given careful consideration in plate girder design. One way of improving the load carrying resistance of a slender plate is to employ stiffeners; the selection of appropriate forms of stiffening is an important aspect of plate girder design.

1.1 Types

There are several forms of plate girder; Figure 2 illustrates three different types - unstiffened, transversely stiffened, and transversely and longitudinally stiffened. The three girders shown have bisymmetric I-profile cross-sections, although flanges of different size are sometimes used, as already shown in Figure 1. Other types of cross-section (see Figure 3) are monosymmetric I-profiles, which are popular in composite construction with the smaller flange on top or as crane girders with the larger flange on top. Figure 3 also shows two other (less common) variations - the "delta girder" and the tubular-top-flange girder - both being possible solutions in cases of long laterally-unsupported top compression flanges prone to lateral-torsional buckling.

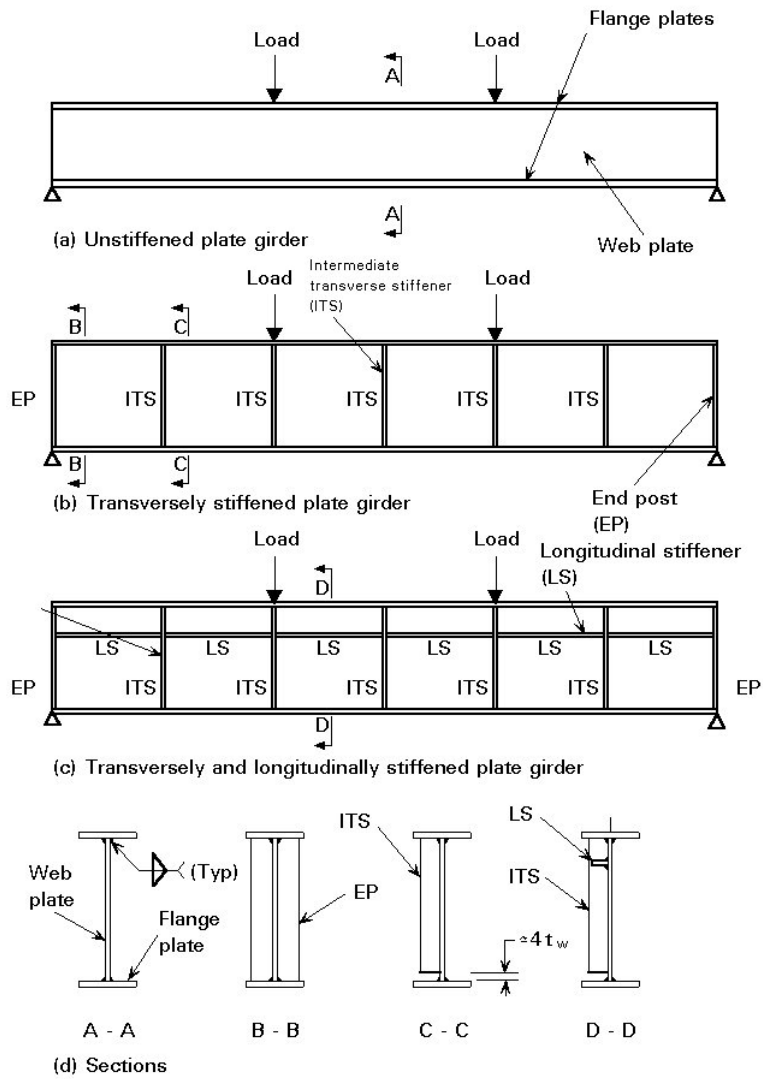


Figure 2 Stiffened and unstiffened plate girders

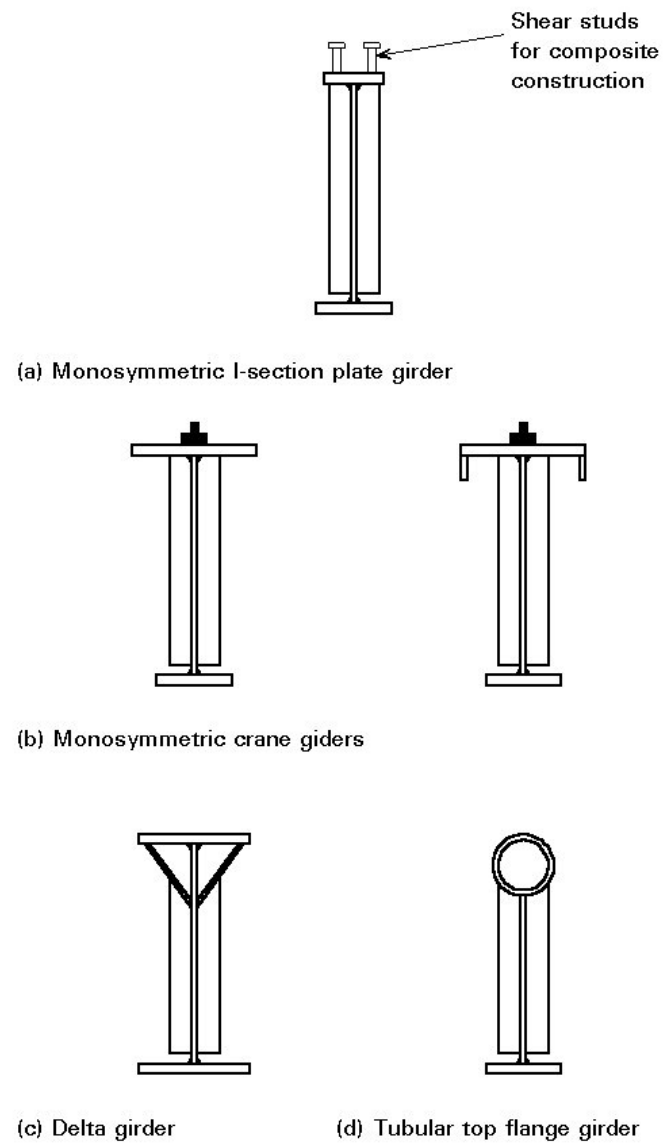


Figure 3 Plate girder cross-sections

There is also considerable scope for variation of cross-section in the longitudinal direction. A designer may choose to reduce the flange thickness (or breadth) in a zone of low applied moment, especially when a field-splice facilitates the change. Equally, in a zone of high shear, the designer might choose to thicken the web plate (see Figure 4). Alternatively, higher grade Fe E355 steel might be employed for zones of high applied moment and shear, while standard grade Fe E235 would be used elsewhere. So-called "hybrid" girders with different strength material in the flanges and the web offer another possible means of more closely matching resistance to requirements. More unusual variations are adopted in special circumstances, such as bridgework e.g. tapered girders, cranked girders, haunched girders (see Figure 5), and of course, plate girders with web holes to accommodate services, see Figure 6.

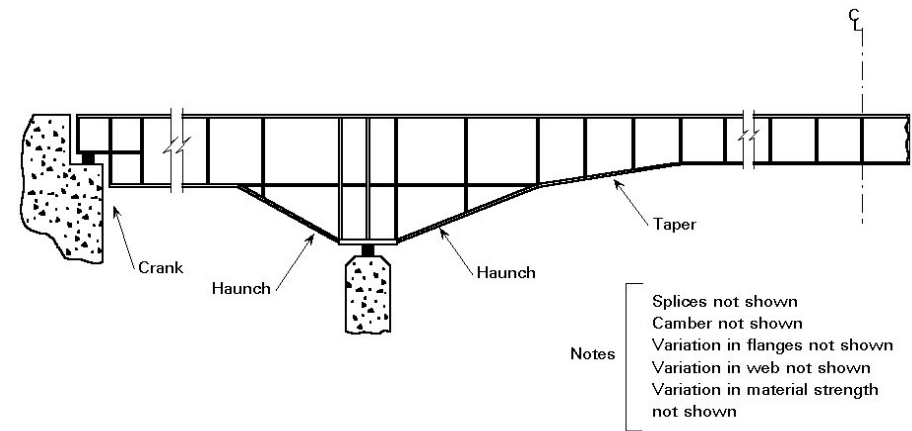


Figure 5 Plate girder with haunches, tapers and cranks

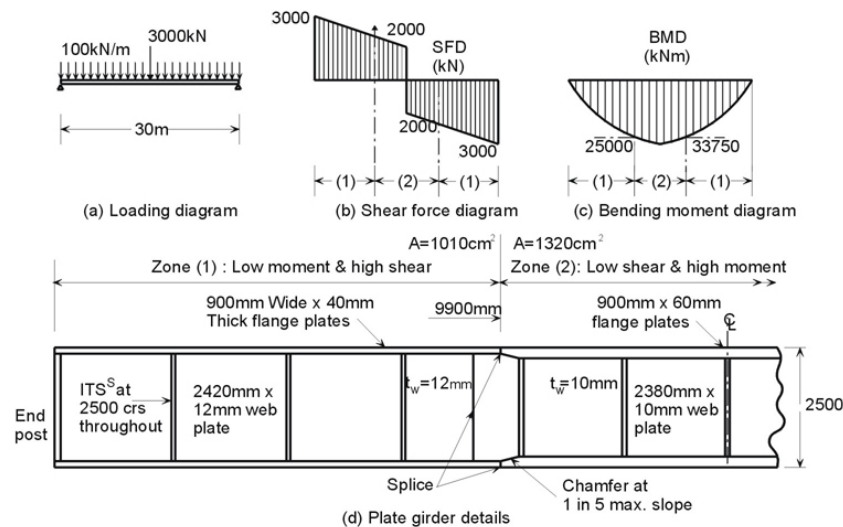


Figure 4 Plate girder with splice and variable cross-section

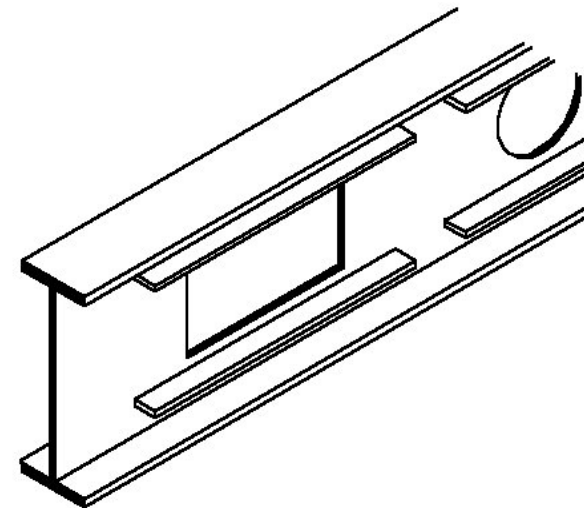


Figure 6 Plate girder with hole for services

1.2 Proportions

Since the designer, in principle, is quite free to choose all the dimensions of a plate girder, some indication of the more usual proportions is now given (see also Figure 7):

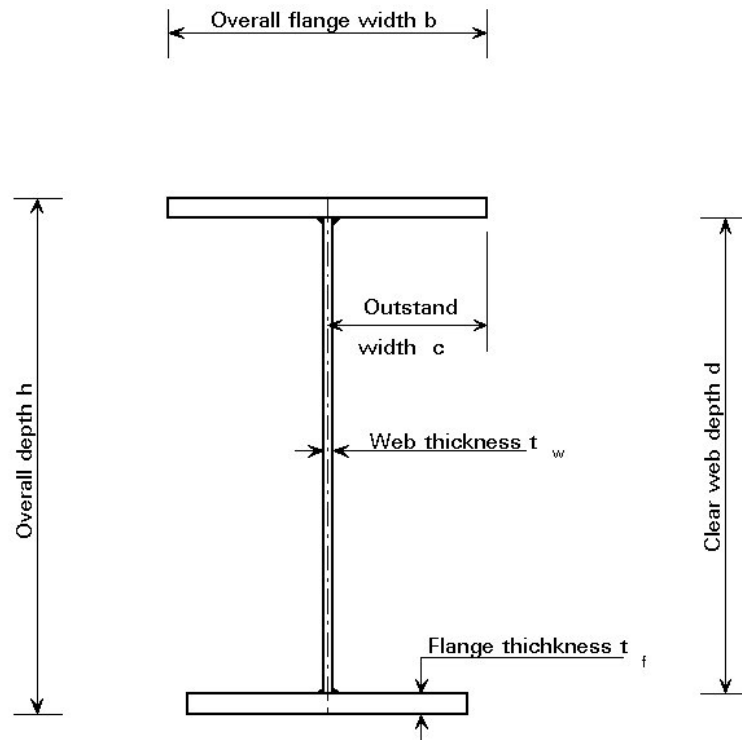


Figure 7 Plate girder proportions

Depth: Overall girder depth, h , will usually be in the range $L_o/12 \leq h \leq L_o/8$, where L_o is the length between points of zero moment. However, for plate girder bridges the range will extend to approximately $L_o/20$.

Flange breadth: The breadth, b , will usually be in the range $h/5 \leq b \leq h/3$, b being in multiples of 25mm. 'Wide flats' may be used unless the flange is very wide.

Flange thickness: The flange thickness, t_f , will usually at least satisfy the requirements of Eurocode 3 (Table 5.3.1) for Class 3 (semi-compact) sections, i.e. $c/t_f \leq 14\epsilon$. The thickness will usually be chosen from the standard plate thicknesses.

Web thickness: Web thickness, t_w , will determine the exact basis for web design, depending on whether the web is classified with regard to shear buckling as "thick" or "thin" (see later). Thin webs will often require stiffening; this may take the form of transverse stiffeners, longitudinal stiffeners or a combination, see Figure 2. Longitudinally stiffened girders are more likely to be found in large bridge construction where high d/t_w ratios are appropriate, e.g. $200 \leq d/t_w \leq 500$, due to the need to minimise self-weight.

Clearly, depending on the particular loading pattern, and on depth and breadth restrictions, one can expect wide variations within all the above limits which should be regarded as indicative only.

2. DESIGN CONCEPTS

Under static loading, ultimate limit states such as strength and stability will normally govern most plate girder design, with serviceability limit states such as deflection or vibration being less critical. Some absolute limits on plate slenderness are advisable so as to ensure sufficient robustness during erection. A generally accepted method [2] for designing plate girders (which is permitted by Eurocode 3) subject to a moment M_{ad}

and a coincident shear V_{ad} is to proportion the flanges to carry all the moment with the web taking all the shear. This provides a particularly convenient means for obtaining an initial estimate of girder proportions.

Thus, at any particular cross-section along a laterally-restrained plate girder, subject to specific values of bending moment and shear force, the flange and web plates can be sized separately. The required flange plate area may readily be obtained as follows:

$$A_f = M / [(h - t_f)f_y / \gamma_{MO}] \cong M / (hf_y / \gamma_{MO}) \quad (1)$$

(An iteration or two may be required depending on an assumed value of t_f and its corresponding f_y value from Table 3.1, Eurocode 3). Because the (normally) slender web will prevent the plastic moment of resistance of the cross-section from being attained, the flange b/t_f ratio need only comply with the Eurocode 3 (Table 5.3.1) requirements for a Class 3 (semi-compact) flange. The cross-sectional moment of resistance may then be checked using:

$$M_{f,Rd} = b t_f (h - t_f)f_y / \gamma_{MO} \quad (2)$$

Unfortunately, economic sizing of the web plate is not quite as straightforward, although where a thick web (defined later) is acceptable it can be rapidly sized by assuming uniform shear stress τ_y over its whole area. The web-to-flange fillet welds must be designed to transmit the longitudinal shear at the flange/web interface.

3. INFLUENCE OF BUCKLING ON DESIGN

Provided that the individual plate elements in a girder are each kept sufficiently stocky, the design may be based on straightforward yield strength considerations. Economic and practical considerations will, however, dictate that not all of these conditions will always be met. In most cases various forms of buckling must be taken into account. Figure 8 lists the different phenomena.

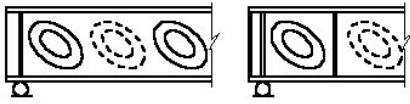




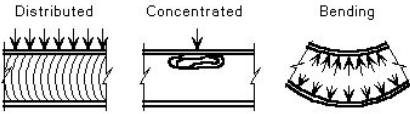
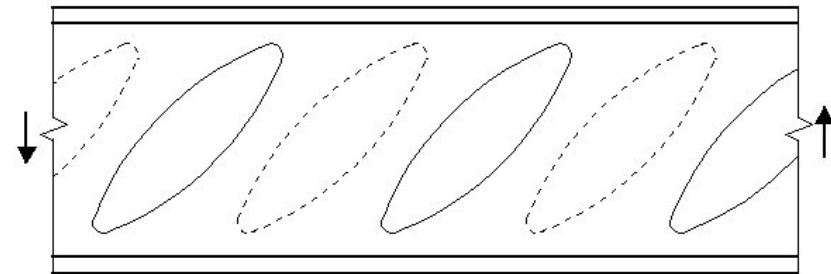
Buckling type	Illustration	[Eurocode 3 clauses] and ESDEP lectures
Shear buckling of web		[5.6] Lecture 8.4.2 & 8.4.3
Lateral-torsional buckling of girder		[5.5.2 and Annex F] Lecture 7.9
Local buckling of compression flange		[5.3 and table 5.3.1 (sheet 3)] Lecture 7.3
Compression buckling of web		[5.3 and table 5.3.1 (sheet 1)] Lecture 7.6
Flange induced buckling of the web		[5.7.7]
Local buckling of web (due to vertical load)		[5.7.4, 5.7.5, 5.3.6, 5.4.10]

Figure 8 Buckling phenomena in plate girders

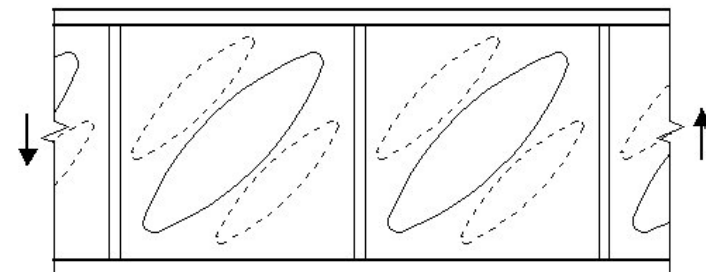
3.1 Shear Buckling of the Web

Once the d/t_w value for an unstiffened web exceeds a limiting figure (69ϵ in Eurocode 3) the web will buckle in shear before

it reaches its full shear capacity $A_w\tau_y$. Diagonal buckles, of the type shown in Figure 9(a), resulting from the diagonal compression associated with the web shear will form. Their appearance may be delayed through the use of vertical stiffeners, see Figure 9(b) since the load at which shear buckling is initiated is a function of both d/t_w and panel aspect ratio a/d .



(a) Unstiffened web



(b) stiffened web

Figure 9 Buckling of slender web under shear

3.3 Local Buckling of the Compression Flange

Provided that outstand proportions c/t_f are suitably restricted, local buckling will have no effect on the girder's load carrying resistance.

3.4 Compression Buckling of the Web

Webs for which $d/t_w \leq 124\epsilon$ and which are not subject to any axial load will permit the full elastic moment resistances of the girder to be attained. If this limit of d/t_w (or a lower one if axial compression in the girder as a whole is also present) is exceeded, then moment resistance must be reduced accordingly. If it is desired to reach the girder's full plastic moment resistance a stricter limit will be appropriate.

3.5 Flange Induced Buckling of the Web

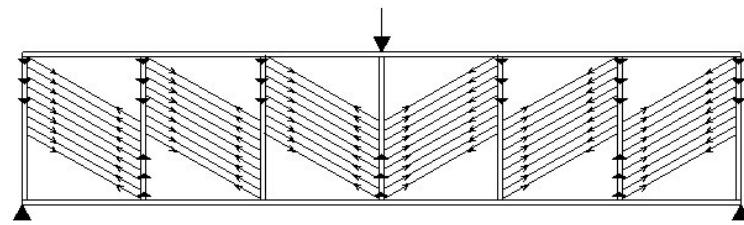
If particularly slender webs are used, the compression flange may not receive enough support to prevent it from buckling vertically rather like an isolated strut buckling about its minor axis. This possibility may be eliminated by placing a suitable limit on d/t_w . Transverse stiffeners also assist in resisting this form of buckling.

3.6 Local Buckling of the Web

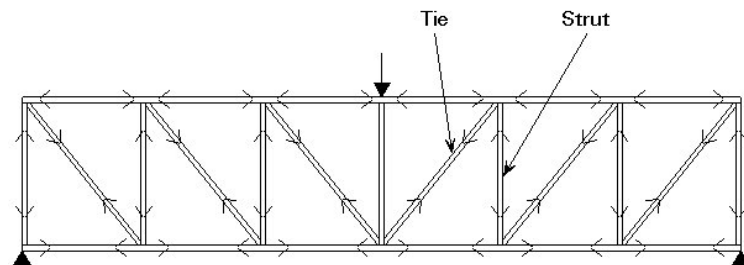
Vertical loads may cause buckling of the web in the region directly under the load as for a vertical strut. The level of loading that may safely be carried before this happens will depend upon the exact way in which the load is transmitted to the web, the web proportions, and the level of overall bending present.

4. POST-BUCKLING STRENGTH OF WEB

Owing to the post-buckling behaviour plates, unlike struts, are often able to support loads considerably in excess of their initial buckling load. In plate girder webs a special form of post-buckling termed "tension field action" is possible. Tension field action involves a change in the way in which the girder resists shear loading from the development of uniform shear in the web at low shear loads, to the equivalent truss action, shown in Figure 10, at much higher loads. In this action the elements equivalent to truss members are: the flanges, which form the chords; the vertical stiffeners which form the struts; and the diagonal tension bands which form the ties. The compressive resistance of the other diagonal of each web panel is virtually eliminated by the shear buckling.



(a) Tension field action



(b) Pratt or N-truss

Figure 10 Post-buckling tension field action

5. DESIGN CONSIDERATIONS

The principal functions of the main components found in plate girders may be summarised as follows:

Flanges	resist moment
Web	resists shear
Web/flange welds	resist longitudinal shear at interface
Vertical stiffeners	improve shear buckling resistance
Longitudinal stiffeners	improve shear and/or bending resistance.

6. CONCLUDING SUMMARY

- The main components in a plate girder have been identified and their principal functions noted.
- Initial sizing may be made on the basis that the flanges carry all of the moment and the web takes all of the shear.
- Shear buckling is likely to prevent the full web shear resistance from being attained in slender webs. Its appearance need not imply failure since additional load may be carried through tension field action.
- Web stiffeners (transverse and/or longitudinal) enhance both initial buckling and post-buckling resistance.

7. REFERENCES

- [1] Eurocode 3: "Design of Steel Structures": European Prestandard ENV1993-1-1: Part 1, General rules and rules for buildings, CEN, 1992.
- [2] Narayanan, R. (ed)., "Plated Structures; Stability and Strength", Applied Science Publishers, London, 1983.

8. ADDITIONAL READING

1. Dubas, P. and Gehri, E. (eds), "Behaviour and Design of Plated Steel Structures", Publication No 44, ECCS, 1986.

Chapters 4 and 5 provide more detailed accounts of the main features of plate girder behaviour and design.

2. Plate Girder Behaviour and Design II

SUMMARY

The design methods for plate girders subject to bending and shear, according to the methods of Eurocode 3[1], are presented. For shear loading two methods are described: the "simple post-critical method", and the "tension field method"; interaction diagrams can be used with both methods to allow for the effect of coincident moments.

1. INTRODUCTION

Any cross-section of a plate girder is normally subjected to a combination of shear force and bending moment. The primary function of the top and bottom flange plates of the girder is to resist the axial compressive and tensile forces arising from the applied bending moment. The primary function of the web plate is to resist the applied shear force.

Plate girders are normally designed to support heavy loads over long spans in situations where it is necessary to produce an efficient design by providing girders of high strength to weight ratio. The search for an efficient design produces conflicting requirements, particularly in the case of the web plate. To produce the lowest axial flange force for a given bending moment, the web depth (d) must be made as large as possible. To reduce the self weight, the web thickness (t_w) must be reduced to a minimum. As a consequence, in many instances the web plate is of slender proportions and is therefore prone to buckling at relatively low values of applied shear. A similar conflict may exist for the flange proportions. The required flange area is defined by the flange force and material yield

stress. The desire to increase weak axis second moment of area encourages wide, thin flanges. Such flanges are prone to local buckling.

Plate elements do not collapse when they buckle; they can possess a substantial post-buckling reserve of resistance. For an efficient design, any calculation relating to the ultimate limit state should take the post-buckling action into account. This is particularly so in the case of a web plate in shear where the post-buckling resistance arising from tension field action can be very significant.

Thus, in designing a plate girder it is necessary to evaluate the buckling and post-buckling action of webs in shear, and of flange plates in compression. The design of web plates operating in the post-buckling range is very different and will be discussed here in some detail. The section will start by concentrating upon the resistance of plate girders to predominantly shear loading. The effects of high co-existent bending moments will be considered.

The section will concentrate only on the main aspects of girder design assuming a basic cross-section. In particular, it is assumed that:

1. Only transverse web stiffeners are present (i.e. there are no longitudinal stiffeners).
2. Transverse web stiffeners possess sufficient stiffness and strength to resist the actions transmitted to them by the web.
3. An appropriate means is available to anchor the tension field.

4. No vertical patch loads are applied between the positions of the transverse web stiffeners.
5. Only solid webs are considered (i.e. there are no web openings or holes).

2. SHEAR BUCKLING RESISTANCE

A typical transversely stiffened plate girder is shown diagrammatically in Figure 1, which also defines the notation used. The shear buckling resistance of the web depends mainly on the depth to thickness ratio (d/t_w), and upon the spacing (a) of the transverse web stiffeners.

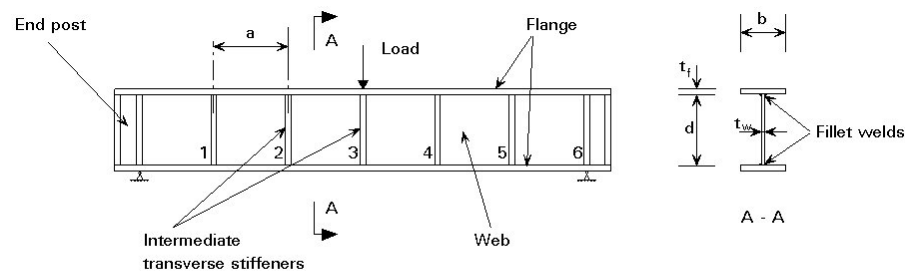


Figure 1 Elevation of a typical plate girder

Intermediate transverse stiffeners are normally employed to increase the shear buckling resistance of the web, although designers may sometimes choose to use a thicker web plate rather than incur the additional fabrication costs arising from the use of intermediate stiffeners. Girders without intermediate stiffeners are normally termed "unstiffened" girders, even though they will normally have stiffeners at points of support and possibly at the position of load application.

Web buckling should be checked in all cases where the depth to thickness ratio, (d/t_w), of the web exceeds 69ϵ . Eurocode 3

then offers the choice of 2 methods for plate girder design. The methods are:

a) **simple post-critical method**, which may be applied to both stiffened and unstiffened girders and is therefore of general application.

b) **tension field method**, which may only be applied to girders with intermediate transverse stiffeners. Even for such girders its range of application is limited to a range of stiffener spacing defined by:

$$1,0 \leq a/d \leq 3,0$$

There is now considerable evidence [2] that tension field action does develop in girders where the stiffener spacing lies outside this range, and also in unstiffened girders; such evidence, however, has yet to be presented in a form that is suitable for inclusion in a design code.

The simple post-critical method is seen as a general-purpose method which can be applied to the design of all girders. The tension field method, on the other hand, can be applied to a certain range of girders only, but will lead to considerably more efficient designs for these girders, because it takes full account of the post-buckling reserve of resistance. Each method will now be discussed.

2.1 Calculation of the Shear Buckling Resistance by the Simple Post-Critical Method

This simple approach allows the design shear buckling resistance ($V_{ba,Rd}$) to be determined directly as follows:

$$V_{ba,Rd} = d t_w \tau_{ba} / \gamma_{M1} \quad (1)$$

where all the terms in the expression are familiar, except the post-critical shear strength, τ_{ba} . The calculation of this term depends upon the slenderness of the web which may be conveniently expressed by the following parameter:

$$\bar{\lambda}_w = \frac{d / t_w}{37,4 \sqrt{k_\tau}} \quad (2)$$

Here, k_τ is a shear buckling factor calculated from elastic buckling theory [3]. For simplicity, it is conservatively assumed in this calculation that the boundaries of the web panel are simply supported, since the true degree of restraint offered by the flanges and adjacent web panels is not known. The resulting expression obtained for the shear buckling factor is dependent upon the spacing of the transverse web stiffeners as follows:

for closely spaced intermediate stiffeners ($a/d < 1,0$):

$$k_\tau = 4 + \frac{5,34}{(a/d)^2}$$

for widely spaced intermediate stiffeners ($a/d \geq 1,0$):

$$k_\tau = 5,34 + \frac{4}{(a/d)^2}$$

for unstiffened webs: $k_\tau = 5,34$

Knowing the shear buckling factor, the slenderness parameter is determined from Equation (2) and the calculation of the post-critical shear strength then depends, as illustrated in Figure 2, upon whether the web is:

a) **stocky or thick** ($\bar{\lambda}_w \leq 0,8$, region AB in Figure 2) in which case the web will not buckle and the shear stress at failure will reach the shear yield stress of the web material:

$$\tau_{ba} = f_{yw} / \sqrt{3}$$

where f_{yw} is the tensile yield strength

b) **intermediate** ($0,8 < \bar{\lambda}_w < 1,2$, region BC in Figure 2) which represents a transition stage from yielding to buckling action with the shear strength being evaluated empirically from the following:

$$\tau_{ba} = [1 - 0,625 (\bar{\lambda}_w - 0,8)] (f_{yw} / \sqrt{3})$$

c) **slender or thin** ($\bar{\lambda}_w \geq 1,2$, region CD in Figure 2) in which case the web will buckle before it yields and a certain amount of post-buckling action is taken into account empirically:

$$\tau_{ba} = \left[\frac{0,9}{\bar{\lambda}_w} \right] (f_{yw} / \sqrt{3})$$

The calculation of the shear buckling resistance by the simple post-critical method is then completed by substitution of the appropriate value of τ_{ba} into Equation (1).

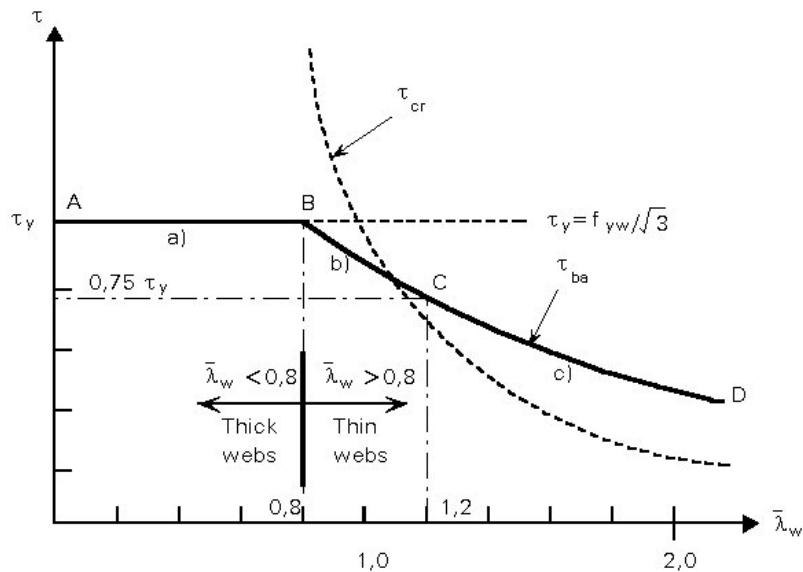


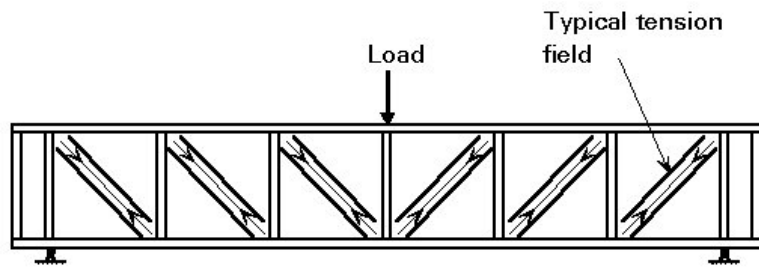
Figure 2 Simple post-critical shear strength

compressive stresses. In this post-buckling range, a new load-carrying mechanism is developed, whereby any additional shear load is carried by an inclined tensile membrane stress field. This tension field anchors against the top and bottom flanges and against the transverse stiffeners on either side of the web panel, as shown. The load-carrying action of the plate girder then becomes similar to that of the N-truss in Figure 3b. In the post-buckling range, the resistance offered by the web plates is analogous to that of the diagonal tie bars in the truss.

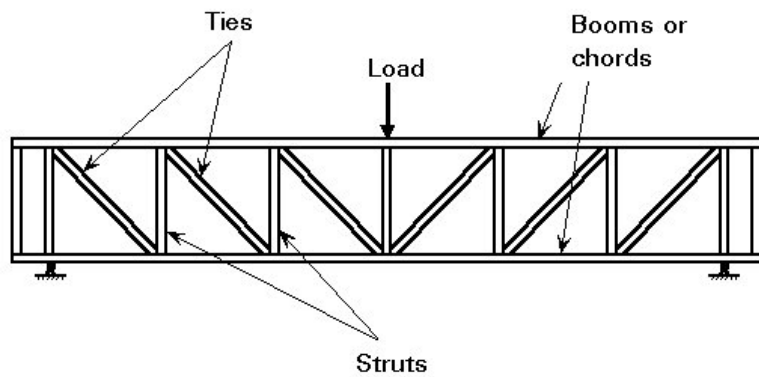
2.2 Calculation of the Shear Buckling Resistance by the Tension Field Method

For transversely stiffened girders where the transverse stiffener spacing lies within the range $1.0 \leq a/d \leq 3.0$, full account may be taken of the considerable reserve of post-buckling resistance. This reserve arises from the development of "tension field action" within the girder.

Figure 3a shows the development of tension field action in the individual web panels of a typical girder. Once a web panel has buckled in shear, it loses its resistance to carry additional



(a) Tension field action in individual sub panels of a girder with transverse stiffeners



(b) Typical N-truss for comparison

Figure 3 Tension field action

The total shear buckling resistance for design ($V_{bb,Rd}$) is calculated in Eurocode 3, by superimposing the post-buckling resistance upon the initial elastic buckling resistance as follows:

Total shear resistance = elastic buckling resistance + post-buckling resistance:

$$V_{bb,Rd} = (d t_w \tau_{bb})/\gamma_{M1} + 0,9 (g t_w \sigma_{bb} \sin \phi)/\gamma_{M1} \quad (3)$$

The basis for this assumed behaviour is shown diagrammatically in Figure 4.

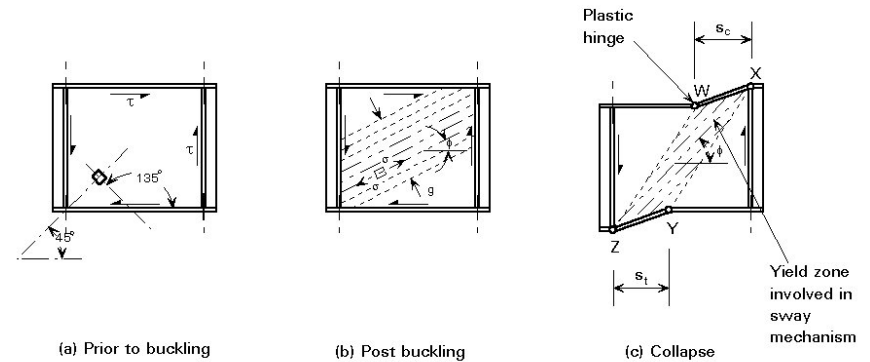


Figure 4 Phases in behaviour up to collapse of a typical panel in shear

Figure 4a shows the situation prior to buckling, as represented by the first term in Equation (3). At this stage, equal tensile and compressive principal stresses are developed in the web. The shear buckling strength, τ_{bb} , is calculated from elastic buckling theory and leads to equations similar, but not identical, to those given earlier in Section 2.1 for the simple post-critical shear strength τ_{ba} . Thus, the calculation of the shear buckling resistance again depends, as shown in Figure 5, upon whether the web is:

a) **stocky or thick** ($\bar{\lambda}_w = 0,8$, region AB in Figure 5) in which case the web will not buckle and the shear yield stress is again taken:

$$\tau_{bb} = f_{yw}/\sqrt{3}$$

where f_{yw} is the tensile yield strength

b) **intermediate** ($0,8 < \bar{\lambda}_w < 1,25$, region BC in Figure 5) where, in the transition from yield to buckling:

$$\tau_{bb} = [1 - 0,8 (\bar{\lambda}_w - 0,8)] (f_{yw}/\sqrt{3})$$

c) **slender or thin** ($\bar{\lambda}_w \geq 1,25$, region CD in Figure 5) where the web will buckle and, from elastic buckling theory:

$$\tau_{bb} = [1/\bar{\lambda}_w^2][f_{yw}/\sqrt{3}]$$

Thus, knowing τ_{bb} , the first term of the expression in Equation (3) can be evaluated.

The evaluation of the second term, corresponding to the post-buckling action, is more complex although it may still be reduced to a convenient design procedure, as described below.

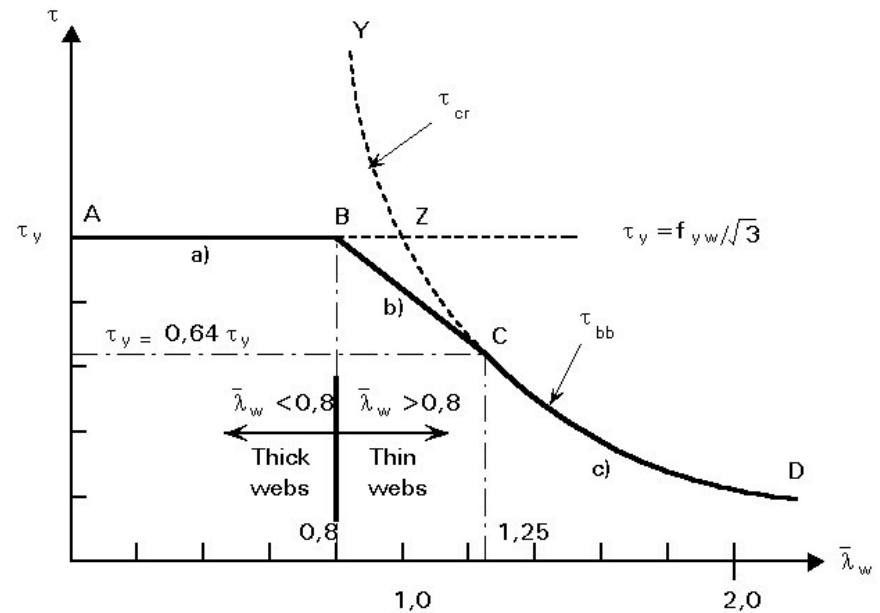


Figure 5 Shear buckling of webs

In the post-buckling range, as shown in Figure 4b, an inclined tensile membrane stress field is developed, at an inclination ϕ to the horizontal. Since the flanges of the girder are flexible, they will begin to bend inwards under the pull exerted by the tension field.

Further increase in the load will result in yield occurring in the web under the combined effect of the membrane stress field and the shear stress at buckling. The value of the tension field stress (σ_{bb}) at which yield will occur, termed the "strength of the tension field" in Eurocode 3, may be determined by

applying the Von Mises-Hencky yield criterion [2]. This results in the following expression for the strength of the tension field:

$$\sigma_{bb} = [f_{yw}^2 - 3\tau_{bb}^2 + \psi^2]^{0.5} - \psi$$

where the term $\psi = 1,5 \tau_{bb} \sin 2\phi$ is introduced for convenience only.

Once the web has yielded, final failure of the girder will occur when the mechanism comprising 4 plastic hinges has formed in the flanges, as shown in Figure 4c. A detailed analysis of this collapse mechanism, by considering the internal forces developed in the web and imposed upon the flanges (see [2]) allows the width (g) of the tension field, which appears in the second term of Equation (3), to be evaluated:

$$g = d \cos \phi - (a - s_c - s_t) \sin \phi$$

where, as in Figure 4c, s_c and s_t denote the positions at which the plastic hinges form in the compression and tension flanges respectively.

The hinge positions are calculated [2] from the knowledge that the hinges will form at the point of maximum moment, and therefore zero shear, in the flanges; the appropriate expression is as follows:

$$S = [2/\sin \theta][M_{Nf,Rk}/t_w \sigma_{bb}]^{1/2} \leq a \quad (4)$$

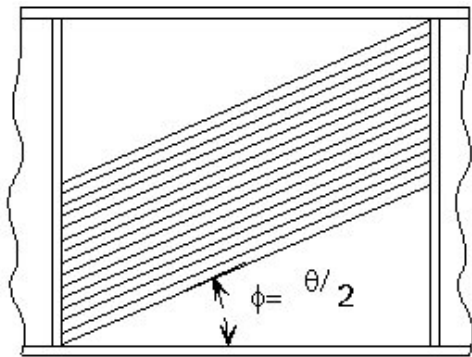
where $M_{Nf,Rk}$ is the plastic moment of resistance of the flange, i.e. $0,25 bt_f^2 f_{yf}$. When high bending moments are applied to the girder, in addition to shear, then axial forces ($N_{f,Sd}$) will be developed in the flanges. Such axial forces will, of course,

reduce the plastic moment of resistance of the flanges. Their effects can be calculated from standard plasticity theory as:

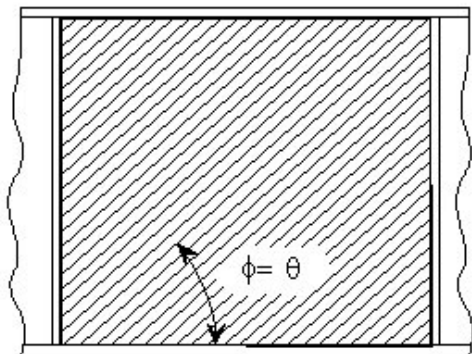
$$M_{Nf,Rk} = 0,25 bt_f^2 f_{yf} \{1 - [N_{f,Sd} / (bt_f f_{yf} / \gamma_{mo})]^2\} \quad (5)$$

All the terms required for the calculation of the total shear resistance from Equation (3), other than the inclination ϕ of the tension field, are now known. Unfortunately, the value of ϕ cannot be determined directly and an iterative procedure has to be adopted in which successive values of ϕ are assumed and the corresponding shear resistance evaluated in each case. The process is repeated until the value of ϕ providing the maximum, and therefore the required, value of the shear resistance has been established. The variation of the shear resistance with ϕ is not very rapid. The correct value of ϕ lies between a minimum of $\theta/2$ and a maximum of θ , where θ is the slope of the panel diagonal $\tan^{-1}(d/a)$, as shown in Figure 6. A parametric study [2] has established that, for girders of normal proportions, the value of ϕ which produces the maximum value of shear resistance is approximately given by:

$$\phi = \theta / 1,5$$



(a) "Web only"
tension field action



(b) Complete
tension field action

Figure 6 Inclination of tension fields

The assumption of this value of ϕ will lead either to the correct value or to an underestimation of the shear resistance. It will therefore give a safe approximation and will also give a good starting value of ϕ if a more accurate process of iteration is to be carried out. The correct value of ϕ is that which gives the maximum value of $V_{bb,Rd}$.

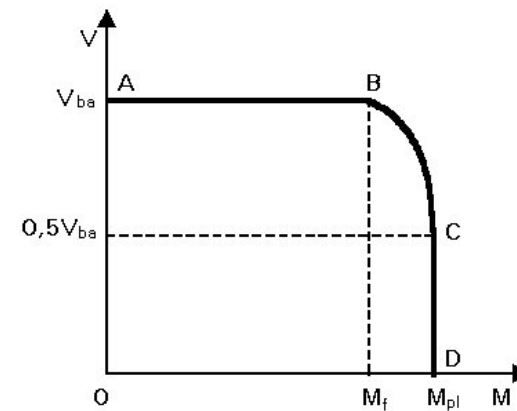
3. INTERACTION BETWEEN SHEAR AND BENDING

In general, any cross-section of a plate girder will be subjected to bending moment in addition to shear. As discussed in [2], this combination makes the stress conditions in the girder web considerably more complex. In the first place, the stresses from the bending moment will combine with the shear stresses to give a lower buckling load. Secondly, in the post-buckling range, the bending stresses will influence the magnitude of the tension field membrane stresses required to produce yield in the web. Finally, as already discussed with reference to Equation (5), the axial flange forces arising from the bending moment will reduce the plastic moment of resistance of the flanges.

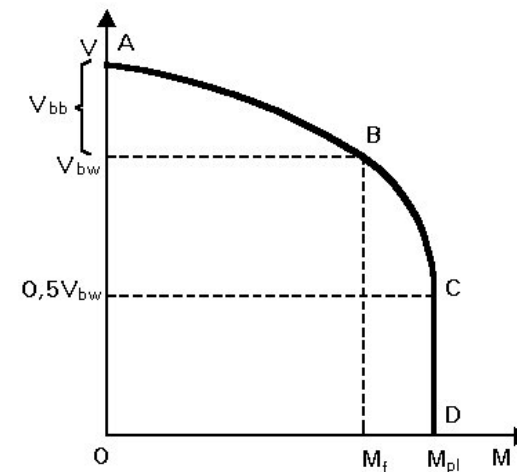
The proper evaluation of all these effects is complex but, as discussed in [2], certain assumptions may be made about the interaction of moment and shear to produce a simple and effective design procedure. In Eurocode 3, the procedure for allowing for moment/shear interaction naturally depends upon whether the simple post-critical method of Section 2.1 or the tension field method of Section 2.2 is being used to calculate the shear buckling resistance. Each case will now be considered separately.

3.1 Interaction between Shear and Bending in the Simple Post-Critical Method

The interaction between shear and bending can be conveniently represented by the diagram shown in Figure 7a (Figure 5.6.4a of Eurocode 3) where the shear resistance of the girder is plotted on the vertical axis and the moment resistance is plotted horizontally. The interaction represents a failure envelope, with any point lying on the curve defining the co-existent values of shear and bending that the girder can just sustain.



(a) Simple post-critical method



(b) Tension field method

Figure 7 Interaction of shear buckling resistance and moment resistance

The interaction diagram can be considered in 3 regions. In region AB, the applied bending moment M_{ad} is low and the girder can then sustain a shear load V_{ad} that is equal to the full value of the shear buckling resistance calculated from the simple post-critical method, as in Equation (1). Thus, in this region:

$$M_{sd} \leq M_{f,Rd}$$

$$V_{sd} \leq V_{ba,Rd} \quad (6)$$

The moment that defines the end of the range at point B ($M_{f,Rd}$) is the plastic moment of resistance of the cross-section consisting of the flanges only, i.e. neglecting any contribution from the web. In this calculation it is necessary to appreciate that the plates of the compression flange may buckle and, if necessary, to take this into account by adopting an effective width b_{eff} for the flange.

At the other extreme of the interaction diagram in region CD, the applied shear V_{ad} is low. Provided it does not exceed the limiting value of $0,5 V_{ba}$ at point C then the plastic moment of resistance of the complete cross-section $M_{pl,Rd}$ need not be reduced to allow for shear.

In the intermediate region BC the co-existent applied moment M_{Sd} and shear V_{Sd} values must satisfy the following relationship:

$$M_{Sd} \leq M_{f,Rd} + (M_{pl,Rd} - M_{f,Rd}) [1 - (2V_{Sd}/V_{ba,Rd} - 1)^2] \quad (7)$$

The complete range of moment/shear interaction has thus been defined for the simple post-critical method.

3.2 Interaction between Shear and Bending in the Tension Field Method

The procedure for the tension field method follows that described above for the simple post-critical method. It leads to the construction of a similar, though not identical, interaction diagram, see Figure 7b (Figure 5.6.4b of Eurocode 3).

In the low moment region AB, again defined by values of the applied moment less than $M_{f,Rd}$, the girder can sustain a shear load V_{ad} that is equal to the "web only" shear resistance $V_{bw,Rd}$ calculated from tension field theory. Thus:

$$M_{Sd} \leq M_{f,Rd}$$

$$V_{Sd} \leq V_{bw,Rd}$$

The "web only" shear resistance is the specific value of the total shear resistance $V_{bw,Rd}$ calculated from Equation (1), for the case when $M_{Nf,Rk}$ in EC3=0 in Equation (5). This is, in effect, a conservative approach which neglects the contribution of the flanges to the tension field action.

At the other extreme in region CD, the procedure remains as for the simple post-critical method except that the limiting value of shear at point C is now taken as $0,5V_{bw}$. Similarly, the procedure for the intermediate region BC remains as before except that the substitution of the tension field value V_{bw} for V_{ba} in Equation (7) gives:

$$M_{Sd} \leq M_{f,Rd} + (M_{pl,Rd} - M_{f,Rd}) [1 - (2V_{Sd}/V_{bw,Rd} - 1)^2] \quad (8)$$

The complete range of moment/shear interaction is thus defined for the tension field method.

4. CONCLUDING SUMMARY

- Procedures for the design of plate girders subject to shear utilize varying degrees of post-buckling resistance and correspond to either the "simple post-critical" or "tension field" methods of Eurocode 3.
- Moment resistance of plate girders may normally be based on the plastic moment resistance of a cross section consisting of the flanges only.
- Design for coincident shear and moment should be undertaken using an interaction diagram. The simplest approach consists of designing the web to carry the whole of the shear, with the flanges resisting the moment.
- The "tension field" method is more restricted in application than the "simple post-critical" method, but gives higher strengths.

5. REFERENCES

[1] Eurocode 3 "Design of Steel Structures": European Prestandard ENV1993-1-1: Part 1.1, General rules and rules for buildings, CEN, 1992.

[2] Narayanan, R. (ed), "Plated Structures; Stability and Strength", Applied Science Publishers, London 1983.

Chapter 1 covers basic aspects of plate girder behaviour and design.

[3] Bulson, P. S. "The Stability of Flat Plates", Chatto & Windus, London, 1970.

General coverage of plate buckling and explanation of k_τ values for numerous cases.

6. ADDITIONAL READING

1. Dubas, P. and Gehri, E. (eds)., "Behaviour and Design of Plated Steel Structures", Publication No. 44, ECCS, 1986.

Chapters 4 and 5 provide a detailed coverage of plate girder design, taking the reader well beyond the content of this section. They also refer to numerous original sources.

2. Galambos, T. V. (ed)., "Guide to Stability Design Criteria for Metal Structures", 4th Edition, John Wiley, 1988.

3. Plate Girder Design - Special Topics

SUMMARY

The detailed design of particular elements of plate girders is considered in this section. The structural action of web panels imposes stringent requirements on adjacent boundary elements. This section considers the design of transverse web stiffeners and end posts according to Eurocode 3 [1] and also considers the particular problems caused by patch loading. Two other aspects of design, not currently covered by Part 1.1 of Eurocode 3, i.e. the design of longitudinally stiffened girders and girders with large web openings, are also discussed.

1. INTRODUCTION

The two design approaches proposed in Eurocode 3 [1] have been outlined; these are the "simple post-critical" method, which is generally applicable, and the "tension field" method which gives significantly higher load resistances by taking the post-buckling resistance of the girders into account.

This section seeks to complete the discussion of plate girder design by considering further aspects of detailed design. For example, the development of post-buckling action in a web plate can only occur when the elements at the boundary of that web plate are able to provide an adequate anchorage for the tension field forces developed within the plate. This section will consider the design of these boundary elements. These elements may be in the form of intermediate transverse stiffeners or end posts.

Girders may be subjected to high loads in localised regions, away from stiffener positions, creating a possibility that crippling of the web plate may occur. An example of this occurs in crane gantry girders subjected to a vertical loading which travels along the flange. The effects of such "patch loading" must be carefully taken into account in design. This aspect is very thoroughly treated in Eurocode 3 [1].

Two other important aspects of plate girder design are the treatment of girders with longitudinal web stiffeners, and of girders with large openings in the web plates. Openings are frequently required, particularly in building construction, to allow access for service ducts, etc. Neither of these two situations is covered in Part 1.1 of Eurocode 3.

2. TRANSVERSE WEB STIFFENERS

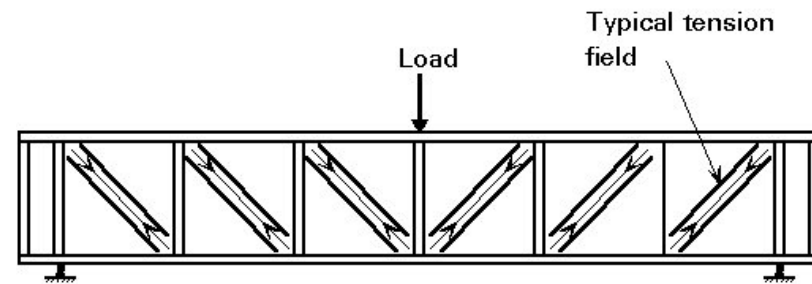
To achieve an effective design, i.e. a plate girder of high strength/weight ratio, it is usually necessary to provide intermediate transverse web stiffeners. Eurocode 3 [1] only allows the application of the tension field method to give a significantly enhanced load resistance, when the web is stiffened. The Eurocode also specifies that such stiffeners must be spaced such that the stiffener spacing/web depth ratio (a/d) is within the following range:

$$1,0 \leq a/d \leq 3,0$$

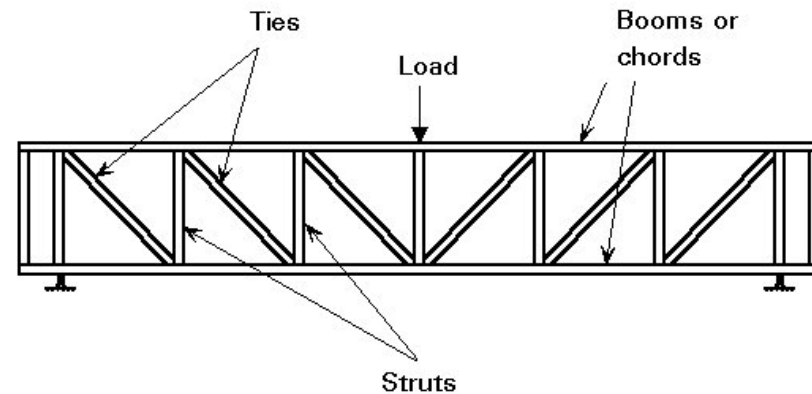
Transverse stiffeners play an important role in allowing the full ultimate load resistance of a plate girder to be achieved. In the first place they increase the buckling resistance of the web; secondly they must continue to remain effective after the web buckles, to provide anchorage for the tension field; finally they

must prevent any tendency for the flanges to move towards one another.

The satisfactory performance of a transverse stiffener can best be illustrated by comparing the girders shown, after testing, in Slides 1 and 2. In Slide 1 the stiffeners have remained straight and have clearly fulfilled the function of vertical struts in the simplified N-truss model of the post-buckling action. In Slide 2 the stiffener has failed and has been unable to limit the buckling to the adjacent sub-panels of the girder; instead, the buckle has run through the stiffener position extending over both panels. Consequently, significant reduction in the failure load of the girder occurred.

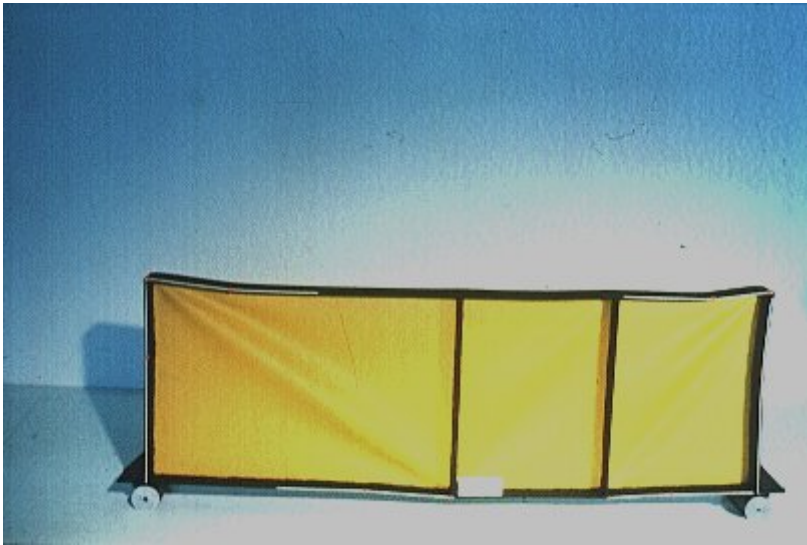


(a) Tension field action in individual sub panels of a girder with transverse stiffeners



(b) Typical N-truss for comparison

Figure 1 Tension field action



Slide 1



Slide 2

The requirements to ensure adequate stiffener performance are given in Section 5.6.5 of Eurocode 3. First, the stiffener must be of adequate rigidity in the direction perpendicular to the plane of the web to prevent web buckling. This condition is satisfied provided the stiffener has a second moment of area I_s that satisfies the following empirical formulae:

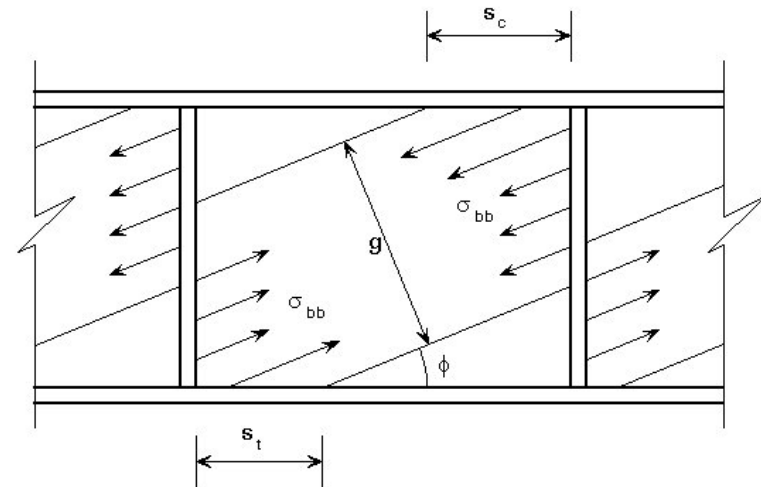
$$I_s \geq 1,5 d^3 t_w^3 / a^2 \text{ when } a/d < \sqrt{2}$$

$$I_s \geq 0,75 d t_w^3 \text{ when } a/d \geq \sqrt{2}$$

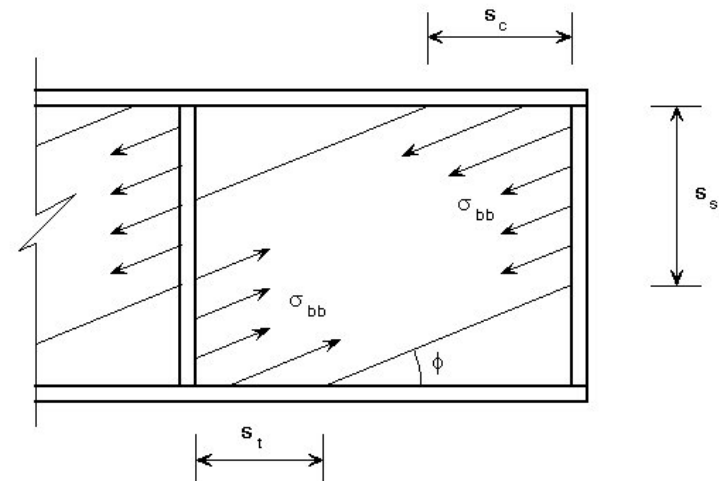
Secondly, the buckling resistance of the vertical "stiffener strut" must be sufficient to support the tension field forces shown in Figure 2a (which has been reproduced from Fig. 5.6.3a of Eurocode 3). It must also resist the resultant axial compressive force N_s that is imposed upon it. This force is calculated as follows:

$$N_s = V_{sd} - d t_w \tau_{bb} / \gamma_{M1}$$

where τ_{bb} is the initial shear buckling resistance of the web panels. When the two web panels adjacent to the particular stiffener being designed are not identical, the lower value of τ_{bb} for the two panels should be taken. As previously, V_{sd} is the design value of the shear force.



(a) Internal panel



(b) End panel

Figure 2 Tension field stresses

The buckling resistance of the stiffener strut to this axial compressive force is then calculated using Section 5.7.6 of Eurocode 3. Since the stiffener is attached to the web plate, a portion of the web acts effectively with the stiffener in resisting the axial compression. It is difficult to calculate the extent of this portion of the web but experimental observations have allowed an empirical effective web width of $30\epsilon t_w$ to be established, as shown in Figure 3 (which has been reproduced from Figure 5.7.4 of Eurocode 3). Having established the effective cross-section of the stiffener strut in this way, its buckling resistance is determined as for any other compression member according to Section 5.5.1 of the Eurocode.

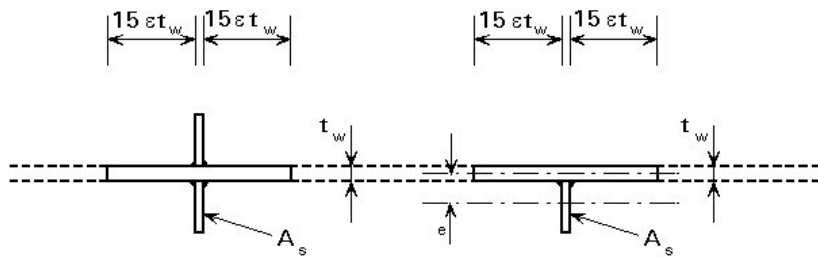
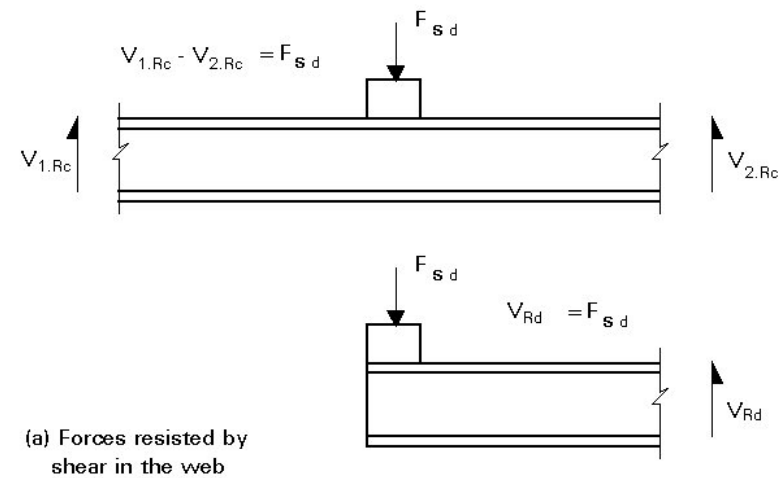
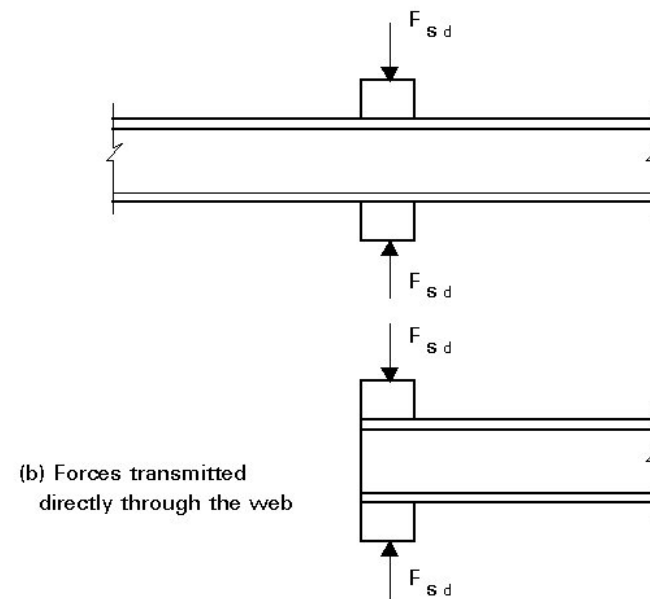


Figure 3 Effective cross-section of stiffeners



(a) Forces resisted by shear in the web



(b) Forces transmitted directly through the web

Figure 5 Forces applied through a flange

For a "load bearing" stiffener, i.e. a transverse stiffener located at a position where an external load is applied to the girder, an additional consideration is necessary. The resistance of the effective cross-section of the load bearing stiffener should also be checked at a position close to the loaded flange.

3. END PANELS AND POSTS

The requirement for adequate boundary members to support the loading imposed by the post-buckling tension field is particularly onerous in the case of the end panel of the girder. The situation of the transverse stiffener at the end of the girder, i.e. the "end post", is very different from that of an intermediate stiffener, compare Figure 2b to Figure 2a. At the end of the girder, the forces imposed by the tension field in the end panel have to be resisted entirely by the end post without support from any further adjacent panels.

Design procedures for end panels and posts are given in Clauses 5.6.4.3 and 5.6.4.4 of Eurocode 3 [1]. Basically, they allow the designer two options.

Firstly, the designer may choose not to design an end post that will provide adequate anchorage for the tension field. As a consequence, the end panel of the web must be designed according to the simple post-critical method so that a tension field does not develop within it. This option offers a simple design procedure but has the disadvantage that the calculated shear resistance of the end panel will be significantly lower than that of the internal web panels in the girder. Since it is probable that the applied shear in the end region will be higher than at any point on the span, this procedure will not provide an effective design solution if the stiffener spacing remains

constant over the complete length of the girder. As shown in Figure 4a, the designer should then reduce the spacing of the stiffeners bounding the end panel so that the shear resistance of that panel as calculated by the simple post-critical method becomes equal to that calculated by the tension field method for the internal panels.

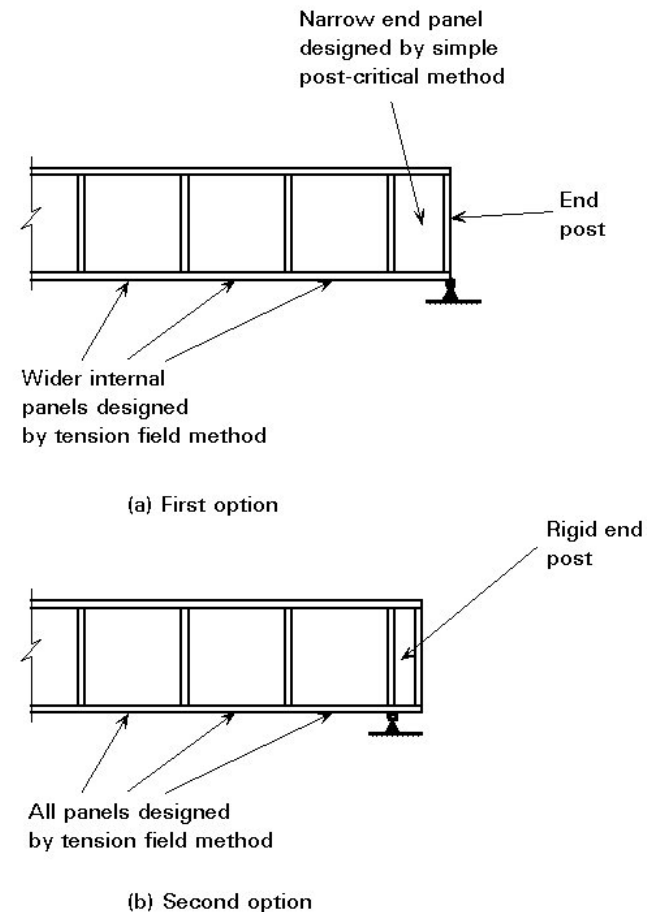


Figure 4 Design of end panels and posts

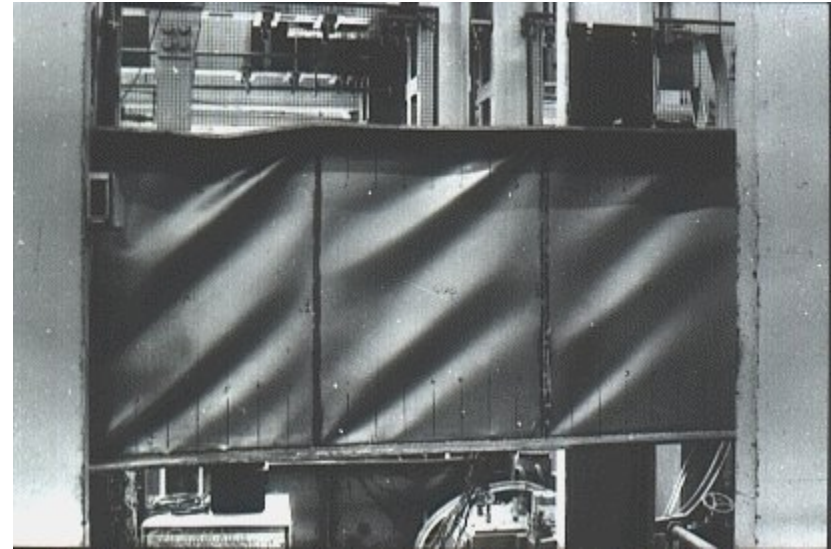
The more effective, but more complex, option is to design the end post to provide an adequate anchorage for the web tension field. The end panel of the web can then be designed according to the tension field method, so that the design shear buckling resistance ($V_{bb,Rd}$) can be calculated for internal web panels, i.e.

$$V_{bb,Rd} = [(d t_w \tau_{bb}) + 0,9 (g t_w \sigma_{bb} \sin \phi)] / \gamma_{M1}$$

The slight difference for the end panel arises in the calculation of the width g of the tension field. For an internal panel the width is given by:

$$g = d \cos \phi - (a - s_c - s_t) \sin \phi$$

where, s_c and s_t denote the lengths over which the tension field anchors onto the compression and tension flanges, see Figure 2a. For an end panel, the failure mechanism may be different since, as shown in Slide 3, a plastic hinge may also form in the end post.



Slide 3

This hinge affects the anchorage length for the compression flange which must now be calculated as:

$$s_c = \frac{2}{\sin \phi} \left[\frac{M_{pl,1} + M_{pl,2}}{2 t_w \sigma_{bb}} \right]^{*}$$

where $M_{pl,1}$ is the reduced plastic moment of the flange at the internal hinge position, allowing for the presence of the axial force (N_{f1}) at that position.

The other plastic hinge will form either at the end of the flange, as in the case of an internal panel, or in the end post. The location of the hinge, as defined by s_s in Figure 2b, will depend

upon which of these two elements has the lower plastic moment of resistance. $M_{pl,2}$ takes the lesser of these two values.

In this way, Clause 5.6.4.3 of Eurocode 3 allows the geometry of the tension field developed in the end panel to be fully defined, see Figure 2b. The design shear buckling resistance $V_{bb,Rd}$ of the panel can then be calculated together with the horizontal component F_{bb} of the anchorage force of the tension field imposed on the end post:

$$F_{bb} = t_w s_s \sigma_{bb} \cos^2 \phi$$

The end post resists this force by acting as a vertical beam spanning between the two flanges. For this purpose it must satisfy the following criterion:

$$M_{pl,2} + M_{pl,3} \geq 0,5 F_{bb} s_s$$

where the reduced plastic moment of the end post:

$$M_{pl,3} = 0,25 b_s t_s^2 f_{ys} \{ 1 - [N_{s3} / (b_s t_s f_{ys})]^2 \}$$

allows for the effect of the axial force in the end post:

$$N_{s3} = V_{sd} - \tau_{bb} t_w (d - s_s)$$

If it proves difficult to provide an end post in the form of a single plate to resist these forces, then the designer may consider providing an end arrangement such as that shown in Figure 4b. In this case, two transverse stiffeners are used. These two stiffeners and the portion of the web projecting beyond the end support form a rigid end post to provide the necessary anchorage for the tension field in the end panel. The

disadvantage of such an arrangement is that adequate space must be available to allow the girder to project beyond its end support.

4. WEB CRIPPLING

There are many situations where it is not possible to provide transverse web stiffeners at all points where vertical loads are applied to the girder. For example, a crane gantry girder is subjected to a vertical loading that travels along the flange; also, girders may be launched during construction so that the flange actually moves over the fixed point of support. In such cases, special consideration must be given to the design of the unstiffened web in the local region underneath, or above, the applied point or "patch" loading to prevent "web crippling". The webs of all beams must be checked for this possible local failure. Plate girders are particularly susceptible to this form of failure because of the slenderness of the web plates that are normally used in their construction.

"Web crippling" is discussed in Section 5.7 of Eurocode 3. It distinguishes between the two different loading cases that are shown in Figure 5 (taken from Figure 5.7.1 of the Eurocode). In Figure 5a, the force is applied to one flange only and is therefore resisted by shear forces developed within the web plate. In this case the web plate has to be checked for its "crushing" and for its "crippling" resistance. In the other case, shown in Figure 5b, the force is applied to one flange, transmitted directly by compressive forces developed in the web, and resisted by a reactive force on the other flange. The web must again be checked for its "crushing" resistance. The "buckling" resistance of the web must also be considered in this case.

There are, therefore, three types of web resistance that must be calculated. In each case the resistance is dependent upon the length over which the applied force is effectively distributed on the flange. This is termed the "stiff bearing length" (s_s). It is calculated on the assumption of a dispersion of load through solid steel material at a slope of 1:1.

The terms "crushing", "crippling" and "buckling" resistance are introduced to differentiate between the phenomena being considered. In each case the appropriate resistance is calculated from empirical formulae:

"Crushing" resistance, where crushing is local yielding of the web without any buckling, is given by:

$$R_{y,Rd} = (s_s + s_y) t_w f_{yw} / \gamma_{M1}$$

"Crippling" resistance, where crippling is localised buckling of the web in the presence of plasticity, is given by:

$$R_{a,Rd} = 0,5 t_w^2 (E f_{yw})^{1/2} [t_f / t_w]^{1/2} + 3 (t_w / t_f) (s_s / d) / \gamma_{M1}$$

"Buckling" of the web occurs with out-of-plane deformation over most of the depth of the web.

The "buckling" resistance ($R_{b,Rd}$) for the compressive loading situation illustrated in Figure 5b is determined simply by considering the web plate as a vertical compression member. First it is necessary to determine the breadth of the web "strut" (b_{eff}) that is effective in resisting the compression. This breadth may be calculated as:

$$b_{eff} = [h^2 + s_s^2]^{1/2}$$

where:

h is the overall depth of the girder.

s_s is the stiff bearing length discussed above.

The buckling resistance of this idealised strut is then determined as for any other compression member according to Section 5.5.1 of Eurocode 3.

5. LONGITUDINAL WEB STIFFENERS

To increase the strength/weight ratio of plate girders, slender webs may be reinforced by longitudinal, as well as transverse, stiffeners. A typical longitudinally stiffened girder is shown after failure in Slide 4. The main function of the longitudinal stiffeners is to increase the buckling resistance of the web with respect of both shear and bending loads. An effective stiffener will remain straight, thereby sub-dividing the web panel and limiting the buckling to the smaller sub-panels. The resulting increase in the ultimate resistance of the girder can be significant.



Slide 4

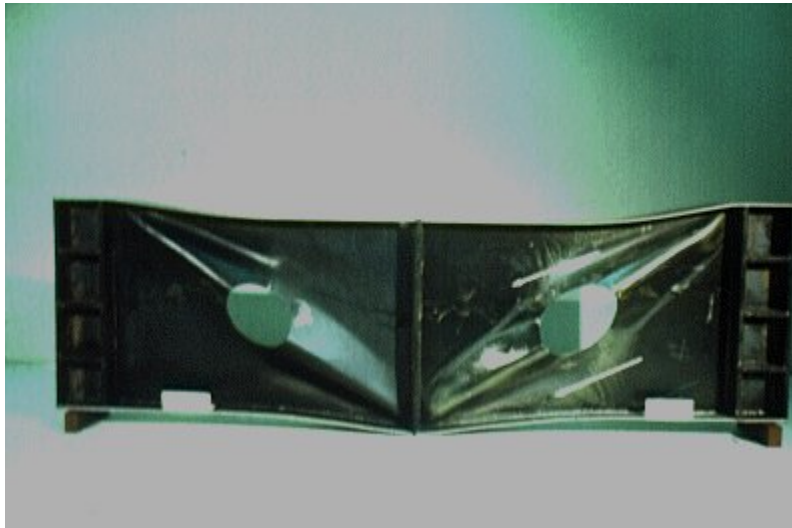
The design of webs with longitudinal stiffeners is not covered in Part 1 of Eurocode 3. It will be addressed in Part 2, for bridges. Because of the greater need for high strength/weight ratios in bridges, girders with longitudinal stiffeners are more commonly encountered in bridge than in building construction.

Design of longitudinal stiffeners is usually based on a number of empirical design curves derived from the results of a parametric study employing numerical modelling techniques. The design procedure is relatively straightforward, although somewhat conservative. Additional information on the behaviour of longitudinally stiffened girders has been presented [2] which will assist the designer to gain a better understanding of the structural action.

6. GIRDERS WITH OPENINGS IN SLENDER WEBS

Holes often have to be cut in the webs of plate girders used in building construction to provide access for service ducts, etc. No mention of such openings is made in Part 1 of Eurocode 3. Such holes have a particular influence on the behaviour of slender webs because the hole interrupts the tension field. (Design methods are available for stocky webs with web openings - see Reference3).

Detailed fundamental work by Narayanan [2] has shown that girders with slender webs and web openings possess a post-buckling reserve of resistance. The collapse mechanism for such girders, illustrated in Slide 5, is similar to the shear sway mechanism that is characteristic of all plate girders. However, some codes adopt a conservative approach. They do not take account of such post-buckling action in any girder which has a web opening with any dimension exceeding some percentage of the minimum dimension of the web panel in which it is located. To provide a simple design procedure, the shear resistance of the perforated panel is calculated as the buckling resistance.



Slide 5

The disadvantage of such a procedure is that the shear resistance of the perforated panel will then be substantially lower than that calculated allowing for the full post-buckling reserve of resistance in adjacent unperforated panels. The designer should therefore reduce the spacing of the transverse stiffeners either side of the web opening so that the initial buckling resistance of the resulting narrow perforated panel is approximately equal to the full post-buckling resistance of adjacent panels.

7. CONCLUDING SUMMARY

- To develop post-buckling action, the elements bounding the web plate of a girder must be designed to provide adequate anchorage for the tension field forces developed in the web panel.

- The tension field method can only be applied when transverse web stiffeners are provided such that the stiffener spacing/web depth ratio lies within the range: $10,0 \leq a/d \leq 3,0$. The stiffeners must have adequate rigidity in the direction perpendicular to the plane of the web to prevent web buckling. The buckling resistance of the vertical stiffener strut must also be sufficient to support the tension field forces.
- Careful consideration must be given to the design of the end post. The designer has the option of either not allowing the development of tension field action in the end panel or, for a more efficient design, providing an end post of sufficient rigidity and strength.
- The possibility of web crippling, web crushing and web buckling must be considered in those localised areas where patch loads are applied to the girder flange.
- Longitudinal web stiffeners allow girders of higher strength/weight ratios to be designed. They are particularly relevant in bridge construction. They are not considered in Eurocode 3: Part 1.1.
- Large web openings are frequently necessary in building construction to allow the passage of services. Plate girders with openings are not considered in Eurocode 3 Part 1.1.

8. REFERENCES

[1] Eurocode 3 "Design of Steel Structures".

European Prestandard ENV1993-1-1: Part 1.1, General rules and rules for buildings, CEN, 1992.

[2] Narayanan, R (Editor), "Plated Structures; Stability and Strength", Applied Science Publishers, London, 1983.

This reference gives detailed information, including the experimental background to the structural action covered in the clauses of Eurocode 3 referred to in this section.

[3] Lawson, R. M., Design for Openings in the Webs of Composite Beams, SCI Publication 068. The Steel Construction Institute, 1987.

9. ADDITIONAL READING

1. "European Recommendations for the Design of Longitudinally Stiffened Webs and of Stiffened Compression Flanges", Publication 60, ECCS, 1990.