UNIT-V

(9)

STRESS DISTRIBUTION IN SOILS

Stress are induced in a soil mass due to weight of overlying soil and due to applied loads. These stresses are required for the stability analysis of the soil mass and in the settlement analysis of the foundation and the determination of earth pressures. The stresses due to self weight of the soil is also called geostatic stresses. The geostatic stresses are two types based on the plane they are acting on. Those are vertical stresses and horizontal stresses.

The vertical stresses are determined by calculating unit weights of the soil layer and porewater pressures in the soil. The stress inducing on soil element at a depth 'Z_0' from the surface of the soilma is given by

$$\sigma_v = \int_0^{Z_0} \gamma_{Z_0}\, dZ$$

Horizontal stresses are formed by multiplying vertic stresses with some co-efficient. These co-efficients are called co-efficient of earth pressure

The horizontal stress $\sigma_x = K_0 \sigma_v$

where K_0 is called co-efficient of static earth Pressure which is given by

$\frac{\nu}{1-\nu}$ (or) $\frac{1-\sin\phi}{1+\sin\phi}$

ν = Poisson's ratio
ϕ = Angle of internal friction.

(1) Vertical stresses due to A concentrated load:-

Boussinesq gave the theoretical solutions for the stress distribution in an elastic medium subjected to a concentrated load on its surface.

The solutions are commonly used to obtained the stresses in a soil mass due to externally applied load.

Assumptions of Boussinesq equation :-

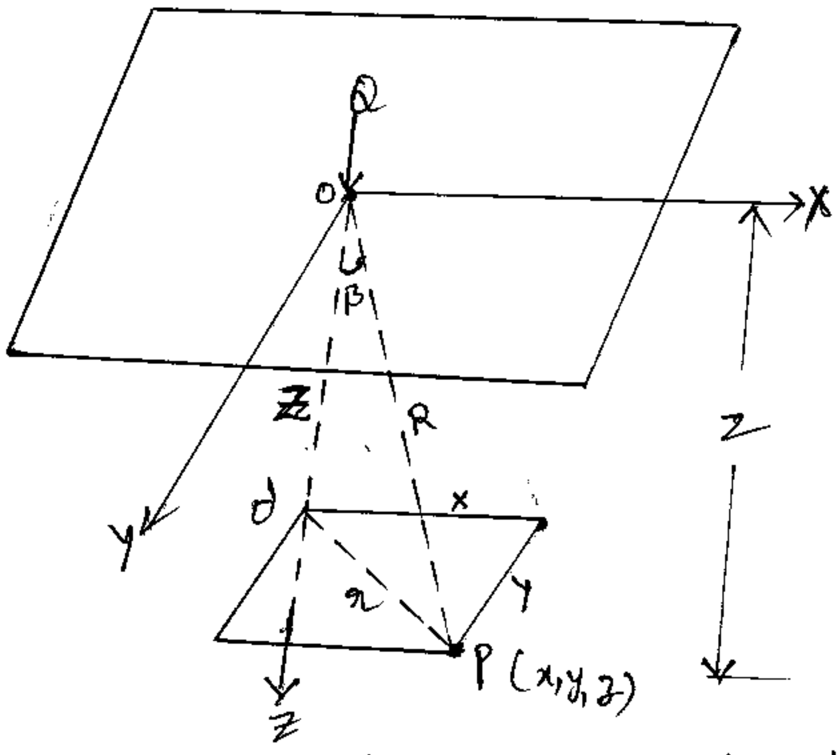
1) The soil mass is an elastic continuum, having a constant value of modulus of elasticity (E), i.e, the ratio b/w the stress and strain is constant.

2) The soil is homogeneous, i.e, it has identical properties at different points.

3) The soil is isotropic i.e it has identical properties in all directions.

4) The soil mass is semi-infinite, i.e it extends to infinity in the downwards. In other words, it is limited on its top by horizontal plane and extends to infinity in all other other directions.

5) The soil is weightless and is free from residual stresses before the application of the load.



Stresses due to concentrated load.

The above fig shows a horizontal surface of the elastic continuum subjected to a point load Q at point O. The origin of the co-ordinates is taken at O. Using logarithmic stress function for the solution of elasticity problem. Bossinesq proved that the polar stress σ_R at point P(x, y, z) is given by,

$$\sigma_R = \frac{3}{2\pi} \frac{Q \cos\beta}{R^2}$$

where R = Polar distance between the origin 'O' & point P

β = Angle which the line OP makes with the vertical (Z-axis)

$$r = \sqrt{x^2+y^2} \longrightarrow (1)$$

$$R = \sqrt{r^2+z^2} \longrightarrow (2)$$

$$OP = R = \sqrt{x^2+y^2+z^2} \longrightarrow (3) \qquad \{\because r^2 = x^2+y^2\}$$

from $\Delta\, OPO'$

$$\sin\beta = \frac{r}{R} \quad \text{and} \quad \cos\beta = \frac{z}{R}$$

The vertical stress (σ_z) at the point P is given by

$$\sigma_z = \sigma_R \cos^2\beta$$

$$= \frac{3}{2\pi}\left(\frac{Q\cos\beta}{R^2}\right)\cos^2\beta$$

$$= \frac{3Q}{2\pi}\,\frac{\cos^3\beta}{R^2}$$

$$= \frac{3Q}{2\pi}\,\frac{z^3/R^3}{R^2}$$

$$= \frac{3Q}{2\pi}\,\frac{z^3}{R^5}$$

$$\sigma_z = \frac{3}{2\pi}\,\frac{Q}{z^2} \times \frac{z^5}{R^5} = \frac{3}{2\pi}\,\frac{Q}{z^2}\left(\frac{z}{R}\right)^5$$

Now sub R value from (2)

$$\sigma_z = \frac{3}{2\pi}\,\frac{Q}{z^2}\left(\frac{z}{\sqrt{r^2+z^2}}\right)^5$$

$(r^2+z^2)^{5/2}$

$z^{5/2}$

$\frac{r}{z}$

$$= \frac{3}{2\pi}\,\frac{Q}{z^2}\,\frac{z^5}{z^5\left(1+\left(\frac{r}{z}\right)^2\right)^{5/2}}$$

$$\sigma_z = \frac{3}{2\pi}\,\frac{Q}{z^2}\left[\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}\right]$$

$$\sigma_z = I_B \times \frac{Q}{z^2}$$

where, $I_B = \dfrac{3}{2\pi\left(1+\left(\frac{r}{z}\right)^2\right)^{5/2}}$

where I_B is known as Boussinesq's influence co-efficient for verticle stress.

The vertical stress exactly below the load than $r=0$

$$I_B = \frac{3}{2\pi} = 0.4775$$

$$\therefore \sigma_z = 0.4775 \frac{Q}{z^2}$$

This obtained by substituting $r=0$ and $z=z$ in the expression.

Observations :-

The following Points are worth noting when using the above expression.

1) The verticle stress does not depending upon the modul of elasticity (E) and poisson's ratio (μ). But the solut has been derived assuming that the soil is linearly elastic. That means the stresses that forms with in the region of elasticity are lesser than the shear strengt of the soil.

2) The intensity of vertical stress just below the load point is given by

$$\sigma_z = 0.4775 \frac{Q}{z^2}$$

3) At the surface ($z=0$). The vertical stress just below the load is theoritically infinite. However in an actual case the soil under the load ~~of~~ yields due to very high stresses. The load point spreads over a small but finite area. There fore only finite stresses will develope.

4) The vertical stress (σ_z) decreases rapidly with an increase in r/z ratio.

5) Boussinesq's solution can be used for negative loads.

limitations of Boussinesq's soloution

(1) The solution was initially obtained for determination of stresses in elastic solids application to soils may be questioned, as the soils are far from purely elastic solids. However experience indicates that the results obtained are ~~st~~ satisfactory.

(2) The application of Boussinesq's solution can be justified when the stress changes are such that only a stress increase occurs in the soil.

(3) when the stress decrease occurs, the relation between stress and strain in not linear and, therefore, the solution is not strictly applicable

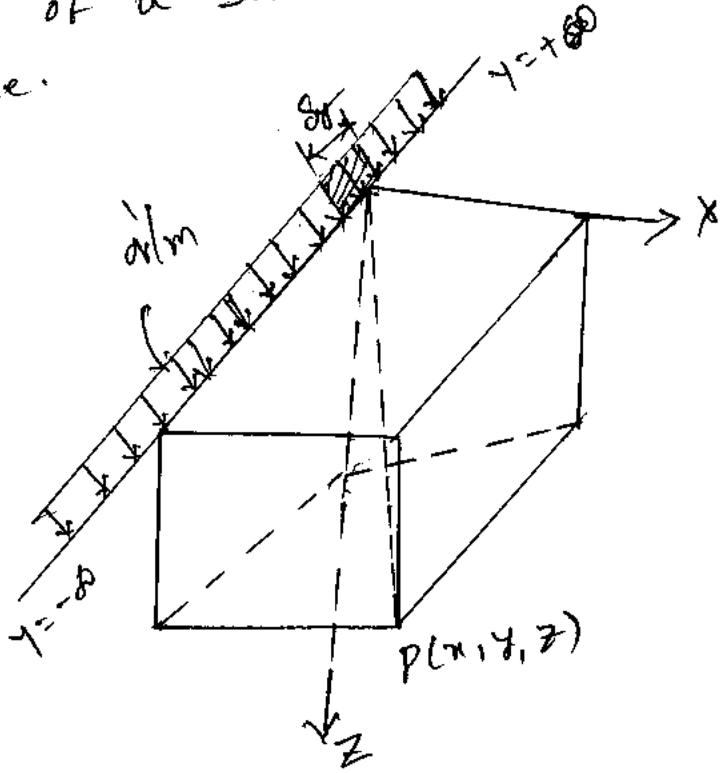
(4) For practical cases, the Boussinesq, solution can be safely used for homogeneous deposits of clay man-made fill and for limited thickness of uniform sand deposits.

(5) The point loads applied below ground surface cause somethat smaller stresses than are caused by surface loads, and, therefore, the Boussinesq solution is not strictly applicable. However, the solution is frequently used for shallow footings in which z is measured below the of the footing.

(ii) Vertical Stress under a line load :-

The expression for vertical stress at any point 'p' under a line load can be obtained by integrating the expression the vertical stress for a point load along the line of load.

Let the vertical line load be of intensity q' for unit length along the y-axis, acting on the surface of a semi infinite soil mass as shown in the figure.



Let us considered the load acting on a small length δy. The load can be taken as a point load of q' δy and boussinesq's solution can be applied to determine the vertical stress at point p(x,y,z) so the normal stresses due to this point load

$$\Delta\sigma_z = \frac{3q'dy}{2\pi}\,\frac{z^3}{(r^2+z^2)^{5/2}}$$

The vertical stress at P due to the line load entending from $-\infty$ to $+\infty$ is obtained by Integrating

$$\therefore\ \sigma_z = \frac{3z^3}{2\pi}\int_{-\infty}^{+\infty}\frac{q'}{(r^2+z^2)^{5/2}}\,dy$$

$$= \frac{3z^3}{2\pi}\int_{-\infty}^{+\infty}\frac{q'}{(x^2+y^2+z^2)^{5/2}}\,dy$$

substitute $x^2+z^2 = u^2$

$$\sigma_z = \frac{3z^3}{2\pi}\int_{-\infty}^{+\infty}\frac{q'}{(u^2+y^2)^{5/2}}\,dy$$

$$= \frac{3q'z^3}{2\pi}\int_{-\infty}^{+\infty}\frac{dy}{(u^2+y^2)^{5/2}}$$

Take $y = u\tan\theta$ $\quad$ $\theta \to -\frac{\pi}{2}$ to $\frac{\pi}{2}$

$dy = u\sec^2\theta\,d\theta$

$$\therefore\ = \frac{3q'z^3}{2\pi}\int_{-\pi/2}^{\pi/2}\frac{u\sec^2\theta\,d\theta}{(u^2+u^2\tan^2\theta)^{5/2}}$$

$$= \frac{3q'z^3}{2\pi}\int_{-\pi/2}^{\pi/2}\frac{u\sec^2\theta\,d\theta}{u^{5}(1+\tan^2\theta)^{5/2}}$$

$$= \frac{3q' z^3}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{u \sec^2\theta \, d\theta}{u^5 \sec^5\theta}$$

$$= \frac{3q' z^3}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{d\theta}{u^4 \sec^3\theta}.$$

$$= \frac{3q' z^3}{2\pi u^4} \int_{-\pi/2}^{\pi/2} \cos^3\theta \, d\theta$$

$t = \sin\theta.$

$$= \frac{3q' z^3}{2\pi u^4} \int_{-\pi/2}^{\pi/2} \cos\theta (1-\sin^2\theta) \, d\theta.$$

$dt = \cos\theta \, d\theta.$

$t \to -1 \text{ to } 1$

$$= \frac{3q' z^3}{2\pi u^4} \int_{-1}^{1} 1-t^2 \, dt.$$

$$= \frac{3q' z^3}{2\pi u^4} \left(t - \frac{t^3}{3} \right)_{-1}^{1}$$

$\left(1-\frac{1}{3}-\left(-1-\frac{-1}{3}\right)\right)$

$\left(1-\frac{1}{3}+1-\frac{1}{3}\right)$

$\left(1-\frac{1}{3}\right)$

$2\left(1-\frac{1}{3}\right)$

$2\left(\frac{2}{3}\right)$

$$= \frac{3q' z^3}{2\pi u^4} \, 2\left(1-\frac{1}{3}\right)$$

$$= \frac{3q' z^3}{\pi u^4} \left(\frac{2}{3}\right) = \frac{2q' z^3}{\pi u^4} = \frac{2q' z^3}{\pi (x^2+z^2)^2}$$

$$= \frac{2q' z^3}{\pi z^4 \left(1+\left(\frac{x}{z}\right)^2\right)^2}$$

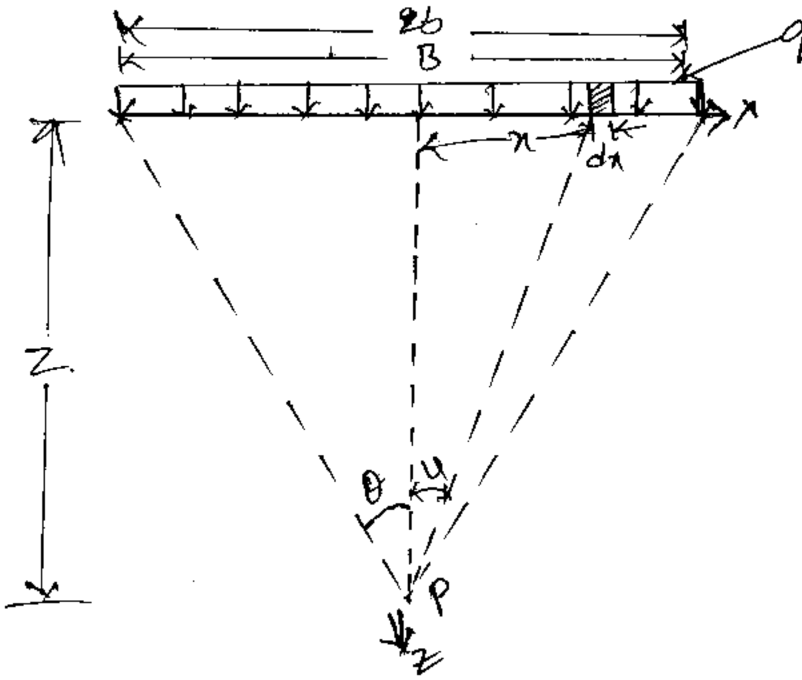
$$\boxed{\sigma_z = \frac{2q'}{\pi z \left(1+\left(\frac{x}{z}\right)^2\right)^2}}$$

(iii) vertical Stress under a strip load:-

The expression for vertical stress at any point P under a strip load can be develope from the expression developed for line load. The expression will depend upon wheather the point P lies below the centre of the strip load or not.

Note:- The length of the strip is very long. For convenience unit length is considered.

Point is below the centre of the strip. The following figur shows a strip load of with B(=2b)



Take an element of width dx from the distance x from the centre point of the strip load. The small load of q dx can be considered as a line load of intensity q'.

The verticle stress acting at the point P.

$$\Delta\sigma_z = \frac{2q\,dx}{\pi z\left(1+\left(\frac{x}{z}\right)^2\right)^2}$$

$$= \frac{2q\,dx}{\pi z}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2$$

The stress due to entire strip load is obtained by integration

$$\sigma_z = \frac{2q}{\pi z}\int_{-b}^{b}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2 dx$$

substitute $\frac{x}{z} = \tan u$ than $\frac{dx}{dz} = \sec^2 u\,du$

$dx = z\sec^2 u\,du$.

Take $\theta = \tan^{-1}\left(\frac{b}{z}\right)$ then

$x=b \quad u=\tan^{-1}\left(\frac{b}{z}\right)=\theta$

$x=-b \quad u=-\tan^{-1}\left(\frac{b}{z}\right)=-\theta$

$$\sigma_z = \frac{2q}{\pi z}\int_{-\theta}^{\theta}\frac{z\sec^2 u}{(1+\tan^2 u)^2}\,du$$

$\frac{1}{\sec\theta} = \cos\theta$

$$= \frac{2q}{\pi z}\cdot 2\int_{0}^{\theta}\frac{z\sec^2 u}{\sec^4 u}\,du$$

$$= \frac{4q}{\pi z}\int_{0}^{\theta} z\cos^2 u\,du = \frac{4q}{\pi}\int_{0}^{\theta}\cos^2 u\,du$$

$$= \frac{4q}{\pi}\int_{0}^{\theta}\left(\frac{1+\cos 2u}{2}\right)du = \frac{2q}{\pi}\left(u+\frac{\sin 2u}{2}\right)_0^{\theta}$$

$$= \frac{2q}{\pi}\left(\theta+\frac{\sin 2\theta}{2}\right)$$

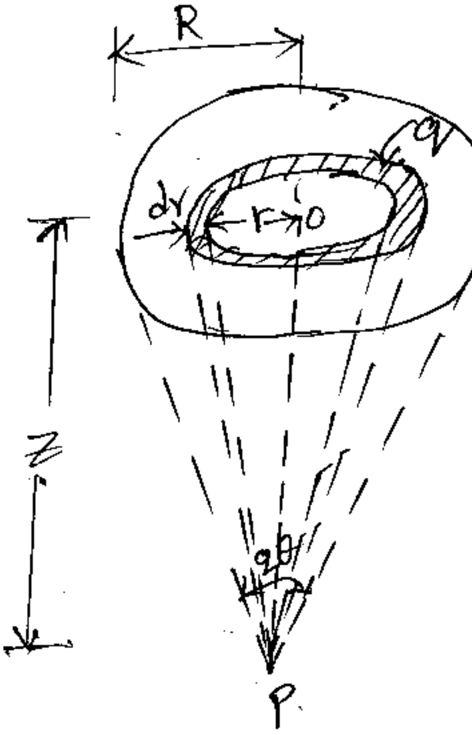
$$\boxed{\sigma_z = \frac{q}{\pi}\left(2\theta+\sin 2\theta\right)}$$

Vertical Stress under a circular Area :-

The load applied to soil surface by footings are not ~~concentrated~~ concentrated loads. These are usually spread over a finite area of footing. It is generally assumed that the footing is flexiable and the contact pressure is uniform. In other words the load is assumed to be uniformly distributed over the area of base of footings.

Let us determine the vertical stress at the point P at depth 'z' below the centre of a uniformly loaded circular area. Let the intensity of the load be 'q' per unit area, and 'R' be the radious of the loaded area.

The load on the elementary ring of radius 'R' and width 'dr' is equal to $q 2\pi r dr$. The load acts at a constant radia distance 'r' form the point 'P'. This load acts as a point load. The vertical stress due to this point load



$$\Delta\sigma_z = \frac{3q\,2\pi r dr}{2\pi z^2}\,\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

$$= \frac{3q}{z^2\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}} \, r\,dr$$

$$\Delta\sigma_z = \frac{3q\,z^3}{(r^2+z^2)^{5/2}} \, r\,dr$$

The vertical stress due to entaire load is given by integrating the above expression with in the limit 0 to R

$$\sigma_z = \int_0^R \frac{3q\,z^3}{(r^2+z^2)^{5/2}} \, r\,dr$$

$$= 3qz^3 \int_0^R \frac{r}{(r^2+z^2)^{5/2}} \, dr$$

substitute $r^2+z^2 = u$

$2r\,dr = du$

$r\,dr = \frac{1}{2} du$

$r = 0, \Rightarrow u = z^2$

$r = R \Rightarrow u = R^2+z^2$

$$\sigma_z = 3qz^3 \int_{z^2}^{R^2+z^2} \frac{\frac{1}{2}du}{u^{5/2}}$$

$$= \frac{3}{2}qz^3 \int_{z^2}^{R^2+z^2} u^{-5/2}\,du = \frac{3}{2}qz^3 \left[\frac{u^{-5/2+1}}{-\frac{5}{2}+1}\right]_{z^2}^{R^2+z^2}$$

$$= \frac{3}{2} q z^3 \times \frac{-2}{3} \left(u^{-3/2} \right]_{z^2}^{R^2+z^2}$$

$$= -q z^3 \left[(R^2+z^2)^{-3/2} - (z^2)^{-3/2} \right]$$

$$\sigma_z = -q z^3 (R^2+z^2)^{-3/2} + q z^3 z^{-3}$$

$$= q \left[1 - z^3 (R^2+z^2)^{-3/2} \right]$$

$$= q \left[1 - z^3 \cdot z^{-3} (1+(R/z)^2)^{-3/2} \right]$$

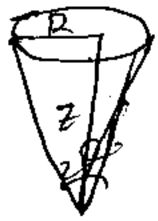
$$= q \left[1 - (1+(\frac{R}{z})^2)^{-3/2} \right]$$

$$\sigma_z = q \left[1 - \frac{1}{(1+(R/z)^2)^{3/2}} \right]$$

$$\sigma_z = I_c q$$

where $I_c = \left[1 - \frac{1}{(1+(R/z)^2)^{3/2}} \right]$

Value of I_c in terms of θ:-



from fig

$$\tan\theta = \frac{R}{z}$$

$$I_c = 1 - \frac{1}{(1+(\frac{R}{z})^2)^{3/2}}$$

$$= 1 - \frac{1}{(1+\tan^2\theta)^{3/2}}$$

$$= 1 - \frac{1}{(\sec^2\theta)^{3/2}}$$

$$\boxed{I_c = 1 - \frac{1}{\sec^3\theta} = 1 - \cos^3\theta}$$

Vertical stress under a corner of Rectangular Area:-

The vertical stress under a corner of rectangular area with a uniformly distributed load of intensity q can be obtained from Boussinesq's solution. From stress due to point load equation the stress at depth z is given by, taking $dQ = q\,dA = q\,dx\,dy$

$$\Delta\sigma_z = \frac{3(q\,dx\,dy)z^3}{2\pi}\frac{1}{(x^2+y^2+z^2)^{5/2}}$$

By integration

$$\sigma_z = \frac{3qz^3}{2\pi}\int_0^L\int_0^B \frac{q\,dx\,dy}{(x^2+y^2+z^2)^{5/2}}$$

Although the integral is quite complicated. Newmark was able to perform it the result were presented as follows.

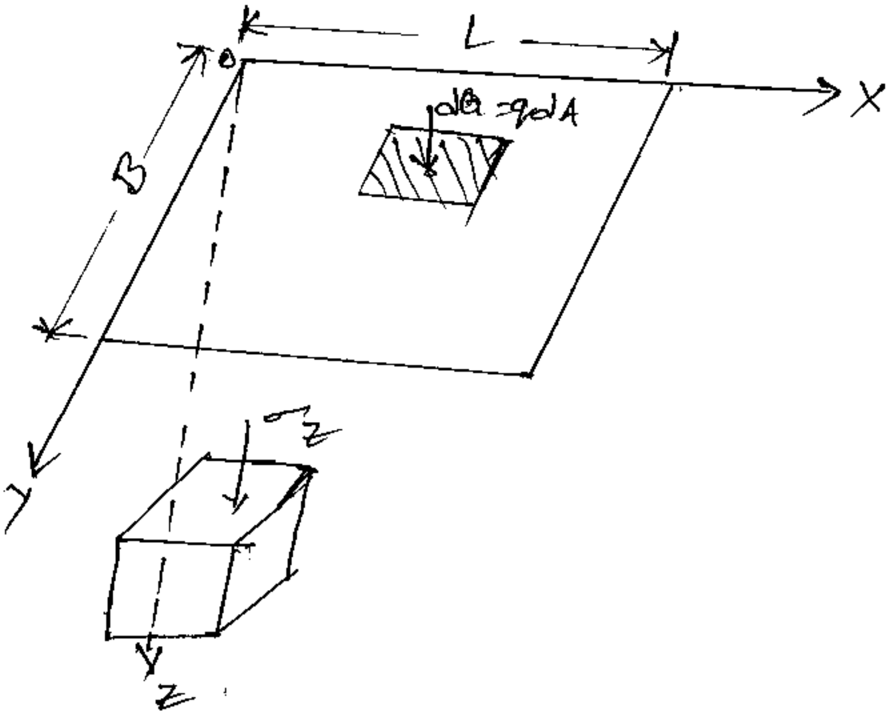
$$\sigma_z = \frac{q}{2\pi}\left[\frac{mn}{\sqrt{m^2+n^2+1}}\,\frac{m^2+n^2+2}{m^2+n^2+m^2n^2+1} + \sin^{-1}\left(\frac{mn}{\sqrt{m^2+n^2+m^2n^2+1}}\right)\right]$$

Where $m = \frac{B}{z}$ and

$n = \frac{L}{z}$

The values of m and n can be interchanged without any effect on the values of σ_z

$$\therefore \boxed{\sigma_z = I_N q}$$



wher I_N is newmark's influence co-efficient, given by

$$I_n = \frac{1}{2\pi}\left[\frac{mn}{\sqrt{m^2+n^2+1}} \cdot \frac{m^2+n^2+2}{m^2+n^2+m^2n^2+1} + \sin^{-1}\left(\frac{mn}{m^2+n^2+m^2n^2 ...}\right)\right]$$

Newmark's Charts:-

Newmarks influence charts are prepared based on boussinesq's solution to find the vertical stresses generated by different types of footings. These vertical stresses due to circular loads are taken as basis for preparing the charts.

In these charts the circles whoes area is in proportion to the vertical stresses which are generated by the footings which are in same size as though circles in the charts.

In practice some times the engineer as to find the vertical stresses under a uniformly loaded area as of other shapes. In such cases. Newmark influence in compressibility, then a permits ~~only downward displacement of the soil mass~~ Charts are entremely used. Newmark's chart in based on the concept of the vertical stress below the centre of circular area.

Let us consider a uniformly loaded circular area of radius R_1 divided into 20 equal sectors.

The vertical stress at point 'p' at depth 'z' just below the centre of loaded area due to load on one sector will be $\frac{1}{20}$th of that due to load on full circle.

$$\sigma_z = \frac{1}{20} q\left[1-\left(\frac{1}{1+(R/z)^2}\right)^{3/2}\right]$$

In the vertical stress σ_z is given an orbitary fined value say $0.005q$

sub. above value in enpression

$$\frac{R_1}{z} = 0.27$$

That means every $1/20$th sector of the circle with a radius R_1 equal to $0.27z$ would give a vertical stress of $0.005q$. at its centre.

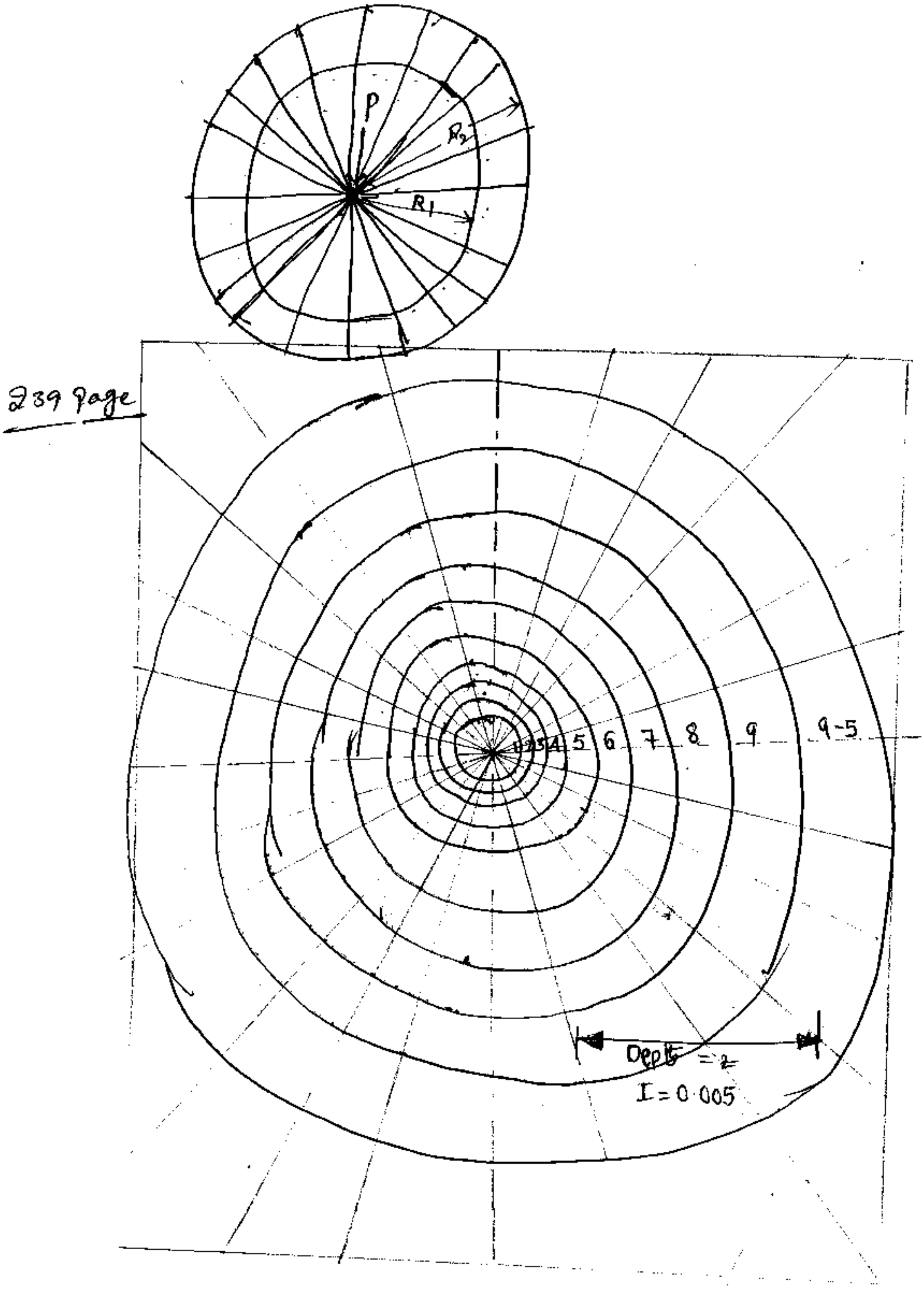
Let us considered another countric circle of radius R_2 and divide it into 20 equal sectors. Each larger sections is divided into two sub area. If the small are a q exerts a stress of $0.005q$ at P. The vertical stress due to both area 1 and area 2 would be equal to $2 \times 0.005q$.

$$2 \times 0.005q = \frac{q}{20}\left[1 - \frac{1}{\left(1+\left(\frac{R}{z}\right)^2\right)^{3/2}}\right]$$

By solving the above expression $\frac{R_2}{z}$ become 0.4

In other words the radius of second circle would be equal to $0.4z$. likewise the radii of the third to the 9th circle can be determined. The values obtained are $0.52z$, $0.64z$, $0.77z$, $0.92z$, $1.11z$, $1.39z$ and $1.91z$ the radius of 9½ circle is $2.54z$ like wise the radius of 10th circular will become Infinity. There for 10th circle cannot be drawn. Like these newmark's charts are prepared for different position ~~differently~~ differently.

while preparing newark charts the co-efficient will multiply with the load is called influence co-efficient. In the above case the influence co-efficient is 0.005.



Westergaard's Solution:-

Boussinesq' solution assumes that the soil deposit is isotropic. Actual sedimentary deposits are generally anisotropic. There are generally thin layers of sand embedded in homogeneous clay strata. Westergaard's solution assumes that there are thin sheets of rigid materials sand-wiched in a homogeneous soil mass. These thin sheets are closely spaced and are of infinite rigidity and are, therfore, incompressible. These permit only downward displacement of the soil mass as a whole without any lateral displacement. Therefore, Westergaard's solution represents more closely the actual sedimentary deposit.

According to Westergaard, the vertical stress at a point P at a depth Z below the concentrated load 'Q' is given by

$$\sigma_z = \frac{c/2\pi}{\left[c^2+\left(\frac{r}{z}\right)^2\right]^{3/2}} \cdot \frac{Q}{z^2}$$

where c depends upon the poisson ratio(ν) and is given by

$$c = \sqrt{(1-2\nu)/(2-2\nu)}$$

For elastic materials ν ~~ratio~~ varies 0 to 0.5

when ν is zero c will become $\frac{1}{\sqrt{2}}$. then

$$\sigma_z = \frac{\frac{1}{8\sqrt{2}\pi}}{\left[\frac{1}{2}+\left(\frac{\sqrt{2}r}{cz}\right)^2\right]^{3/2}} \times \frac{Q}{z^2}$$

$$\sigma_z = \frac{1}{\pi\left[1+2\left(\frac{r}{z}\right)^2\right]^{3/2}} \cdot \frac{Q}{z^2}$$

$$\boxed{\sigma_z = I_W \frac{Q}{z^2}}$$

where I_W is known as westergaard influence coefficien

$$\boxed{I_W = \frac{1}{\pi\left[1+\left(\frac{2r}{z}\right)^2\right]^{3/2}}}$$

The values of I_W are considerably smaller than the Boussinesq influence factor (I_B).

Problems

1) A countrated load of 40KN is applied vertically on a horizontal ground surface. Determine the vertical stress intensities at the following points.

(i) At a depth of 2m below the point of application of the load

(ii) At a depth of 1m and at a radial distance of 3m from the line of action of the load

(iii) At a depth of 3m and at a radial distance of 1m from the line of action of the load.

S9) The Boussinesq's solution $\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}\left(\frac{1}{(1+(r/z)^2)^{5/2}}\right)$

$Q = 40 kN = 4 tones$

(i) $r = 0 \quad z = 2m$

$\frac{r}{z} = 0$

$\sigma_z = \frac{3}{2\pi} \times \frac{40}{2^2} \cdot \frac{1}{1^{5/2}} = 4.77 \; kN/m^2$

(ii) $z = 1m, \; r = 3m$

$\frac{r}{z} = 3$

$\therefore \sigma_z = \frac{3}{2\pi} * \frac{40}{1^2} \cdot \frac{1}{(1+9)^{5/2}} = 0.06 \; kN/m^2$

(iii) $z = 3m, \; r = 1m$

$\frac{r}{z} = \frac{1}{3} = 0.333$

$\therefore \sigma_z = \frac{3}{2\pi} \times \frac{40}{3^2} \times \frac{1}{(1+0.33^2)^{5/2}} = 1.63 \; kN/m^2$

3) A rectangular footing 2m×3m in size has to carry a uniformly distributed load of 100kN/m². plot the distribution of vertical stress intensity on a horizontal plane at a depth of 2m below the base of the footing by the following methods. and compare those two distribution

(i) Boussinesq's Solution

(ii) Two to one dispersion method.

Sq) (a) load = 100 kN/m^2

Point load $Q = 100 \times 2 \times 3$

$= 600$ kN

$$\sigma_z = \frac{3}{2\pi} \cdot \frac{Q}{z^2}\left(\frac{1}{(1+(r/z)^2)^{5/2}}\right)$$

$$= \frac{3}{2\pi} \cdot \frac{600}{2^2}\left(\frac{1}{(1+(r/2)^2)^{5/2}}\right)$$

(i) Boussinesq's $r=0$, $\sigma_z = 71.61$ kN/m^2

Put $r=1$ $\sigma_z = 40.99$ kN/m^2 $r=-1$, $\sigma_z = 40.99$ kN/m^2

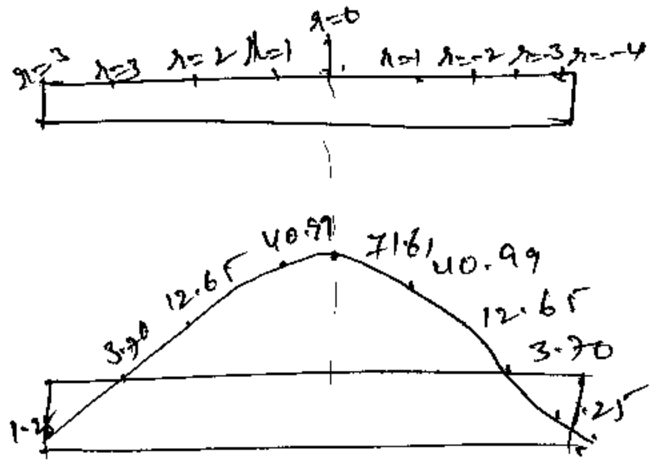
$r=2$ $\sigma_z = 12.65$ kN/m^2 $r=-2$, $\sigma_z = 12.65$ kN/m^2

$r=3$, $\sigma_z = 3.76$ kN/m^2 $r=-3$, $\sigma_z = 3.76$ kN/m^2

$r=4$, $\sigma_z = 1.28$ kN/m^2 $r=-4$, $\sigma_z = 1.28$ kN/m^2

(ii) 2:1 despersion method

$$\sigma_z = \frac{q(B \times L)}{(B+z)(L+z)} = \frac{100 \times 2 \times 3}{(2+2)(2+3)} = 30 \text{ kN/m}^2$$



3) concentra load of 200kN is applied at the ground surface. determin the vertical stress at a point P which is 6m directly below the load. Also calculate the vertical stress at a point R which is at a depth of 6m but a horizontal distance of 5m from the axis of the load.

so) load intensity $Q = 200kN$

$z = 6m$

$r = 5m$

Vertical stress $\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}\frac{1}{(1+(r/z)^2)^{5/2}}$

$= \frac{3 \times 2000}{2 \times 3.142 \times 6^2}\frac{1}{(1+(\frac{5}{6})^2)^{5/2}}$

$= 7.096 \text{ kN/m}^2$

P is directly below the load then

$z = 6m$, $r = 0m$

$\sigma_z = \frac{3 \times 2000}{2 \times 3.142 \times 6^2} \times \frac{1}{(1+(\frac{0}{6})^2)^{5/2}}$

$= 26.52 \text{ kN/m}^2$

4) There is a line load of 120 kN/m acting on the ground surface along y-axis, Determine the vertical stress at Points P, Q whose (x, z) ordinates are as follow.

$$P(x, z) = (2, 3.5)$$
$$Q(x, z) = (3, 4.5)$$

So.) for line load $\sigma_z = \dfrac{2q'}{\pi z\left(1+\left(\frac{x}{z}\right)^2\right)^2}$

$P(x, z) = (2, 3.5)$, $q = 120$ kN/m $= q'$

$$\sigma_z = \frac{2 \times 120}{\pi \times 3.5\left(1+\left(\frac{2}{3.5}\right)^2\right)^2}$$

$$= 12.4 \text{ kN/m}^2$$

$Q(x, z) = (3, 4.5)$

$$\sigma_z = \frac{2 \times 120}{\pi \times 4.5\left(1+\left(\frac{3}{4.5}\right)^2\right)^2}$$

$$= 8.135 \text{ kN/m}^2$$

5) The unit weight of the soil in a uniform deposite of loose Sand is 16.5 kN/m3. Determine the geostatic stresses at a depth of 2m. Take the coefficient of static earth pressure $K_0 = 0.50$

so) $\gamma = 16.5$ kN/m³

$z = 2m, \quad K_0 = 0.50$

$\sigma_z = \gamma z$

$= 16.5 \times 2 = 33$ kN/m²

$\sigma_h = K_0 \sigma_z = 0.50 \times 33 = 16.5$ kN/m²

6) Determine the vertical stress at point P which is 3m below and at a radial distance of 3m from the vertical load of 100kN. used both boussinesq and westergard solution and compare the results and comment.

so.) $z = 3m, \quad r = 3m, \quad Q = 100$ kN

Bossinesq's solution, vertical stress

$$\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}\left(\frac{1}{1+(r/z)^2}\right)^{5/2}$$

$$= \frac{3 \times 100}{2\pi \times 3^2}\frac{1}{(1+(3/3)^2)^{5/2}}$$

$= 0.937$ kN/m²

westergard solution

$$\sigma_z = \frac{1}{\pi(1+2(r/z)^2)^{3/2}} \times \frac{Q}{z^2}$$

$$= \frac{1}{\pi(1+2(3/3)^2)^{3/2}} \times \frac{100}{3^2} = 0.68 \text{ kN/m}^2$$