Chapter 5

Footing Design

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5.1 Introduction

Reinforced concrete foundations, or footings, transmit loads from a structure to the supporting soil. Footings are designed based on the nature of the loading, the properties of the footing and the properties of the soil.

Design of a footing typically consists of the following steps:

1. Determine the requirements for the footing, including the loading and the nature of the supported structure.

2. Select options for the footing and determine the necessary soils parameters. This step is often completed by consulting with a Geotechnical Engineer.

3. The geometry of the foundation is selected so that any minimum requirements based on soils parameters are met. Following are typical requirements:

   - The calculated bearing pressures need to be less than the allowable bearing pressures. Bearing pressures are the pressures that the footing exerts on the supporting soil. Bearing pressures are measured in units of force per unit area, such as pounds per square foot.
   - The calculated settlement of the footing, due to applied loads, needs to be less than the allowable settlement.
   - The footing needs to have sufficient capacity to resist sliding caused by any horizontal loads.
   - The footing needs to be sufficiently stable to resist overturning loads. Overturning loads are commonly caused by horizontal loads applied above the base of the footing.
   - Local conditions.
   - Building code requirements.

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4. Structural design of the footing is completed, including selection and spacing of reinforcing steel in accordance with ACI 318 and any applicable building code. During this step, the previously selected geometry may need to be revised to accommodate the strength requirements of the reinforced concrete sections. Integral to the structural design are the requirements specific to foundations, as defined in ACI 318-05 Chapter 15.

5.2 Types of Foundations

Shallow footings bear directly on the supporting soil. This type of foundation is used when the shallow soils can safely support the foundation loads.

A *deep foundation* may be selected if the shallow soils cannot economically support the foundation loads. Deep foundations consist of a footing that bears on piers or piles. The footing above the piers or piles is typically referred to as a pile cap.

The *piers or piles* are supported by deeper competent soils, or are supported on bedrock. It is commonly assumed that the soil immediately below the pile caps provides no direct support to the pile cap.

5.3 Allowable Stress Design and Strength Design

Traditionally the geometry of a footing or a pile cap is selected using unfactored loads. The structural design of the foundation is then completed using strength design in accordance with ACI 318.

ACI Committee 336 is in the process of developing a methodology for completing the entire footing design using the strength design method.

5.4 Structural Design

The following steps are typically followed for completing the structural design of the footing or pile cap, based on ACI 318-05:

1. Determine footing plan dimensions by comparing the gross soil bearing pressure and the allowable soil bearing pressure.

2. Apply load factors in accordance with Chapter 9 of ACI 318-05.

3. Determine whether the footing or pile cap will be considered as spanning one-way or two-ways.

4. Confirm the thickness of the footing or pile cap by comparing the shear capacity of the concrete section to the factored shear load. ACI 318-05 Chapter 15 provides guidance on selecting the location for the critical cross-section for one-way shear. ACI 318-05 Chapter 11 provides guidance on selecting the location for the critical cross-section for two-way shear. Chapter 2 of this handbook on shear design also provides further design information and design aids.

5. Determine reinforcing bar requirements for the concrete section based on the flexural capacity along with the following requirements in ACI 318-05.

   - Requirements specific to footings
   - Temperature and shrinkage reinforcing requirements
   - Bar spacing requirements
   - Development and splicing requirements
   - Seismic Design provisions
   - Other standards of design and construction, as required

5.5 Footings Subject to Eccentric Loading

Footings are often subjected to lateral loads or overturning moments, in addition to vertical loads. These types of loads are typically seismic or wind loads.

Lateral loads or overturning moments result in a non-uniform soil bearing pressure under the footing, where the soil bearing pressure is larger on one side of the footing than the other. Non-uniform soil bearing can also be caused by a foundation pedestal not being located at the footing center of gravity.

If the lateral loads and overturning moments are small in proportion to the vertical loads, then the entire bottom of the footing is in compression and a $P/A \pm M/S$ type of analysis is appropriate for calculating the soil bearing pressures, where the various parameters are defined as follows:

P = The total vertical load, including any applied loads along with the weight of all of the components of the foundation, and also including the weight of the soil located directly above the footing.

A = The area of the bottom of the footing.

M = The total overturning moment measured at the bottom of the footing, including horizontal loads times the vertical distance from the load application location to the bottom of the footing plus any overturning moments.

S = The section modulus of the bottom of the footing.

If M/S exceeds P/A, then $P/A - M/S$ results in tension, which is generally not possible at the footing/soil interface. This interface is generally only able to transmit compression, not tension. A different method of analysis is required when M/S exceeds P/A.

Following are the typical steps for calculating bearing pressures for a footing, when non-uniform bearing pressures are present. These steps are based on a footing that is rectangular in shape when measured in plan, and assumes that the lateral loads or overturning moments are parallel to one of the principal footing axes. These steps should be completed for as many load combinations as required to confirm compliance with applicable design criteria. For instance, the load combination with the maximum downward vertical load often causes the maximum bearing pressure while the load combination with the minimum downward vertical load often causes the minimum stability.

1. Determine the total vertical load, *P*.

2. Determine the lateral and overturning loads.

3. Calculate the total overturning moment *M*, measured at the bottom of the footing.

4. Determine whether *P/A* exceeds *M/S*. This can be done by calculating and comparing *P/A* and *M/S* or is typically completed by calculating the eccentricity, which equals *M* divided by *P*. If e exceeds the footing length divided by 6, then *M/S* exceed *P/A*.

5. If *P/A* exceeds *M/S*, then the maximum bearing pressure equals *P/A* + *M/S* and the minimum bearing pressure equals *P/A*-*M/S*.

6. If P/A is less than M/S, then the soil bearing pressure is as shown in Fig. 5-1. Such a soil bearing pressure distribution would normally be considered undesirable because it makes the footing structurally ineffective. The maximum bearing pressure, shown in the figure, is calculated as follows:

Maximum Bearing pressure = 2 P / [(B) (X)]
Where X = 3(L/2 - e) and e = M / P

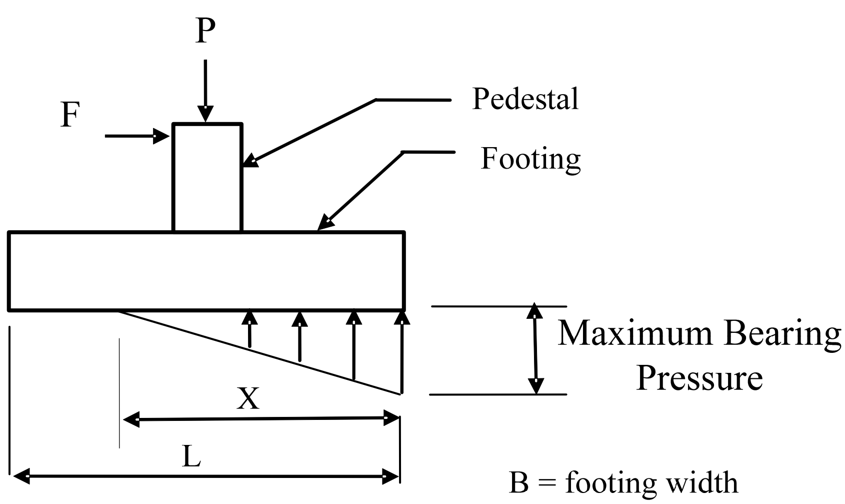


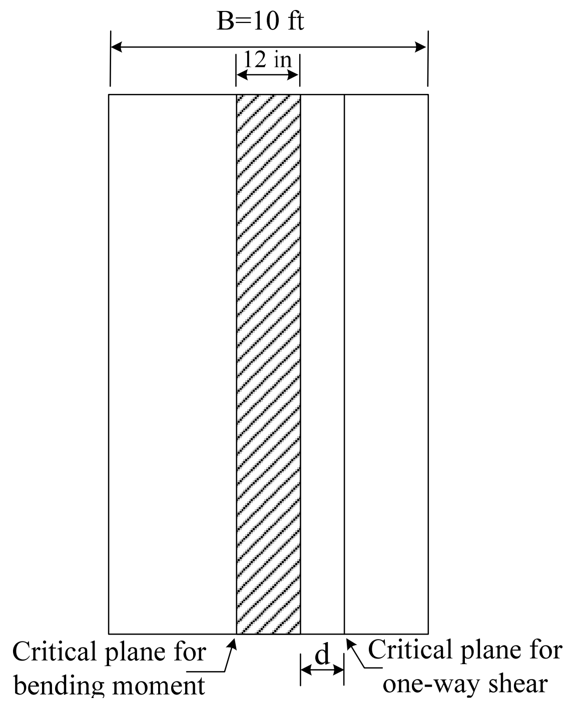
Fig. 5-1 Footing under eccentric loading

5.6 Footing Design Examples

The footing examples in this section illustrate the use of ACI 318-05 for some typical footing designs as well as demonstrate the use of some design aids included in other chapters. However, these examples do not necessarily provide a complete procedure for foundation design as they are not intended to substitute for engineering skills or experience.

FOOTINGS EXAMPLE 1 - Design of a continuous (wall) footing

Determine the size and reinforcement for the continuous footing under a 12 in. bearing wall of a 10 story building founded on soil.



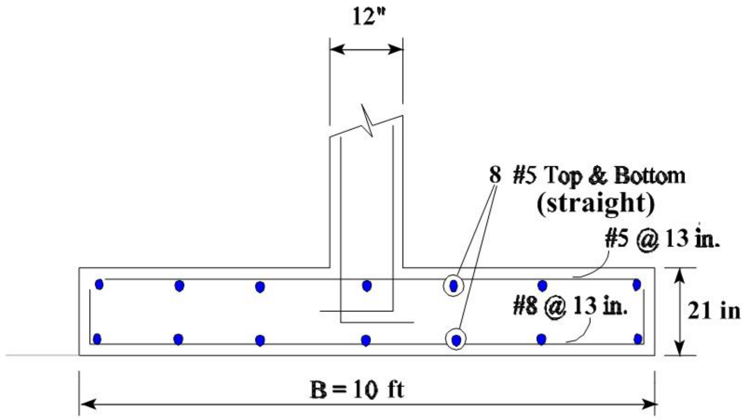
Given:
$f'_c = 4$ ksi
$f_y = 60$ ksi
Dead Load = D = 25 k/ft
Live Load = L = 12.5 k/ft
Wind O.T. = W = 4 k/ft
(axial load due to overturning under wind loading)
Seismic O.T. = E = 5 k/ft
(axial load due to overturning under earthquake loading)

Allowable soil bearing pressures:
D = 3 ksf = "a"
D + L = 4 ksf = "b"
D + L + (W or E) = 5 ksf = "c"

Procedure	Computation	ACI 318-05 Section	Design Aid
Sizing the footing.	Ignoring the footing self-weight; D/a = 25/3 = 8.3 ft (D + L)/b = 37.5/4 = 9.4 ft ← controls (D + L + W)/c = 41.5/5 = 8.3 ft (D + L + E)/c = 42.5/5 = 8.5 ft Use B = 10 ft		
Required strength.	U = 1.4D = 1.4(25) = 35 k/ft or 3.50 ksf U = 1.2D + 1.6L = 1.2(25) + 1.6(12.5) = 50 k/ft or 5.00 ksf (Controls) U = 1.2D + 1.6W + 1.0L = 1.2(25) + 1.6(4) +12.5 = 48.9 k/ft or 4.89 ksf U = 0.9D + 1.6W = 0.9(25) + 1.6(4) = 28.9 k/ft or 2.89 ksf U = 1.2D + 1.0E + 1.0L = 1.2(25) + (5) + 12.5	9.2	

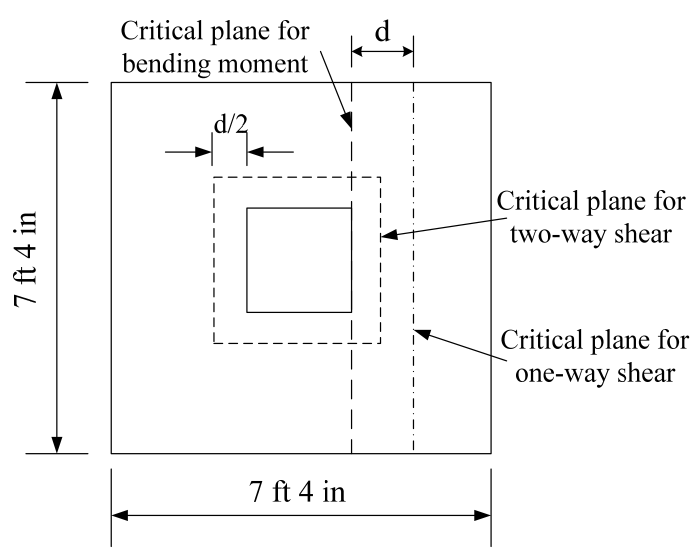
	$= 47.5$ k/ft or 4.75 ksf $U = 0.9D + 1.0E$ $= 0.9(25) + (5)$ $= 27.5$ k/ft or 2.75 ksf		
Design for shear. Calculate V_u at d from the face of the wall	$\phi_{shear} = 0.75$ Assume $V_s = 0$ (no shear reinforcement) $\phi V_n = \phi V_c$ $\phi V_c = \phi(2\sqrt{f'_c} b_w d)$ Try d = 17 in. and h = 21 in. $\phi V_c = 0.75(2\sqrt{4000})(12)(17)/1000$ $=19.35$ k/ft $V_u = (10/2 - 6/12 - 17/12)(5.00) = 15.5$ k/ft $\phi V_n = \phi V_c > V_u$ OK	9.3.2.3 11.1.1 11.3 11.1.3.1	
Calculate moment at the face of the wall Compute flexural tension reinforcement Shrinkage and temperature reinforcement	$M_u = (5)(4.5)^2/2 = 50.6$ ft-k/ft $\phi K_n = M_u (12{,}000)/(bd^2)$ $\phi K_n = 50.6 (12{,}000)/[(12)(17)^2] = 176$ psi For $\phi K_n = 176$ psi, select $\rho = 0.34\%$ $A_s = \rho bd = 0.0034 (12) (17) = 0.70$ in^2/ft Check for $A_{s,min} = 0.0018$ bh $A_{s,min}=0.0018(12)(21)=0.46$ in^2/ft <0.7in^2/ft OK Use bottom bars #8 @ 13 in c/c hooked at ends. If these bars are not hooked, provide calculations to justify the use of straight bars. **Note:** $\varepsilon_t = 0.040 > 0.005$ for tension controlled sections and $\phi = 0.9$ Use top bars #5 @ 13 in c/c arbitrarily designed to take approximately 40% of bending moment due to possible reversal caused by earthquake loads. 8# 5 top and bottom longitudinal bars will satisfy the requirement for shrinkage and	15.4.2 7.12 10.5.4 10.3.4 9.3.2 7.12	 Flexure 1 Flexure 1

	temperature reinforcement in the other direction.		
Check shear for earthquake load effects. For structural members resisting earthquake loads, if the nominal shear strength is less than the shear corresponding to the development of nominal flexural resistance, then; $\phi_{shear} = 0.6$	M_n = 61.9 ft-k/ft and the corresponding V_{fn} = 18.6 k/ft $V_c = 2\sqrt{4000}(12)(17.5)/1000$ = 26.5 k/ft > V_{fn} = 18.6 k/ft Therefore, the use of $\phi_{shear} = 0.75$ above is correct.	9.3.4 (a)	
Final Design			



FOOTINGS EXAMPLE 2 - Design of a square spread footing

Determine the size and reinforcing for a square spread footing that supports a 16 in. square column, founded on soil.



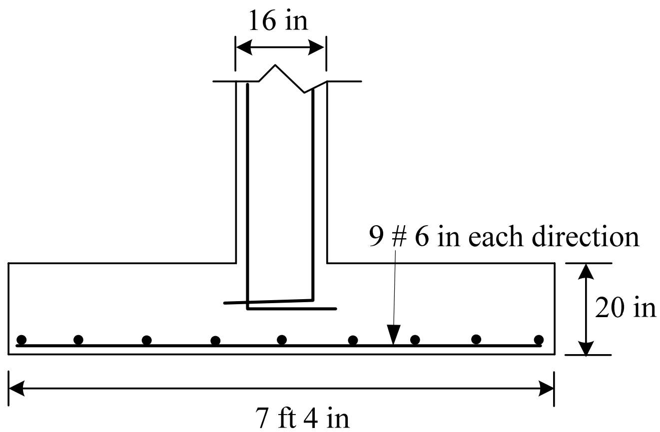
__Given:__

f'_c = 4 ksi
f_y = 60 ksi
Dead Load = D = 200 k
Live Load = L = 100 k
Allowable soil bearing pressures:
Due to D = 4 ksf = "a"
Due to D + L = 7 ksf = "b"

Procedure	Computation	ACI 318-05 Section	Design Aid
Sizing the footing.	Ignoring the footing self-weight; D/a = 200/4 = 50 sq. ft. (Controls) (D+L)/b = 300/7 = 42.9 sq. ft. **Use 7.33 ft x 7.33 ft** A = 53.7 > 50 sq. ft. OK		
Required strength.	U = 1.4D = 1.4(200) = 280 k or (280/53.7) = 5.3 ksf U = 1.2D + 1.6L = 1.2(200) + 1.6(100) = 400k or (400/53.7)= 7.5 ksf (Controls)	9.2	
Design for shear. Two-way action	ϕ_{shear} = 0.75 Assume $V_s = 0$ (no shear reinforcement) $\phi V_n = \phi V_c$ Try d = 16 in. and h = 20 in. $b_o = 4(16 + 16) = 128$ in. $V_c = (2 + \frac{4}{\beta})\sqrt{f'_c}b_o d$ $V_c = (2 + \frac{4}{16/16})\sqrt{f'_c}b_o d = 6\sqrt{f'_c}b_o d$ $V_c = (\frac{\alpha_s d}{b_o} + 2)\sqrt{f'_c}b_o d$ $V_c = (\frac{(40)(16)}{128} + 2)\sqrt{f'_c}b_o d$ $V_c = 7\sqrt{f'_c}b_o d$ $V_c = 4\sqrt{f'_c}b_o d$ (Controls) $\phi V_c = 0.75(4\sqrt{4000}(128)(16))/1000$ = 388.5 k $V_u = [(7.33)^2 - ((16+16)/12)^2](7.5) =$ 349.6 k	9.3.2.3 11.1.1 11.12.1.2 11.12.2.1 (a) 11.12.2.1 (b) 11.12.2.1 (c)	

One-way action	$\phi V_n = \phi V_c > V_u$ OK b_w = 7.33 (12) = 88 in. and d = 15.5 in. $V_c = 2\sqrt{f'_c}b_w d$ $\phi V_c = 0.75(2\sqrt{4000})(88)(15.5)/1000$ = 129.4 k V_u =7.33 [(7.33/2) – (8+15.5)/12](7.5) = 94.0 k $\phi V_n = \phi V_c > V_u$ OK	11.12.1.1 11.3.1.1	
Bearing Bearing resistance of footing	$\phi_{bearing}$ = 0.65 $\sqrt{A_2 / A_1} = 2$ $B_r = \phi(0.85 f'_c A_1)\sqrt{A_2 / A_1}$ B_r = 0.65(0.85)(4)(16)^2 (2) B_r = 1131 k > 400 k □ OK	9.3.2.4 10.17.1	
Calculate moment at the column face Compute flexural tension reinforcement (bottom bars) using design aids in Chapter 1	M_u = (7.5)(3)^2 (7.33)/2 = 248 ft-k $\phi K_n = M_u$ (12,000)/(bd^2) ϕK_n = 248 (12,000)/[(7.33)(12)(15.5)^2] = 141 psi For ϕK_n = 141 psi, select ρ = 0.27% $A_s = \rho bd$ = 0.0027 (7.33)(12)(15.5) = 3.7 in^2 Check for $A_{s,min}$= 0.0018 bh $A_{s,min}$=0.0018(7.33)(12)(20)= 3.2 in^2 < 3.7in^2 OK Use 9 #6 straight bars in both directions **Note:** ε_t = 0.050 > 0.005 for tension controlled sections and ϕ = 0.9.	15.4.2 7.12 10.5.4 10.3.4 9.3.2	Flexure 1 Flexure 1
Development length: Critical sections for development length occur at the column face.	$\ell_d = \left(f_y \Psi_t \Psi_e \lambda / (25\sqrt{f'_c})\right) d_b$ $\ell_d = \left(\frac{(60{,}000)(1.0)(1.0)(1.0)}{25\sqrt{4{,}000}}\right) 0.75$ P_d =29 in. < P_d (provided) = (3)(12) – 3 = 33 in □ OK	15.6.3 15.4.2 12.2.2	

Final Design

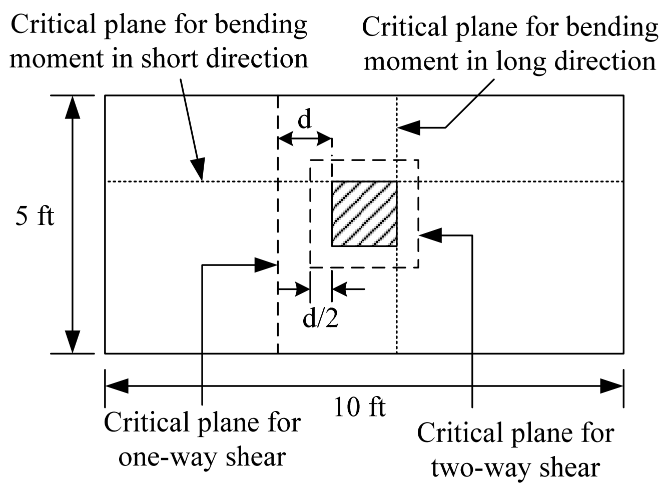


FOOTINGS EXAMPLE 3 - Design of a rectangular spread footing.

Determine the size and reinforcing for a rectangular spread footing that supports a 16 in. square column, founded on soil.

<u>**Given:**</u>
f'_c = 4 ksi
f_y = 60 ksi
Dead Load = D = 180 k
Live Load = L = 100 k
Wind O.T. = W = 120 k
(axial load due to overturning under wind loading)
Allowable soil bearing pressures:
Due to D = 4 ksf = "a"
Due to D + L = 6 ksf = "b"
Due to D + L + W = 8.4 ksf = "c"

Design a rectangular footing with an aspect ratio ≤ 0.6



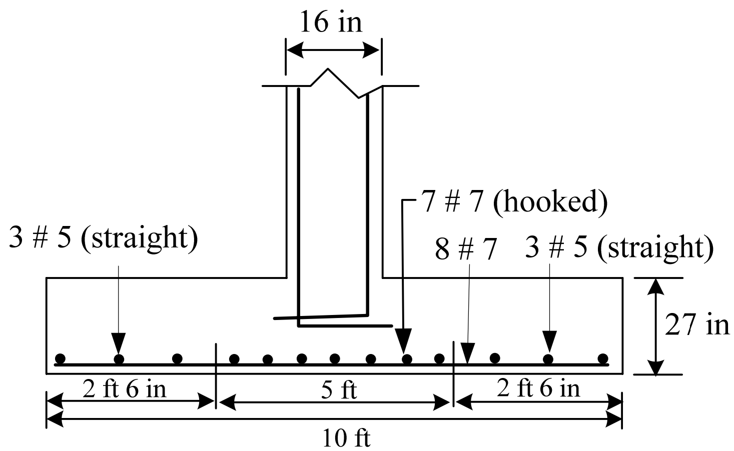
Procedure	Computation	ACI 318-05 Section	Design Aid
Sizing the footing.	Ignoring the self-weight of the footing; D/a = 180/4 = 45 sq.ft. (D+L)/b = 280/6 = 46.7 sq.ft. (D + L + W)/c = 400/8.4 = 47.6 sq.ft. Controls Use 5 ft x 10 ft A = 50 sq.ft. is OK		
Required Strength	U = 1.4D = 1.4(180) = 252 k or (252/50) = 5.1 ksf U = 1.2D + 1.6L	9.2	

	= 1.2(180) + 1.6(100) = 376 k or (376/50) = 7.6 ksf U = 1.2D + 1.6W + 1.0L = 1.2(180) + 1.6(120) + 1.0(100) = 508 k or 10.2 ksf (Controls) U = 0.9D + 1.6W = 0.9(180) + 1.6(120) = 354 k or 7.1 ksf		
Design for shear. Two-way action One-way action (in short direction)	$\phi_{shear} = 0.75$ Assume $V_s = 0$ (no shear reinforcement) $\phi V_n = \phi V_c$ Try d = 23 in. and h = 27 in. $b_o = 4(16 + 23) = 156$ in. $V_c = (2 + \frac{4}{\beta})\sqrt{f'_c}b_o d$ $V_c = (2 + \frac{4}{16/16})\sqrt{f'_c}b_o d = 6\sqrt{f'_c}b_o d$ $V_c = (\frac{\alpha_s d}{b_o} + 2)\sqrt{f'_c}b_o d$ $V_c = (\frac{(40)(23)}{156} + 2)\sqrt{f'_c}b_o d$ $V_c = 7.9\sqrt{f'_c}b_o d$ $V_c = 4\sqrt{f'_c}b_o d$ (Controls) $\phi V_c = 0.75(4\sqrt{4000}(156)(23))/1000$ = 680.7 k $V_u = [(10)(5) - (16+23)/12)^2](10.2)$ = 402.3 k $\phi V_n = \phi V_c > V_u$ □ OK $b_w = 5(12) = 60$ in. and d = 23.5 in. $V_c = 2\sqrt{f'_c}b_w d$	9.3.2.3 11.1.1 11.12.1.2 11.12.2.1 (a) 11.12.2.1 (b) 11.12.2.1 (c) 11.12.1.1 11.3.1.1	

	$\phi V_c = 0.75(2\sqrt{4000})(60)(23.5)/1000$ $= 133.7$ k V_u=5[(10/2) – (8+23.5)/12](10.2)=121.2 k $\phi V_n = \phi V_c > V_u$ OK One-way action in the long direction is not a problem because the footing edge is located within the potential critical section for one-way shear.		
Bearing Bearing resistance of footing	$\phi_{bearing} = 0.65$ $\sqrt{A_2 / A_1} = 2$ $B_r = \phi(0.85 f'_c A_1)\sqrt{A_2 / A_1}$ $B_r = 0.65(0.85)(4)(16)^2 (2)$ $B_r = 1131 \text{ k} > 508 \text{ k}$ OK	9.3.2.4 10.17.1	
Calculate moment in the long direction, at the column face. Compute flexural tension reinforcement (bottom bars) using design aids in Chapter 1.	$M_u = (10.2)(4.33)^2 (5)/2 = 479$ ft-k $\phi K_n = M_u (12,000)/(bd^2)$ $\phi K_n = 479 (12,000)/[(5)(12)(23.5)^2]$ $= 173.5$ psi For ϕK_n = 173.5 psi, select ρ = 0.335% $A_s = \rho bd = 0.00335 (5)(12)(23.5) = 4.72$ in^2 Check for $A_{s,min}$= 0.0018 bh $A_{s,min}$= 0.0018(5)(12)(27) = 2.92 in^2 < 4.72 in^2 OK Use 8 #7 bars distributed uniformly across the entire 5ft width of footing **Note:** $\varepsilon_t = 0.041 > 0.005$ for tension controlled sections and $\phi = 0.9$.	15.4.2 7.12 10.5.4 15.4.4.1 10.3.4 9.3.2	Flexure 1 Flexure 1
Calculate moment in the short direction, at the column face. Compute flexural tension reinforcement (bottom bars) using design aids in Chapter 1.	$M_u = (10.2)(1.83)^2 (10)/2 = 171.4$ ft-k $\phi K_n = M_u (12,000)/(bd^2)$ $\phi K_n = 171.4 (12,000)/[(10)(12)(22.5)^2]$ $= 33.9$ psi For ϕK_n = 33.9 psi, select ρ = 0.07% $A_s = \rho bd = 0.0007 (10)(12)(22.5)$ $= 1.89$ in^2	15.4.2	Flexure 1

Check for minimum reinforcement	$A_{s,min}$= 0.0018 bh $A_{s,min}$= 0.0018(10)(12)(27) = 5.83 in^2 > 1.89 in^2 Use A_s = 5.83 in^2 (Reinf. In central 5-ft band) / (total reinf.) = 2/(β+1) β = 10/5 = 2; and 2/(β+1) = 2/3 Reinf. In central 5-ft band = 5.83(2/3) = 3.89 in^2 Use 7 #7 bars distributed uniformly across the entire 5ft band. Reinforcement outside the central band = 5.83 – 7(0.6) = 1.63 in^2 Use 6 #5 bars (3 each side) distributed uniformly outside the central band.	7.12 10.5.4 15.4.4.2	
Development length: Critical sections for development length occur at the column face.	$\ell_d = (3/40)(f_y/\sqrt{f'_c})$ $[(\Psi_t\Psi_e\Psi_s\lambda)/((c_b + K_{tr})/d_b)]d_b$ K_{tr} = 0; and ((c_b+ K_{tr})/d_b) = 2.5 $\ell_d = (3/40)(60{,}000/\sqrt{4{,}000})$ $[(1.0)(1.0)(1.0)(1.0)/2.5]0.875$ P_d =25 in. for # 7 bars P_d =25 in < P_d (provided) = (4.33)(12) – 3 = 49 in in the long direction: use straight # 7 bars P_d =25 in > P_d (provided) = (1.83)(12) – 3 = 19 in in the short direction: use hooked # 7 bars $\ell_d = (3/40)(60{,}000/\sqrt{4{,}000})$ $[(1.0)(1.0)(0.8)(1.0)/2.5]0.625$ P_d =15 in. for # 5 bars P_d = 15 in < P_d (provided) = 19 in in the short direction: use straight # 5 bars	15.6.3 15.4.2 12.2.3 12.2.4	

Final Design



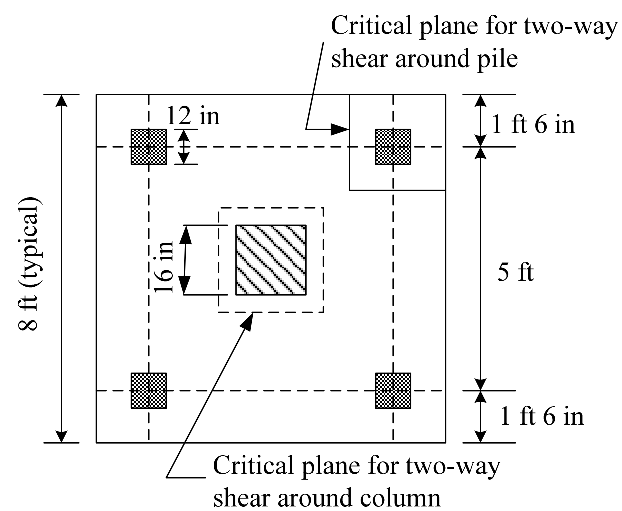
FOOTINGS EXAMPLE 4 - Design of a pile cap.

Determine the size and reinforcing for a square pile cap that supports a 16 in. square column and is placed on 4 piles.

<u>Given:</u>

f'_c = 5 ksi
f_y = 60 ksi
Dead Load = D = 250 k
Live Load = L = 150 k

16 x 16 in. reinforced concrete column
12 x 12 in. reinforced concrete piles
(4 piles each @ 5 ft on centers)

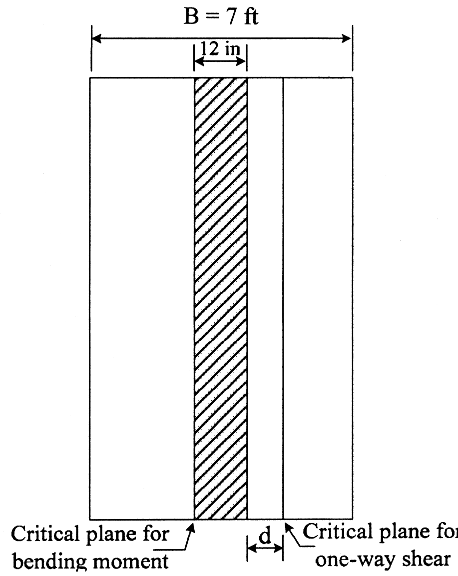


Procedure	Computation	ACI 318-05 Section	Design Aid
Factored Loads:	<u>Column:</u> U = 1.4D = 1.4(250) = 350 k U = 1.2D + 1.6L = 1.2(250) + 1.6(150) = 540 k = V_u (Controls) <u>Piles:</u> P_u = 540/4 = 135 k = V_u ignoring the self-weight of pile cap	9.2	

Design for shear.	$\phi_{shear} = 0.75$ Assume $V_s = 0$, (no shear reinforcement)	9.3.2.3	
	$\phi V_n = \phi V_c$	11.1.1	
	Try d = 26 in. and h = 33 in.		
Two-way action	<u>Around Column:</u>		
	$b_o = 4(16 + 26) = 168$ in.	11.12.1.2	
	$V_c = (2 + \frac{4}{\beta})\sqrt{f'_c}b_o d$	11.12.2.1 (a)	
	$V_c = (2 + \frac{4}{16/16})\sqrt{f'_c}b_o d = 6\sqrt{f'_c}b_o d$		
	$V_c = (\frac{\alpha_s d}{b_o} + 2)\sqrt{f'_c}b_o d$	11.12.2.1 (b)	
	$V_c = (\frac{(40)(26)}{168} + 2)\sqrt{f'_c}b_o d$		
	$V_c = 8.2\sqrt{f'_c}b_o d$		
	$V_c = 4\sqrt{f'_c}b_o d$ (Controls)	11.12.2.1 (c)	
	$\phi V_c = 0.75(4\sqrt{5000}(168)(26))/1000$ $= 926$ k		
	$V_u = 540$ k		
	$\phi V_n = \phi V_c > V_u$ OK		
	<u>Around Piles</u>		
	$b_o = 2(18 + 6 + 13) = 74$ in.	11.12.1.2	
	$V_c = (2 + \frac{4}{12/12})\sqrt{f'_c}b_o d = 6\sqrt{f'_c}b_o d$	11.12.2.1 (a)	
	$V_c = (\frac{(20)(26)}{74} + 2)\sqrt{f'_c}b_o d$	11.12.2.1 (b)	
	$V_c = 9\sqrt{f'_c}b_o d$		
	$V_c = 4\sqrt{f'_c}b_o d$ (Controls)	11.12.2.1 (c)	

One-way action	$\phi V_c = 0.75(4\sqrt{5000}(74)(26))/1000$ $= 408$ k $V_u = 135$ k $\phi V_n = \phi V_c > V_u$ OK Note: The effective depth is conservative for the two-way action but is O.K. considering the overlapping of the critical sections around the column and the piles One-way action will not be a problem because the piles are located within potential critical sections for one-way shear.		
Design for flexure Find flexural tension reinforcement (bottom bars) Top reinforcement:	$M_u = 2(135)(2.5 - 0.67) = 495$ ft-k $\phi K_n = M_u (12{,}000)/(bd^2)$ $\phi K_n = 495 (12{,}000)/[(8)(12)(25.5)^2]$ $= 95.2$ psi For $\phi K_n = 95.2$ psi, select $\rho = 0.19\%$ $A_s = \rho bd = 0.0019 (8)(12)(25.5) = 4.7$ in^2 Check for $A_{s,min} = 0.0018$ bh $A_{s,min} = 0.0018(8)(12)(33) = 5.7$ in^2 > 4.7 in^2 A_s (required) $= 5.7$ in^2 Use 10 #7 each way (bottom reinforcement) Not required.	7.12 10.5.4	Flexure 1

FOOTINGS EXAMPLE 5 - Design of a continuous footing with an overturning moment



Determine the size and reinforcing bars for a continuous footing under a 12-in. bearing wall, founded on soil, and subject to loading that includes an overturning moment.

Given:
$f'_c = 4$ ksi
$f_y = 60$ ksi

Depth from top of grade to bottom of footing = 3 ft
Density of soil above footing = 100 pcf
Density of footing concrete = 150 pcf

Vertical Dead Load = 15 k/ft (including wall weight)
Horizontal wind shear = V = 2.3 k/ft (applied at 1 ft above grade)

Allowable soil bearing pressure based on unfactored loads = 4 ksf

Procedure	Computation	ACI 318-05 Section	Design Aid
Sizing the footing	Try footing width = B = 7 ft Area = A = 1(7) =7 ft^2/ft Section Modulus = S =1(7)(7)/6=8.167 ft^3/ft Try a 14 inch thick footing Weight of footing + soil above footing = (14/12)(0.150) + (36-14)(0.100/12) = 0.175 + 0.183 = 0.358 ksf Total weight of footing + soil above footing + wall from top of grade to top of footing = (0.175)(7)+(.183)(7-1)+(36-14)(0.150/12) = 2.60 kips/ft Total Vertical Load = P = 15 + 2.6 =17.6k/ft (Dead Load) Vertical distance from bottom of footing to location of applied shear = H = 3 + 1 = 4 ft. Overturning moment measured at base of footing = M = (V)(H) = (2.3)(4) = 9.2 ft-kips/ft (Wind Load)		

	Eccentricity = e = M/P = 9.2/17.6 = 0.52 ft B/6 = 7/6 = 1.17 ft Since e < B/6, bearing pressure = P/A ± M/S Maximum bearing pressure =P/A + M/S = (15 + 2.6)/7 + 9.2/8.167 = 3.64 ksf Minimum bearing pressure =P/A – M/S = (15 + 2.6)/7 – 9.2/8.167 = 1.39 ksf Max bearing pressure < allowable: OK		
Required Strength	U = 1.4D = 1.4(17.6)/7 = 3.52 ksf U = 1.2D + 1.6W + 1.0L 1.2D = 1.2(17.6)/7 = 3.02 ksf 1.6W = 1.6(9.2)/8.167 = 1.80 ksf 1.0L = 0 e = 1.6(M)/(1.2(P)) = 1.6(9.2)/(1.2(17.6) = 0.70 ft Since e < B/6, bearing pressure = 1.2(P/A) ± 1.6(M/S) U = 4.82 ksf (maximum) U = 1.22 ksf (minimum) U = 0.9D + 1.6W 0.9D = 0.9(17.6)/7 = 2.27 ksf 1.6W =1.6(9.2)/8.167 = 1.80 ksf e = 1.6(M)/(0.9(P)) = 1.6(9.2)/(0.9(17.6) = 0.93 ft Since e < B/6, bearing pressure = 0.9(P/A) ± 1.6(M/S) U = 4.07 ksf (maximum) U = 0.47 ksf (minimum)	9.2	

Design for Shear	$\phi_{shear} = 0.75$ Assume $V_s = 0$ (i.e. no shear reinforcement) $\phi V_n = \phi V_c$ $\phi V_c = \phi\left(2\sqrt{f_c^{'}}b_w d\right)$ Try d = 10 in. and h = 14 in. $\phi V_c = 0.75\left(2\sqrt{4000}\times 12\times 10\right)/1000$ $= 11.38$ k/ft Calculate V_u for the different load combinations that may control. Calculate at the location d from the face of the wall. Delete the portion of bearing pressure caused by weight of footing and soil above footing. Distance d from face of wall = (7/2 – 6/12 – 10/12) = 2.17 ft measured from the edge of the footing U = 1.4D V_u = (3.52 – (1.4)(0.358))(2.17) = 6.55 k/ft U = 1.2D + 1.6W + 1.0L Bearing pressure measured at distance d from face of wall = 4.82 – (4.82 – 1.22)(2.17/7) = 3.70 ksf V_u = (3.70 – 1.2(0.358))(2.17) + (4.82 – 3.70)(2.17/2) = 8.31 k/ft (controls) $\phi V_n = \phi V_c > V_u$ OK	9.3.2.3 11.1.1 11.3 11.1.3.1	

Moment	Calculate the moment at the face of the wall = (7/2 – 6/12) = 3.0 ft measured from the edge of the footing U = 1.4D $M_u = (3.52 - 1.4(0.358))(3.0)^2 / 2$ $= 13.58$ ft-k/ft U = 1.2D + 1.6W + 1.0L Bearing pressure measured at face of wall = 4.82 − (4.82 − 1.22)(3/7) = 3.28 ksf $M_u = (3.28 - 1.2\times0.358)(3.0)^2 / 2$ $+ (4.82 - 3.28)(3.0)^2 / 3$ = 17.45 ft-k/ft (controls) Compute $\phi K_n = M_u (12{,}000)/(bd^2)$ $\phi K_n = 17.45(12{,}000)/(12\times10^2) = 175\, psi$ For $\phi K_n = 175\, psi$, select Δ = 0.34% Compute flexural tension reinforcement $A_s = \rho b d = 0.0034(12)(10) = 0.41 in^2/ft$ Check for A_{smin} = 0.0018 bh A_{smin} = 0.0018(12)(14) = 0.30 in^2/ft < 0.41 in^2/ft OK Use bottom bars #6 @12 in. Use 7#5 bottom longitudinal bars to satisfy the requirements for shrinkage and temperature reinforcement in the other direction.	15.4.2 7.12 10.5.4 7.12	Flexure 1

CHAPTER 11: FOOTINGS

11.1 Introduction

Footings are structural elements that transmit column or wall loads to the underlying soil below the structure. Footings are designed to transmit these loads to the soil without exceeding its safe bearing capacity, to prevent excessive settlement of the structure to a tolerable limit, to minimize differential settlement, and to prevent sliding and overturning. The settlement depends upon the intensity of the load, type of soil, and foundation level. Where possibility of differential settlement occurs, the different footings should be designed in such away to settle independently of each other.

Foundation design involves a soil study to establish the most appropriate type of foundation and a structural design to determine footing dimensions and required amount of reinforcement.

Because compressive strength of the soil is generally much weaker than that of the concrete, the contact area between the soil and the footing is much larger than that of the columns and walls.

11.2 Footing Types

The type of footing chosen for a particular structure is affected by the following:

1. The bearing capacity of the underlying soil.
2. The magnitude of the column loads.
3. The position of the water table.
4. The depth of foundations of adjacent buildings.

Footings may be classified as deep or shallow. If depth of the footing is equal to or greater than its width, it is called deep footing, otherwise it is called shallow footing. Shallow footings comprise the following types:

1- Isolated Footings:

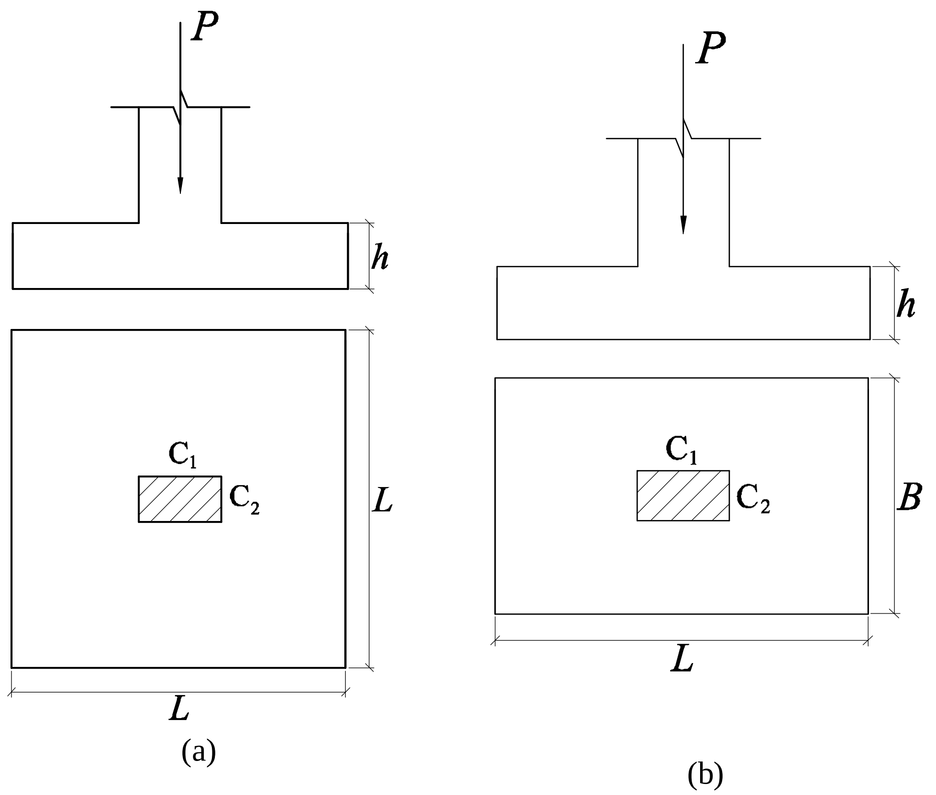


Figure 11.1: (a) Square isolated footing; (b) Rectangular isolated footing

An isolated footing is used to support the load on a single column. It is usually either square or rectangular in plan. It represents the simplest, most economical type and most widely used footing. Whenever possible, square footings are provided so as to reduce the bending moments and shearing forces at their critical sections. Isolated footings are used in case of light column loads, when columns are not closely spaced, and in case of good homogeneous soil. Under the effect of upward soil pressure, the footing bends in a dish shaped form. An isolated footing must, therefore, be provided by two sets of reinforcement bars placed on top of the other near the bottom of the footing. In case of property line restrictions, footings may be designed for eccentric loading or combined footing is used as an alternative to isolated footing. Figure 11.1 shows square and rectangular isolated footings.

11.3 Depth of Footing

The depth to which foundations shall be carried is to satisfy the following:

a. Ensuring adequate bearing capacity.

b. In the case of clay soils, footings are to penetrate below the zone where shrinkage and swelling due to seasonal weather changes are likely to cause appreciable movement.

c. The footing should be located sufficiently below maximum scouring depth.

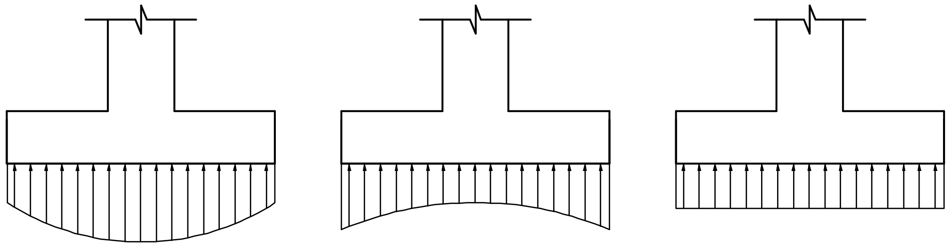
d. The footing should be located away from top soils containing organic materials.

e. The footing should be located away from unconsolidated materials such as garbage.

All footings shall extend to a depth of at least 0.50 meter below natural ground level. On rock or such other weather-resisting natural ground, removal of the top soil may be all that is required. In such cases, the surface shall be cleaned, so as to provide a suitable bearing. Usually footings are located at depths of 1.5 to 2.0 meters below natural ground level.

11.4 Pressure Distribution Below Footings

The distribution of soil pressure under a footing is a function of the type of soil, the relative rigidity of the soil and the footing, and the depth of foundation at level of contact between footing and soil. A concrete footing on sand will have a pressure distribution similar to Figure 11.2.a. When a rigid footing is resting on sandy soil, the sand near the edges of the footing tends to displace laterally when the footing is loaded. This tends to decrease in soil pressure near the edges, whereas soil away from the edges of footing is relatively confined. On the other hand, the pressure distribution under a footing on clay is similar to Figure 11.2.b. As the footing is loaded, the soil under the footing deflects in a bowl-shaped depression, relieving the pressure under the middle of the footing. For design purposes, it is common to assume the soil pressures are linearly distributed. The pressure distribution will be uniform if the centroid of the footing coincides with the resultant of the applied loads, as shown in Figure 11.2.



(a) (b) (c)

Figure 11.2: Pressure distribution under footing; (a) footing on sand; (b) footing on clay; (c) equivalent uniform distribution

11.4.1 Ultimate Bearing Capacity of Soil

The maximum intensity of loading at the base of a foundation which causes shear failure of soil is called *ultimate bearing capacity of soil,* denoted by q_u.

11.4.2 Allowable Bearing capacity of Soil

The intensity of loading that the soil carries without causing shear failure and without causing excessive settlement is called *allowable bearing capacity of soil,* denoted by q_a. It should be noted that q_a is a service load stress. The allowable bearing capacity of soil is obtained by dividing the ultimate bearing capacity of soil by a factor of safety on the order of 2.50 to 3.0.

The allowable soil pressure for soil may be either gross or net pressure permitted on the soil directly under the base of the footing. The gross pressure represents the total stress in the soil created by all the loads above the base of the footing. These loads include: (a) column service loads; (b) the weight of the footing; and (c) the weight of the soil on the top of the footing, or

$$q_{gross} = q_{soil} + q_{footing} + q_{column} \tag{11.1}$$

For moment and shear calculations, the upward and downward pressures of the footing mass and the soil mass get cancelled. Thus, a net soil pressure is used instead of the gross pressure value, or

$$q_{net} = q_{gross} - q_{footing} - q_{soil} \tag{11.2}$$

Figure 11.3 shows schematic representation of allowable gross and net soil pressures.

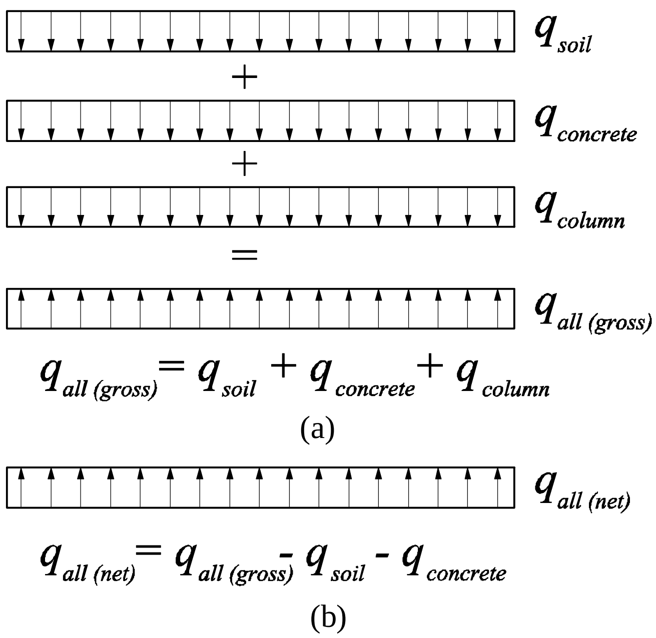


Figure 11.3: Gross and net soil pressures; (a) gross soil pressure; (b) net soil pressure

11.5 Concentrically loaded Footings

If the resultant of the loads acting at the base of the footing coincides with the centroid of the footing area, the footing is concentrically loaded and a uniform distribution of soil pressure is assumed in design, as shown in Figure 11.4. The magnitude of the pressure intensity is given by

$$q = \frac{P}{A} \qquad (11.3)$$

where A is the bearing area of the footing, and P is the applied load.

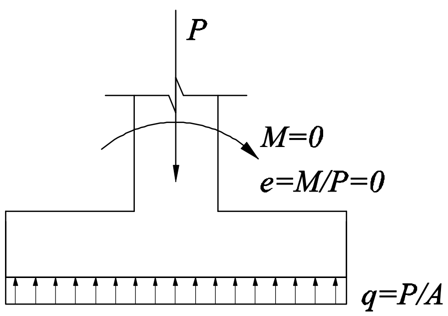


Figure 11.4: Concentrically loaded footing

11.6 Design of Isolated Footings

Design of isolated rectangular footings is detailed in the following steps.

1- Select a trial footing depth.

According to *ACI Code 15.7*, depth of footing above reinforcement is not to be less than 15 *cm* for footings on soil. Noting that 7.5 *cm* of clear concrete cover is required if concrete is cast against soil, a practical minimum depth is taken as 25 *cm*.

2- Establish the required base area of the footing.

The allowable net soil pressure is

$$q_{all}\,(net) = q_{all}\,(gross) - g_c\,(h_c) - g_s\,(d_f - h_c)$$

where h_c is assumed footing depth, d_f is distance from ground surface to the contact surface between footing base and soil, g_c is weight density of concrete, and g_s is weight density of soil on top of footing.

Based on *ACI Code 15.2.2*, base area of footing is determined from unfactored forces and moments transmitted by footing to soil and the allowable soil pressure evaluated through principles of soil mechanics. The required base area of the footing is obtained by dividing the column service loads by the allowable net soil pressure of the soil, or

$$A_{req} = \frac{P_D + P_L}{q_{all}\,(net)} \qquad (11.4)$$

where P_D and P_L are column service dead and live loads respectively.

Select appropriate *L*, and *B* values, if possible, use a square footing to achieve greatest economy.

3- Evaluate the net factored soil pressure.

Evaluate the net factored soil pressure by dividing the factored column loads by the chosen footing area, or

$$q_u\,(net) = \frac{1.2\,P_D + 1.6\,P_L}{L \times B} \qquad (11.5)$$

4- Check footing thickness for punching shear.

Since large soil pressures are present under footings, high shear stresses are produced and since shear reinforcement is not normally used, shear rather than moment commonly determines the minimum required depth of footing. The depth of the footing must be set so that the shear capacity of the concrete equals or exceeds the critical shear forces produced by factored loads.

As discussed in Chapter 4, the critical section for punching shear is located at distance $d/2$ from column faces and usually takes the shape of the column. Footing thickness is adequate for resisting punching shear once $V_u \le \Phi V_C$

The critical punching shear force can be evaluated using one of the two following methods:

$$V_u = q_u\,(net)\left[(L)B - (C_1 + d)(C_2 + d)\right] \tag{11.6.a}$$

$$V_u = (1.2\,P_D + 1.6\,P_L) - q_u\,(net)(C_1 + d)(C_2 + d) \tag{11.6.b}$$

where C_1 *and* C_2 are column cross sectional dimensions, shown in Figure 1.5.

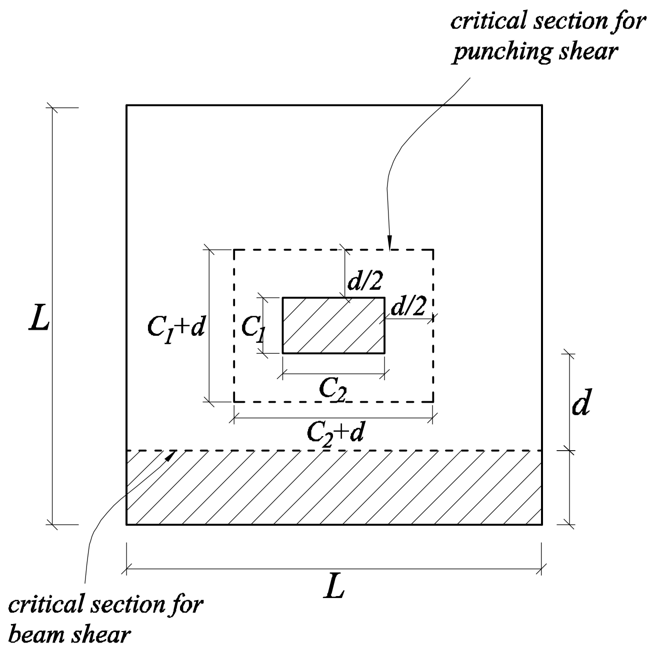


Figure 11.5.a: Critical sections for punching and beam shears (square footings)

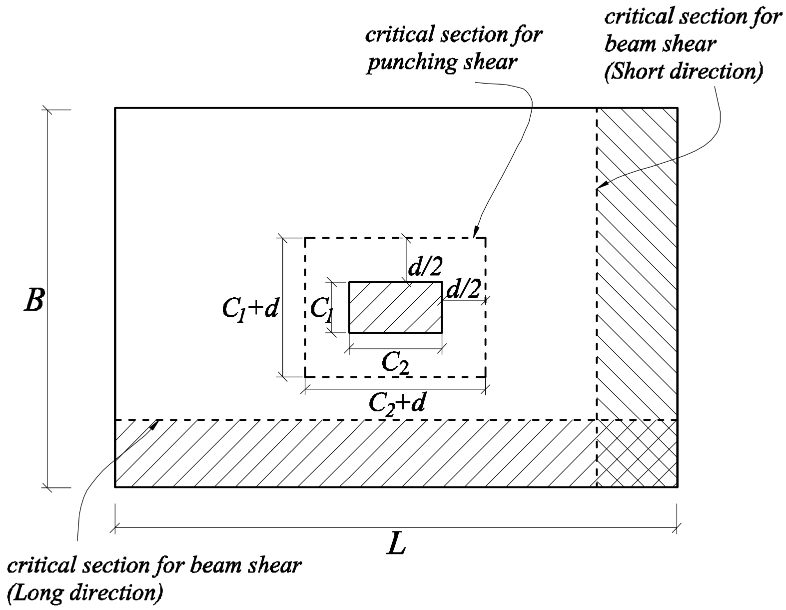


Figure 11.5.b: Critical sections for punching and beam shears (rectangular footings)

Punching shear force resisted by concrete V_c is given as the smallest of:

$$V_c = 0.53\sqrt{f'_c}\left(1+\frac{2}{b}\right) l\ b_0\ d \qquad (11.7)$$

$$V_c = l\ \sqrt{f'_c}\ b_0\ d \qquad (11.8)$$

$$V_c = 0.27\left(\frac{a_s\ d}{b_0}+2\right) l\ \sqrt{f'_c}\ b_0\ d \qquad (11.9)$$

When b = 2, equations (11.7) and (11.8) give the same value, if $b > 2$ *Eq.* (11.7) gives smaller value than that evaluated using *Eq.* (11.8).

Since there are two layers of reinforcement, an average value of d may be used. The average effective depth is given as

$d_{avg} = h_c - 7.5\,cm - d_b$, where d_b is bar diameter.

Increase footing thickness if additional shear strength is required.

5- Check footing thickness for beam shear in each direction.

If $V_u \le \Phi V_C$, thickness will be adequate for resisting beam shear without using shear reinforcement. The critical section for beam shear is located at distance d from column faces.

a- In the short direction:

The factored shear force is given by

$$V_u = q_u\,(net)\;B\left[\left(\frac{L-C_2}{2}\right)-d\right] \tag{11.10}$$

The factored shearing force resisted by concrete is given as

$$V_c = 0.53\sqrt{f_c'}\;B\,d \tag{11.11}$$

b- In the long direction:

The factored shear force is given by

$$V_u = q_u\,(net)\;L\left[\left(\frac{B-C_1}{2}\right)-d\right] \tag{11.12}$$

The factored shearing force resisted by concrete is given as

$$V_c = 0.53\sqrt{f_c'}\;L\,d \tag{11.13}$$

Increase footing thickness if necessary until the condition $V_u \le \Phi V_C$ is satisfied.

6- Compute the area of flexural reinforcement in each direction.

The critical section for bending is located at face of column, or wall, for footings supporting a concrete column or wall, as specified by *ACI Code 15.4.2*. Figure 11.6 shows critical sections for flexure for footings supporting concrete columns, masonry walls, and columns with steel base plates.

a- Reinforcement in the long direction:

$$M_u = q_u\,(net)\;\frac{B}{2}\left(\frac{L-C_2}{2}\right)^2 \tag{11.14}$$

b- Reinforcement in the short direction:

$$M_u = q_u\,(net)\;\frac{L}{2}\left(\frac{B-C_1}{2}\right)^2 \tag{11.15}$$

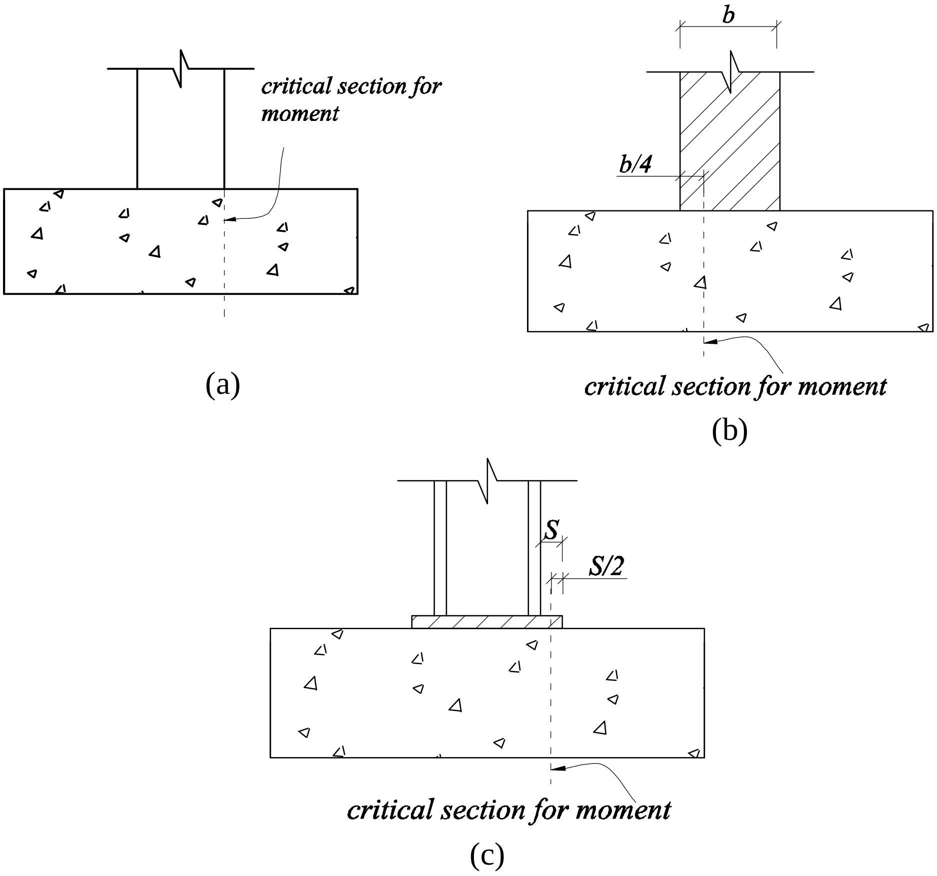
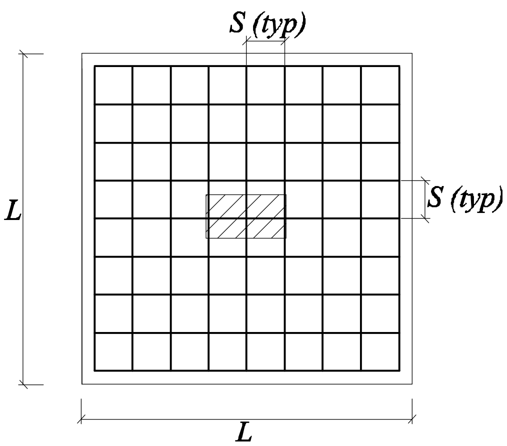
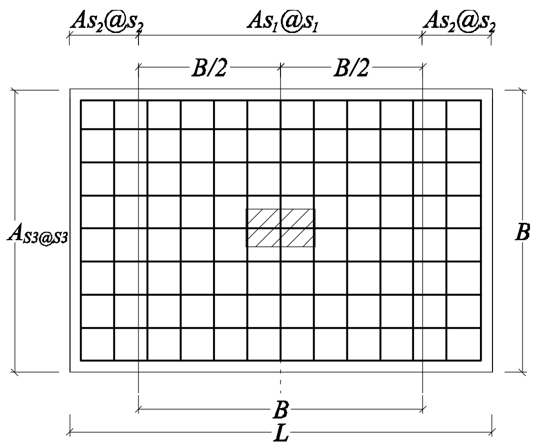


Figure 11.6: Critical section for moment (a) concrete column or wall; (b) masonry wall; (c) column with steel base plate

The reinforcement ratio is calculated based on rectangular section design, where the minimum reinforcement ratio r_{min} is not to be less than *0.0018*.



(a)



(b)

Figure 11.6: Flexural reinforcement; (a) square footing; (b) rectangular footing

According to *ACI Code 15.4.3*, for square footings, the reinforcement is identical in both directions as shown in Figure 11.6.a, neglecting the slight difference in effective depth values in the two directions. For rectangular footings, *ACI Code 15.4.4* specifies that the reinforcement in the long direction is uniformly distributed while portion of the total reinforcement in the short direction, $g_S A_S$ is to be distributed uniformly over a band width, centered on centerline of column, equal to the length of the short side of footing. Remainder of reinforcement required in short direction, $(1-g_S)A_S$ is to be distributed uniformly outside center band width of footing as shown in Figure 11.6.b.

where A_s is the total reinforcement required in the short direction, b equals the ratio of the long side to the short side of the footing and g_S is given as

$$g_S = \frac{2}{1+b} \qquad (11.16)$$

7- Check for bearing strength of column and footing concrete.

All forces applied at the base of a column or wall must be transferred to the footing by bearing on concrete and/or by reinforcement. Tensile forces must be resisted entirely by the reinforcement. Bearing on concrete for column and footing must not exceed the concrete bearing strength.

The joint could fail by crushing of the concrete at the bottom of the column where the column bars are no longer effective or by crushing the concrete in the footing under the column.

For a supported column, the bearing capacity ΦP_n is

$$\Phi P_n = \Phi\left(0.85\, f_c'\, A_1\right) \tag{11.17}$$

where

f_c' = compressive strength of the column concrete

A_1 = column cross-sectional area

Φ = strength reduction factor for bearing = 0.65

For a supporting footing,

$$\Phi P_n = \Phi\left(0.85\, f_c'\, A_1\right)\sqrt{\frac{A_2}{A_1}} \le 2.0\,\Phi\left(0.85\, f_c'\, A_1\right) \tag{1.18}$$

where

f_c' = compressive strength of the footing concrete

A_2 = area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the footing and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal.

When bearing strength is exceeded, reinforcement in the form of dowel bars must be provided to transfer the excess load. A minimum area of reinforcement must be provided across the interface of column or wall and footing, even where concrete bearing strength is not exceeded.

For columns, minimum dowel reinforcement is given by *ACI Code 15.8.2.1* as

$$A_{s,\,\min} = 0.005\, A_g \tag{11.19}$$

where A_g = column gross cross-sectional area

Required dowel reinforcement is given by

$$A_{s,\,req} = \frac{\left(P_u - \Phi\, P_n\right)}{\Phi\, f_y} \tag{11.20}$$

8- Check for anchorage of the reinforcement.

Both flexural and dowel reinforcement lengths are checked for anchorage to prevent bond failure of the dowels in the footing and to prevent failure of the lap splices between the dowels and the column bars, as shown in Figure 11.7.

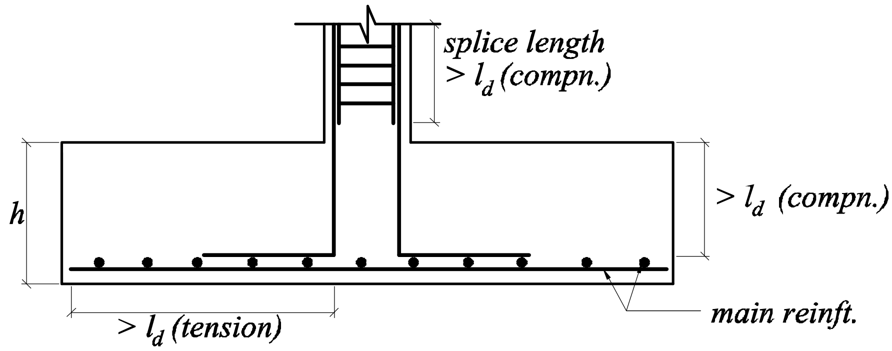


Figure 11.7: Anchorage of reinforcement

9- Prepare neat design drawings showing footing dimensions and provided reinforcement.

Example (11.1):

Design an isolated square footing to support an interior column 40 *cm* × 40 *cm* in cross section and carries a dead load of 80 *tons* and a live load of 60 *tons*.

Use $f_c' = 250\, kg/cm^2$, $f_y = 4200\, kg/cm^2$, $q_{all}\,(gross) = 2.0\, kg/cm^2$, $g_{soil} = 1.7\, t/m^3$, and $D_f = 1.25\, m$.

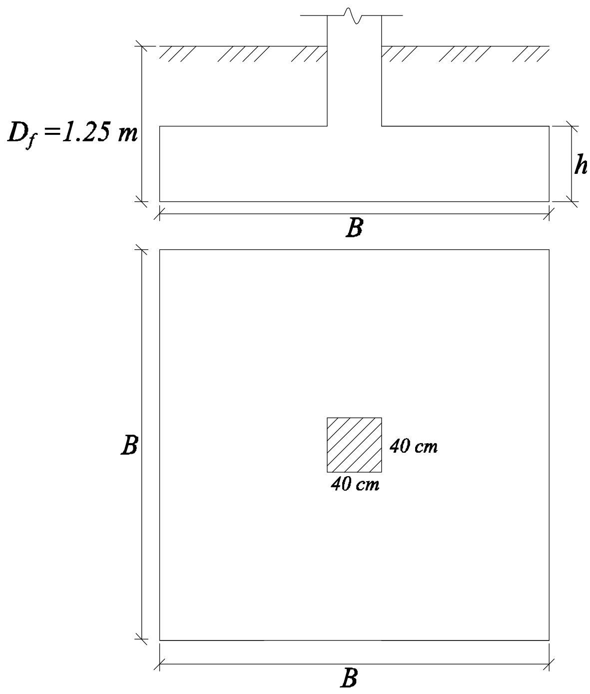


Figure 11.8.a: Footing dimensions

Solution:

1- Select a trial footing depth:

Assume that the footing is 50 *cm* thick.

2- Establish the required base area of the footing:

$$q_{all}\,(net) = 20 - 0.75\,(1.7) - 0.5\,(2.5) = 17.475\; t/m^2$$

$$A_{req} = \frac{P}{q_{all}\,(net)} = \frac{80+60}{17.475} = 8.011\, m^2$$

For a square footing, $B = \sqrt{8.011} = 2.83\; m$

Use 285 *cm* × 285 *cm* × 50 *cm* footing, as shown in Figure 11.8.a.

3- Evaluate the net factored soil pressure:

$$P_u = 1.20\,(80) + 1.60\,(60) = 192\; tons$$

$$q_u\,(net) = \frac{P_u}{B^2} = \frac{192}{(2.85)^2} = 23.64\; t/m^2$$

4- Check footing thickness for punching shear:

Average effective depth d = 50 – 7.5 – 1.6 = 40.9 *cm*

The factored shear force

$$V_u = (23.64)\left[(2.85)(2.85) - (0.809)(0.809)\right] = 176.54\; tons$$

$$b_0 = 4\,(40 + 40.9) = 323.6\; cm$$

$F\,V_c$ is the smallest of:

$$F\;V_c = 0.53\,F\;\sqrt{f'_c}\left(1 + \frac{2}{b}\right) l\; b_0\; d$$

$$= 0.53\,(0.75)\sqrt{250}\left(1 + \frac{2}{1}\right)(1)\,(323.6)\,(40.9)/\,1000 = 249.55\; tons$$

$$F\;V_c = F\;\;l\;\sqrt{f'_c}\; b_0\; d$$

$$= 0.75\sqrt{250}\;(323.6)\,(40.9) = 156.95\; tons$$

$$F\;V_c = 0.27\,F\left(\frac{a_s\; d}{b_0} + 2\right) l\;\sqrt{f'_c}\; b_0\; d$$

$$=0.27(0.75)\left(\frac{40(40.9)}{323.6}+2\right)\sqrt{250}\,(323.6)(40.9)/1000=299\ tons$$

$F\ V_c=156.95\ tons \quad <176.54\ tons$

Increase footing thickness to 55 *cm*, and repeat punching shear check.

Average effective depth d = 55 – 7.5 – 1.6 = 45.9 *cm*

The factored shear force

$V_u=(23.64)[(2.85)(2.85)-(0.859)(0.859)]=174.57\ tons$

$b_{\mathbf{0}}=4(40+45.9)=343.6\ cm$

$F\ V_c=F\ \ l\ \sqrt{f'_c}\ b_{\mathbf{0}}\ d$

$$=0.75\sqrt{250}\,(343.6)(45.9)=187.02\ tons$$

$F\ V_c=187.02\ tons \quad >174.57\ tons$

i.e. footing thickness is adequate for resisting punching shear.

5- Check footing thickness for beam shear in each direction:

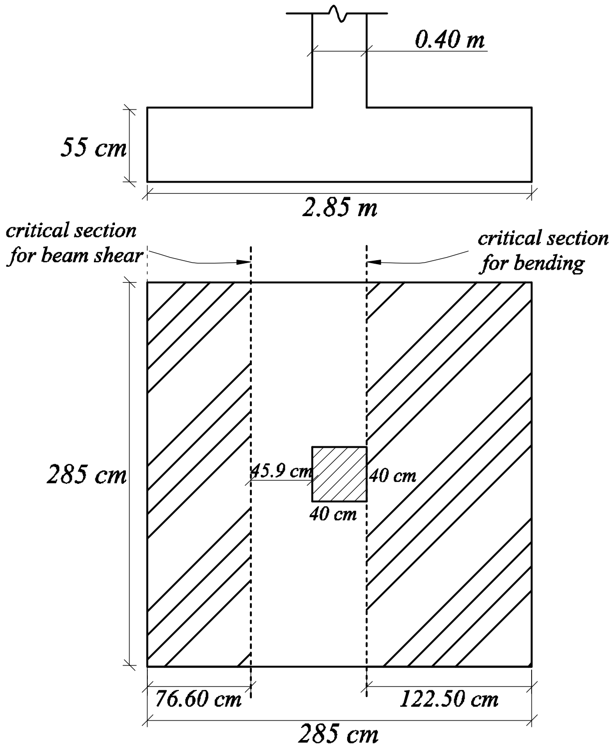


Figure 11.8.b: Critical sections for beam shear and moment

$$\Phi V_c = 0.75(0.53)\sqrt{250}(285)(45.9)/1000 = 82.22 \ tons$$

Maximum factored shear force V_u is located at distance d from faces of column, as shown in Figure 11.8.b, or:

$$V_u = (23.64)(2.85)(0.766) = 51.61 \ tons < 82.22 \ tons$$

6- Compute the area of flexural reinforcement in each direction:

The critical section for moment is located at column faces, as shown in Figure 11.8.b, or:

$$M_u = (23.64)(2.85)\frac{(1.225)^2}{2} = 50.55 \ t.m$$

$$\rho = \frac{0.85(250)}{4200}\left[1 - \sqrt{1 - \frac{2.353(10)^5(50.55)}{(0.9)(285)(45.9)^2(250)}}\right] = 0.00228$$

$A_s = 0.00228(285)(45.9) = 29.82 \ cm^2$, use $15 \ \phi \ 16 \ mm$ in both directions.

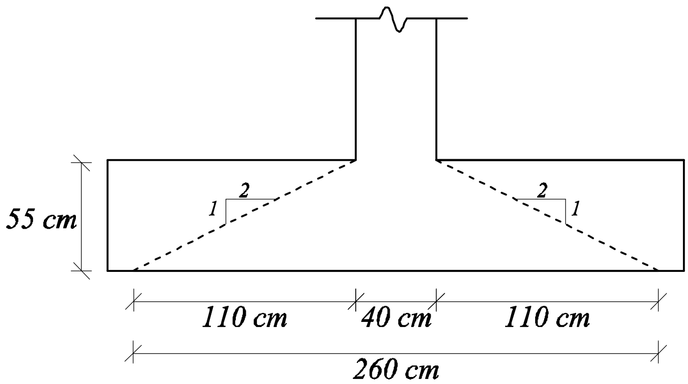


Figure 11.8.c: Critical section for bearing

7- Check for bearing strength of column and footing concrete:

For column,

$$\Phi P_n = 0.65(0.85)(250)(40)(40)/1000 = 221 \ tons \quad > 192 \ tons$$

i.e. use minimum dowel reinforcement, $A_s = 0.005(40)(40) = 8.0 \ cm^2$

Use $4 \ \phi \ 16 \ mm$ for dowel reinforcing bars.

For footing,

$$\Phi P_n = 0.65(0.85)(250)(40)(40)\sqrt{\frac{(260)^2}{(40)^2}}/1000 = 1436.5 \ tons \quad > 442 \ tons$$

i.e. $F\ P_n = 442\ tons \quad > 192\ tons$

8- Check for anchorage of the reinforcement:

Bottom reinforcement (f 16 mm):

$y_t = y_e = l = 1$ and $y_s = 0.8$

c_b is the smaller of:

$7.5 + 0.8 = 8.3\,cm$, or $\dfrac{285 - 15 - 1.6}{14(2)} = 9.58\ cm$, i.e., c_b = 8.3 cm

$$\frac{c_b + K_{tr}}{d_b} = \frac{8.3 + 0}{1.6} = 5.1875 > 2.5\text{, taken as } 2.5$$

$$l_d = d_b \left(\frac{f_y y_t y_e y_s}{3.5\ l \left(\dfrac{c_b + K_{tr}}{d_b} \right) \sqrt{f'_c}} \right) = 1.6 \left(\frac{4200\,(0.8)}{3.5\,(2.5)\sqrt{250}} \right) = 38.86\ cm$$

Available length = 122.5 – 7.5 = 115.0 cm > 38.86 cm

a- Dowel reinforcement (f 16 mm):

$$\frac{0.075\ f_y}{l\ \sqrt{f'_c}}\, d_b \geq 0.0044\ f_y\ d_b$$

$l_d = \dfrac{0.075(1.6)(4200)}{\sqrt{250}} = 31.88\ cm$, or

$l_d = 0.0044(1.6)(4200) = 29.57\ cm\ > 20\ cm$

Available length = 55 –7.5 –1.6 –1.6 –1.6 = 42.7 cm > 31.88 cm

b- Column reinforcement splices:

$l_{sp} = 0.0073(1.6)(4200) = 49.06\ cm$, taken as 50 cm.

9- Prepare neat design drawings showing footing dimensions and provided reinforcement:

Design drawings are shown in Figure 11.8.d.

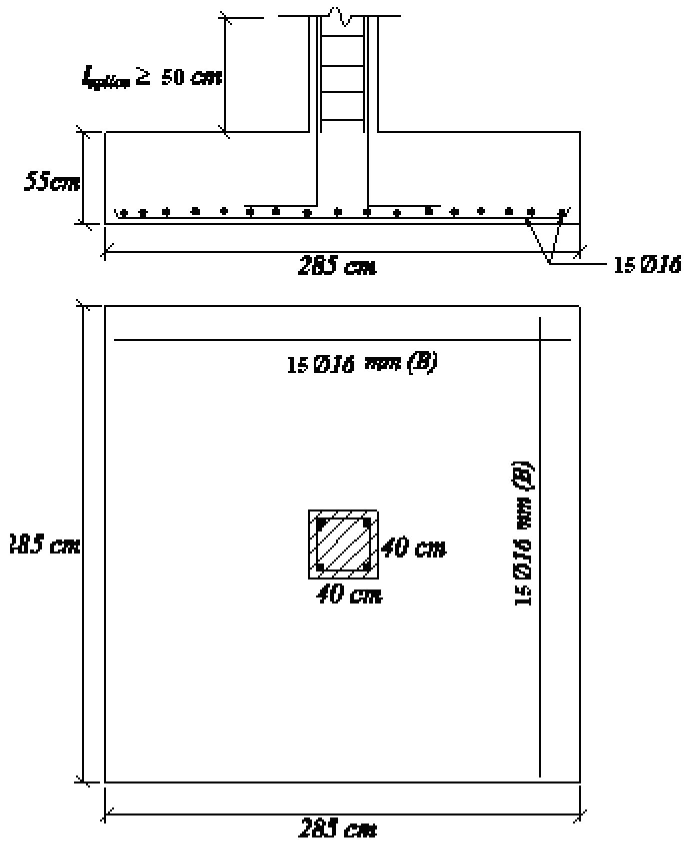


Figure 11.8.d: Design drawings

Example (11.2):

Design an isolated rectangular footing to support an interior column 40 *cm* × 40 *cm* in cross section and carries a dead load of 80 *tons* and a live load of 60 *tons*. Use $f_c' = 250\, kg/cm^2$, $f_y = 4200\, kg/cm^2$, $q_{all}(gross) = 2.0\, kg/cm^2$, $\gamma_{soil} = 1.7\ t/m^3$, and $D_f = 1.25\ m$.

Space limitations are such that one lateral dimension cannot exceed 2.5*m*.

Solution:

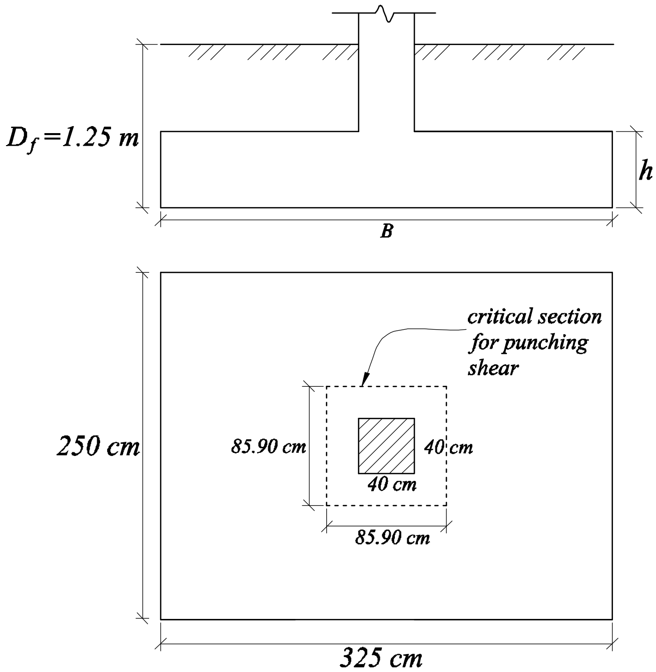


Figure 11.9.a: Critical section for punching shear

1- Select a trial footing depth:

Assume that the footing is 55 *cm* thick.

2- Establish the required base area of the footing:

$$q_{all}(net) = 20 - 0.7(1.7) - 0.55(2.5) = 17.435\ t/m^2$$

$$A_{req} = \frac{P}{q_{all}(net)} = \frac{80+60}{17.435} = 8.03\,m^2$$

Let B = 2.5 *m*, $L = \frac{8.03}{2.5} = 3.21\,m$

Use 325 *cm* × 250 *cm* × 55 *cm* footing.

3- Evaluate the net factored soil pressure:

$$P_u = 1.20(80) + 1.60(60) = 192\ tons$$

$$q_u(net) = \frac{P_u}{L \times B} = \frac{192}{3.25 \times 2.5} = 23.63\ t/m^2$$

4- Check footing thickness for punching shear:

The critical section for punching shear is shown in Figure 11.9.a.

Average effective depth d = 55 – 7.5 – 1.6 = 45.9 *cm*

The factored shear force

$$V_u = (23.63)[(3.25)(2.5) - 0.859(0.859)] = 174.56\ tons$$

$$b_o = 4(40 + 45.9) = 343.6\ cm$$

ΦV_c is the smallest of:

$$F\ V_c = 0.53\, F\ \sqrt{f'_c}\left(1 + \frac{2}{b}\right) l\ b_0\ d$$

$$= 0.53\,(0.75)\sqrt{250}\left(1 + \frac{2}{1}\right)(1)(343.6)(45.9)/1000 = 297.37\ tons$$

$$F\ V_c = F\ \ l\ \sqrt{f'_c}\ b_0\ d$$

$$= 0.75\sqrt{250}\ (343.6)(45.9) = 187.02\ tons$$

$$F\ V_c = 0.27\, F\left(\frac{a_s\ d}{b_0} + 2\right) l\ \sqrt{f'_c}\ b_0\ d$$

$$= 0.27(0.75)\left(\frac{40\,(45.9)}{343.6} + 2\right)\sqrt{250}\ (343.6)(45.9)/1000 = 370.82\ tons$$

$$F\ V_c = 187.02\, tons \quad > 174.56\ tons$$

i.e. footing thickness is adequate for resisting punching shear.

5- Check footing thickness for beam shear in each direction:

a- In the short direction:

$$F\ V_c = 0.75(0.53)\sqrt{250}\,(250)(45.9)/1000\ = 72.12\ tons$$

Maximum factored shear force V_u is located at distance d from faces of column,

$$V_u = (23.63)(2.5)(0.966) = 57.07\ tons < 72.12\ tons$$

b- In the long direction:

$$F\ V_c = 0.75(0.53)\sqrt{250}\,(325)(45.9)/1000\ = 93.76\ tons$$

Maximum factored shear force V_u is located at distance d from faces of column,

$$V_u = (23.63)(3.25)(0.591) = 45.39\ tons < 93.76\ tons$$

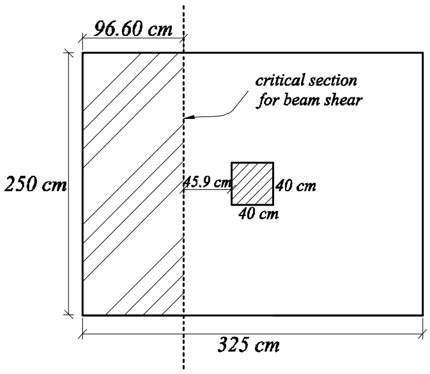


Figure 11.9.b: Critical section for beam shear (short direction)

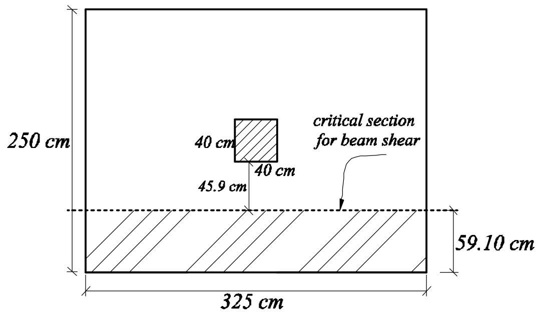


Figure 11.9.b: Critical section for beam shear (long direction)

6- Compute the area of flexural reinforcement in each direction:

a- Reinforcement in long direction:

The critical section for bending is shown in Figure 11.9.d.

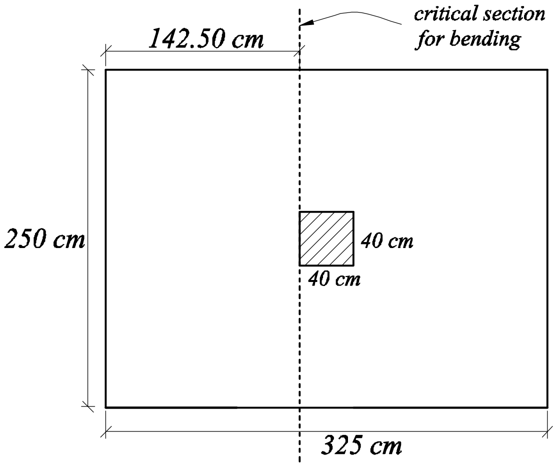


Figure 11.9.d: Critical section for bending (long direction)

$$M_u = (23.63)(2.5)\frac{(1.425)^2}{2} = 59.98\ t.m$$

$$r = \frac{0.85(250)}{4200}\left[1 - \sqrt{1 - \frac{2.353(10)^5(59.98)}{(0.9)(250)(45.9)^2(250)}}\right] = 0.00311$$

$A_s = 0.00311(250)(45.9) = 35.69\,cm^2$, use $15\ f\ 18\ mm$ in the long direction.

b- Reinforcement in short direction:

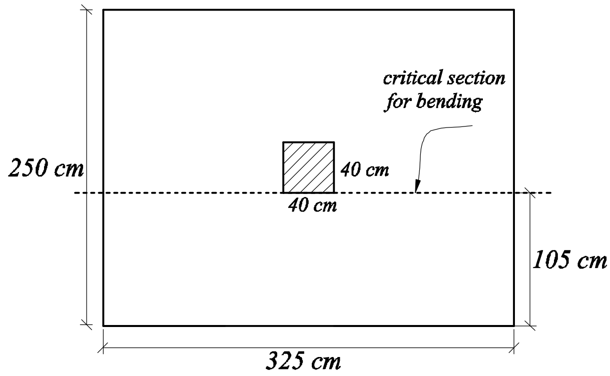


Figure 11.9.e: Critical section for bending (short direction)

The critical section for bending is shown in Figure 11.9.e.

$$M_u = (23.63)(3.25)\frac{(1.05)^2}{2} = 42.33\ t.m$$

$$r = \frac{0.85(250)}{4200}\left[1 - \sqrt{1 - \frac{2.353(10)^5\,(42.33)}{(0.9)(325)(45.9)^2\,(250)}}\right] = 0.00166$$

$$A_{s,min} = 0.0018\ (325)(55) = 32.17\,cm^2$$

Central band reinforcement $= g_s\ A_s = \left(\frac{2}{1+325/250}\right)(32.17) = 27.97\ cm^2$

Use $14\ f\ 16\ mm$ in the central band.

For each of the side bands, $A_s = \left(\frac{32.17 - 27.97}{2}\right) = 2.10\ cm^2$

Use $2\ f\ 16\ mm$ in each of the two side bands.

7- Check for bearing strength of column and footing concrete:

For column,

$$F\ P_n = 0.65(0.85)(250)(40)(40)/1000 = 221\ \ tons \quad > 192\ tons$$

i.e. use minimum dowel reinforcement, $A_s = 0.005(40)(40) = 8.0\ cm^2$.

Use $4\ f\ 16\ mm$ for dowel reinforcing bars.

For footing,

$$F\ P_n = 0.65(0.85)(250)(40)(40)\sqrt{\frac{(260)^2}{(40)^2}}\,/\,1000 = 1436.5\ tons > 476\ tons$$

i.e. $F\ P_n = 476\ tons \quad > 192\ tons$

8- Check for anchorage of the reinforcement:

a- Reinforcement in long direction (f 18 mm ***):***

$y_t = y_e = l = 1$ and $y_s = 0.8$

c_b is the smaller of:

$7.5 + 0.9 = 8.4\ cm$, or $\frac{325 - 15 - 1.8}{15(2)} = 10.27\ cm$, i.e., c_b= 8.40 cm

$$\frac{c_b + K_{tr}}{d_b} = \frac{8.40 + 0}{1.8} = 4.67 > 2.5\text{, take it equal to } 2.5$$

$$l_d = d_b\left(\frac{f_y\, y_t\, y_e\, y_s}{3.5\ l\left(\frac{c_b + K_{tr}}{d_b}\right)\sqrt{f'_c}}\right) = 1.8\left(\frac{4200\,(0.8)}{3.5\,(2.5)\sqrt{250}}\right) = 43.72\ cm$$

Available length = 142.5 – 7.5 = 135.0 cm > 43.72 cm

b- Reinforcement in short direction (f 16 mm ***):***

$y_t = y_e = l = 1$ and $y_s = 0.8$

c_b is the smaller of:

$7.5 + 0.8 = 8.3\ cm$, or $\frac{250 - 15 - 1.6}{14(2)} = 8.34\ cm$, i.e., c_b= 8.3 cm

$$\frac{c_b + K_{tr}}{d_b} = \frac{8.30 + 0}{1.6} = 5.19 > 2.5\text{, take it equal to } 2.5$$

$$l_d = d_b\left(\frac{f_y\, y_t\, y_e\, y_s}{3.5\ l\left(\frac{c_b + K_{tr}}{d_b}\right)\sqrt{f'_c}}\right) = 1.6\left(\frac{4200\,(0.8)}{3.5\,(2.5)\sqrt{250}}\right) = 38.86\ cm$$

Available length = 105.0 – 7.5 = 97.5 cm > 38.86 cm

c- Column reinforcement splices:

$l_{sp} = 0.0073(1.6)(4200) = 49.06\ cm$, taken as 50 *cm*.

9- Prepare neat design drawings showing footing dimensions and provided reinforcement:

Design drawings are shown in Figure 11.9.f.

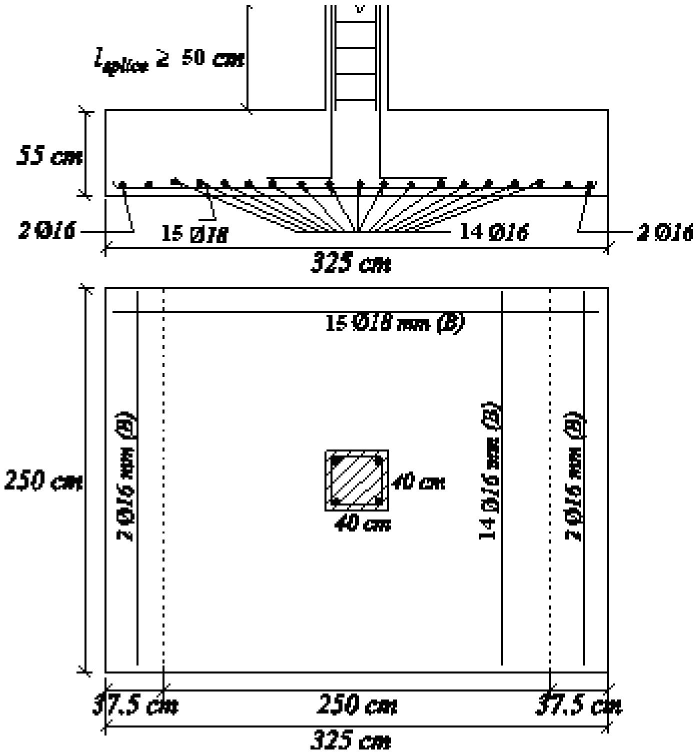


Figure 11.9.f: Design drawings

11.7 Problems

P11.1 Design a circular footing to support a column 40 *cm* in diameter, and carries a service dead load of 60 *tons*, and a service live load of 20 *tons*.

Use $f_c' = 300\ kg/cm^2$, $f_y = 4200\ kg/cm^2$, $q_{all}(gross) = 1.7\ kg/cm^2$, $g_{soil} = 1.7\ t/m^3$, and $D_f = 1.5\ m$.

P11.2 Design an isolated footing to support a column 25 *cm* × 60 *cm* in cross section. The column carries a service dead load of 60 *tons*, and a service live load of 30 *tons*, in addition to a service dead load moment of 8 *t.m*, and a service live load moment of 4 *t.m*.

Use $f_c' = 300\, kg/cm^2$, $f_y = 4200\, kg/cm^2$, and $q_{all}(net) = 1.7\, kg/cm^2$.