(1)

UNIT - VIII

13

Shear Strength

13.1. INTRODUCTION

The shear strength of a soil is its maximum resistance to shear stresses just before the failure. Soils are seldom subjected to direct shear. However, the shear stresses develop when the soil is subjected to direct compression. Although shear stresses may also develop when the soil is subjected to direct tension, but these shear stresses are not relevant, as the soil in this case fails in tension and does not fail in shear. In field, soils are seldom subjected to tension, as it causes opening of the cracks and fissures. These cracks are not only undesirable, but are also detrimental to the stability of the soil masses. Thus, the shear failure of a soil mass occurs when the shear stresses induced due to the applied compressive loads exceed the shear strength of the soil. It may be noted that the failure in soil occurs by relative movements of the particles and not by breaking of the particles.

Shear strength is the principal engineering property which controls the stability of a soil mass under loads. It governs the bearing capacity of soils, the stability of slopes in soils, the earth pressure against retaining structures and many other problems, as explained in later chapters. All the problems of soil engineering are related in one way or the other with the shear strength of the soil. Unfortunately, the shear strength is one of the most complex engineering properties of the soil. The current research is giving new concepts and theories. This chapter presents the basic concepts and the accepted theories of the shear strength.

13.2. STRESS-SYSTEM WITH PRINCIPAL PLANES PARALLEL TO THE COORDINATE AXES

In general, a soil mass is subjected to a three—dimensional stress system. However, in many soil engineering problems, the stresses in the third direction are not relevant and the stress system is simplified as two-dimensional. The plane strain conditions are generally assumed, in which the strain in the third (longitudinal) direction is zero. Such conditions exist, for example, under a strip footing of a long retaining wall.

At every point in a stressed body, there are three planes on which the shear stresses are zero. These planes are known as *principal planes*. The plane with the maximum compressive stress (σ_1) is called the major principal plane, and that with the minimum compressive (σ_3) as the minor principal plane. The third principal plane is subjected to a stress which has the value intermediate between σ_1 and σ_3, and is known as the intermediate principal plane. Generally, the stresses on a plane perpendicular to the intermediate principal plane are required in the analysis. Therefore, the stresses on the intermediate principal plane are not much relevant. Only the major principal stress (σ_1) and the minor principal stress (σ_3) are generally important.

In solid mechanics, the tensile stresses are taken as positive. In soil engineering problems, tensile stresses rarely occur. To avoid many negative signs, compressive stresses are taken as positive and the tensile stresses as negative in soil engineering.

Fig. 13.1 shows a plane which is perpendicular to the intermediate principal plane. The major and minor principal stresses act on this plane. The major principal plane is horizontal and the minor principal plane is

$$\sin 2\theta_p = \pm \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}}$$

$$\cos 2\theta_p = \pm \frac{(\sigma_y - \sigma_x)/2}{\sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}}$$

Substituting these values of $\sin 2\theta_p$ and the $\cos 2\theta_p$ in Eq. 13.3,

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \left(\frac{\sigma_y - \sigma_x}{2}\right) \times \frac{(\sigma_y - \sigma_x)/2}{\sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}} \pm \frac{\tau_{xy} \times \tau_{xy}}{\sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}}$$

or

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

Therefore, the two principal stresses are as under.

Major principal stress,
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2} \quad \text{...(13.7)}$$

The point U gives the major principal stress (σ_1).

Minor principal stress,
$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2} \quad \text{...(13.8)}$$

The point V gives the minor principal stress (σ_3)

Also, because $\tan 2\theta_p = \tan (2\theta_p + 180°)$, the second principal plane is indicated by the line CV.

13.6. IMPORTANT CHARACTERISTICS OF MOHR'S CIRCLE

The following important characteristics of Mohr's circle should be carefully noted, as these are required for further study.

(1) The maximum shear stress τ_{max} is numerically equal to $(\sigma_1 - \sigma_3)/2$ and it occurs on a plane inclined at 45° to the principal planes (Fig. 13.5).

(2) Point D on the Mohr circle represents the stresses (σ, τ) on a plane make an angle θ with the major principal plane.

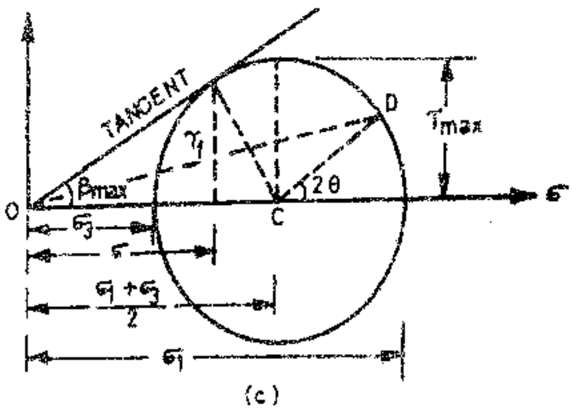


Fig. 13.5. Characteristics of Mohr's Circle.

The resultant stress on that plane is equal to $\sqrt{\sigma^2 + \tau^2}$ and its angle of obliquity with the normal of the plane is equal to angle β, given by

$$\beta = \tan^{-1}(\tau/\sigma) \qquad \text{...(13.9)}$$

(3) The maximum angle of obliquity β_{max} is obtained by drawing a tangent to the circle from the origin *O*.

$$\beta_{max} = \sin^{-1}\frac{(\sigma_1 - \sigma_3)/2}{(\sigma_1 + \sigma_2)/2} = \sin^{-1}\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right) \qquad \text{...(13.9)}$$

(4) The shear stress τ_f on the plane of the maximum obliquity is less than the maximum shear stress τ_{max}.

(5) Shear stresses on planes at right angles to each other are numerically equal but are of opposite signs, as shown in Fig. 13.4 (*c*).

(6) As the Mohr circle is symmetrical about σ-axis, it is usual practice to draw only the top half circle for convenience.

(7) There is no need to be rigid about sign convention for plotting the shear stresses in Mohr's circle. These can be plotted either upward or downward. Although the sign convention is required for locating the orientation of the planes, the numerical results are not affected.

13.7. MOHR-COULOMB THEORY

The soil is a particulate material. The shear failure occurs in soils by slippage of particles due to shear stresses. The failure is essentially by shear, but shear stresses at failure depend upon the normal stresses on the potential failure plane. According to Mohr, the failure is caused by a critical combination of the normal and shear stresses.

The soil fails when the shear stress (τ_f) on the failure plane at failure is a unique function of the normal stress (σ) acting on that plane.

$$\tau_f = f(\sigma)$$

Since the shear stress on the failure plane at failure is defined as the shear strength (s), the above equation can be written as

$$s = f(\sigma) \qquad \text{...(13.11)}$$

The Mohr theory is concerned with the shear stress at failure plane at failure. A plot can be made between the shear stress τ and the normal stress σ at failure. The curve defined by Eq. 13.11 is known as the Mohr envelope [Fig. 13.6 (*a*)]. There is a unique failure envelope for each material.

Failure of the material occurs when the Mohr circle of the stresses touches the Mohr envelope. As discussed in the preceding sections, the Mohr circle represents all possible combinations of shear and normal stresses at the stressed point. At the point of contact (*D*) of the failure envelope and the Mohr circle, the critical combination of shear and normal stresses is reached and the failure occurs. The plane indicated by the line *PD* is, therefore, the failure plane. Any Mohr's circle which does not cross the failure envelope and

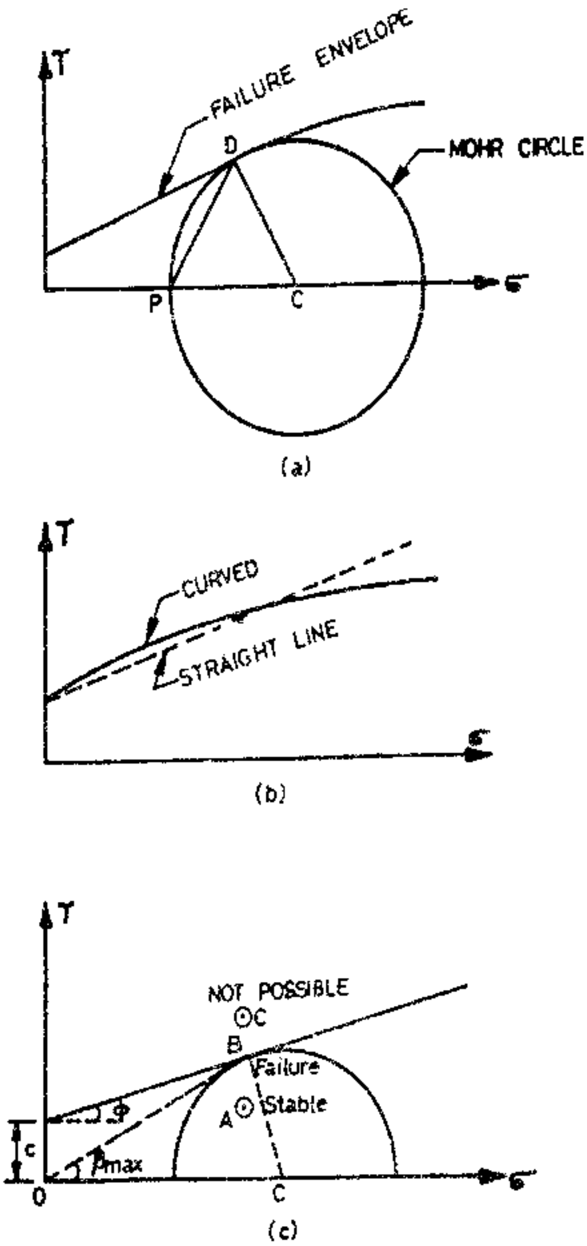


Fig. 13.6. Failure Envelopes.

lies below the envelope represents a (non-failure) stable condition. The Mohr circle cannot cross the Mohr envelope, as the failure would have already occurred as soon as the Mohr circle touched the envelope.

The shear strength (s) of a soil at a point on a particular plane was expressed by Coulomb as a linear function of the normal stress on that plane, as

$$s = c + \sigma \tan \phi \qquad \text{...(13.12)}$$

In other words, the Mohr envelope is replaced by a straight line by Coulomb as shown in Fig. 13.6 (b).

In Eq. 13.12, c is equal to the intercept on τ-axis and ϕ is the angle which the envelope makes with σ-axis [Fig. 13.6 (c)]. The component c of the shear strength is known as *cohesion.* Cohesion holds the particles of the soil together in a soil mass, and is independent of the normal stress. The angle ϕ is called the *angle of internal friction.* It represents the frictional resistance between the particles, which is directly proportional to the normal stress.

As mentioned before, the failure occurs when the stresses are such that the Mohr circle just touches the failure envelope, as shown by point B in Fig. 13.6 (c). In other words, shear failure occurs if the stresses σ and τ on the failure plane plot as point B. If the stresses plot as point A below the failure envelope, it represents a stable, non-failure condition. On the other hand, a state of stress represented by point C above the failure envelope is not possible. It may be noted that a material fails along a plane when the critical combination of the stresses σ and τ gives the resultant with a maximum obliquity (β_{max}), in which case the resultant just touches the Mohr circle.

13.8. REVISED MOHR-COULOMB EQUATION

Later research showed that the parameters c and ϕ in Eq. 13.12 are not necessarily fundamental properties of the soil as was originally assumed by Coulomb. These parameters depend upon a number of factors, such as the water content, drainage conditions, conditions of testing. The current practice is to consider c and ϕ as mathematical parameters which represent the failure conditions for a particular soil under given conditions. That is the reason why c and ϕ are now called cohesion intercept and the angle of shearing resistance. These indicate the intercept and the slope of the failure envelope, respectively.

Terzaghi established that the normal stresses which control the shear strength of a soil are the effective stresses and not the total stresses. In terms of effective stresses, Eq. 13.12 is written as

$$s = c' + \bar{\sigma} \tan \phi' \qquad \text{...(13.13)}$$

where c' and ϕ' are the cohesion intercept and the angle of shearing resistance in terms of the effective stresses.

Eq. 13.13 is known as the *Revised Mohr—Coulomb* equation for the shear strength of the soil. The equation has replaced the original equation (Eq. 13.12). It is one of the most important equations of soil engineering.

The Mohr—Coulomb theory shows a reasonably good agreement with the observed failures in the field and in the laboratory. The theory is ideally suited for studying the behaviour of soils at failure. The theory is used for estimation of the shear strength of soils. However, even this theory is not perfect. It has the following main limitations :

(1) It neglects the effect of the intermediate principal stress (σ_2).

(2) It approximates the curved failure envelope by a straight line, which may not give correct results.

(3) When the Mohr envelope is curved, the actual obliquity of the failure plane is slightly smaller than the maximum obliquity. Therefore, the angle of the failure plane, as found, is not correct.

(4) For some clayey soils, there is no fixed relationship between the normal and shear stresses on the plane of failure. The theory cannot be used for such soils.

13.9. DIFFERENT TYPES OF TESTS AND DRAINAGE CONDITIONS

The following tests are used to measure the shear strength of a soil.

(1) Direct shear test (2) Triaxial Compression test

(3) Unconfined Compression test (4) Shear Vane test.

The shear test must be conducted under appropriate drainage conditions that simulate the actual field problem. In shear tests, there are two stages :

(1) Consolidation stage in which the normal stress (or confining pressure) is applied to the specimen and it is allowed to consolidate.

(2) Shear stage in which the shear stress (or deviator stress) is applied to the specimen to shear it.

Depending upon the drainage conditions, there are three types of tests as explained below :

(1) Unconsolidated—Undrained Condition. In this type of test, no drainage is permitted during the consolidation stage. The drainage is also not permitted in the shear stage.

As no time is allowed for consolidation or dissipation of excess pore water pressure, the test can be conducted quickly in a few minutes. The test is known as unconsolidated—undrained test (*UU* test) or quick test (*Q*-test).

(2) Consolidated—Undrained Condition. In a consolidated—undrained test, the specimen is allowed to consolidate in the first stage. The drainage is permitted until the consolidation is complete.

In the second stage when the specimen is sheared, no drainage is permitted. The test is known as consolidated—undrained test (*CU* test) It is also called a *R*-test, as the alphabet *R* falls between the alphabet *Q* used for quick test, and the alphabet *S* used for slow test.

The pore water pressure can be measured in the second stage if the facilities for its measurement are available. In that case, the test is known as $\overline{CU}$ test.

(3) Consolidated—Drained Condition. In a consolidated—drained test, the drainage of the specimen is permitted in both the stages. The sample is allowed to consolidate in the first stage. When the consolidation is complete, it is sheared at a very slow rate to ensure that fully drained conditions exist and the excess pore water is zero.

The test is known as a consolidated—drained test (*CD* test) or drained test. It is also known as the slow test (*S*—test).

13.10. MODE OF APPLICATION OF SHEAR FORCE

The shear force in a shear test is applied either by increasing the shear displacement at a given rate or by increasing the shearing force at a given rate. Accordingly, the shear tests are either strain—controlled or stress-controlled.

(1) Strain controlled tests. In a strain-controlled test, the test is conducted in such a way that the shearing strain increases at a given rate. Generally, the rate of increase of the shearing strain is kept constant, and the specimen is sheared at a uniform strain rate.

The shear force acting on the specimen is measured indirectly using a proving ring. The rate of shearing strain is controlled manually or by a gear system attached to an electric motor.

Most of the shear tests are conducted as strain—controlled. The stress—strain characteristic are easily obtained in these tests, as the shape of the stress—strain curve beyond the peak point can be observed only in a strain—controlled test. A strain—controlled test is easier to perform than a stress- controlled test.

(2) Stress—Controlled tests. In a stress—controlled test, the shear force is increased at a given rate. Usually, the rate of increase of the shear force is maintained constant. The shear load is increased such that the shear stresses increase at a uniform rate. The resulting shear displacements are obtained by means of a dial gauge.

Stress—controlled tests are preferred for conducting shear tests at a very low rate, because an applied load can easily be kept constant for any given period of time. Further, the loads can be conveniently applied and removed. The stress-controlled test represents the field conditions more closely.

13.11 DIRECT SHEAR TEST

(*a*) Apparatus. A direct shear test is conducted on a soil specimen in a shear box which is split into two halves along a horizontal plane at its middle (Fig. 13.7). The shear box is made of brass or gunmetal. It is

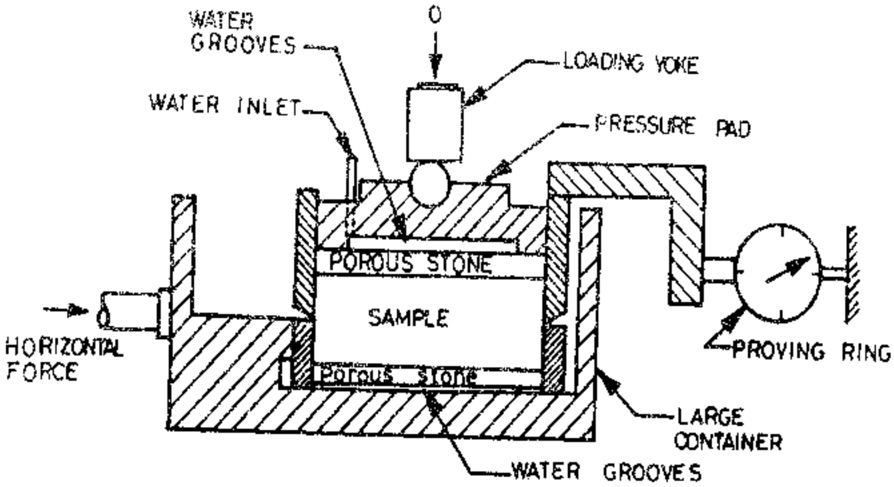


Fig. 13.7. Direct Shear Test.

either square or circular in plan. A square box of size 60 × 60 × 50mm is commonly used. The box is divided horizontally such that the dividing plane passes through the centre. The two halves of the box are held together by locking pins. Suitable spacing screws to separate the two halves are also provided. The spacing screws are fixed to the upper half and they butt against the top of the lower half.

The box is provided with the gripper or the grid plates which are toothed and fitted inside it. The gripper plates are plain (without perforations) for undrained tests and perforated for drained tests. Porous stones are placed at the top and the bottom of the specimen in drained tests. A pressure pad of brass or gun metal is fitted into the box at its top to transmit the normal load to the sample. The normal load from the loading yoke is applied on the top of the specimen through a steel ball bearing upon the pressure pad.

The lower half of the box is fixed to the base plate which is rigidly held in position in a large container. The large container is supported on rollers (rollers not shown). The container can be pushed forward at a constant rate by a geared jack which works as a strain-controlled device. The jack may be operated manually or by an electric motor.

A loading frame is used to support the large container. It has the arrangement of a loading yoke and a lever system for applying the normal load.

A proving ring is fitted to the upper half of the box to measure the shear force.The proving ring butts against a fixed support. As the box moves, the proving ring records the shear force. The shear displacement is measured with a dial gauge fitted to the container. Another dial gauge is fitted to the top of the pressure pad to measure the change in the thickness of the specimen.

(*b*) Test. A soil specimen of size 60 × 60 × 25 mm is taken. It may be either an undisturbed sample or made from compacted and remoulded soil. The specimen may be prepared directly in the box and compacted. The base plate is attached to the lower half of the box. A porous stone is placed in the box. For undrained tests, a plain grid is kept on the porous stone, keeping its segregations at right angles to the direction of shear. For drained tests, perforated grids are used instead of plain grids. The mass of the base plate, porous stone and grid is taken. The specimen if made separately is transferred to the box and its mass taken.

The upper grid, porous stone and the pressure pad are placed on the specimen. The box is placed inside the large container and mounted on the loading frame. The upper half of the box is brought in contact with the proving ring. The loading yoke is mounted on the steel ball placed on the pressure pad. The dial gauge is fitted to the container to give the shear displacement. The other dial gauge is mounted on the loading yoke to record the vertical movement.

The locking pins are removed and the upper half box is slightly raised with the help of spacing screws. The space between the two halves is adjusted, depending upon the maximum particle size. The space should be such that the top half of the box does not ride on soil grains which come between the edges.

The normal load is applied to give a normal stress of 25 kN/m^2. Shear load is then applied at a constant rate of strain. For undrained tests, the rate is generally between 1.0 mm to 2.00 mm per minute. For drained tests, the strain rate depends upon the type of soil. For sandy soils, it may be taken as 0.2 mm/minute; whereas for clayey soils, it is generally between 0.005 to 0.02 mm/min. The sample shears along the horizontal plane between the two halves. The readings of the proving-ring and the dial gauges are taken every 30 seconds. The test is continued till the specimen fails. The failure is indicated when the proving ring dial gauge begins to recede after having reached the maximum. For the soils which do not give a peak point, the failure is assumed to have occurred when a shearing strain of 20% is reached. At the end of the test, the specimen is removed from the box and its water content found.

The test is repeated under the normal stress of 50, 100, 200 and 400 kN/m^2. The range of the normal stress should cover the range of loading in the field problem for which the shear parameters are required. The shear stress at any stage during shear is equal to the shear force indicated by the proving ring divided by the area of the specimen. A plot can be made between the shear stress and the shear strain. The shear strain is equal to the shear displacement (ΔH) divided by the length of the specimen (L). The shear stress is obtained from the shear load indicated by the proving ring and the cross-sectional area.

Direct shear tests can be conducted for any one of the three drainage conditions. For *U-U* test, plain grids are used and the sample is sheared rapidly. For *CU* test, perforated grids are used. The sample is consolidated under the normal load and after the completion of consolidation, it is sheared rapidly in about 5—10 minutes. In a *CD* test, the sample is consolidated under the normal load and then sheared slowly so that excess pore water pressure is dissipated. A *CD* test may take a few hours for cohesionless soils. For cohesive soils, it may take 2 to 5 days.

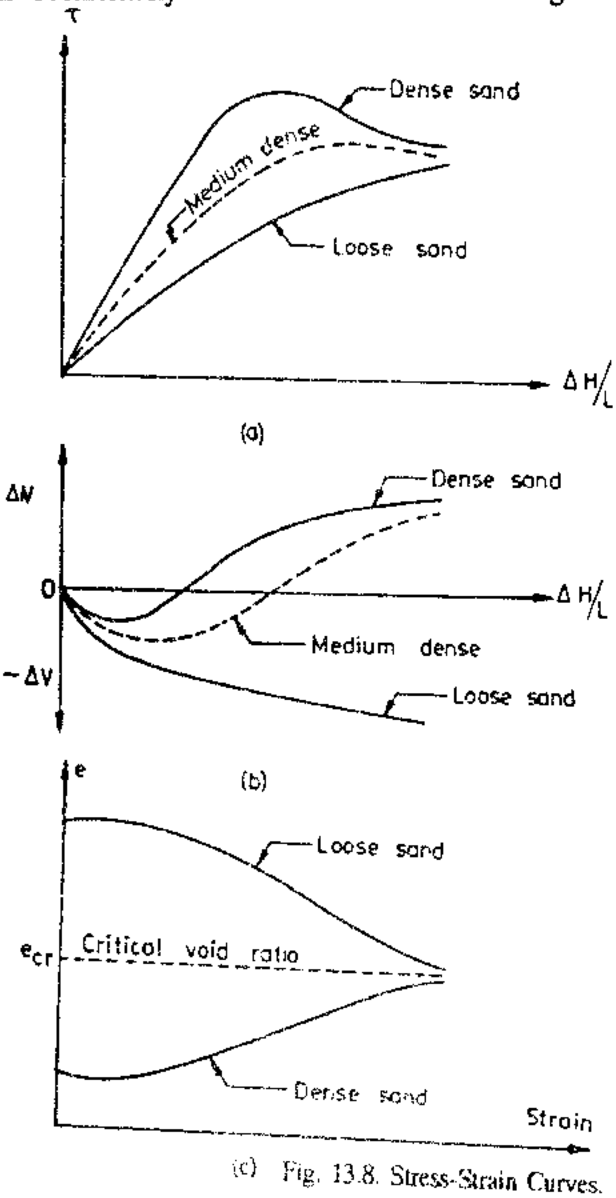
The direct shear test is generally conducted on cohesionless soils as *CD* test. It is convenient to perform and it gives good results for the strength parameters. It is occasionally used to determine the strength parameters of silt and clay under unconsolidated—undrained and consolidated drained conditions, but it does not offer the flexibility of a triaxial compression test, as explained later.

13.12. PRESENTATION OF RESULTS OF DIRECT SHEAR TEST

(a) Stress-Strain Curve. A stress-strain curve is a plot between the shear stress τ and the shear displacement ($\Delta H/L$) [Fig. 13.8 (*a*)]. In case of dense sand (and also over-consolidated clays), the shear stress attains a *peak value* at a small strain. With further increase in strain, the shear stress decreases slightly and becomes more or less constant, known as ultimate stress. In case of loose sands (and normally consolidated clays), the shear stress increases gradually and finally attains a constant value, known as the *ultimate stress* or residual strength. It has been observed that the ultimate shear stress attained by both dense and loose sands tested under similar conditions is approximately the same. The figure also shows the stress-strain curve of a medium dense sand.

Generally, the failure strain is 2 to 4% for dense sand and 12 to 16% for loose sand.

Fig. 13.8 (*b*) shows the volume changes with an increase in shear strain for *CD* tests. Since the cross-sectional area of the specimen remains unchanged, the volume change is proportional to the change in thickness measured by the dial gauge. In case of dense sands (and over-consolidated clays), the volume first decreases slightly,



(c) Fig. 13.8. Stress-Strain Curves.

but it increases with further increase in strain. In the case of loose sands (and normally consolidated clays), the volume decreases with an increase in shear strain. The figure also shows the curve for medium dense sand.

It may be observed that the void ratio of an initial loose sand decreases with an increase in shear strain, whereas that for the initially dense sand increases with an increase in strain [Fig. 13.8 (c)]. The void ratio at which there is no change in it with an increase in strain is known as the *critical void ratio.* If the sand initially is at the critical void ratio, there would be practically no change in volume with an increase in shear strain.

(*b*) **Failure Envelope.** For obtaining a failure envelope, a number of identical specimens are tested under different normal stresses. The shear stress required to cause failure is determined for each normal stress. The failure envelope is obtained by plotting the points corresponding to shear strength at different normal stresses and joining them by a straight line [Fig. 13.9 (*a*)]. The inclination of the failure envelope to

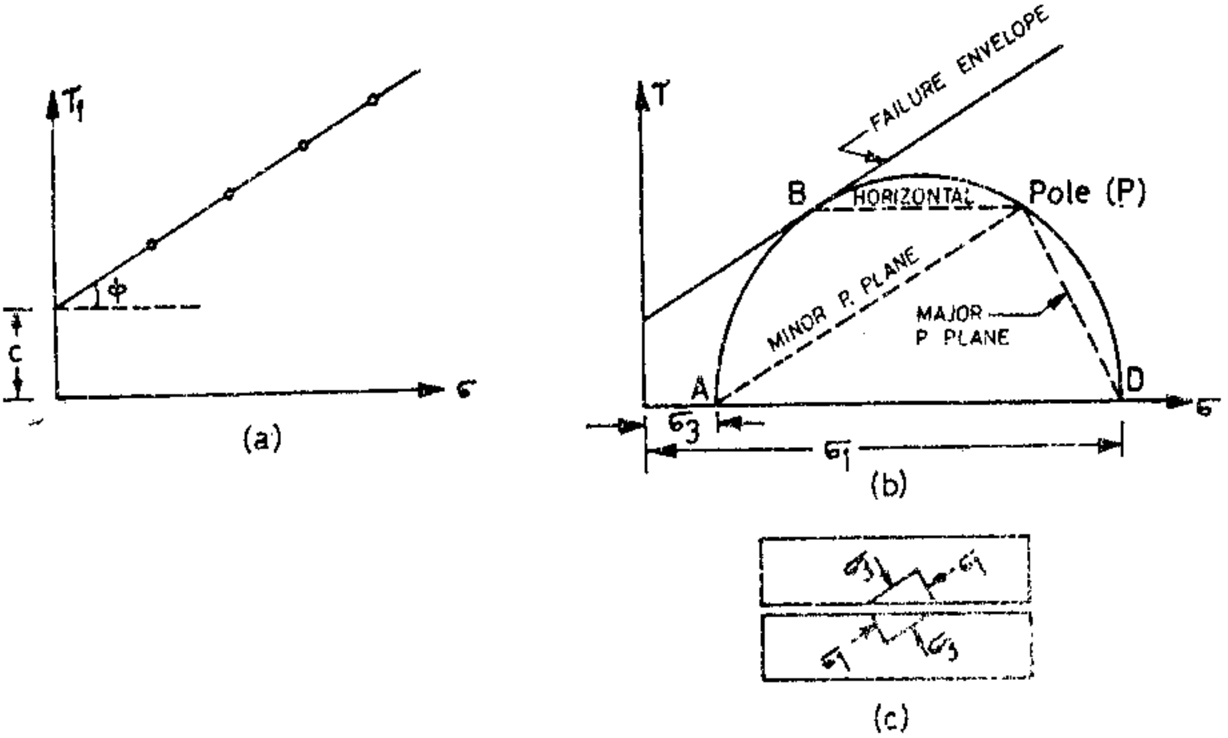


Fig. 13.9. Failure Envelope.

the horizontal gives the angle of shearing resistance ϕ and its intercept on the vertical axis is equal to the cohesion intercept c.

For dense sands, the failure envelope can be drawn either for peak stress or for ultimate stress. The values of the parameters ϕ and c for the two envelopes will be different. For loose sands, the failure envelope is drawn for ultimate stress, which is usually taken as the shear stress at 20% shear strain.

(c) **Mohr-Circle.** In a direct shear test, the stresses on planes other than the horizontal plane are not known. It is, therefore, not possible to draw Mohr stress circle at different shear loads. However, the Mohr circle can be drawn at the failure condition assuming that the failure plane is horizontal.

In Fig. 13.9 (*b*), the point *B* represents the failure condition for a particular normal stress. The Mohr circle at failure is drawn such that it is tangential to the failure envelope at *B*. The horizontal line *BP* gives the direction of the failure plane. The point *P* is the pole. The lines *PD* and *PA* give the directions of the major and minor principal planes, respectively. The principal planes are also shown in Fig. 13.9 (*c*).

Merits and Demerits of Direct Shear Test

The direct shear test has the following merits and demerits as compared to the triaxial compression test (described in the following section).

Merits.

(1) The sample preparation is easy. The test is simple and convenient.

(2) As the thickness of the sample is relatively small, the drainage is quick and the pore pressure dissipates very rapidly. Consequently, the consolidated-drained and the consolidated- undrained tests take relatively small period.

(3) It is ideally suited for conducting drained tests on cohesionless soils.

(4) The apparatus is relatively cheap.

Demerits.

(1) The stress conditions are known only at failure. The conditions prior to failure are indeterminate and, therefore, the Mohr circle cannot be drawn.

(2) The stress distribution on the failure plane (horizontal plane) is not uniform. The stresses are more at the edges and lead to the progressive failure, like tearing of a paper. Consequently, the full strength of the soil is not mobilised simultaneously on the entire failure plane.

(3) The area under shear gradually decreases as the test progresses. But the corrected area cannot be determined and, therefore, the original area is taken for the computation of stresses.

(4) The orientation of the failure plane is fixed. This plane may not be the weakest plane.

(5) Control on the drainage conditions is very difficult. Consequently, only drained tests can be conducted on highly permeable soils.

(6) The measurement of pore water pressure is not possible.

(7) The side walls of the shear box cause lateral restraint on the specimen and do not allow it to deform laterally.

13.13. DIFFERENT TYPES OF SOILS

On the basis of shear strength, soils can be divided into three types.

(1) Cohesionless soils. (2) Purely cohesive soils and

(3) Cohesive-frictional soils.

1. Cohesionless soils. These are the soils which do not have cohesion *i.e.*, $c' = 0$. These soils derive the shear strength from the intergranular friction. These soils are also called *frictional soils*. For example, sands and gravels.

2. Purely cohesive soils. These are the soils which exhibit cohesion but the angle of shearing resistance $\phi = 0$. For example, saturated clays and silts under undrained conditions. These soils are also called $\phi_u = 0$ soils.

3. Cohesive-frictional soils. These are composite soils having both c' and ϕ'. These are also called c—ϕ soils. For example, clayey sand, silty sand, sandy clay, etc.

[**Note.** Sometimes, cohesive-frictional soils are also called cohesive soils. Thus any soil having a value of c' is called a cohesive soil.]

13.14. TRIAXIAL COMPRESSION TEST APPARATUS

The triaxial compression test, or simply triaxial test, is used for the determination of shear characteristics of all types of soils under different drainage conditions. In this test, a cylindrical specimen is stressed under conditions of axial symmetry, as shown in Fg. 13.10. In the first stage of the test, the specimen is subjected to an all round confining pressure (σ_c) on the sides and at the top and the bottom. This stage is known as the consolidation stage.

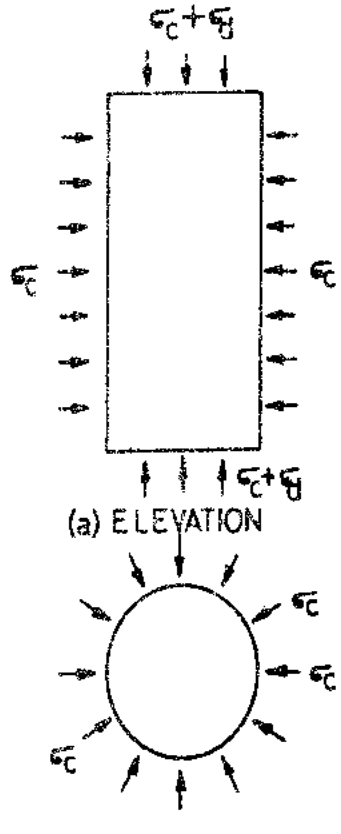


Fig. 13.10.

In the second stage of the test, called the shearing stage, an additional axial stress, known as the deviator stress (σ_d), is applied on the top of the specimen through a ram. Thus, the total stress in the axial direction at the time of shearing is equal to $(\sigma_c + \sigma_d)$. It may be noted that when the axial stress is increased, the shear stresses develop on inclined planes due to compressive stresses on the top.

The vertical sides of the specimen are principal planes, as there are no shear stresses on the sides. The confining pressure σ_c is equal to the minor principal stress (σ_3). The top and bottom planes are the major principal planes. The total axial stress which is equal to the sum of the confining pressure and the deviator stress, is the major principal stress (σ_1). Because of axial symmetry, the intermediate principal stress (σ_2) is also equal to the confining pressure (σ_c).

[**Note.** The above interpretation of the stress conditions in the triaxial test is not strictly correct according to the theory of elasticity. In the case of cylindrical specimens, the three principal stresses are the axial, radial and the circumferential stresses. The state of stress is statically indeterminate throughout the specimen. For convenience, in the triaxial test, the circumferential stress is taken equal to the radial stress and the principal stresses σ_2 and σ_3 are assumed to be equal].

The main features of a triaxial test apparatus are shown in Fig. 13.11. It consists of a circular base that has a central pedestal. The pedestal has one or two holes which are used for the drainage of the specimen in a drained test or for the pore pressure measurement in an undrained test. A triaxial cell is fitted to the top of the base plate with the help of 3 wing nuts (not shown in the figure) after the specimen has been placed on the pedestal. The triaxial cell is a perspex cylinder which is permanently fixed to the top cap and the bottom brass collar. There are three tie rods which support the cell. The top cap is a bronze casting with its central boss forming a bush through which a stainless steel ram can slide. The ram is so designed that it has minimum of friction and at the same time does not permit any leakage. There is an air-release valve in the top cap which is kept open when the cell is filled with water (or glycerine) for applying the confining pressure. An oil valve is also provided in the top cap to fill light machine oil in the cell to reduce the leakage of water past the ram in long duration tests. The apparatus is mounted on a loading frame. The deviator stress is applied to the specimen from a strain-controlled loading machine. The loading system consists of either a screw jack operated by an electric motor and gear box or a hydraulic ram operated by a pump.

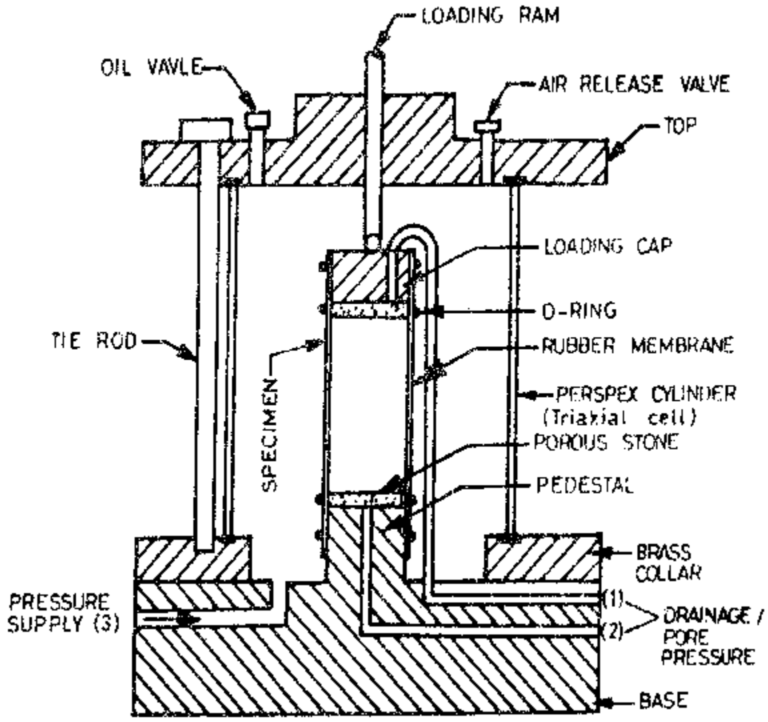


Fig. 13.11. Triaxial Tes Apparatus.

The triaxial test apparatus has the following special attachments.

1. Mercury Control System. The cell pressure is a triaxial test in maintained constant with a self-compensating mercury control system, developed by Bishop and Henkel. It consists of two limbs of a water-mercury manometer (Fig. 13.12). The pressure in the water of the triaxial cell develops due to the difference in levels of the mercury in the two pots. The water pressure at the centre of the specimen in the triaxial cell, at a height of h_3 above the datum, can be calculated using the theory of manometers. As the mercury surface in the upper pot is open to atmosphere, the (gauge) pressure there is zero. From the manometer equation,

$$0 + \gamma_m h_1 - \gamma_m h_2 - (h_3 - h_2)\, \gamma_w = \sigma_c$$

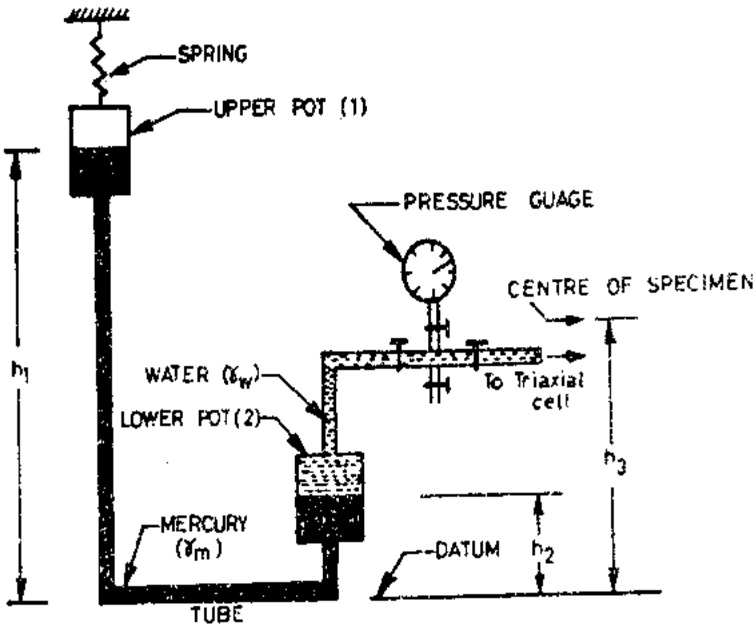


Fig. 13.12. Mercury Control System.

where σ_c is the cell pressure at the centre of the specimen,

γ_w is the unit weight of water, and

γ_m is the unit weight of mercury.

The above equation can be simplified as

$$\sigma_c = \gamma_m (h_1 - h_2) + (h_2 - h_3)\gamma_w \qquad ...(13.14)$$

The upper pot is supported by a spring. When the volume of the specimen decreases due to consolidation or when the water leaks past the ram, water flows from the lower pot to the cell and the mercury level in the lower pot rises by a small amount Δh. The mercury level in the upper pot would also fall by the same amount if the two pots are of the same cross-sectional area. However, the difference of mercury levels in the two pots is maintained constant by the spring. The stiffness (k) of the spring is selected such that it reduces in length and causes a rise of the upper pot as soon as its weight decreases due to flow of mercury. The stiffness of the spring is given by

$$k = A\ \gamma_m \left[\frac{1}{2 - (\gamma_w/\gamma_m)}\right] - W \qquad ...(13.15)$$

where A = cross-sectional area of the mercury pot,

and W = weight of unit length of the tube filled with mercury which is also lifted above the floor.

2. Pore water Pressure Measurement Device. The pore water pressure in the triaxial specimen is measured by attaching it to the device shown in Fig. 13.13. It consists of a null indicator in which no- flow condition is maintained. For accurate measurement, *no- flow condition is essential because* the flow of water from the sample to the gauge would modify the actual magnitude of the pore water pressure. Further, the flow of water leads to a time lag in the attainment of a steady state in samples of cohesive soils because of low permeability.

The null indicator is essentially a U-tube partly filled with mercury. One limb of the null indicator is connected to the specimen in the triaxial cell and the other limb is connected to a pressure gauge. A control cylinder, which is filled with water, is attached to the system. The water can be displaced by a screw-controlled piston in the control cylinder. The whole system is filled with deaired water. The tubes connecting the specimen and the null-indicator should be such that these undergo negligible volume changes under pressure and are free from leakage.

Any change in the pore-water pressure in the specimen tends to cause a movement of the mercury level in the null-indicator. However, the no-flow condition is maintained by making a corresponding change in the

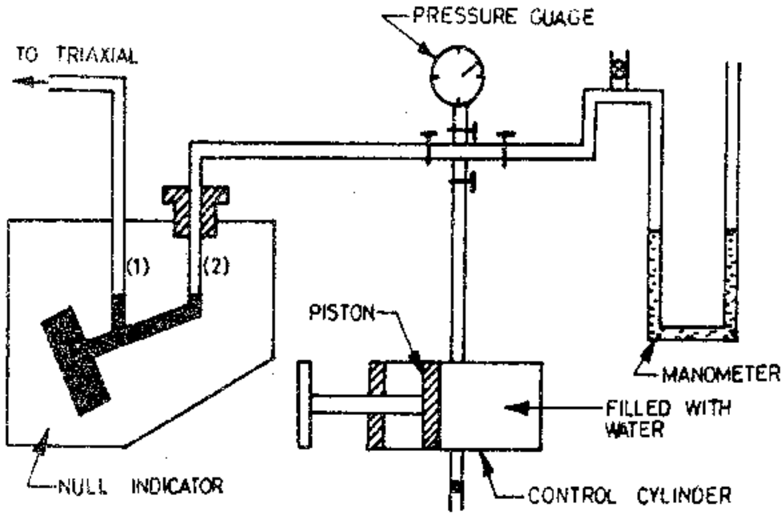


Fig. 13.13. Pore Water Pressure Measurement Device.

other limbs by means of the control cylinder. Thus the mercury levels in the two limbs remain constant. The pressure applied by the control cylinder is recorded by pressure gauge or the manometer.

If the specimen is partially saturated, a special fine, porous ceramic disc is placed below the sample in the triaxial cell. The ceramic disc permits only pore water to flow, provided the difference between the pore air pressure and pore water pressure is below a certain value, known as *the air-entry value* of the ceramic disc. Under undrained conditions, the ceramic disc will remain fully saturated, provided the air-entry value is high. It may be mentioned that if the required ceramic disc is not used and instead the usual coarse, porous disc is used, the device would measure air pressure and not water pressure in a partially saturated soil.

In modern equipment, sometimes the pore water pressure is measured by means of a transducer and not by conventional null indicator.

3. Volume Changes Measurement. Volume changes in a drained test and during consolidation stage of a consolidated undrained test are measured by means of a burette connected to the specimen in the triaxial cell. For accurate measurements, the water level in the burette should be approximately at the level of the centre of the specimen (Fig. 13.14).

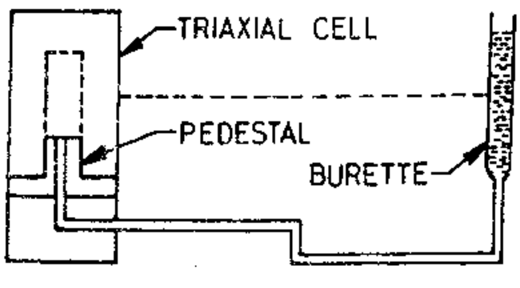


Fig. 13.14. Volume Change Measurement.

During consolidation stage, the volume of the specimen decreases and the water level in the burette rises. The change in the volume of the specimen is equal to the volume of the water increased in the burette. During shearing of specimens of dense sand when the volume of the sample increases, the water flows from the burette to the specimen. The increase in volume of the specimen is equal to the volume of water decreased in the burette.

13.15. TRIAXIAL TESTS ON COHESIVE SOILS

The following procedure is used for conducting the triaxial tests an cohesive soils.

(a) **Consolidated-undrained test.** A deaired, coarse porous disc or stone is placed on the top of the pedestal in the triaxial test apparatus. A filter paper disc is kept over the porous stone. The specimen of the cohesive soil is then placed over the filter paper disc. The usual size of the specimen is about 37.5 mm diameter and 75.0 mm height. A porous stone is also placed on the top of the specimen. Deaired vertical filter strips are placed at regular spacing around the entire periphery such that these touch both the porous stones. The sample in then enclosed in a rubber membrane, which is slid over the specimen with the help of a membrane stretcher. The membrane is sealed to the specimen with O-rings.

The triaxial cell is placed over the base and fixed to it by tightening the nuts. The cell is then filled with water by connecting it to the pressure supply. Some space in the top portion of the cell is filled by injecting oil through the oil valve. When excess oil begins to spill out through the air-vent valve, both the valves (oil valve and air-vent valve) are closed. Pressure is applied to the water filled in the cell by connecting it to the mercury-pot system. As soon as the pressure acts on the specimen, it starts consolidating. The specimen is connected to the burette through pressure connections for measurement of volume changes. The consolidation is complete when there is no more volume change.

When the consolidation is complete, the specimen is ready for being sheared. The drainage valve is closed. The pore water pressure measurement device is attached to the specimen through the pressure connections. The proving ring dial gauge is set to zero. Using the manual control provided in the loading frame, the ram is pushed into the cell but not allowed to touch the loading cap. The loading machine is then run at the selected speed. The proving ring records the force due to friction and the upward thrust acting on the ram. The machine is stopped, and with the manual control, the ram is pushed further into the cell bringing it in contact with the loading cap. The dial gauge for the measuring axial deformation of the specimen is set to zero.

The sample is sheared by applying the deviator stress by the loading machine. The proving ring readings are generally taken corresponding to axial strains of 1/3%, 2/3%, 1%, 2%, 3%, 4%, 5%, ...until failure or 20% axial strain.

Upon completion of the test, the loading is shut off. Using the manual control, all additional axial stress is removed. The cell pressure is then reduced to zero, and the cell is emptied. The triaxial cell is unscrewed and removed from the base. O-rings are taken out, and the membrane is removed. The specimen is then recovered after removing the loading cap and the top porous stone. The filter paper strips are peeled off. The post- shear mass and length are determined. The water content of the specimen is also found.

(*b*) **Unconsolidated Undrained test.** The procedure is similar to that for a consolidated-undrained test, with one basic difference that the specimen is not allowed to consolidate in the first stage. The drainage valve during the test is kept closed. However, the specimen can be connected to the pore-water pressure measurement device if required.

Shearing of the specimen is started just after the application of the cell pressure. The second stage is exactly the same as in the consolidated-undrained test described above.

(*c*) **Consolidated Drained test.** The procedure is similar to that for a consolidated-undrained test, with one basic difference that the specimen is sheared slowly in the second stage. After the consolidation of the specimen in the first stage, the drainage valve is not closed. It remains connected to the burette throughout the test. The volume changes during the shearing stage are measured with the help of the burette. As the permeability of cohesive soils is very low, it takes 4-5 days for the consolidated drained test.

13.16. TRIAXIAL TESTS ON COHESIONLESS SOILS

Triaxial tests on specimens of cohesionless soils can be conducted using the procedure as described for cohesive soils. As the samples of cohesionless soils cannot stand of their own, a special procedure is used for preparation of the sample as described below.

A metal former, which is a split mould of about 38.5 mm internal diameter, is used for the preparation of the sample (Fig. 13.15). A coarse porous stone is placed on the top of the pedestal of the triaxial base, and the pressure connection is attached to a burette (not shown). One end of a membrane is sealed to the pedestal by O-rings. The metal former is clamped to the base. The upper metal ring of the former is kept inside the top end of the rubber membrane and is held with the help of a clamp before placing the funnel and the rubber bung in position as shown in figure.

The membrane and the funnel are filled with deaired water. The cohesionless soil which is to be tested is saturated by mixing it with enough water in a beaker. The mixture is boiled to remove the entrapped air. The saturated soil is deposited in the funnel, with a stopper in position, in the required quantity. The glass rod is then removed and the sample builds up by a continuous rapid flow of saturated soil in the former. The funnel is then removed. The sample may be compacted if required. The surface of the sample is leveled and a porous stone is placed on its top. The loading cap is placed gently on the top porous stone. O-rings are fixed over the top of the rubber membrane.

A small negative pressure is applied to the sample by lowering the burette. The negative pressure gives rigidity to the sample and it can stand without any lateral support. For sample of 37.5 mm diameter, a negative pressure of 20 cm of water (or 2 kN/m^2) is sufficient. As soon as the negative pressure is applied, the consolidation of the sample occurs and it slightly shortens. The diameter of the upper porous stone should be slightly smaller than that of the specimen so that it can go inside when the sample shortens; otherwise, a neck is formed.

The split mould is then removed, and the diameter and the height of the sample are measured. The thickness of the membrane is deducted from the total diameter to get the net diameter of the sample. The cell is then placed over the base and clamped to the base. It is then filled with water.

The rest of the procedure is the same as for cohesive soils.

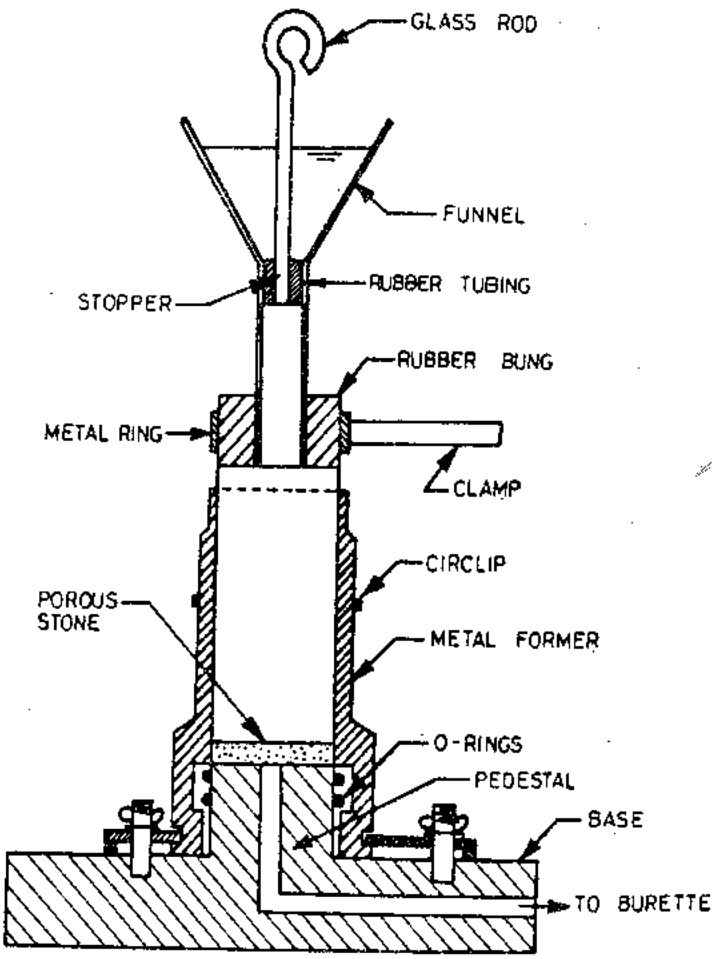


Fig. 13.15. Preparation of Sample of Cohesionless Soil.

13.17. MERITS AND DEMERITS OF TRIAXIAL TEST

The triaxial test has the following merits and demerits.

Merits.

(1) There is complete control over the drainage conditions. Tests can be easily conducted for all three types of drainage conditions.

(2) Pore pressure changes and the volumetric changes can be measured directly.

(3) The stress distribution on the failure plane is uniform.

(4) The specimen is free to fail on the weakest plane.

(5) The state of stress at all intermediate stages upto failure is known. The Mohr circle can be drawn at any stage of shear.

(6) The test is suitable for accurate research work. The apparatus is adaptable to special requirements such as extension test and tests for different stress paths.

Demerits.

(1) The apparatus is elaborate, costly and bulky.

(2) The drained test takes a longer period as compared with that in a direct shear test.

(3) The strain condition in the specimen are not uniform due to frictional restraint produced by the loading cap and the pedestal disc. This leads to the formation of the dead zones at each end of the specimen.

The non-uniform distribution of stresses can be largely eliminated by lubrication of end surfaces. However, non-uniform distribution of stresses has practically no effect on the measured strength if length/diameter ratio is equal to or more than 2.0.

(4) It is not possible to determine the cross-sectional area of the specimen accurately at large strains, as the assumption that the specimen remains cylindrical does not hold good.

(5) The test simulates only axis-symmetrical problems. In the field, the problem is generally 3-dimensional. A general test in which all the three stresses are varied would be more useful.

(6) The consolidation of the specimen in the test is isotropic; whereas in the field, the consolidation is generally anisotropic.

Despite the above-mentioned demerits, the triaxial test is extremely useful. It is the only reliable test for accurate determination of the shear characteristics of all types of soils and under all the drainage conditions.

13.18. COMPUTATION OF VARIOUS PARAMETERS

(a) Post-Consolidation Dimensions. In consolidated-drained and consolidated-undrained tests, the consolidation of the specimen takes place during the first stage. As the volume of the specimen decreases, its post-consolidation dimensions are different from the initial dimensions. The post consolidation dimensions can be determined approximately assuming that the sample remains cylindrical and it behaves isotropically. Let L_i, D_i, and V_i be the length, diameter and the volume of the specimen before consolidation. Let L_0, D_0 and V_0 be the corresponding quantities after consolidation.

Therefore, volumetric change, $\Delta V_i = V_i - V_0$

The volumetric change (ΔV_i) is measured with the help of burette.

Volumetric strain, $$\varepsilon_v = \frac{\Delta V_i}{V_i}$$

For isotropic consolidation, the volumetric strain is three times the linear strain (ε_l). Thus

$$\varepsilon_l = \varepsilon_v/3$$

Thus $$L_0 = L_i - \Delta L_i = L_i - L_i \times \varepsilon_l$$

or $$L_0 = L_i(1 - \varepsilon_l) = L_i\,(1 - \varepsilon_v/3) \qquad \ldots(13.16)$$

Likewise, $$D_0 = D_i(1 - \varepsilon_v/3)$$

The post consolidation diameter D_0 can also be computed after L_0 has been determined from the relation,

$$(\pi/4 \cdot D_0^2) \times L_0 = V_0$$

or $$D_0 = \sqrt{\frac{V_0}{(\pi/4) \times L_0}} \qquad \ldots(13.17)$$

(b) Cross-sectional Area During Shear Stage. As the sample is sheared, its length decreases and the diameter increases. The cross-sectional area A at any stage during shear can be determined assuming that the sample remains cylindrical in shape. Let ΔL_0 be the change in length and ΔV_0 be the change in volume. The volume of the specimen at any stage is given by $V_0 \pm \Delta V_0$.

Therefore, $$A\,(L_0 - \Delta L_0) = V_0 \pm \Delta V_0$$

or $$A = \frac{V_0 \pm \Delta V_0}{L_0 - \Delta L_0} = \frac{V_0\left(1 \pm \dfrac{\Delta V_0}{V_0}\right)}{L_0\left(1 - \dfrac{\Delta L_0}{L_0}\right)} \qquad \ldots(13.18)$$

Eq. 13.18 is the general equation which gives the cross-sectional area of the specimen.

The above equation can be written as

(*PI*) of the soil remains constant (Fig. 13.26). An approximate value of the undrained shear strength of a normally consolidated deposit can be obtained from Fig. 13.26, if the plasticity index has been determined. The relationship is expressed as (Skempton, 1957).

$$\frac{c_u}{\bar{\sigma}} = 0.11 + 0.0037\ PI$$

where c_u = undrained cohesion intercept,

$\bar{\sigma}$ = effective over-burden pressure

PI = plasticity index (%)

The value of the ratio $(c_u / \bar{\sigma})$ determined in a consolidated-undrained test on undisturbed samples is generally greater than actual value because of anisotropic consolidation in the field. The actual value is best determined by in-situ shear vane test, as explained later.

13.22. UNCONFINED COMPRESSION TEST

The unconfined compression test is a special form of a triaxial test in which the confining pressure is zero. The test can be conducted only on clayey soils which can stand without confinement. The test is generally performed on intact (non- fissured), saturated clay specimens. Although the test can be conducted in a triaxial test apparatus as a *U—U* test, it is more convenient to perform it in an unconfined compression testing machine. There are two types of machines, as described below.

(1) Machine with a spring. Fig. 13.27 shows the unconfined compression testing machine in which a loaded spring is used. It consists of two metal cones which are fixed on horizontal loading plates *B* and *C* supported on the vertical posts *D*. The upper loading plate *B* is fixed in position, whereas the lower plate *C* can slide on the vertical posts The soil specimen is placed between the two metal cones.

When the handle is turned, the plate *A* is lifted upward. As the plate *A* is attached to the plate *C*, the latter plate is also lifted. When the handle is turned slowly, at a speed of about half a turn per second, a compressive force acts on the specimen. Eventually, the specimen fails in shear. The compressive load is proportional to the extension of the spring.

The strain in the specimen is indicated on a chart fixed to the machine. As the lower plate *C* moves upward, the pen attached to this plate swings sideways. The lateral movement of the pen (in arc) is proportional to the strain in the specimen.

The chart plate is attached to the yoke *Y*. As the yoke moves upward when the handle is rotated, the chart plate moves upward. The pivot of the arm of the pen also moves upward with the lower plate. The vertical movement of the pen relative to the chart is equal to the extension of the spring and hence the compressive force. Thus the chart gives a plot between the deformation and the compressive force. Springs of different stiffnesses can be used depending upon the expected compressive strength of the specimen.

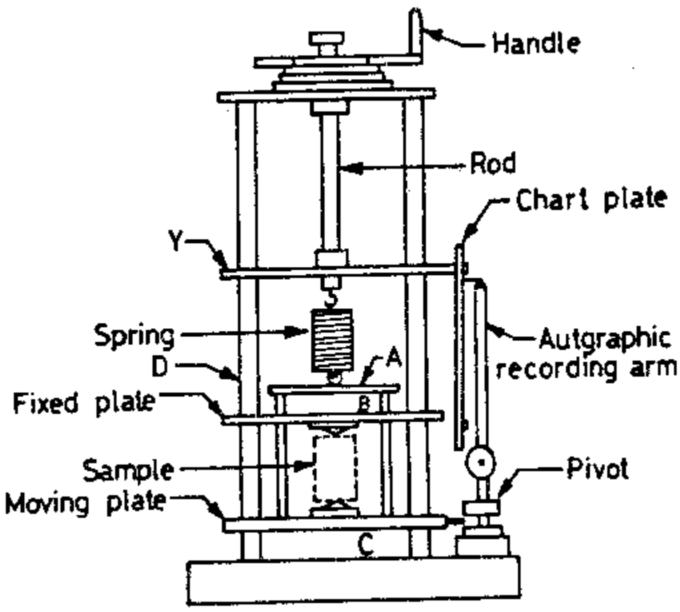


Fig. 13.27. Unconfined Compression Testing Machine (Spring Type).

(2) Machine with a Proving Ring. In this type of the unconfined compression testing machine, a proving ring is used to measure the compressive force (Fig. 13.28). There are two plates, having cone seatings for the specimen. The specimen is placed on the bottom plate so that it makes contact with the upper plate. The dial gauge and proving ring are set to zero.

The compressive load is applied to the specimen by turning the handle. As the handle is turned, the upper plate moves downward and causes compression. (In some machines, the upper plate is fixed and the compressive load is applied by raising the lower plate). The handle is turned gradually so as to produce an axial strain of 1/2% to 2% per minute. The shearing is continued till the specimen fails or till 20% of the axial strain occurs, whichever is earlier.

The compressive force is determined from the proving ring reading, and the axial strain is found from the dial gauge reading.

Presentation of Results. In an unconfined compression test, the minor principal stress (σ_3) is zero. The major principal stress (σ_1) is equal to the deviator stress, and is found from Eq. 13.21.

$$\sigma_1 = P/A$$

where P = axial load,

and A = area of cross-section.

The axial stress at which the specimen fails is known as the unconfined compressive strength (q_u). The stress- strain curve can be plotted between the axial stress and the axial strain at different stages before failure.

While calculating the axial stress, the area of cross-section of the specimen at that axial strain should be used. The corrected area can be obtained from Eq. 13.20 as

$$A = A_0/(1-\varepsilon)$$

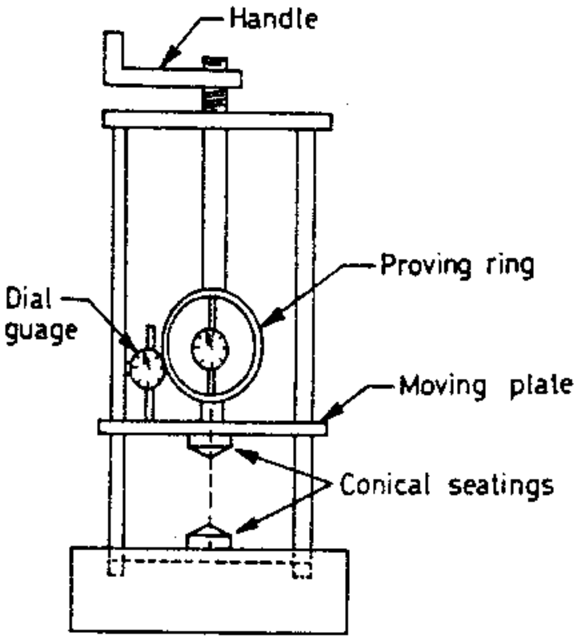


Fig. 13.28. Unconfined Compression Testing Machine (Proving Ring Type).

The Mohr circle can be drawn for stress conditions at failure. As the minor principal stress is zero, the Mohr circle passes through the origin (Fig. 13.29). The failure envelope is horizontal ($\phi_u = 0$). The cohesion intercept is equal to the radius of the circle, *i.e.*

$$s = c_u = \frac{\sigma_1}{2} = \frac{q_u}{2} \qquad \text{...(13.25)}$$

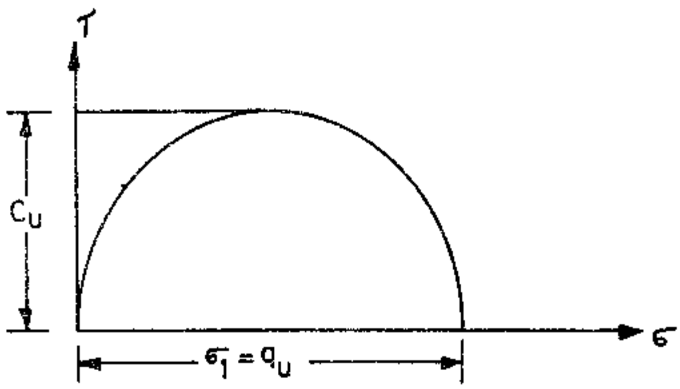


Fig. 13.29. Mohr Circle for Unconfined Compression Test.

Merits and Demerits of the test

Merits

(1) The test is convenient, simple and quick.

(2) It is ideally suited for measuring the unconsolidated-undrained shear strength of intact, saturated clays.

(3) The sensitivity of the soil may be easily determined by conducting the test on an undisturbed sample and then on the remoulded sample.

Demerits

(1) The test cannot be conducted on fissured clays.

(2) The test may be misleading for soils for which the angle of shearing resistance is not zero. For such soils, the shear strength is not equal to half the compressive strength.

(See Chapter 30, Sect. 30.17 for the laboratory experiment).

13.23. VANE SHEAR TEST

The undrained shear strength of soft clays can be determined in a laboratory by a vane shear test. The test can also be conducted in the field on the soil at the bottom of a bore hole. The field test can be performed even without drilling a bore hole by direct penetration of the vane from the ground surface if it is provided with a strong shoe to protect it.

The apparatus consists of a vertical steel rod having four thin stainless steel blades (vanes) fixed at its bottom end. IS : 2720—XXX—1980 recommends that the height H of the vane should be equal to twice the overall diameter D. The diameter and the length of the rod are recommended as 2.5 mm and 60 mm respectively. Fig. 13.30 (*a*) shows a vane shear test apparatus.

For conducting the test in the laboratory, a specimen of the size 38 mm diameter and 75 mm height is taken in a container which is fixed securely to the base. The vane is gradually lowered into the specimen till the top of the vane is at a depth of 10 to 20 mm below the top of the specimen. The readings of the strain indicator and torque indicator are taken.

Torque is applied gradually to the upper end of the rod at the rate of about 6° per minute (*i.e.* 0.1° per second). The torque acting on the specimen is indicated by a pointer fixed to the spring. The torque is continued till the soil fails in shear. The shear strength of the soil is determined using the formula derived below.

Derivation of Formula. In the deviation of the formula, it is assumed that the shear strength (s) of the soil is constant on the cylindrical sheared surface and at the top and bottom faces of the sheared cylinder. The torque applied (T) must be equal to the sum of the resisting torque at the sides (T_1) and that at the top and bottom (T_2). Thus,

$$T = T_1 + T_2 \qquad \ldots(a)$$

The resisting torque on the sides is equal to the resisting force developed on the cylindrical surface multiplied by the radial distance. Thus,

$$T_1 = (s\pi DH) \times D/2 \qquad \ldots(b)$$

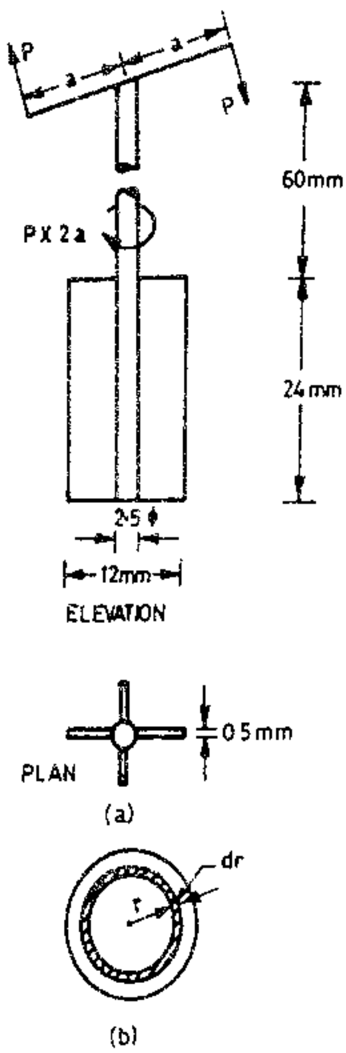


Fig. 13.30. Vane Shear Test.

The resisting torque T_2 due to the resisting forces at the top and bottom of the sheared cylinder can be determined by the integration of the torque developed on a circular ring of radius r and width dr [Fig. 13.30 (*b*)]. Thus.

$$T_2 = 2\int_0^{D/2} [s\,(2\pi r)\,dr]\,r = 4\pi s\left[\frac{r^3}{3}\right]_0^{D/2}$$

or

$$T_2 = \pi s\,\frac{D^3}{6} \qquad \ldots(c)$$

From Eqs. (*a*), (*b*) and (*c*), $\quad T = \pi\, s\,[D^2 H/2 + D^3/6]$

or
$$s = \frac{T}{\pi (D^2 H/2 + D^3/6)} \qquad \text{...(13.27)}$$

For example, if D = 1.2 cm, and H = 2.4 cm, $s = 0.158\ T$

where T is in N-cm and s in N/cm^2.

Eq. 13.27 is modified if the top of the vane is above the soil surface and the depth of the vane inside the sample is H_1. In such a case.

$$s = \frac{T}{\pi (D^2 H_1/2 + D^3/12)} \qquad \text{...(13.28)}$$

The shear strength of the soil under undrained conditions is equal to the apparent cohesion c_u.

The vane shear test can be used to determine the sensitivity of the soil. After the initial test, the vane is rotated rapidly through several revolutions such that the soil becomes remoulded. The test is repeated on the remoulded soils and the shear strength in remoulded state is determined. Thus,

$$\text{Sensitivity } (S_t) = \frac{(s) \text{ undisturbed}}{(s) \text{ remoulded}}$$

Merits and Demerits of Shear Vane Test

Merits.

(1) The test is simple and quick.

(2) It is ideally suited for the determination of the in-situ undrained shear strength of non-fissured, fully saturated clay.

(3) The test can be conveniently used to determine the sensitivity of the soil.

Demerits.

(1) The test cannot be conducted on the fissured clay or the clay containing sand or silt laminations.

(2) The test does not give accurate results when the failure envelope is not horizontal.

13.24. PORE PRESSURE PARAMETERS

A knowledge of the pore water pressure is essential for the determination of effective stresses from the total stresses. The pore water pressure is usually measured in the field by installing piezometers. However, in some cases, it becomes difficult and impractical to install the piezometers and measure the pore water pressure directly in the field. For such cases, a theoretical method for the determination of the pore water pressure is useful. Skempton gave the pore pressure parameters which express the response of pore pressure due to changes in the total stresses under undrained conditions. These parameters are used to predict pore water pressure in the field under similar conditions. The expressions for pore pressure parameters are derived separately for isotropic consolidation, for deviatoric stress and for the combined effect.

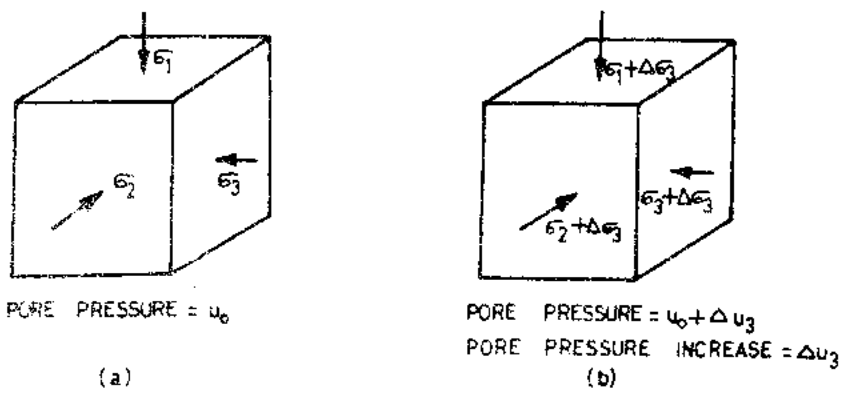


Fig. 13.31. Pore Pressure Under Isotropic Consolidation.

where K is known as Hvorslev coefficient of cohesion. Accordingly, the shear strength can be expressed as

$$s = K\bar{\sigma}_e + \bar{\sigma}\tan\varphi_e \qquad ...(13.52)$$

Bishop and Henkel (1962) suggested a method for determination of c_e and ϕ_e from a series of consolidated-undrained triaxial tests on normally consolidated and over-consolidated specimens. The two failure envelopes are obtained as usual and are shown in Fig. 13.42 (*b*). The water content at failure for the two types of specimens is plotted against the maximum principal stress as shown in Fig. 13.42 (*a*).

For determination of the true failure envelope, any circle (say left circle I) for the over-consolidated clay in Fig. 13.42 (*b*) is chosen. The point corresponding to its maximum stress $(\bar{\sigma}_1)_I$ is projected upward to the $\bar{\sigma}_1 - w_f$ curve in Fig. 13.42 (*a*) to get the point 1 on the curve for over-consolidated clay. The point 1 is projected horizontally across at constant water content to obtain point 2 on the curve for the normally consolidated clay. The point 2 is projected downward to obtain the point $(\bar{\sigma}_1)_{II}$ in Fig. 13.42 (*c*). Through this point, a Mohr circle II is drawn to touch the failure envelope for normally consolidated clay. In Fig. 13.42 (*c*), the left circle I is the same as the circle I in Fig. 13.42 (*b*). The common tangent to the circle I and II in Fig. 13.42 (*c*) is the true failure envelope. The parameters c_e and ϕ_e are obtained from this envelope.

The true failure envelope has been obtained using the concept that two samples can exist at the same water content, one as normally consolidated and one as over-consolidated. As the water contents at points 1 and 2 are equal, the true cohesion is the same and the difference between the shear strength of the two samples is due to the internal friction only.

The fundamental properties of soils can be studied in terms of Hvorslev shear strength parameter. However, the theory is generally used only for research purposes. For practical use in engineering problems, the Mohr-Coulomb theory is commonly used.

13.30. LIQUEFACTION OF SANDS

As discussed earlier, the shear strength of sandy soils is given by the Mohr-Coulomb equation (Eq. 13.13), taking the cohesion intercept as zero.

Thus
$$s = \bar{\sigma}\tan\phi' \qquad ....(13.53)$$

If the sand deposit is at a depth of z below the ground and the water table is at the ground surface, the effective stress is given by (see Chapter 10),

$$\bar{\sigma} = \gamma_{sat}\, z - \gamma_w z = \gamma' z$$

Therefore,
$$s = \gamma' z \tan\phi'$$

If the sand deposit is shaken due to an earth-quake or any other oscillatory load, extra pore water pressure (u') develops, and the strength equation becomes

$$s = (\gamma' z - u')\tan\phi'$$

It can also be expressed in the term of extra pore pressure head h, where $u' = \gamma_w h$. Thus

$$s = (\gamma' z - \gamma_w h)\tan\phi' \qquad ...(13.54)$$

As indicated by Eq. 13.54, the shear strength of sand decreases as the pore water increases. Ultimately, a stage is reached when the soil loses all its strength. In which case,

$$\gamma' z - \gamma_w h = 0$$

or
$$\frac{h}{z} = \frac{\gamma'}{\gamma_w}$$

Expressing h/z as critical gradient,

$$i_{cr} = \frac{(G-1)\gamma_w}{1+e}\cdot\frac{1}{\gamma_w}$$

or
$$i_{cr} = \frac{G-1}{1+e} \qquad ...(13.55)$$

The phenomenon when the sand loses its shear strength due to oscillatory motion is known as *liquefaction of sand*. The structures resting on such soils sink. In the case of partial liquefaction, the structure may undergo excessive settlement and the complete failure may not occur.

The soils most susceptible to liquefaction are the saturated, fine and medium sands of uniform particle size. When such deposits have a void ratio greater than the critical void ratio and are subjected to a sudden shearing stresses, these decrease in volume and the pore pressure u' increases. The soil momentarily liquefies and behaves as a dense fluid. Extreme care shall be taken while constructing structures on such soils. If the deposits are compacted to a void ratio smaller than the critical void ratio, the chances of liquefaction are reduced. (See Chapter 32 for more details on liquefaction of sand.)

13.31. SHEAR CHARACTERISTICS OF COHESIONLESS SOILS

The shear characteristics of cohesionless soils can be summarized as given below.

The shear strength of cohesionless soils, such as sands and non-plastic silts, is mainly due to friction between particles. In dense sands, interlocking between particles also contributes significantly to the strength.

The stress-strain curve for dense sands exhibits a relatively high initial tangent modulus. The stress reaches a maximum value at its peak at a comparatively low strain and then decreases rapidly with an increasing strain and eventually becomes more or less constant, as discussed earlier. The stress-strain curve for loose sands exhibits a relatively low initial tangent modulus. At large strains, the stress becomes more or less constant.

The dense sand shows initially a volume decrease in a drained test, but as the strain increases, the volume starts increasing. The loose sand shows a volume decrease throughout.

In the case of loose sand, the specimen bulges and ultimately fails by sliding simultaneously on numerous planes. The failure is known as the *plastic failure* [Fig. 13.43 (*a*)]. In the case of dense sand, the specimen shows a clear failure plane and the failure is known as the *brittle failure* [Fig. 13.43 (*b*)].

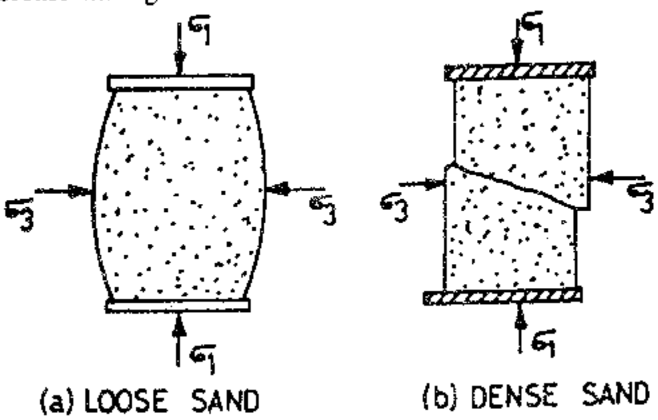


Fig. 13.43. Types of Failure.

The failure envelope for dense sand can be drawn either for the peak stresses or for the ultimate stresses. The value of the angle of shearing resistance (ϕ') for the failure envelope for peak stresses is considerably greater than that for the ultimate stresses. In the case of loose sands, as the peak stress and the ultimate stress are identical, there is only one failure envelope. The angle of shearing resistance in very loose state is approximately equal to the angle of repose. The angle of repose is the angle at which a heap of dry sand stands without any support. It has been established that air-dry sand gives approximately the same value of ϕ' as the saturated sand. As it is easier to perform tests on dry sand, tests can be performed on dry sand instead of saturated sand.

If the failure envelope is slightly non-linear, a straight line may be drawn for the given pressure range and the angle of shearing resistance is taken as the slope of this line. The cohesion intercept, if any, is usually neglected.

The angle of shearing resistance of sands in the field can be determined indirectly by conducting in-situ tests, such as the standard penetration test (SPT) as explained in chapter 17.

The factors that affect the shear strength of cohesionless soils are summarized below:

(1) **Shape of particles.** The shearing strength of sands with angular particles having sharp edges is greater than that with rounded particles, other parameters being identical.

(2) **Gradation.** A well-graded sand exhibits greater shear strength than a uniform sand.

(3) **Denseness.** The degree of interlocking increases with an increase in density. Consequently, the greater the denseness, the greater the strength. The value of ϕ' is related to the relative density (D_r) as $\phi' = 26° + 0.2\,D_r$. However, the ultimate value of ϕ' is not affected by denseness.

(4) Confining pressure. The shear strength increases with an increase in confining pressure. However, for the range of pressures in the common field problems, the effect of confining pressure on the angle of shearing resistance is not significant.

(5) Deviator stress. The angle ϕ' decreases under very high stresses. As the maximum deviator stress is increased from 500 to 5000 kN/m^2, the value of ϕ' decreases by about 10%. This is due to the crushing of particles.

(6) Intermediate principal stress. The intermediate principal stress affects the shear strength to a small extent. The friction angle for dense sands in the plane strain case is about 2° to 4° greater than that obtained from a standard triaxial test. However, for loose sand, there is practically no difference in the two values.

(7) Loading. The angle of shearing resistance of sand is independent of the rate of loading. The increase in the value of ϕ' from the slowest to the fastest possible rate of loading is only about 1 to 2%.

The angle of shearing resistance in loading is approximately equal to that in unloading.

(8) Vibrations and Repeated loading. Repeated loading can cause significant changes. A stress much smaller than the static failure stress if repeated a large number of times can cause a very large strain and hence the failure.

(9) Type of minerals. If the sand contains mica, it will have a large void ratio and a lower value of ϕ'. However, it makes no difference whether the sand is composed of quartz or feldspar minerals.

(10) Capillary moisture. The sand may have apparent cohesion due to capillary moisture. The apparent cohesion is destroyed as soon as the sand becomes saturated.

A person can easily walk on damp sand near the sea beach because it possesses strength due to capillary moisture. On the same sand in saturated conditions, it becomes difficult to walk as the capillary action is destroyed.

Table 13.2 gives the representative values of ϕ' for different types of cohesionless soils.

Table 13.2. Representative Values of ϕ' for Sands and Silts

S. No.	Soil	ϕ'
1.	Sand, round grains, uniform	27° to 34°
2.	Sand, angular, well-graded	33° to 45°
3.	Sandy gravels	35° to 50°
4.	Silty sand	27° to 34°
5.	Inorganic silt	27° to 35°

Note. Smaller values are for loose conditions and larger values are for dense conditions.

13.32. SHEAR CHARACTERISTICS OF COHESIVE SOILS

The shear characteristics of cohesive soils are summarized below :

The shear characteristics of a cohesive soil depend upon whether a soil is normally consolidated or over-consolidated. The stress- strain curve of an over-consolidated clay is similar to that of a dense sand and that of a normally consolidated clay is identical to that of a loose sand. However, the strain required to reach peak stress are generally greater in clay than in sand. The high strength at the peak point in an over-consolidated clay is due to structural strength; whereas in the dense sand, it is mainly due to interlocking. In over-consolidated clay, strong structural bonds develop between the particles. Loose sands tend to increase in volume at large strains whereas normally consolidated clays show no tendency to expand after a decrease in volume.

The effective stress parameters (c', ϕ') for an overconsolidated clay are determined from the failure envelope,

$$s = c' + \bar{\sigma} \tan \phi'$$

However, for a normally consolidated clay, the failure envelope passes through the origin and hence $c' = 0$.