AREA COMPUTATION

Objects of Land Surveying

- To determine the area of the tract surveyed.
- To determine the quantities of earth work.

The units of measurement of area in English units are sq. ft or acres, while in metric units, the units are sq. metres or hectares.

General Methods of Finding Areas

The following are the general methods of calculating areas:

1. By computations based directly on field measurements:
   These include:

(a)By dividing the area into a number of triangles

(b)By offsets to base line

(c)By latitudes and departures:
   (i)by double meridian distance (D.M.D. method)
   (ii)by double parallel distance (D.P.D. method)

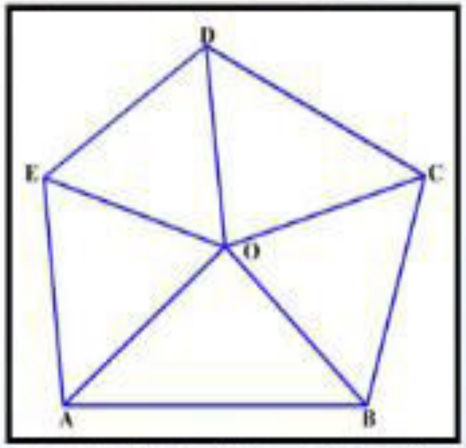
(d)By co-ordinates.

2. By computation based on measurements scaled from a map

3. By mechanical method: usually by means of a planimeter.

Areas Computed by Sub-Division into Triangles

- In this method, the area is divided into a number of triangles and the area of each triangle is calculated.



- The total area of the tract will be equal to the sum of areas of individual triangles.
- Fig.4.1 shows an area divided into several triangles.
- For field work, a transit may be set up at O, and the lengths and directions of each of the lines OA, OB ...etc. may be measured.
- The area of each triangle can be computed.
- In addition, the sides AB, BC,....etc can also be measured and a check may be applied by calculating the area from the three known sides of a triangle.

Areas Computed by Sub-Division into Triangles

- Thus, if two sides and one included angle of a triangle is measured, the area of the triangle is given by

$$\text{Area} = \frac{1}{2} ab \sin C$$

- When the lengths of the three sides of a triangle are measured, its area is computed by the equation

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where s = half perimeter = $\frac{1}{2}(a+b+c)$

- The method is suitable only for work of small nature where the determination of the closing error of the figure is not important and hence the computation of latitudes and departure is unnecessary.

- The accuracy in such cases, may be determined by measuring the diagonal in the field and comparing its length to the computed length.

Areas from Offsets to a Base Line

The areas from offsets to a base line can computed using two methods:

- Offsets at regular intervals,
- Offsets at irregular intervals.

Areas from Offsets to a Base Line

Offsets at regular intervals

- This method is suitable for long narrow strips of land.
- The offsets are measured from the boundary to the base line or a survey line at regular intervals.
- The method can also be applied to a plotted plan from which the offsets to a line can be scaled off.
- The area may be calculated by the following rules:

  (i)mid-ordinate rule

  (ii)Average ordinate rule

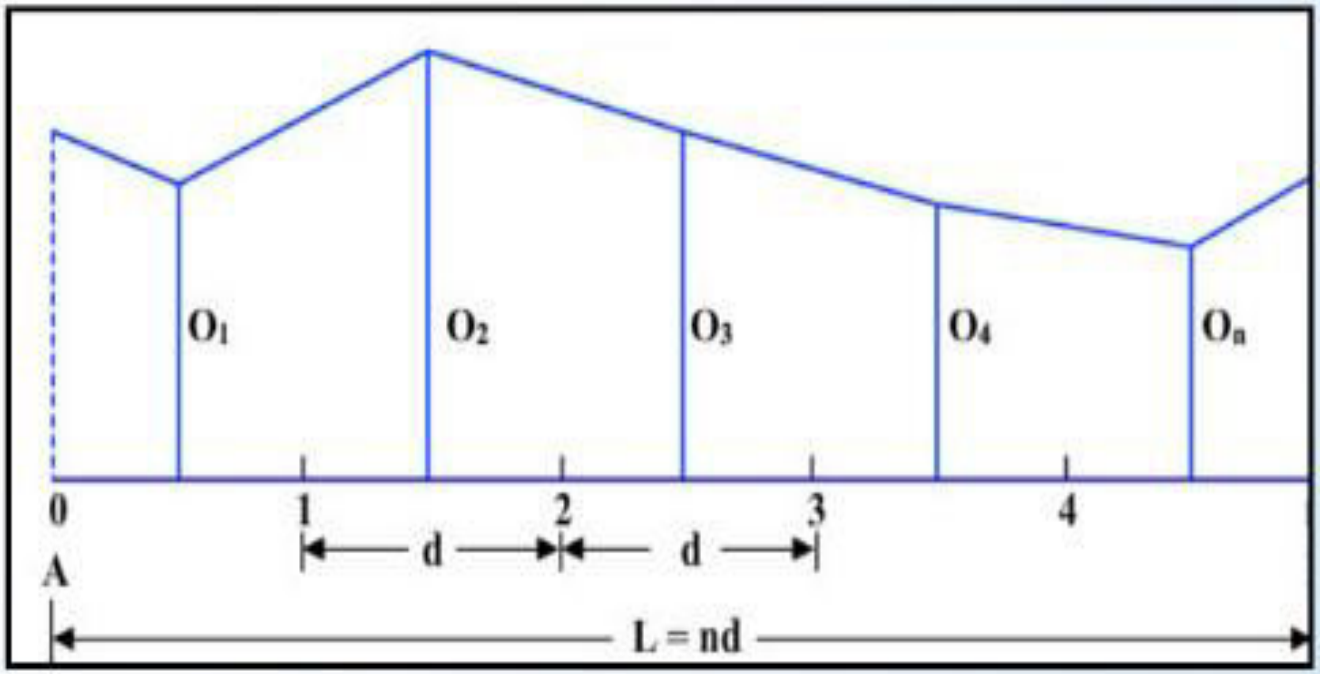
  (iii)Trapezoidal rule

  (iv)Simpson's one-third rule

Areas from Offsets to a Base Line

(1)mid-ordinate rule

- The method is used with the assumption that the boundaries between the extremities of the ordinates (or offsets) are straight lines.



- The base line is divided into a number of divisions and the ordinates are measured at the mid-points of each division, as illustrated in fig.4.2

Areas from Offsets to a Base Line

The area is calculated by the formula

Area = average ordinate × Length of base

$$= \frac{O_1 + O_2 + O_3 + ... + O_n}{n} L = (O_1 + O_2 + O_3 + ... + O_n) d = d \Sigma O$$

where O_1 , O_2 ....= the ordinates at the mid points of each division.

ΣO = sum of the mid-ordinates;

n = number of divisions;

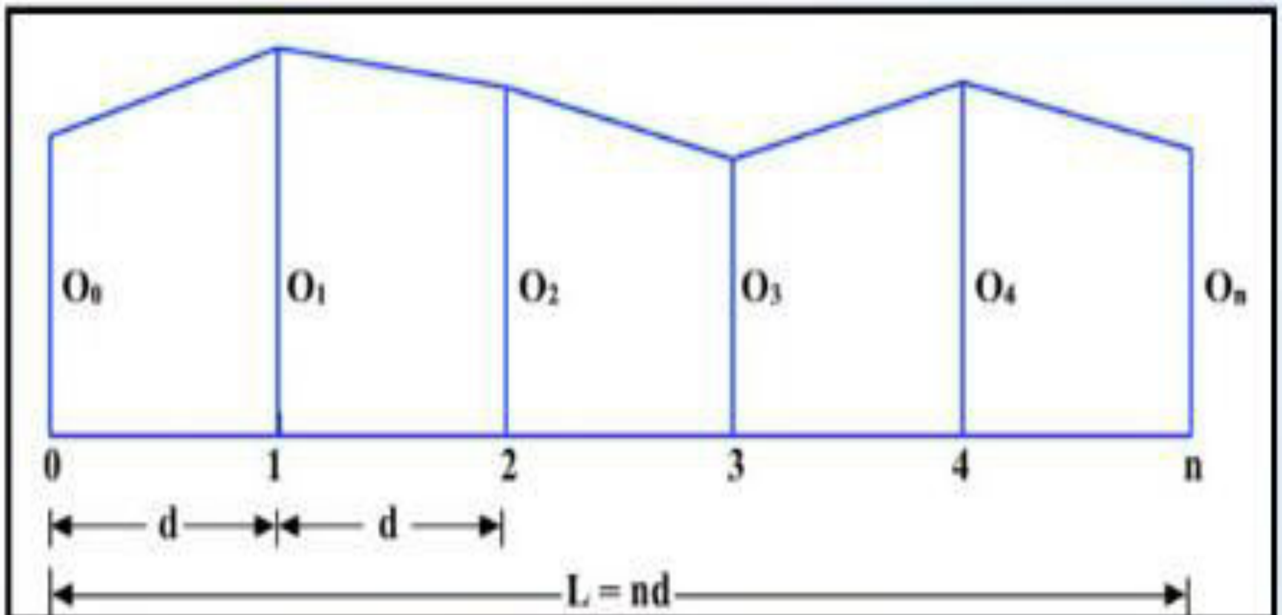
L = length of base line = nd;

d = distance of each division.

Areas from Offsets to a Base Line

(2)Average ordinate rule(fig.4.3)

- This rule also assumes that the boundaries between the extremities of the ordinates are straight lines.



- The offsets are measured to each of the points of the divisions of the base line.

Areas from Offsets to a Base Line

The area is given by

Δ = Average ordinate × Length of the base

$$= \left(\frac{O_0 + O_1 + \ldots + O_n}{n+1}\right) L = \frac{L}{(n+1)} \Sigma O$$

where O_0 = ordinate at one end of the base.

O_n = ordinate at the other end of the base divided into n equal divisions.

O_1, O_2 ....= ordinates at the end of each division

Areas from Offsets to a Base Line

(3)Trapezoidal rule

- This rule is based on the assumption that the figures are trapezoids.
- The rule is more accurate than the previous two rules which are approximate versions of the trapezoidal rule.
- Referring to the fig.4.3 the area of the first trapezoid is given by

$$\Delta_1 = \frac{O_0 + O_1}{2} d \quad \text{- - - - - - - - -(1)}$$

- Similarly, the area of the second trapezoid is given by

$$\Delta_2 = \frac{O_1 + O_2}{2} d \quad \text{- - - - - - -(2)}$$

- Area of the last trapezoid (nth) is given by

$$\Delta_n = \frac{O_{n-1} + O_n}{2} d \quad \text{- - - - - - - - - (3)}$$

Areas from Offsets to a Base Line

- To calculate the area of the segment of the curve, we will utilize the property of the parabola that area of a segment (such as DFC) is equal to two-third the area of the enclosing parallelogram (such as CDEG)

- Hence the total area of the figure is given by

$$\Delta = \Delta_1 + \Delta_2 + \dots \Delta_n = \frac{O_0 + O_1}{2} d + \frac{O_1 + O_2}{2} d + \dots + \frac{O_{n-1} + O_n}{2} d \quad ---- (4)$$

$$\Delta = \left(\frac{O_0 + O_n}{2} + O_1 + O_2 + \dots + O_{n-1} \right) d \quad ---- (5)$$

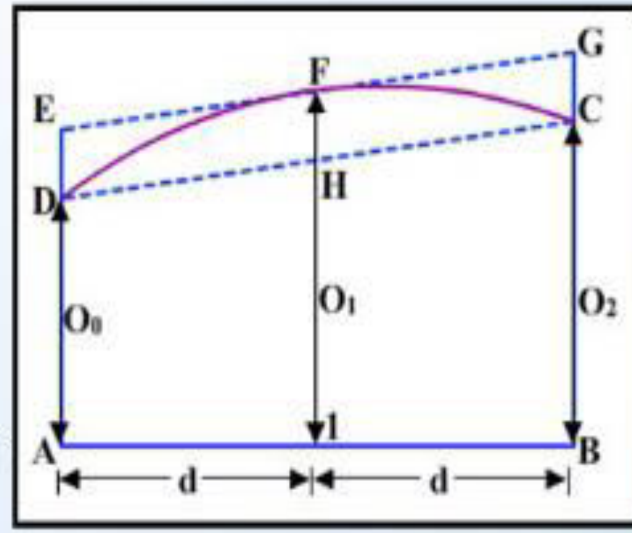
Equation gives the trapezoidal rule which may be expressed as below:

Add the average of the end offsets to the sum of the intermediate offsets. Multiply the total sum thus obtained by the common distance between the ordinates to get the required area.

Areas from Offsets to a Base Line

(4) Simpson's one third rule

- This rule assumes that the short lengths of boundary between the ordinates are parabolic arcs.
- This method is more useful when the boundary line departs considerably from the straight line.
- Thus, in fig.4.4 the area between the line AB and the curve DFC may be considered to be equal to the area of the trapezoid $ABCD$ plus the area of the segment between the parabolic arc DFC and the corresponding chord DC.



- Let O_0 , O_1 , O_2 = any three consecutive ordinates taken at regular interval of d.
- Through F, draw a line EG parallel to the chord DG to cut the ordinates in E and G.

Areas from Offsets to a Base Line

- Area of trapezoid

$$ABCD = \frac{O_0 + O_2}{2}.2d \text{ --------- (6)}$$

- Thus, area of segment

$$DFC = \frac{2}{3}(FH \times AB) = \frac{2}{3}\left\{\left(O_1 - \frac{O_0 + O_2}{2}\right)2d\right\} \text{ -------- (7)}$$

- Adding (6) and (7), we get the required area ($\Delta_{1,2}$) of first two intervals.

- Thus,

$$\Delta_{1,2} = \frac{O_0 + O_2}{2}.2d + \frac{2}{3}\left\{\left(O_1 - \frac{O_0 + O_2}{2}\right)2d\right\} = \frac{d}{3}\left(O_0 + 4O_1 + O_2\right) \text{ --- (8)}$$

- Similarly, the area of next two intervals ($\Delta_{3,4}$) is given by

$$\Delta_{3,4} = \frac{d}{3}\left(O_2 + 4O_3 + O_4\right) \text{ -------- (9)}$$

Areas from Offsets to a Base Line

- Area of the last two intervals (Δ_{n-1} , Δ_n) is given by

$$O_{n-1,n} = \frac{d}{3}\left(O_{n-2} + 4O_{n-1} + O_n\right) \quad ------(10)$$

- Adding all these to get the total area (Δ), we get

$$\Delta = \frac{d}{3}\left(O_0 + 4O_1 + 2O_2 + 4O_3 .... + 2O_{n-2} + 4O_{n-1} + O_n\right) \quad ---(11)$$

$$\Delta = \frac{d}{3}\left[\left(O_0 + O_n\right) + 4\left(O_1 + O_3 + ... + O_{n-1}\right) + 2\left(O_2 + O_4 + ... + O_{n-2}\right)\right] \quad ---(12)$$

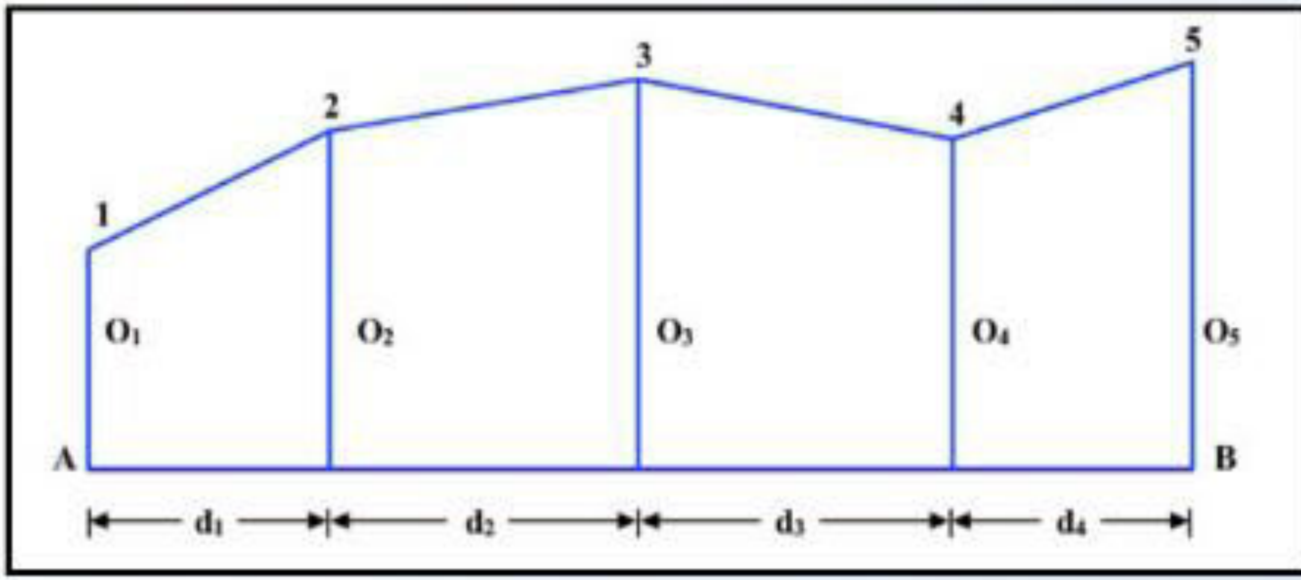
- Simpson's one third rule may be stated as follows:

The area is equal to the sum of the two end ordinates plus four times the sum of the even intermediate ordinates + twice the sum of the odd intermediate ordinates, the whole multiplied by one-third the common interval between them.

Offsets at Irregular Intervals

(a) First method

- In this method, the area of each trapezoid is calculated separately and then added together to calculate the total area.



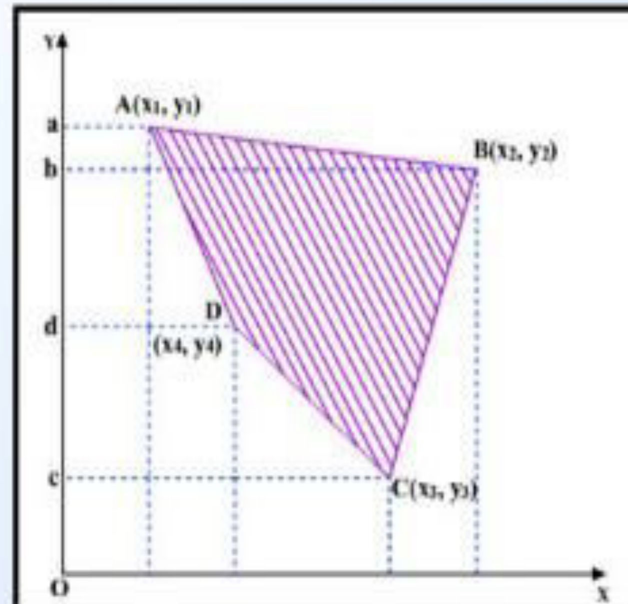
Thus from fig.4.5

$$\Delta = \frac{d_1}{2}(O_1 + O_2) + \frac{d_2}{2}(O_2 + O_3) + \frac{d_3}{2}(O_3 + O_4) \text{ - - - - - - -(13)}$$

Offsets at Irregular Intervals

(b)Second method:

By method of coordinates:



- Let (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) be the coordinates of the stations A, B, C, D respectively, of a traverse $ABCD$.

- If A is the total area of the traverse, we have
  A = (area aABb) + (area bBCc) – (area cCDd) – (area dDAa)
  $= ½[(y_1 - y_2)(x_1 + x_2) + (y_2 - y_3)(x_2 + x_3) - (y_4 - y_3)(x_4 + x_3) - (y_1 - y_4)(x_1 + x_4) = ½[y_1(x_2 - x_4) + y_2(x_3 - x_1) + y_3(x_4 - x_2) + y_4(x_1 - x_3)]$

- In general, if we have n stations, we get
  $A = ½[y_1(x_2 + x_n) + y_2(x_3 - x_1) + y_3(x_4 - x_2) + \ldots\ldots\ldots + y_n(x_1 - x_{n-1})]$.

Exercise

- The following perpendicular offsets were taken at 10 metres intervals from a survey line to an irregular boundary line:
  3.35 5.50 4.10 6.55 8.65 6.30 3.15 4.30 5.55

- Calculate the area enclosed between the survey line, the irregular boundary line, and the first and last offsets, by the application of (a)average ordinate rule, (b) trapezoidal rule and (c)Simpson's rule.

To find:

Area enclosed between the survey line, the irregular boundary line and the first and last offsets by average ordinate rule, trapezoidal rule and Simpson's rule.

Exercise

Hint:

Average ordinate rule: $\Delta = \frac{L}{(n+1)}\Sigma O$

n = number of divisions, n + 1 = number of ordinates, L = length of base.

Trapezoidal rule:

$$\Delta = \left(\frac{O_0 + O_n}{2} + O_1 + O_2 + \ldots + O_{n-1}\right)d$$

Simpson's rule:

$$\Delta = \frac{d}{3}\left[\left(O_0 + O_n\right) + 4\left(O_1 + O_3 + \ldots + O_{n-1}\right) + 2\left(O_2 + O_4 + \ldots + O_{n-2}\right)\right]$$

Exercise

Solution:

a)By average ordinate rule

$n = 8$, n+1 = 8 +1 = 9, L = 10 * 8 = 80 m

ΣO = 3.35 + 5.50 + 4.10 + 6.55 + 8.65 + 6.30 + 3.15 + 4.30 + 5.55 = 47.45 m

Δ = (80/9) * 47.45 = 421.77 sq.metres = 4.2177 ares.

(b)By trapezoidal rule:

d = 10 m, $(O_0 + O_n)/2$ = (3.35 + 5.55)/2 = 4.45 m

$O_1 + O_2 \ldots. O_{n-1}$ = 5.50 + 4.10 + 6.55 + 8.65 + 6.30 + 3.15 + 4.30 = 38.55 m

Δ = (4.45 + 38.55)10 = 430 sq.metres = 4.30 ares.

(c)By Simpson's rule

d = 10 m, $O_0 + O_n$ = 3.35 + 5.55 = 8.9 m

$4(O_1 + O_3 + \ldots. O_{n-1})$ = 4(5.55 + 6.55 + 6.30+ 4.30) = 90.80

$2(O_2 + O_4 + \ldots. O_{n-2})$ = 2(4.10 + 8.65 + 3.35) = 32.20

Δ = 10/3 (8.9 + 90.80 + 32.20) = 439.66 sq.metres = 4.3966 ares.

Exercise

The following perpendicular offsets were taken from a chain line to an irregular boundary:

Chainage	0	10	25	42	60	75 *m*
Offset	15	26.3	31.9	25.7	29	32

Calculate the area between the chain line, the boundary and the end offsets.

To find:

Area between the chain line, the boundary line and the end offsets.

Formula:

$$\Delta = \frac{d_1}{2}(O_1 + O_2) + \frac{d_2}{2}(O_2 + O_3) + \frac{d_3}{2}(O_3 + O_4)$$

Solution:

Area of first trapezoid (Δ_1)= 10/2 (15 + 26.3) = 206.5 m^2

Area of second trapezoid (Δ_2)= 25 – 10 / 2 (26.3 + 31.9) = 436.5 m^2

Area of third trapezoid (Δ_3)= 42 – 25 / 2 (31.9 + 25.7) = 489.6 m^2

Area of fourth trapezoid (Δ_4) = 60 – 42 / 2 (25.7 + 29) = 492.3 m^2

Area of fifth trapezoid (Δ_5)= 75 – 60 / 2 (29 + 31.9) = 456.75 m^2

Total area = $\Delta = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5$

= 206.5 + 436.5 + 489.6 + 492.3 + 456.75

= 2081.65 m^2 = 20.816 ares.

Summary

Lets summarize the topic:

- In areas computed by sub-division of triangles, the area is divided into a number of triangles and the area of each triangle is calculated.

- When the lengths of the three sides of a triangle are measured, its area is computed by the equation

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

- The formula for area according to Average ordinate rule is given by

$$\Delta = \frac{L}{(n+1)}\sum O$$

- The formula for area according to Trapezoidal rule is given by

$$\Delta = \left(\frac{O_0 + O_n}{2} + O_1 + O_2 + \ldots + O_{n-1}\right)d$$

- The formula for area according to Simpson's rule is given by

$$\Delta = \frac{d}{3}\left[\left(O_0 + O_n\right) + 4\left(O_1 + O_3 + \ldots + O_{n-1}\right) + 2\left(O_2 + O_4 + \ldots + O_{n-2}\right)\right]$$

- The formula for area according to offsets at regular intervals is given by

$$\Delta = \frac{d_1}{2}\left(O_1 + O_2\right) + \frac{d_2}{2}\left(O_2 + O_3\right) + \frac{d_3}{2}\left(O_3 + O_4\right)$$

Calculation of Volumes

Introduction

There are three methods generally adopted for measuring the volume. They are:

(i)From cross-sections

(ii)From spot levels

(iii)From contours

The first two methods are commonly used for the calculation of earth work while the third method is generally adopted for the calculation of reservoir capacities.

Measurement from Cross-Sections

- In this method, the total volume is divided into a series of solids by the planes of cross-sections.

- The fundamental solids on which measurements is based are the prism, wedge and prismoid.

- The spacing of sections depend upon the character of the ground and the accuracy required in the measurement.

- The area of cross-section taken along the line are first calculated by standard formulae developed below, and the volumes of the prismoids between successive cross-sections are then calculated by either trapezoidal formula or by prismoidal formula.

Measurement from Cross-Sections

The various cross-sections may be classed as

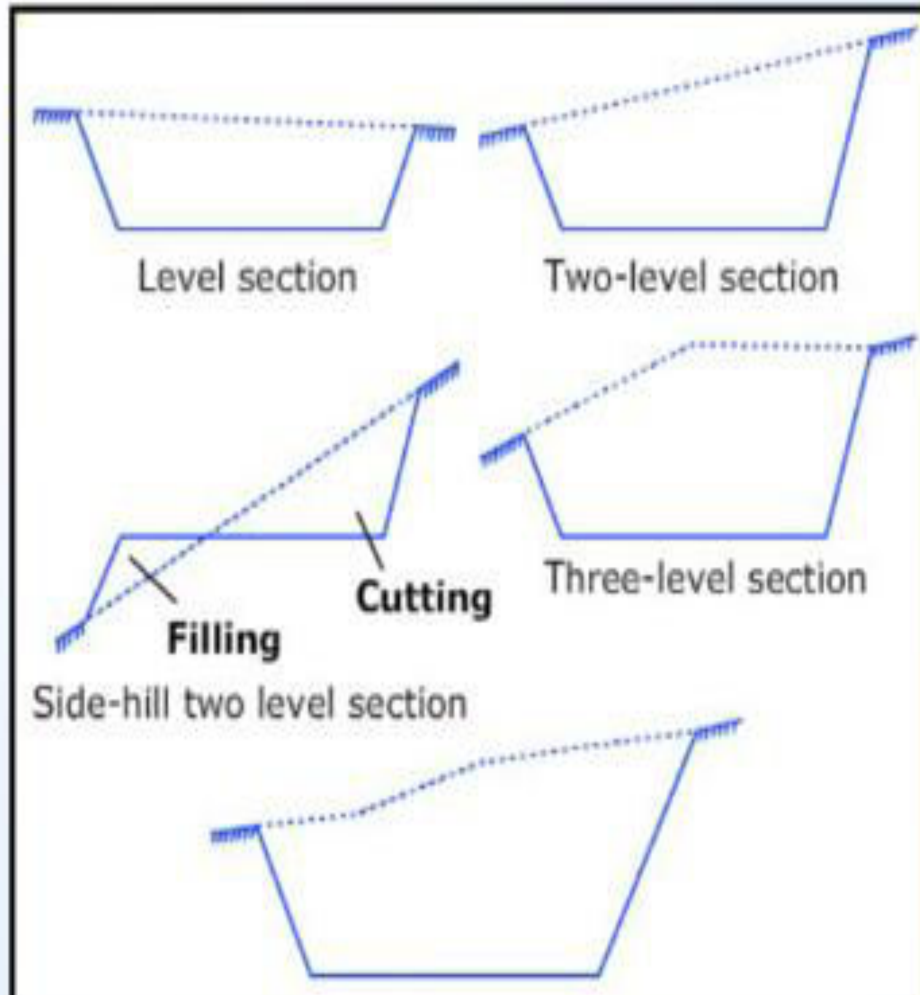
(1)Level section,

(2)two-level section,

(3)Side hill two-level section,

(4)three-level section,

(5)multi-level section.



Measurement from Cross-Sections

General notation

Let

b = the constant formation (or sub-grade) width.

h = the depth of cutting on the centre line.

w_1 and w_2 = the side widths, or half breadths, i.e. the horizontal distances from the centre to the intersection of the side slopes with original ground level.

h_1 and h_2 = the side heights, i.e. the vertical distances from formation level to the intersections of the slope with the original surface.

n horizontal to 1 vertical = inclination of the side slopes.

m horizontal to 1 vertical = the transverse slope of the original ground.

A = the area of the cross-section.

Measurement from Cross-Sections

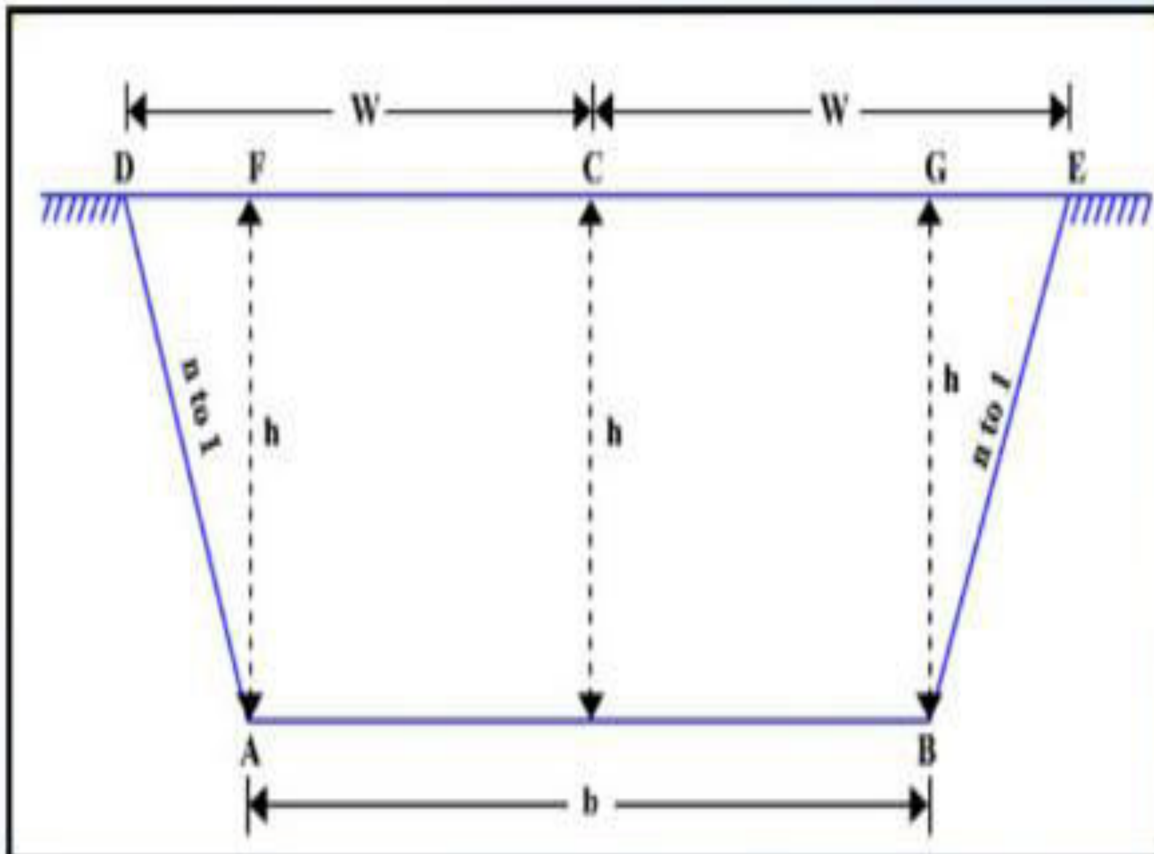
(1)Level section

In this case the ground is level transversely.

Therefore $h_1 = h_2 = h$

$w_1 = w_2 = w = b/2 + nh$

$A = \{b/2 + [b/2 + nh]\}h.$



Measurement from Cross-Sections

(2) Two-level section

- Let O be the point on the centre line at which the two side slopes intersect.

- Hence $BH : HO : : n : 1$ or $HO = b/2n$

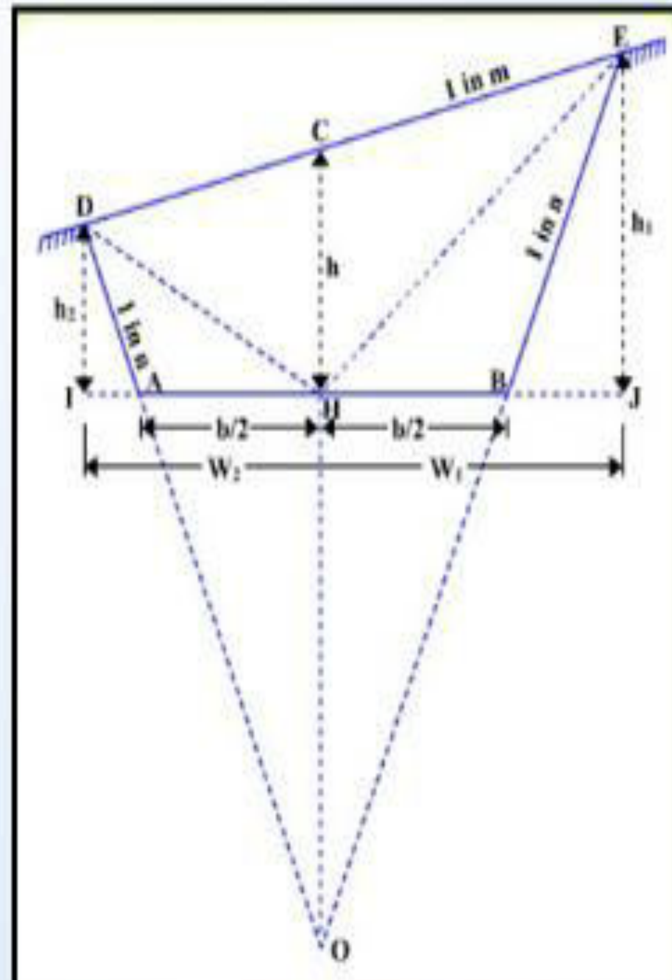
- Then area $DCEBA = \Delta DCO + \Delta ECO - \Delta ABO$

  $= \frac{1}{2}\{[h + b/2n]w_1 + [h + b/2n]w_2 - b^2/2n\}$

  $= \frac{1}{2}\{(w_1 + w_2)[h + b/2n] - b^2/2n\}$

- The above formula has been derived in terms of w_1 and w_2 and does not contain the term m.

- The formula can also be expressed in terms of h_1 and h_2 .



Measurement from Cross-Sections

- Thus, Area $DCEBA = \Delta DAH + \Delta EBH + \Delta DCH + \Delta ECH$

  $= ½\{b/2\ h_2 + b/2\ h_1 + hw_2 + hw_1\}$

  $= ½\ \{b/2(h_1 + h_2) + h(w_1 + w_2\ )\}$

- The above expression is independent of m and n. Let us now find the expression for w_1 , w_2 , h_1 and h_2 in terms of b, h, m and n.

  $BJ = nh_1$

- Also $BJ = HJ - HB = w_1 - b/2$

- Therefore $nh_1 = w_1 - b/2$ - - - - - -(i)

- Also $w_1 = (h_1 - h)m$- - - - - - - - - - -(ii)

- Substituting the value of w_1 in (i), we get $nh_1 = (\ h1 - h)m - b/2$

  or $h1\ (m - n) = mh + b/2$

  or

$$h_1 = \frac{m}{m-n}\left(h + \frac{b}{2m}\right)$$

Measurement from Cross-Sections

- Substituting the value of h_1 in (i), we get

$$w_1 = \frac{b}{2} + nh_1 = \frac{b}{2} + \frac{mn}{m-n}\left(h + \frac{b}{2m}\right)$$

- Proceeding in similar manner, it can be shown that

$$h_2 = \frac{m}{m+n}\left(h - \frac{b}{2m}\right) \quad \text{and} \quad w_2 = \frac{b}{2} + \frac{mn}{m+n}\left(h - \frac{b}{2m}\right)$$

- Substituting the values of w_1 and w_2 in equation and simplifying, we get

$$\text{Area} = \frac{m^2 n}{m^2 - n^2}\left(h + \frac{b}{2m}\right)^2 - \frac{b^2}{4n}$$

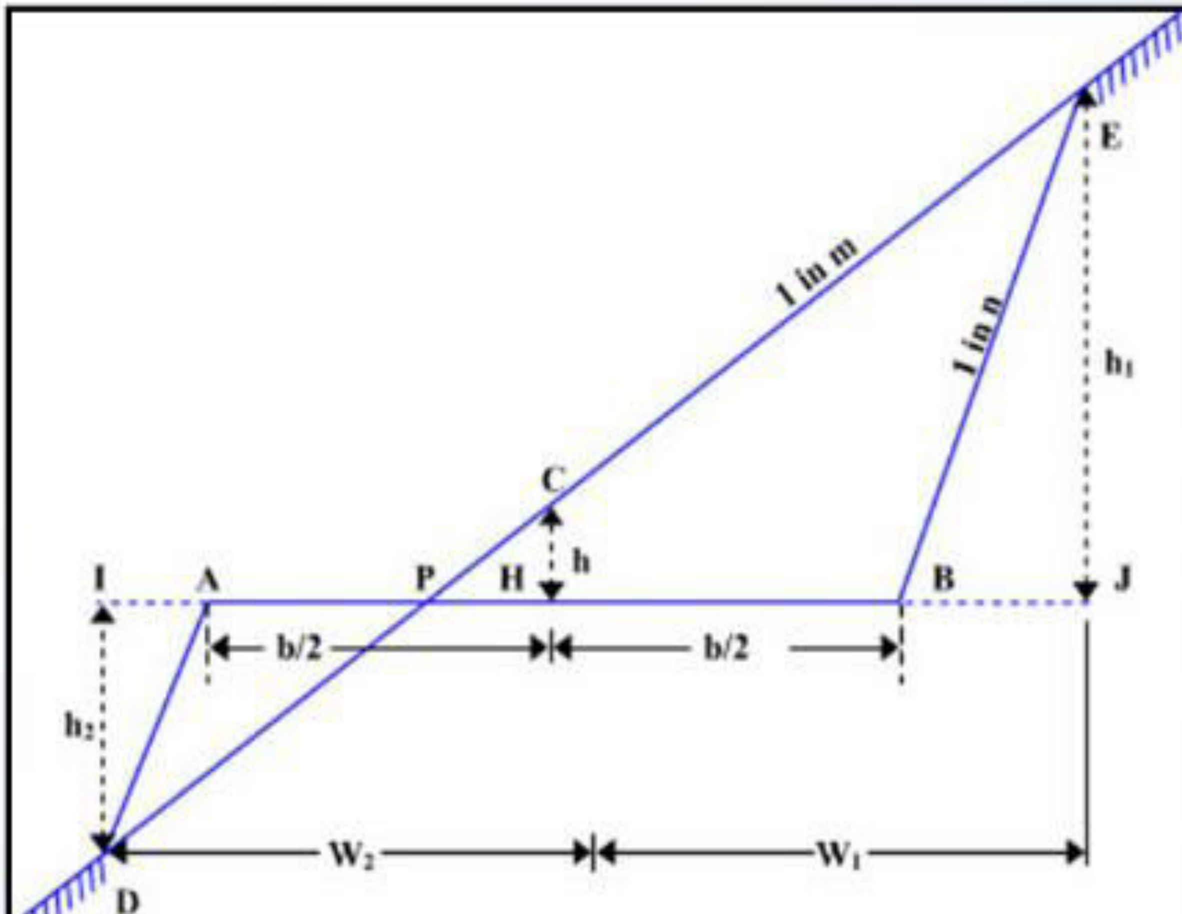
- Similarly, substituting the values of w_1 , w_2 , h_1 and h_2 in equation, we get

$$\text{Area} = \frac{n\left(\frac{b}{2}\right)^2 + m^2\left(bh + nh^2\right)}{\left(m^2 - n^2\right)}$$

Measurement from Cross-Sections

(3)Side hill two-level section

In this case, the ground slope crosses the formation level so that one portion of the area is in cutting and the other in filling.



Measurement from Cross-Sections

Now, $BJ = nh_1$

Also, $BJ = HJ - HB = w_1 - b/2$

Therefore $nh_1 = w_1 - b/2$ - - - - -(i)

But $w_1 = (h_1 - h)\ m$ - - - - - - - -(ii)

Solving (i) and (ii) as before, we get $h_1 = \frac{mn}{m-n}\left(h + \frac{b}{2m}\right)$

$$w_1 = \frac{b}{2} + \frac{mn}{m-n}\left(h + \frac{b}{2m}\right)$$

Let us now derive expressions for w_2 and h_2 :

$IA = nh_2$

Also $IA = IH - AH = w_2 - b/2$

Therefore $nh_2 = w_2 - b/2$

Also $w_2 = (h + h_2)\ m$

Therefore $nh_2 = (h + h_2)m - b/2$ or $h_2\ (m - n) = b/2 - mh$

or

$$h_2 = \frac{m}{m-n}\left(\frac{b}{2m} - h\right)$$

Measurement from Cross-Sections

Hence

$$w_2 = \frac{b}{2} + nh_2 = \frac{b}{2} + \frac{mn}{m-n}\left(\frac{b}{2m} - h\right)$$

- By inspection, it is clear that the expressions for w_1 and w_2 are similar; also expression of h_1 and h_2 are similar, except for $-h$ in place of $+h$.

$$A_1 = \frac{1}{2}(PB)(EJ) = \frac{1}{2}\left(\frac{b}{2} + mh\right)\left\{\frac{m}{m-n}\left(\frac{b}{2m} + h\right)\right\} = \frac{\left(\frac{b}{2} + mh\right)^2}{2(m-n)}$$

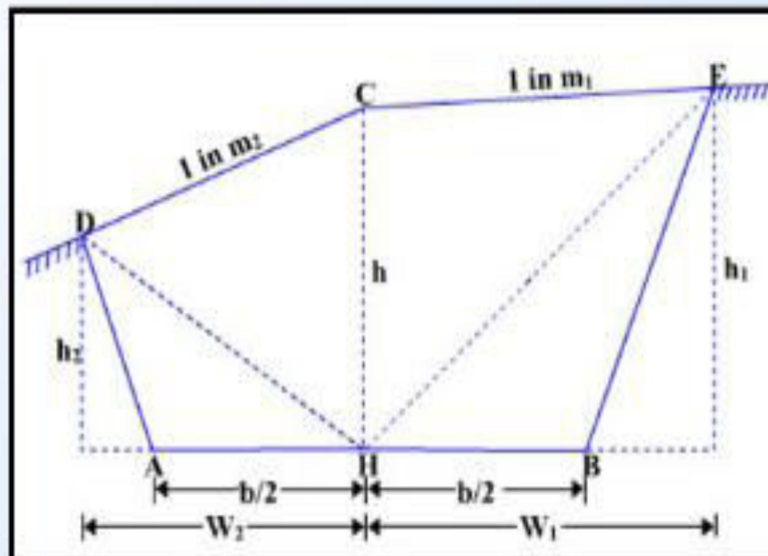
$$A_2 = \frac{1}{2}(AP)(ID) = \frac{1}{2}\left(\frac{b}{2} - mh\right)\left\{\frac{m}{m-n}\left(\frac{b}{2m} - h\right)\right\} = \frac{\left(\frac{b}{2} - mh\right)^2}{2(m-n)}$$

Measurement from Cross-Sections

(4)Three level section:

Let 1 in m_1 be the transverse slope of the ground to one side and 1 in m_2 be the slope to the other side of the centre line of the cross section.

The expressions for w_1, w_2, h_1, and h_2 can be derived in the similar way as for two level section. Thus,



$$w_1 = \frac{m_1 n}{m_1 - n}\left(h + \frac{b}{2n}\right)$$

$$w_2 = \frac{m_2 n}{m_2 - n}\left(h + \frac{b}{2n}\right)$$

$$h_1 = \left(h + \frac{w_1}{m_1}\right) = \frac{m_1}{m_1 - n}\left(h + \frac{b}{2m_1}\right)$$

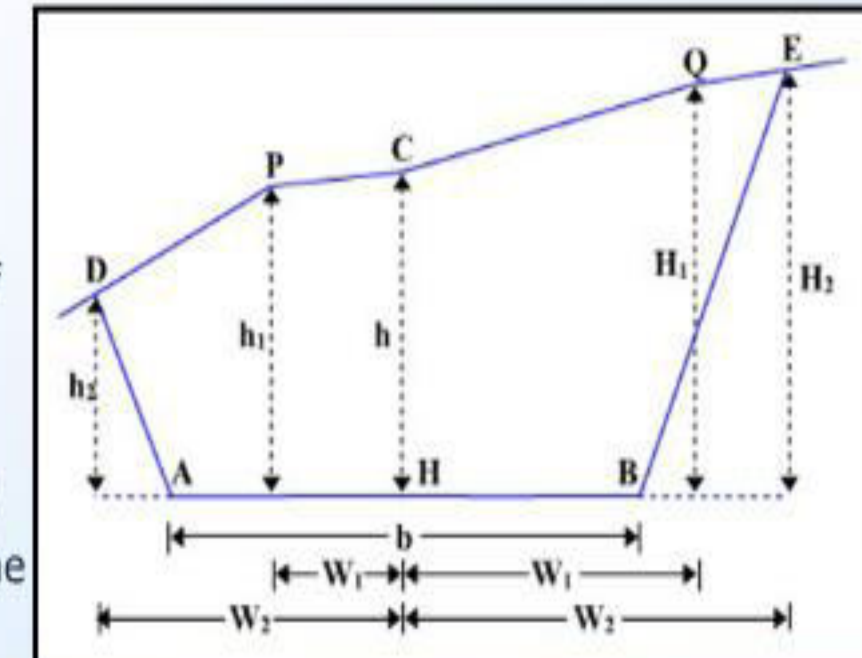
$$h_2 = \left(h - \frac{w_2}{m_2}\right) = \frac{m_2}{m_2 + n}\left(h - \frac{b}{2m_2}\right)$$

Measurement from Cross-Sections

The area $ABECD = \Delta AHD + \Delta BHE + \Delta CDH + \Delta CEH$

$$= \frac{1}{2}\left[\left(h_2 \times \frac{b}{2}\right) + \left(h_1 \times \frac{b}{2}\right) + hw_2 + hw_1\right] = \left[\frac{b}{4}(h_1 + h_2) + \frac{h}{2}(w_1 + w_2)\right]$$

(5)Multi level section:



- In the multi level section the coordinate system provides the most general method of calculating the area.
- The cross section notes provide with x and y coordinates for each vertex of the section, the origin being at the central point (H).
- The x coordinates are measured positive to the right and negative to the left of H. similarly, the y coordinates are measured positive for cuts and negative for fills.

Measurement from Cross-Sections

- In usual form, the notes are recorded as below:

$$\frac{h_2}{w_2} \quad \frac{h_1}{w_1} \quad \frac{h}{0} \quad \frac{H_1}{W_1} \quad \frac{H_2}{W_2}$$

- If the coordinates are given proper sign and if the coordinates of formation points A and B are also included (one at extreme left and other at extreme right), they appear as follows:

$$\frac{0}{-b/2} \quad \frac{h_2}{-w_2} \quad \frac{h_1}{-w_1} \quad \frac{h}{0} \quad \frac{H_1}{+w_1} \quad \frac{H_2}{+w_2} \quad \frac{0}{+b/2}$$

- There are several other methods to calculate the area. In one of the methods, the opposite algebraic sign is placed on the opposite side of each lower term. The coordinates then appear as:

$$\frac{0}{-b/2+} \quad \frac{h_2}{-w_2+} \quad \frac{h_1}{-w_1+} \quad \frac{h}{0} \quad \frac{H_1}{+w_1-} \quad \frac{H_2}{+w_2-} \quad \frac{0}{+b/2-}$$

Measurement from Cross-Sections

- The area can now be computed by multiplying each upper term by the algebraic sum of the two adjacent lower terms, using the signs facing the upper term. The algebraic sum of these products will be double the area of the cross-section. Thus,

$$A = \frac{1}{2}\left[h_2\left(+b/2 - w_1\right) + h_1\left(+w_2 + 0\right) + h\left(+w_1 + W_1\right) + H_1\left(0 + W_2\right) + H_2\left(-W_1 + b/2\right)\right]$$

Exercise

A railway embankment is 10 *m* wide with side slopes 1½ to 1. Assuming the ground to be level in a direction transverse to the centre line, calculate the volume contained in a length of 120 metres, the centre heights at 20 m intervals being in metres 2.3, 3.6, 3.7, 4.2, 3.7, 2.7, 2.3.

To find:

Volume by trapezoidal rule and prismoidal rule

Hint:

Volume by trapezoidal rule = $V = d\left\{\frac{A_1 + A_2}{2} + A_2 + A_3 + \ldots A_{n-1}\right\}$

Volume by prismoidal rule

$$V = \frac{d}{3}\left[\left(A_1 + A_n\right) + 4(A_2 + A_4 \ldots A_{n-1}) + 2(A_3 + A_5 \ldots A_{n-2})\right]$$

Solution:

For a level section, the area is given by $A = (b + nh)h$

Slope is 1½ : 1 , hence $n = 1.5$.

Exercise

The areas at different sections will be as under:

$A_1 = (10 + 1.5 \times 2.3)\ 2.3 = 30.935\ m^2$,

$A_2 = 55.44\ m^2$

$A_3 = 57.535\ m^2$

$A_4 = 68.46\ m^2$

$A_5 = 57.535\ m^2$

$A_6 = 37.935\ m^2$

$A_7 = 30.935\ m^2$

Exercise

and

Volume by trapezoidal rule is given by

$$V = d\left\{\frac{A_1 + A_n}{2} + A_2 + A_3 + \ldots.A_{n-1}\right\}$$

= 20[30.935 + 30.935 /2 + 55.44 + 57.535+ 68.46+ 57.535 + 37.935] = 6156.8 m^3

Volume by prismoidal rule is given by

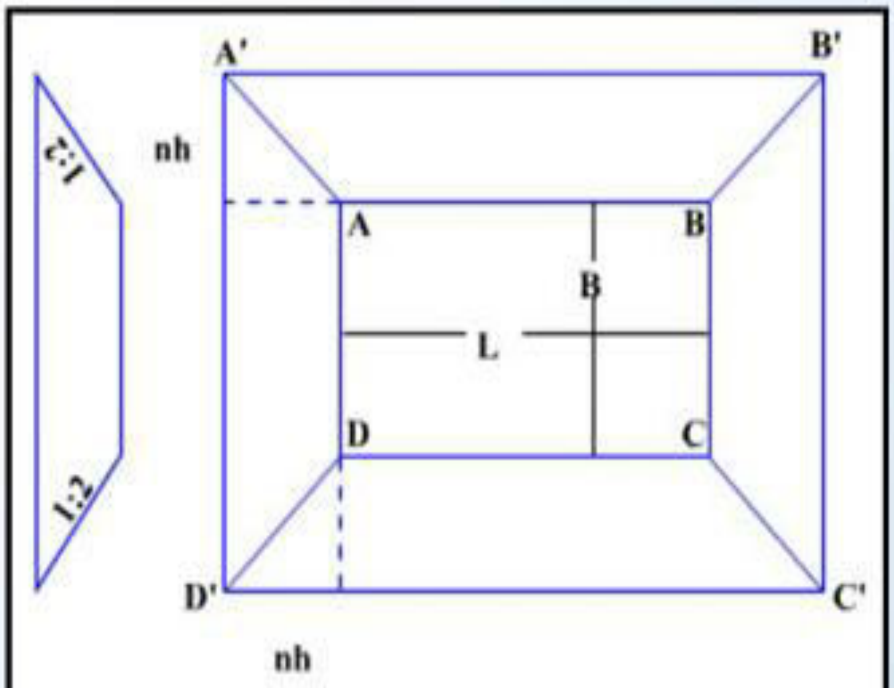
$$V = \frac{d}{3}\left[(A_1 + A_n) + 4(A_2 + A_4 \ldots.A_{n-1}) + 2(A_3 + A_5 \ldots A_{n-2})\right]$$

= 20/3[(30.935 + 30.935) + 4(55.44 + 68.46+ 37.935) + 2(57.535 + 57.535)]

=6262.333 m^3

Capacity of Reservoirs

- Reservoirs are made for water supply and for power or irrigation projects.
- A contour map is very useful to study the possible location of dams and the volume of water to be confined.
- All the contours are closed lines within the reservoir area. This is a typical case of volume in which the finished surface (i.e. surface of water) is level surface.



Capacity of Reservoirs

- The volume is calculated by assuming it as being divided up into a number of horizontal slices by contour planes.

- The whole area lying within a contour line is measured by planimeter and the volume can be calculated.

- Let A_1 , A_2 , A_3 ......A_n = the area of successive contours,

  h = contour interval,

  V = capacity of reservoir.

  Then by trapezoidal formula

$$V = h\left\{\frac{A_1 + A_2}{2} + A_2 + A_3 + \ldots A_{n-1}\right\}$$

Capacity of Reservoirs

By the prismoidal rule,

$$V = \frac{h}{3}\left[\left(A_1 + A_n\right) + 4\left(A_2 + A_4 \ldots\ldots A_{n-1}\right) + 2\left(A_3 + A_5 \ldots A_{n-2}\right)\right]$$

where n is an odd number.

Length of the reservoir at the top = $L + 2\,(nh)$

where

h = depth of the reservoir

n = slope.

Width of the reservoir at the top = $B + 2(nh)$

Exercise

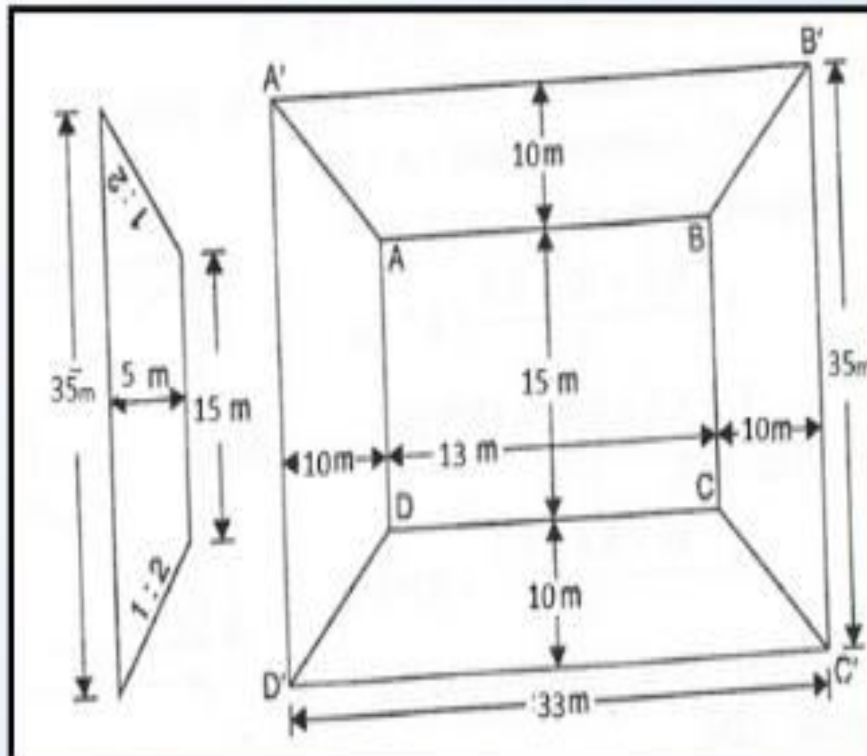
An excavation is to be made for a reservoir 13 m long 15 m wide at the bottom, having the side of the excavation slope at 2 horizontal to 1 vertical. Calculate the volume of excavation if the depth is 5 metres. The ground surface is level before excavation.

To find:

Volume of excavation

Hint:

Volume of excavation $V = \frac{h}{6}(A_1 + 4A_m + A_2)$



Solution:

Length of the reservoir at the top = $L + 2nh$

= 13 + (2 * 2* 5) = 33 m

Exercise

Width of the reservoir at the top = $B + 2nh$ = 15 + (2 * 2 * 5) = 35 m

Length of the reservoir at mid-height = (13 + 33)/2 = 23 m

Width of the reservoir at mid-height = (15 + 35)/2 = 25 m

Area of the bottom of the reservoir = 13 * 15 = 195 m^2

Area of the top of the reservoir = 23 * 25 = 575 m^2

Area of the reservoir at mid-height = 25 * 13 = 325 m^2

Since the areas $ABCD$ and $A'B'C'D'$ are in parallel planes spaced 5 m part, prismoidal formula can be used.

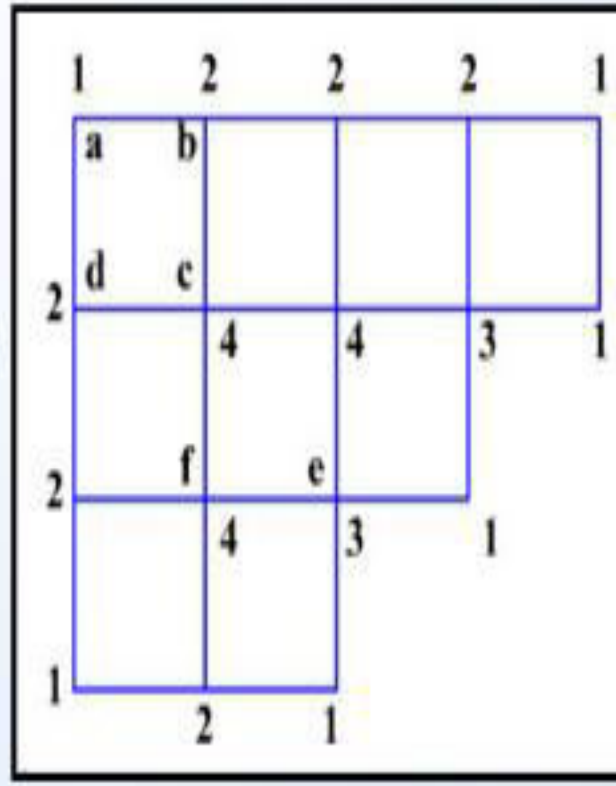
V = 4/6 (195 + 4(325) + 575) = 1380 m^3

Volume from Spot Levels (Volume of Borrow Pits)

- In this method, the field work consists in dividing the area into a number of squares, rectangles or triangles and measuring the levels of their corners before and after the construction.
- Thus, the depth of excavation or height of filling at every corner is known.
- Let us assume that the four corners of any one square or rectangle are at different elevations but lie in the same inclined plane
- Assume that it is desired to grade down to a level surface a certain distance below the lowest corner.
- The earth to be moved will be a right truncated prism, with vertical edges at a, b, c and d (fig.4.12)

Volume from Spot Levels (Volume of Borrow Pits)

- The rectangle abcd represents the horizontal projection of the upper inclined base of the prism and also the lower horizontal base.



- Let us consider the rectangle abcd of fig. If h_a, h_b, h_c and h_d represent the depth of excavation of the four corners, the volume of the right truncated prism will be given by

$$V = \left(\frac{h_a + h_b + h_c + h_d}{4} \right) \times A$$

= average height × the horizontal area of the rectangle

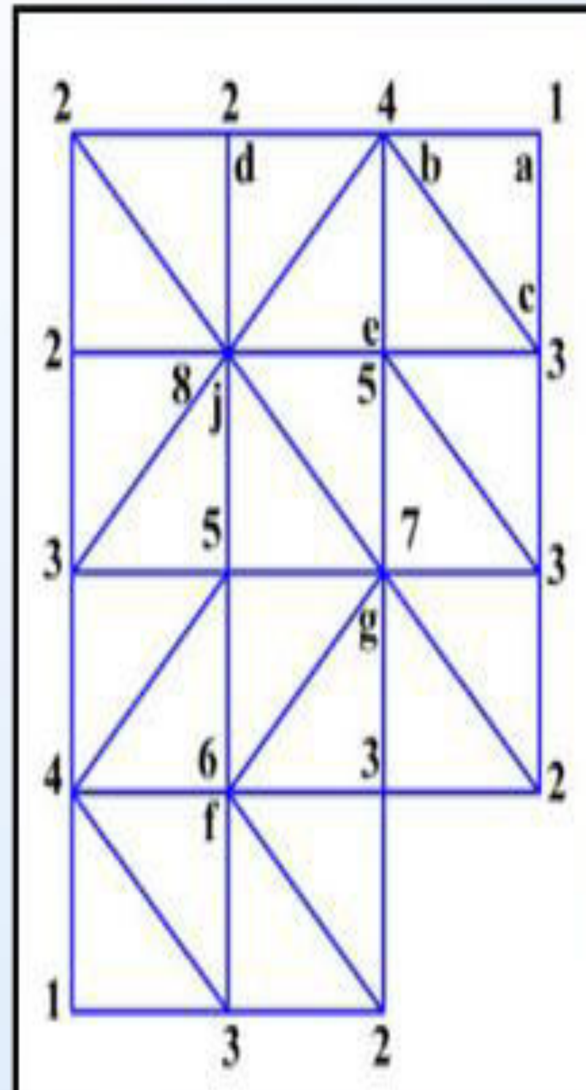
Volume from Spot Levels (Volume of Borrow Pits)

- Similarly, let us consider the triangle abc of fig.4.13.

- If h_a, h_b, h_c are the depths of excavation of the three corners, the volume of the truncated triangular prism is given by

$$V = \left(\frac{h_a + h_b + h_c}{3} \right) \times A$$

= *average depth × the horizontal* area of the triangle



Volume from Spot Levels (Volume of Borrow Pits)

Volume of a group of rectangles or squares having the same area

- Let us now consider a group of rectangles of the same area, arranged as shown in fig. 4.12.
- It will be seen by inspection that some of the heights are used once only, some heights are common to two rectangles (such as at b), some heights are common to three rectangles (such as at e), and some heights are common to four rectangles (such as at f).
- Thus, in fig.4.12 each corner height will be used as many times as there are rectangles joining at the corner

Volume from Spot Levels (Volume of Borrow Pits)

- Let Σh_1 = the sum of heights used once.

  Σh_2 = the sum of heights used twice.

  Σh_3 = the sum of heights used thrice.

  Σh_4 = the sum of heights used four times.

  A = horizontal area of the cross-section of one prism.

- Then the total volume is given by

$$V = \frac{A(1\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4)}{4}$$

Volume from Spot Levels (Volume of Borrow Pits)

Volume of a group of triangles having equal area

- If the ground is very much undulating, the area may be divided into a number of triangles having equal area.

- In this case, some corner heights will be used once (such as point a of fig.4.13), some twice (such as at d), some thrice (such as at c), some four times (such as at b), some five times (such as at e), some six times (such as at f), and some seven times (such as at j).

- The maximum number of times a corner height can be used is eight. Thus, in fig.4.13. each corner height will be used as many times as there are triangles joining at the corner.

Volume from Spot Levels (Volume of Borrow Pits)

- Let Σh_1 = the sum of heights used once.
  Σh_2 = the sum of heights used twice.
  Σh_3 = the sum of heights used thrice.
  Σh_4 = the sum of heights used four times.
  Σh_5 = the sum of heights used five times.
  Σh_6 = the sum of heights used six times.
  Σh_7 = the sum of heights used seven times.
  Σh_8 = the sum of heights used eight times.

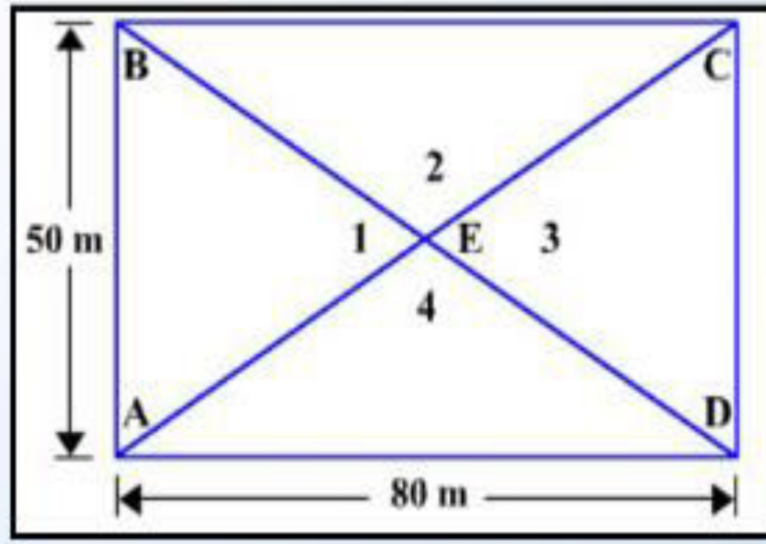
- The total volume of the group is given by

$$V = \frac{A}{3}(1\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4 + 5\Sigma h_5 + 6\Sigma h_6 + 7\Sigma h_7 + 8\Sigma h_8)$$

Exercise

A rectangular plot ABCD forms the plane of a pit excavated for road work. E is point of intersection of the diagonals. Calculate the volume of the excavation in cubic metres from the following data:

Point	A	B	C	D	E
Original level	45.2	49.8	51.2	47.2	52.0
Final level	38.6	39.8	42.6	40.8	42.5



Length of AB = 50 *m* and BC = 80 *m*.

To find:

Volume of excavation.

Hint:

The total volume of the group is given by

$$V = \frac{A}{3}(1\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4 + 5\Sigma h_5 + 6\Sigma h_6 + 7\Sigma h_7 + 8\Sigma h_8)$$

Exercise

Solution:

Area of each triangle = ½ * 50 * 40 = 1000 $sq.m$.

Take the vertices of each triangle and find the mean depth at each triangle. Thus,

Depth of cutting at A = 45.2 – 38.6 = 6.6 m

Depth of cutting at B = 49.8 – 39.8 = 10.0 m

Depth of cutting at C = 51.2 – 42.6 = 8.6 m

Depth of cutting at D = 47.2 – 40.8 = 6.4 m

Depth of cutting at E = 52.0 – 42.5 = 9.5 m

Now volume of any truncated triangular prism is given by

V = (average height) * $A = hA$

For the triangular prism ABE $h = \dfrac{6.6 + 10 + 9.5}{3} = 8.7\ m$

$$V_1 = 8.7 \times 1000 = 8700\ m^3$$

Exercise

From the prism BCE,

$$h = \frac{10 + 8.6 + 9.5}{3} = 9.367\ m$$

$$V_1 = 9.367 \times 1000 = 9367\ m^3$$

From the prism CDE,

$$h = \frac{8.6 + 6.4 + 9.5}{3} = 8.167\ m$$

$$V_3 = 8.167 \times 1000 = 8167\ m^3$$

From the prism DAE,

$$h = \frac{6.4 + 6.6 + 9.5}{3} = 7.5\ m$$

$$V_3 = 7.5 \times 1000 = 7500\ m^3$$

Therefore, volume = $V_1 + V_2 + V_3 + V_4 = 8700 + 9367 + 8167 + 7500 = 33734\ m^3$

Exercise

Alternatively, the total volume can be obtained by

$$V = \frac{A}{3}(1\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4 + 5\Sigma h_5 + 6\Sigma h_6 + 7\Sigma h_7 + 8\Sigma h_8)$$

Here

$$1\Sigma h_1 = 0$$

$$2\Sigma h_2 = 2(6.6 + 10 + 8.6 + 6.4) = 63.2$$

Since height of every outer corner is utilized in two triangles $3\Sigma h_3$, $5\Sigma h_5$, $6\Sigma h_6$, $7\Sigma h_7$ and $8\Sigma h_8$ are each zero.

$4\Sigma h_4 = 4(9.5) = 38$

Substituting the values in the above formula, we get

$$V = \frac{1000}{3} \times (63.2 + 38) = 33733\ m^3$$

Summary

Lets summarize the topic:

- There are three methods generally adopted for measuring the volume. They are:

  (i)From cross-sections

  (ii)From spot levels

  (iii)From contours

- The various cross-sections may be classified as Level section, two-level section, Side hill two-level section, three-level section, multi-level section.

- Volume by trapezoidal rule is given by

$$V = d\left\{\frac{A_1 + A_n}{2} + A_2 + A_3 + \ldots A_{n-1}\right\}$$

- Volume by prismoidal rule is given by

$$V = \frac{d}{3}\left[\left(A_1 + A_n\right) + 4(A_2 + A_4 \ldots A_{n-1}) + 2(A_3 + A_5 \ldots A_{n-2})\right]$$

- The volume of the right truncated prism is given by

$$V = \left(\frac{h_a + h_b + h_c + h_d}{4}\right) \times A$$

= average height × the horizontal area of the rectangle

- The volume of the truncated triangular prism is given by

$$V = \frac{A(1\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4)}{4}$$

- Volume of a group of rectangles or squares having the same area is given by

$$V = \left(\frac{h_a + h_b + h_c}{3}\right) \times A$$

= average depth × the horizontal area of the triangle

- Volume of a group of triangles having equal area is given by

$$V = \frac{A}{3}(1\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4 + 5\Sigma h_5 + 6\Sigma h_6 + 7\Sigma h_7 + 8\Sigma h_8)$$