

## UNIT - III

### PERMEABILITY

#### Soil Water :-

Water present in the voids of soil mass is called Soil Water.

The soil water is broadly classified into two categories

1. free water.
2. Held water.

free water moves in the pores of the soil under the influence of gravity. The held water is retained in the pores of the soil, and it can not move under the influence of gravitational force.

free water flows from one point to the other wherever there is a difference of total head.

Held water is further divided into three types.

(i) Structural Water :- The structural water is chemically combined water in the crystal structure of the mineral of the soil.

This water can not be removed without breaking the structure of the mineral.

(ii) Adsorbed Water :- The water held by electrochemical forces existing on the soil surface is known as adsorbed water (or) hygroscopic water.

(iii) Capillary Water :- The water held in the interstices of soil due to capillary forces is called capillary water.

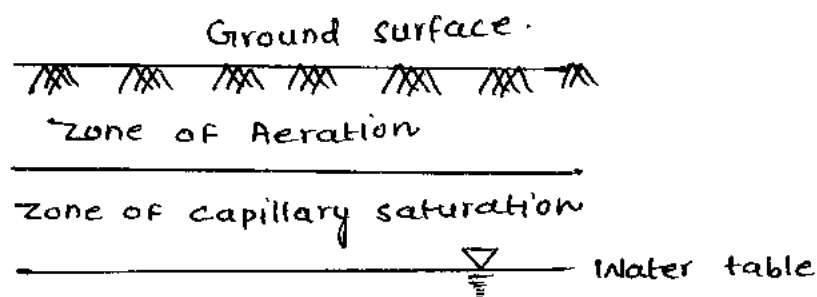
## Capillary rise in soils

capillary rise in soils depends upon the size and grading of the particles. The diameter ( $d$ ) of channel in pore passage depends upon the diameter of the particle. it is generally taken as  $\frac{1}{5}$ th diameter of the effective diameter ( $D_{10}$ ) in the case of coarse-grained.

$$\text{Thus } d = 0.2 D_{10}$$

The space above the water table can be divided into two regions.

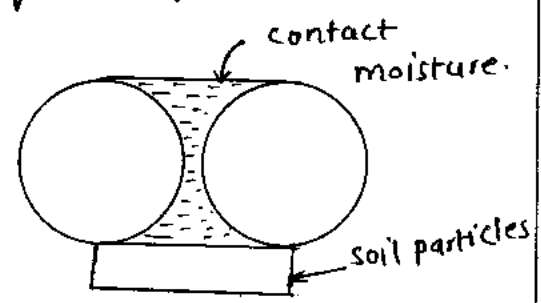
1. Zone of capillary saturation
2. Zone of Aeration.



- \* In zone of capillary saturation the soil is fully saturated
- \* In zone of Aeration the soil is not saturation.
- \* The height to which capillary water rises in soils is known as "capillary fringe".
- \* The soil above the capillary fringe may contain water in the form of contact water.

\* Terzaghi and Pick (1948) gave a relation between the maximum height of capillary fringe and the effective size as.

$$(h_c)_{max} = \frac{c}{e D_{10}}$$



Where  $c$  = constant, depending upon the shape of the grain and impurities

$e$  = void ratio.

$D_{10}$  = effective diameter, the size corresponding to 10% finer.

if  $D_{10}$  is in mm, the value of  $c$  varies between 10 to 50 mm<sup>2</sup>, and the height  $(h)_{max}$  is also given in mm. if  $D_{10}$  and  $(h_c)_{max}$  are in centimeters,

$$c = 0.1 \text{ to } 0.5 \text{ cm}^2$$

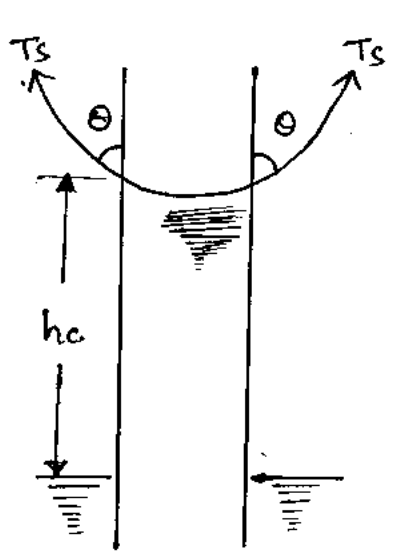
Representative heights of capillary rise.

S.No	Soil type	capillary rise (m)
1.	fine gravel	0.02 to 0.10
2.	coarse sand	0.10 to 0.15
3.	fin sand	0.30 to 1.00
4.	silt	1.0 to 10.0
5.	clay	10.0 to 30.0
6.	colloid	More than 30.0

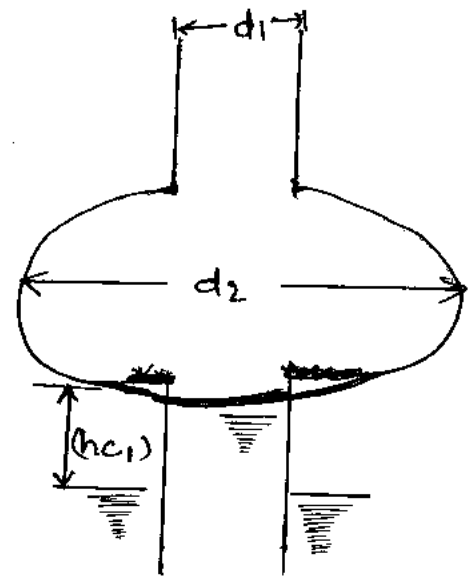
## Capillary Rise in small diameter tubes $\div$

Water rises in small diameter, capillary tubes, because of adhesion and cohesion. Adhesion occurs because water adheres or sticks to the solid walls of the tubes. Cohesion is due to mutual attraction of water molecules. If the effect of cohesion is less significant than the effect of adhesion, the liquid wets the surface and the liquid rises at the point of contact. However if the effect of cohesion is more predominant than adhesion, the liquid level is depressed at the point of contact as in the case of mercury.

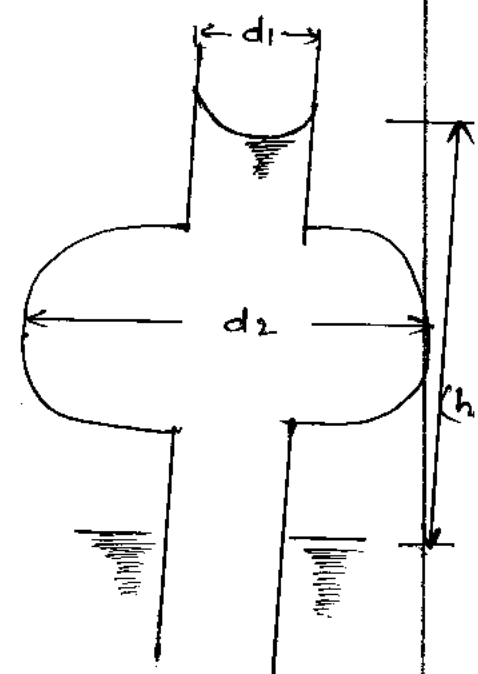
If a glass tube of small diameter, open at both ends, is lowered into water, the water level rises in the tube, as the water wets the tube. Let ' $\theta$ ' be the angle of contact between the water and the wall of the tube.



(a)



(b)



(c)

$F_u =$  up ward pull due to surface tension  $= (T_s \cos \theta) \pi d$

Where  $T_s =$  surface tension and 'd' diameter of the tube

$f_d =$  Down word force due to mass of water in the tube.

$$= \gamma_w \left( \frac{\pi}{4} d^2 \right) \times h_c$$

where  $h_c =$  height of capillary rise.

for equilibrium

$$f_u = f_d$$

$$(T_s \cos \theta) \cdot \pi d = \gamma_w \left( \frac{\pi}{4} d^2 \right) h_c$$

$$h_c = \frac{4 T_s \cos \theta}{\gamma_w} = \frac{4 T_s \cos \theta}{\rho_w g d}$$

for a clean glass tube and pure water, the meniscus is approximately hemispherical, i.e.  $\theta = 0$

Therefore,

$$h_c = \frac{4T_s}{\gamma_w d}$$

Taking  $T_s = 0.073 \text{ N/m}$ ,  $\gamma_w = 9810 \text{ N/m}^3$

$$h_c = \frac{4 \times 0.073}{9810 d} = \frac{3 \times 10^{-5}}{d} \text{ meters.}$$

Where  $d$  is in meters.

if  $d$  is in centimeters

$$h_c = \frac{3 \times 10^{-3}}{d} \text{ meters.}$$

if  $h_c$  and  $d$  both are in cm

$$h_c = \frac{0.3}{d} \text{ cm.}$$

Formulas:

$$1. \quad h_c = \frac{4 T_s \cos \theta}{\rho_w d} \quad (\text{or}) \quad \frac{4 T_s \cos \theta}{\gamma_w d}$$

$$2. \quad h_c = \frac{c}{e D_{10}}$$

$$3. \quad h_c = \frac{0.30}{d} \text{ cm.}$$

→ ① What is the -ve pressure in the water just below the meniscus in a capillary tube of diameter 0.1 mm, filled with water, the surface tension is 0.075 N/m, and wetting angle is  $10^\circ$ .

Sol:

$$h_c = \frac{4T_s \cos \theta}{\gamma_w d}$$

$$= \frac{4 \times 0.075 \times 0.9848}{9810 \times 0.1 \times 10^{-3}} = 0.301 \text{ m.}$$

$$\begin{aligned} \text{-ve pressure} &= \gamma_w h_c = 9810 \times 1000 \times 0.301 \\ &= 2952.81 \text{ N/m}^2. \end{aligned}$$

→ ② Estimate the capillary rise in a soil with a void ratio of 0.60 and effective size of 0.01 mm. Take  $c = 15 \text{ mm}^2$ .

$$\begin{aligned} \text{Sol: } h_c &= \frac{c}{e D_{10}} = \frac{15}{0.6 \times 0.01} \\ &= 2500 \text{ mm} \\ &= 2.5 \text{ m.} \end{aligned}$$

③ The capillary rise in a soil A with an effective size of  $0.02 \text{ mm}$ ;  $60 \text{ cm}$ . Estimate the capillary rise in a similar soil B with an effective size of  $0.04 \text{ mm}$ .

Sol: 
$$\frac{(hc)_1}{(hc)_2} = \frac{(D_{10})_2}{(D_{10})_1}$$

$$\frac{60}{(hc)_2} = \frac{0.04}{0.02} = 2 \quad \text{or} \quad (hc)_2 = 30 \text{ cm}$$

→ ④ The capillary rise in silt is  $50 \text{ cm}$  and that in fine sand is  $30 \text{ cm}$ . What is the difference in the pore size of the two soils?

Sol: 
$$hc = \frac{0.30}{d} \text{ cm}$$

for silt  $(hc)_1 \Rightarrow 50 = \frac{0.30}{d_1} \Rightarrow d_1 = 6.0 \times 10^{-3} \text{ cm}$

for fine sand  $(hc)_2 \Rightarrow 30 = \frac{0.30}{d_2} \Rightarrow d_2 = 10.0 \times 10^{-3} \text{ cm}$

Difference in pore size =  $(10.00 - 6.0) \times 10^{-3}$

$$= 4.00 \times 10^{-3} \text{ cm}$$

≡



### DARCY'S LAW:-

The law of flow of water through soil was first studied by Darcy (1856) who demonstrated experimentally that for laminar flow conditions in a saturated soil, the rate of flow or the ~~the~~ velocity per unit time is proportional to the hydrolic gradient.

$$v \propto i$$

$$v = ki$$

$k$  = co-eff of permeability

$i$  = hydrolic gradient.

The discharge ' $q$ ' is obtained by multiplying the velocity of flow ( $v$ ) by the total cross-section area of soil ( $A$ ) normal to the direction of flow.

$$\text{Thus } q = vA$$

$$q = kiA$$

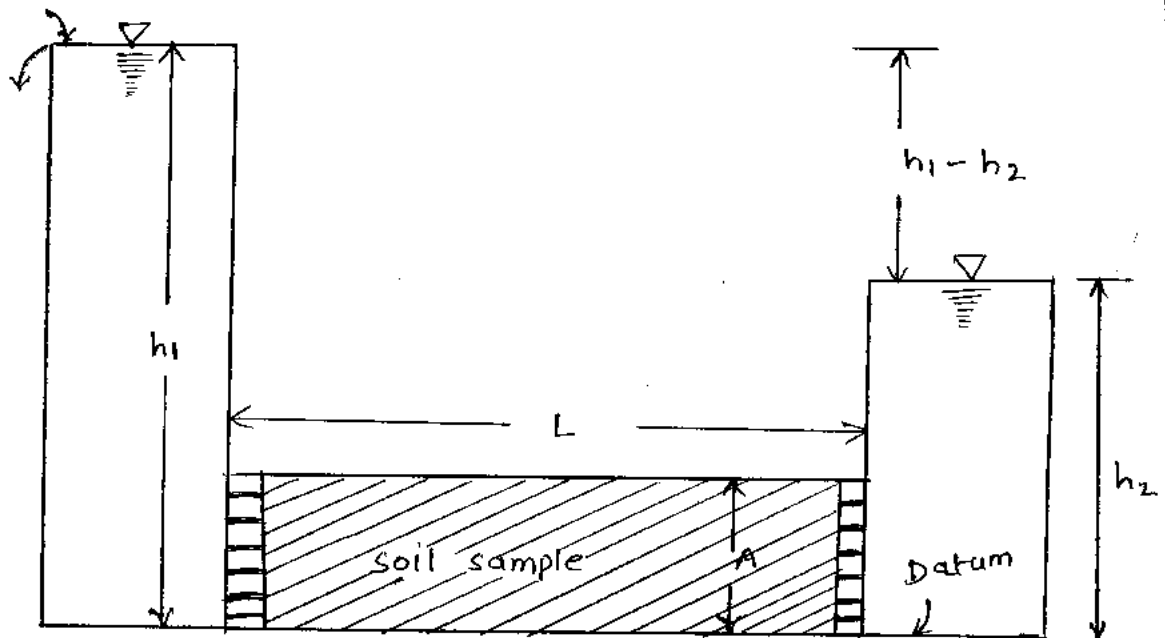
Where,  $q$  = discharge per unit time

$A$  = total c/s area of soil mass.

if a soil sample of length ~~l~~ and c/s area 'A' is subjected to difference head of water  $h_1 - h_2$ , the hydrolic gradient  $i$  will be equal to  $\frac{h_1 - h_2}{L}$  and, we have

$$q = K \frac{h_1 - h_2}{L} A.$$

Where,  $i$  is unity,  $K$  is equal to the v. the co-eff of Permiability is defined as Avg velocity of Flow that will occur through the total c/s area of soil under unit hydrolic gradient.



\* flow of water Through soil \*

## Permeability of soil :-

" The property of a soil which permits flow of water (or only other liquid) through it is called the permeability of soil " A soil is highly pervious when water can flow through it easily.

In an impervious soil, the permeability is very low and water can not easily flow through it.

Permeability is a very important engineering property of soils. A knowledge of permeability is essential in a number of soil engineering problems.

Such as settlement of buildings, yield of wells, seepage through and below the earth structures.

It controls the hydrolic stability of soil masses.

The permeability of soil is also required in the design of filters used to prevent piping in hydrolic structures.

## factors affecting permeability of soil :

In the laminar flow through porous media.

$$k = c \left( \frac{\gamma_w}{\mu} \right) \left( \frac{e^3}{1+e} \right) D^2$$

Where, -

$c$  = co-efficient

$\gamma_w$  = unit weight of water

$e$  = void ratio.

$\mu$  = viscosity

$D$  = particle size.

The flowing factors affects the permeability of soil.

1. particle size : The co-efficient of permeability of a soil is proportional to the square of the particle size ( $D$ ). The permeability of coarse-grain soil is very large as compared to that of fine-grained soils. The permeability of coarse sand may be more than one million times as much that of clay.

2. structure of soil mass :

The size of the flow passage depends upon the structural arrangement. For the same void ratio

The permeability is more in the case of flocculated structure as compared to that in the dispersed structure. Stratified soil deposits have greater permeability parallel to the plane of stratification than that of '⊥' to this plane. permeability of soil ~~deposit~~ ~~also~~ deposit also depends upon shrinkage cracks, joints, fissures and shear zones. Loos deposits have greater permeability in the vertical direction than in the horizontal direction.

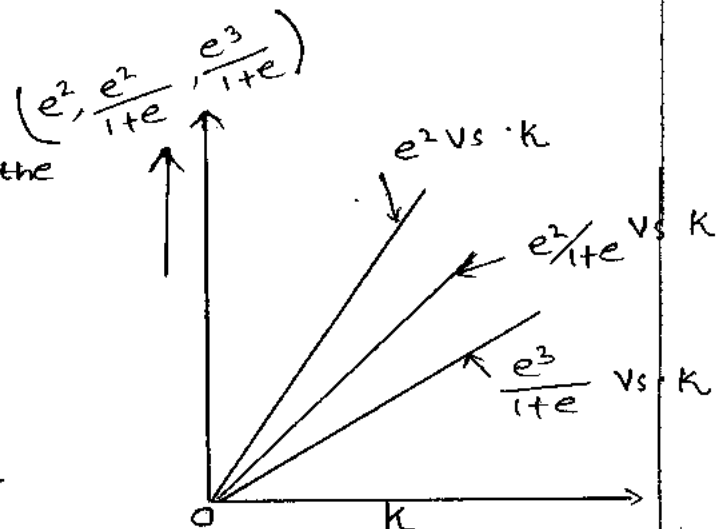
→ 3. Shape of particles

The permeability of a soil depends upon the shape of particles. Angular particles have greater specific surface area as compared with the rounded particles. For the same void ratio, the soil with angular particles are less permeable than those with rounded particles.

→ 4. Void ratio

The above equation indicated the co-eff of permeability  $\frac{e^3}{1+e}$

So the void ratio is greater for the given sample them.



The permeability is also higher the graph plot between the void ratio and co-eff. of permeability is coming a straight line.

→ 5. Property of water

The co-efficient of permeability is directly proportional to the unit weight of water ( $\gamma_w$ ) and is inversely proportional to the viscosity ( $\mu$ ). The unit weight of water does not vary much over the range of temperature ordinarily encountered in soil engg. problems. However there is a large variation in the value of the co-efficient of viscosity ( $\mu$ ).

The  $K$  increases with an increase in temperature due to reduction in viscosity.

→ 6. Degree of saturation

If the soil is not fully saturated, it contains air pockets formed due to entrapped air (or) due to air liberated from percolating water. Whatever may be the cause of the presence of air in soil, the permeability is reduced due to presence of air which

causes blockage of passage.

Consequently, the permeability of a partially saturated soil is smaller than the fully saturated soil.

#### 7. Adsorbed Water

The fine-grained soil have a layer of adsorbed water strongly attracted to their surface.

This adsorbed water layer is not free to move under gravity. It causes an obstruction to flow of water in the pores and hence reduces the permeability of soil.

#### 8. Impurities in Water

Any foreign matter in water has a tendency to plug the flow passage and reduce the effective voids and hence the permeability of soils.

## Laboratory Methods For permeability test

The co-efficient of permeability of a soil sample can be determined by the following Methods.

[1] constant head Method.

[2]. Variable head Method.

### 1. constant head Method

The co-efficient of permeability of a relatively more permeable soil can be determined in a laboratory by the constant-head permeability test.

The test is conducted in an instrument known as constant-head permeameter test.

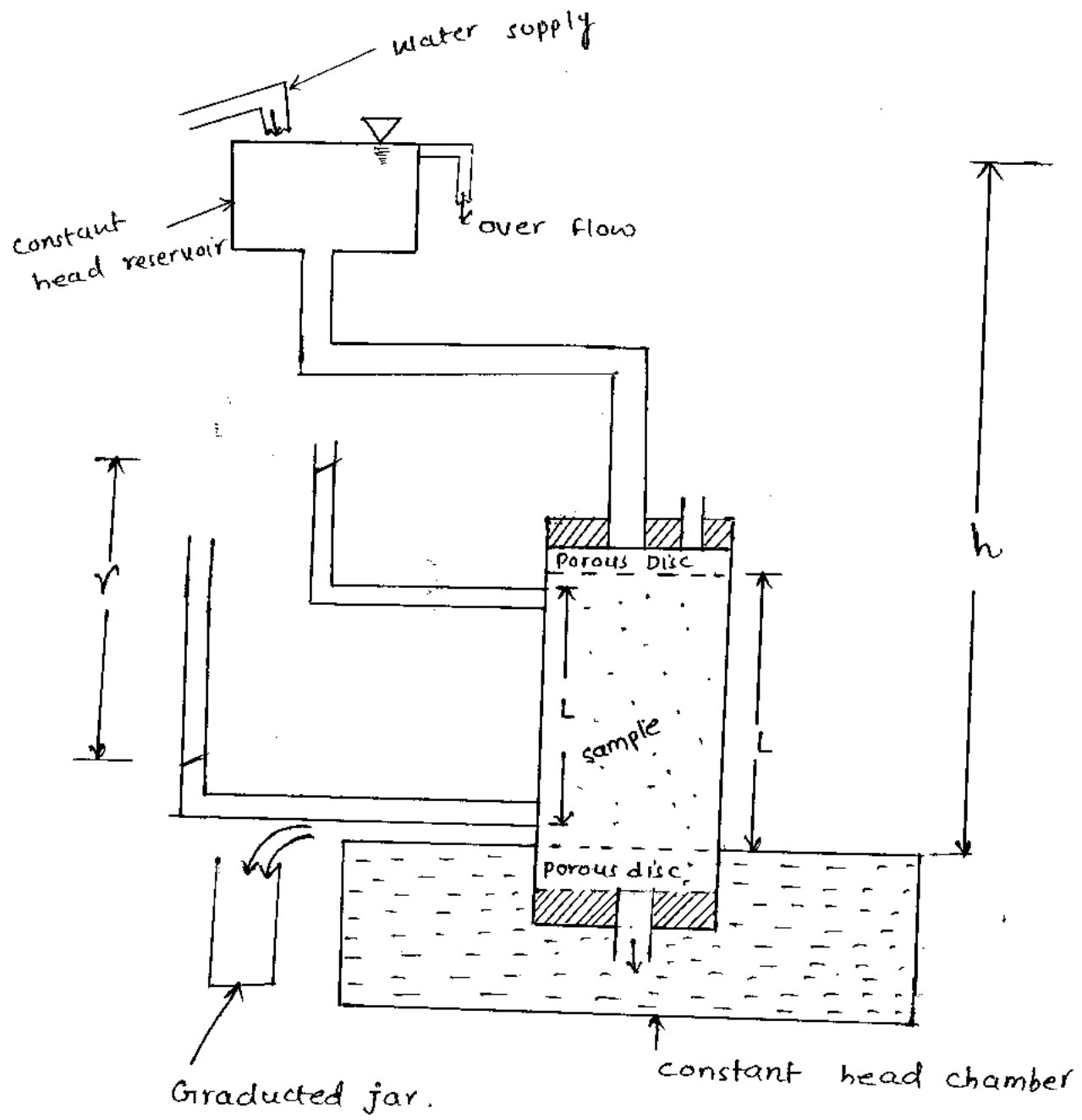
It consist of a metallic mould, 100 mm internal diameter, 127.3 mm effective height and 1000 ml capacity according to IS : 2720 (part XVII).

The mould is provided with a detachable extension collar, 100 mm diameter and 60 mm height, required during compaction of soil. The mould is provided with a drainage base plate with a recess for porous stone. the mould is fitted with a



a discharge cap having an inlet valve and an air release valve.

The discharge base and cap have fitting for clamping to the mould.



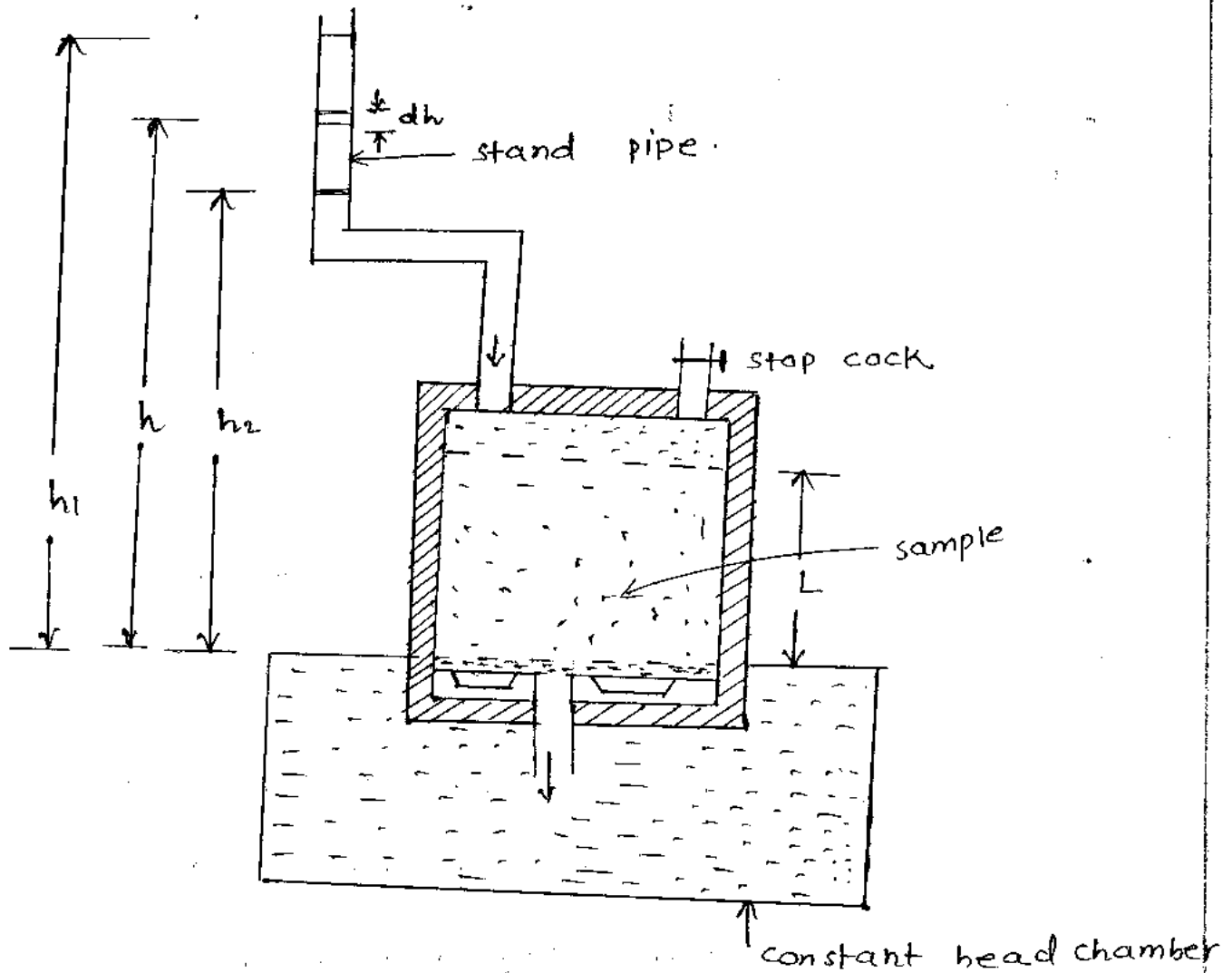
## Procedure:

- ① Remove the cover of the mould and apply a little grease on the sides of the mould.
- ② Measure the internal diameter and effective height of the mould and then attach the collar and the base plate.
- ③ Compact the soil at required density and moisture content.
- ④ Remove the collar and base plate. trim off the excess soil level with the top of the mould.
- ⑤ Put the porous plate and a filter paper both at top and bottom of the soil sample.
- ⑥ Place this assembly with washer on the porous stone.
- ⑦ Connect the reservoir with water to the inlet at the top of the mould and allow water to flow in till the sample gets saturated.
- ⑧ Allow the water to flow through the soil and establish a steady flow.
- ⑨ Collect the water in a measuring jar for a convenient time interval 't' see
- ⑩ Repeat step (9) for four or five times and tabulate the result as follows.

S.No	Quantity of water collected (V) in c.c	Time of collection t (sec)	$q = \frac{V}{t}$	Head over the sample (H)	$K = \frac{qL}{AH}$ cm/sec
1.					
2.					
3.					
4.					
5.					

variable - head permeability test

For relatively less permeable soils, the quantity of water collected in the graduated jar of the constant - head permeability test is very small and can not be measured accurately. For such soils, the variable - head permeability test is used. A vertical, graduated stand pipe of known diameter is fitted to the top of permeameter. The sample is placed between two porous discs. The whole assembly is placed in a constant head chamber filled with water to the brim at the start of the test.



\* Variable head permeameter \*

Let us consider the instant when the head is 'h'  
 for the infinitesimal small time 'dt' the head  
 falls by dh. let the discharge through the sample  
 be q. from continuity of flow.

$$a dh = - q dt$$

$$a dh = - q dt$$

Where  $a$  is cross-sectional area of the stand pipe

$$a dh = - (A \times K \times i) \times dt$$

$$a dh = - AK \times \frac{h}{L} \times dt$$

$$\frac{AK dt}{aL} = \frac{-dh}{h}$$

integrating  $\frac{AK}{aL} \int_{t_1}^{t_2} dt = - \int_{h_1}^{h_2} \frac{dh}{h}$

$$\frac{AK}{aL} (t_2 - t_1) = \log_e (h_1/h_2)$$

$$K = \frac{aL}{At} \log_e (h_1/h_2)$$

$$(or) \quad K = \frac{2.30aL}{At} \log_{10} (h_1/h_2)$$

Procedure :

Step ① to step ⑥ is same as the constant-head Method procedure.

→ ⑦ connect the stand pipe to the inlet at the top plate and fill the stand pipe with water.

- ⑧ open the stop clock at the top and allow water to flow out so that all the air in the cylinder is removed
- ⑨ Allow water to flow through the soil and establish a steady flow.
- ⑩ Record the time intervals for the head to fall from  $h_1$  to  $h_2$  for five times, and tabulate the results as follows.

<u>S.No</u>	$h_1$	$h_2$	Time interval 't'	$k = 2.3 \frac{aL}{At} \log_{10} \left( \frac{h_1}{h_2} \right)$

$a =$  c/s area of stand pipe.

$A =$  c/s area of soil sample

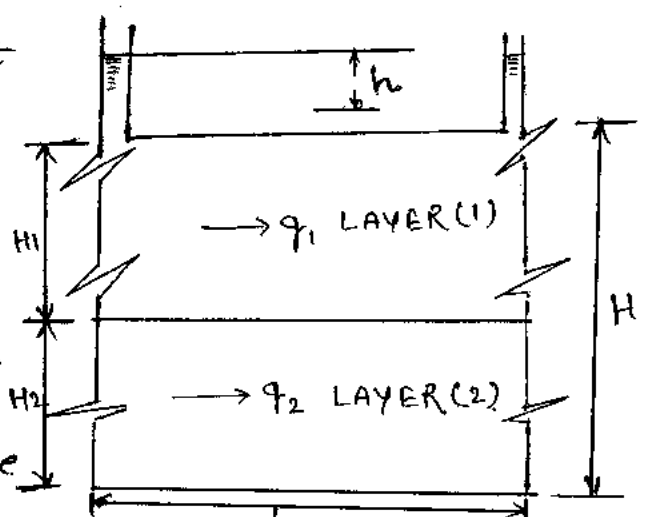
$L =$  length of the soil sample.

Permeability of layered systems

A stratified soil deposit consists of a number of soil layers having different permeabilities. The Avg permeability of deposit as a whole parallel to the planes of stratification and that normal of the planes of below.

① Flow parallel to planes of stratification

Let us consider a deposit consisting of two horizontal layers of soil of thickness  $H_1$  and  $H_2$  as shown in fig.



for flow parallel to the planes of stratification, the loss of head (h) over a length L is the same for the both the layers. therefore, the hydrolic gradient (i) for each layer is equal to the hydrolic gradient of entire deposite. the system is analogous to the two resistance in parallel in an electrical circuit, where in the potential drop is the same in both the resistance.

From the continuity equation, the total discharge ( $q$ ) per unit width is equal to the sum of the discharges in the individual layers i.e.,

$$q = q_1 + q_2$$

Let  $(k_h)_1$  and  $(k_h)_2$  be the permeability of the layers 1 and 2 respectively, parallel to the plane of stratification direction.

from equation (a) using Darcy's law.

$$k_h \times i \times (H_1 + H_2) = (k_h)_1 \times i \times H_1 + (k_h)_2 \times i \times H_2$$

$$k_h = \frac{(k_h)_1 \times H_1 + (k_h)_2 \times H_2}{H_1 + H_2}$$

if there are  $n$  layers

$$k_n = \frac{(k_n)_1 \times H_1 + (k_n)_2 \times H_2 + \dots + (k_n)_n \times H_n}{H_1 + H_2 + \dots + H_n}$$

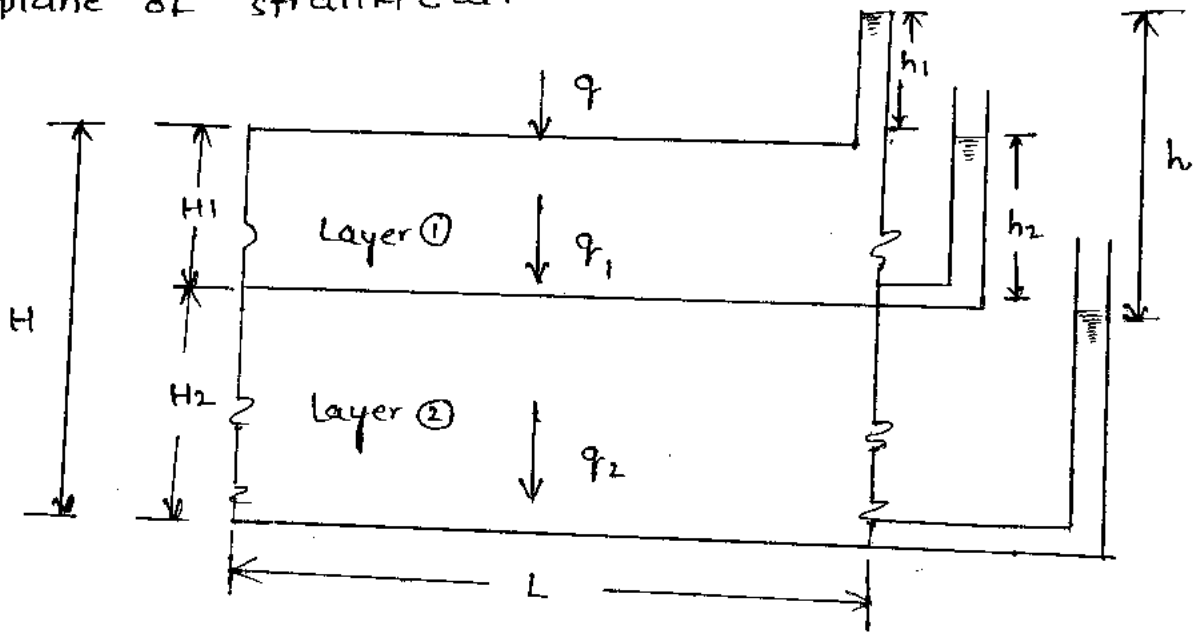
⑥ Flow normal to the plane of stratification

Let us consider a soil deposit consisting of two layers of thickness  $H_1$  and  $H_2$

in which the flow occurs normal to the



the plane of stratification.



\* flow normal to plane of stratification \*

Let  $(kV)_1$  and  $(kV)_2$  be the co-efficient of permeability of the layer ~~1 and 2~~ ① & ② in the direction perpendicular to the plane of stratification and  $kV$  be the average co-eff. of the entire deposit in that direction.

For each layer discharge is equal

$$q = q_1 = q_2 \longrightarrow \text{①}$$

using Darcy's law, considering unit area  $\perp$  to flow,

$$k_v \times i_v \times l = (k_v)_1 \times (i_v)_1 \times l = (k_v)_2 \times (i_v)_2 \times l \longrightarrow \textcircled{2}$$

where,  $i_v$  = overall hydrolic gradient.

$(i_v)_1$  = hydrolic gradient in layer ①

$(i_v)_2$  = " " " in layer ②

from equation ②.

$$(i_v)_1 = \left[ \frac{k_v}{(k_v)_1} \right] \times i_v \longrightarrow \textcircled{3}$$

$$(i_v)_2 = \left[ \frac{k_v}{(k_v)_2} \right] \times i_v \longrightarrow \textcircled{4}$$

As the total loss of head ( $h$ ) over the entire deposite is equal to the sum of the loss of heads in the individual layers

$$h = h_1 + h_2$$

writing in terms of hydrolic gradient (1) and the distance of flow remembering

$$h = i \times e$$

$$\dot{i}_v \times H = (\dot{i}_v)_1 \times H_1 + (\dot{i}_v)_2 \times H_2$$

using equation ③ & ④

$$\dot{i}_v \times H = \frac{K_v}{(K_v)_1} \times \dot{i}_v \times H_1 + \frac{K_v}{(K_v)_2} \times \dot{i}_v \times H_2$$

$$K_v \left[ \frac{H_1}{(K_v)_1} + \frac{H_2}{(K_v)_2} \right] = H = H_1 + H_2$$

$$K_v = \frac{H_1 + H_2}{\frac{H_1}{(K_v)_1} + \frac{H_2}{(K_v)_2}}$$

formulas:

$$\textcircled{1} \quad k = \frac{qL}{Ah}$$

$$\textcircled{2} \quad k = \frac{qL}{At} \log_e \left( \frac{h_1}{h_2} \right)$$

$$\textcircled{3} \quad k = c \left( \frac{r_w}{\mu} \right) \left( \frac{e^3}{1+e} \right) D^2$$

$$\textcircled{4} \quad k_h = \frac{(k_h)_1 \times H_1 + (k_h)_2 \times H_2 + d (k_h)_3 \times H_3}{H_1 + H_2 + H_3}$$

### Problems:

\* In a constant head permeameter test, the following observations were taken:

Distance b/w piezometer toppings = 100 mm.

Difference of water levels in piezometers = 60 mm.

Diameter of the test sample = 100 mm.

Quantity of the ~~test sample~~ ~~water~~ water collected = 350 ml<sup>3</sup>

Duration of the test = 270 sec.

Determine the co-eff of permeability of the soil.

Sol<sup>n</sup>

We know -  $k = \frac{qL}{Ah}$

$$q = \frac{V}{t} = \frac{350}{270} = 1.296 \text{ ml}^3/\text{sec}$$

$$k = \frac{1.296 \times 10.0}{\frac{\pi}{4} \times 10^2 \times 6.0} = 0.0275 \text{ cm/sec.}$$

\*② The falling-head permeability test was conducted on a soil sample of 4 cm diameter and 18 cm length, the head fell from 1.0 m, 0.40 m in 20 min. if the c/s area of the stand pipe was  $1 \text{ cm}^2$ , determine the co-efficient of permeability.

Sol<sup>n</sup>

$$k = \frac{qL}{At} \log_e \left( \frac{h_1}{h_2} \right)$$

$$= \frac{1.0 \times 18.0}{\frac{\pi}{4} \times 4^2 \times 20 \times 60} \log_e \left( \frac{1.0}{0.40} \right)$$

$$= 1.09 \times 10^{-3} \text{ cm/sec.}$$

\*③- The co-efficient of permeability of a soil at a void ratio of 0.7 is  $4 \times 10^{-4} \text{ cm/sec}$ .

Estimate its value at a void ratio of 0.50.

Sol<sup>n</sup>

$$k = c \left( \frac{r_w}{\mu} \right) \left( \frac{e^3}{1+e} \right) D^2$$

As all the parameters remain constant, except,

$$\frac{k_{0.7}}{k_{0.5}} = \frac{(0.70)^3}{(0+0.70)} \times \left( \frac{1+0.50}{(0.50)^3} \right)$$

$$= \frac{4 \times 10^{-4}}{k_{0.5}} = 2.421$$

$$k_{0.5} = 1.65 \times 10^{-4} \text{ cm/sec.}$$

SEEPAGE THROUGH SOILS

Introduction:-

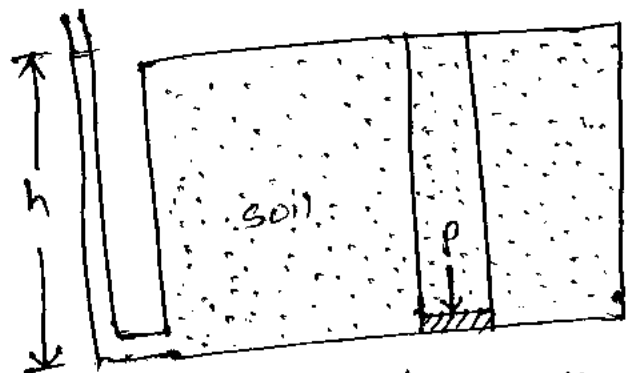
seepage is the flow of water under gravitational forces in a permeable medium. Flow of water takes place from a point of high head to a point of low head. The flow generally laminar.

The path taken by a water particle is represented by a flow line. Although an infinite number of flow lines can be drawn, for convenience, only a few are drawn. At certain points on different flow lines, the total head will be same. The lines connecting points of equal total head can be drawn. These lines are known as equipotential lines.

Effective stress principle:-

• definition of effective stress:-

The fig. shows a soil mass which is fully saturated. Let us consider a prism of



saturated soil mass

soil with a cross-sectional area  $A$ . The weight  $P$  of the soil in the prism is given by

$$P = \gamma_{sat} h A$$

where  $\gamma_{sat}$  is the saturated weight of the soil, and  $h$  is the height of the prism.

Total stress ( $\sigma$ ) on the base of the prism is equal to the force per unit area. Thus,

$$\sigma = \frac{P}{A} = \frac{\gamma_{sat} h A}{A}$$

$$\sigma = \gamma_{sat} h \rightarrow \textcircled{1}$$

While dealing with stresses, it is more convenient to work in terms of unit weights rather than density.

$$r = \rho g$$

where  $r$  is in  $N/m^3$  and  $\rho$  is in  $kg/m^3$ ,  $g = 9.81 m/s^2$

$$\text{Thus, } \gamma_{sat} = \rho_{sat} \times g = 9.81 \rho_{sat}$$

Generally, the unit weights are expressed in  $kN/m^3$  and the mass density in  $kg/m^3$ . In that case,

$$\gamma_{sat} = \frac{\rho_{sat} \times g}{1000} = 9.81 \times 10^{-3} \rho_{sat}$$



For example, if  $P_{sat} = 2000 \text{ kg/m}^3$

$$\gamma_{sat} = 9.81 \times 10^{-3} \times 2000 = 19.62 \text{ kN/m}^3$$

Pore water pressure ( $u$ ) is the pressure due to pore water filling the voids of the soil. Thus

$$u = \gamma_w h \longrightarrow \textcircled{2}$$

Pore water pressure is also known as neutral pressure & neutral stress, because it cannot resist shear stresses.

Pore water pressure is taken as zero when it is equal to atmospheric pressure, because in soil engineering the pressures used are generally gauge pressure and not absolute pressures.

The effective stress ( $\bar{\sigma}$ ) at a point in the soil mass is equal to the total stress minus the pore water pressure. Thus.

$$\bar{\sigma} = \sigma - u \longrightarrow \textcircled{3}$$

For saturated soils, it is obtained as

$$\bar{\sigma} = \gamma_{sat} h - \gamma_w h$$

$$\bar{\sigma} = (\gamma_{sat} - \gamma_w) h$$

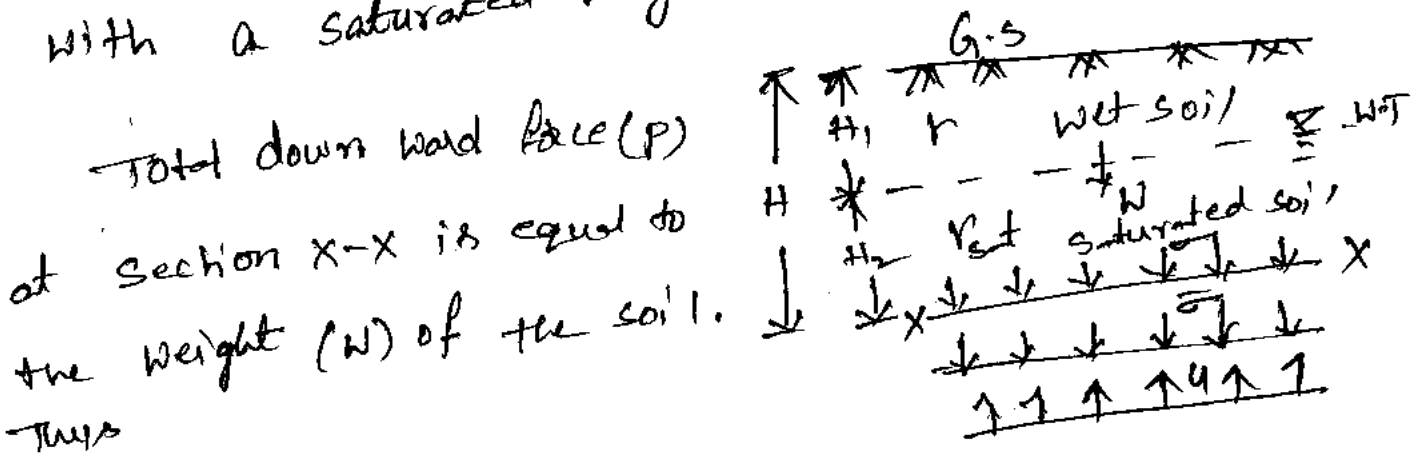
$$\bar{\sigma} = \gamma' h$$

where  $\gamma'$  is the submerged unit weight.

The effective stress is also represented by  $\sigma'$  in some texts.

### Effect of Water Table Fluctuations on Effective Stress:

Let us consider a soil mass shown in the below fig. The depth of the water table (W.T) is  $H_2$  below the ground surface. The soil above the water table is assumed to be wet, with a bulk unit weight of  $\gamma$ . The soil below the water table is saturated, with a saturated weight of  $\gamma_{sat}$ .



Total down ward force (P) at section x-x is equal to the weight (W) of the soil.

$$P = W = \gamma H_1 A + \gamma_{sat} H_2 A$$

where A is the area of c/s of the soil mass considering by x throughout,

$$\frac{P}{A} = \gamma H_1 + \gamma_{sat} H_2$$

The left-hand side is equal to the total stress from eqn (1)

$$\therefore \sigma = \gamma H_1 + \gamma_{sat} H_2$$

The pore water pressure ( $u$ ) is given by

$$u = \gamma_w H_2$$

The effective stress  $\bar{\sigma} = \sigma - u$

$$= (\gamma H_1 + \gamma_{sat} H_2) - \gamma_w H_2$$

$$= \gamma H_1 + (\gamma_{sat} - \gamma_w) H_2$$

$$= \gamma H_1 + \gamma' H_2$$

a) If the water table rises to the ground surface, the whole of the soil is saturated, and

$$\bar{\sigma} = \gamma' (H_1 + H_2) = \gamma' H$$

As  $\gamma' < \gamma$ , the effective stress is reduced due to rise of water table.

b) If the water table is depressed below the section  $x-x$ .

$$\bar{\sigma} = \gamma H$$

In this case, the effective stress is increased.

Thus, it is observed that the fluctuations in water table level cause changes in the pore water pressure and the corresponding changes in the effective stress.

# Effective stress in a soil mass under hydrostatic conditions

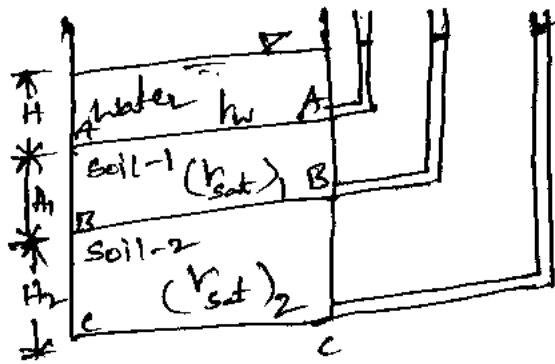


Fig (a)

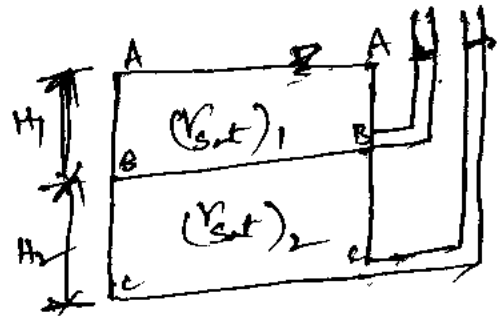


Fig (b)

Fig(a) shows a soil mass under hydrostatic conditions, where in the water level remains constant. As the interstices in the soil mass are interconnected water rises to the same elevation in different piezometers fixed to the soil mass. The effective stress at various sections can be determined using  $\bar{\sigma} = \sigma - u$

1) Water table above the soil surface A-A :-

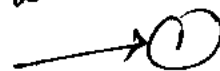
(a) Section A-A :-

$$\sigma = \gamma_w H, \quad u = \gamma_w H$$

$$\therefore \bar{\sigma} = \sigma - u$$

$$= \gamma_w H - \gamma_w H$$

$$\boxed{\bar{\sigma} = 0}$$





② Water table at the soil surface A-A:-

Fig (b) shows the condition when the depth  $H$  of water above the section A-A is reduced to zero. In this case, the effective stresses at various sections are determined as under.

a) At Section A-A:-

$$\sigma = 0, u = 0$$

$$\bar{\sigma} = \sigma - u$$

$$\boxed{\sigma = 0} \longrightarrow \textcircled{4}$$

b) At section B-B:-

$$\sigma = (\gamma_{sat})_1 H_1$$

$$u = \gamma_w H_1$$

$$\bar{\sigma} = \sigma - u = (\gamma_{sat})_1 H_1 - \gamma_w H_1$$

$$= [(\gamma_{sat})_1 - \gamma_w] H_1$$

$$\boxed{\bar{\sigma} = \gamma'_1 H_1} \longrightarrow \textcircled{5}$$

c) At section C-C:-

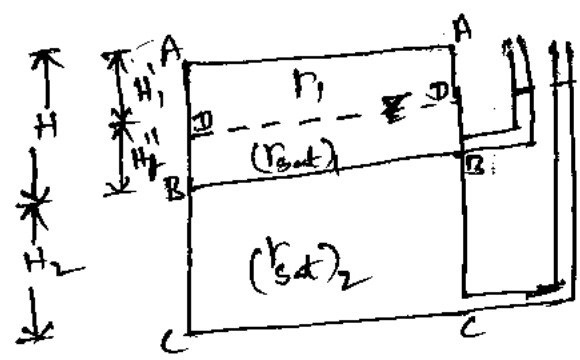
$$\sigma = (\gamma_{sat})_1 H_1 + (\gamma_{sat})_2 H_2$$

$$u = \gamma_w (H_1 + H_2)$$

$$\bar{\sigma} = \sigma - u = (\gamma_{sat})_1 H_1 + (\gamma_{sat})_2 H_2 - \gamma_w H_1 - \gamma_w H_2$$

$$\therefore \boxed{\bar{\sigma} = \gamma'_1 H_1 + \gamma'_2 H_2} \longrightarrow \textcircled{6}$$

③ Water Table in Soil ①:-



The fig show the case when the water table is at  $\textcircled{1}-\textcircled{1}$  in the soil-1 at depth  $H_1'$ . The effective stresses at various sections are determined as follows.

② At section A-A :-  $\sigma = 0, u = 0$   
 $\therefore \bar{\sigma} = \sigma - u = 0$   
 $\bar{\sigma} = 0 \rightarrow \textcircled{7}$

③ At section  $\textcircled{1}-\textcircled{1}$  :-  
 $\sigma = \gamma_1 H_1', u = 0$   
 $\bar{\sigma} = \gamma_1 H_1' - 0$   
 $\bar{\sigma} = \gamma_1 H_1' \rightarrow \textcircled{8}$

where  $\gamma_1$  is unit weight of soil above  $\textcircled{1}-\textcircled{1}$ .

④ At section B-B :-  
 $\sigma = \gamma_1 H_1' + (\gamma_{sat})_1 H_1''$        $\left\{ \because H_1' + H_1'' = H_1 \right\}$   
 $u = \gamma_w H_1''$   
 $\therefore \bar{\sigma} = \sigma - u = \gamma_1 H_1' + (\gamma_{sat})_1 H_1'' - \gamma_w H_1''$   
 $= \gamma_1 H_1' + [(\gamma_{sat})_1 - \gamma_w] H_1''$   
 $\bar{\sigma} = \gamma_1 H_1' + \gamma_1' H_1'' \rightarrow \textcircled{9}$

(d) At section C-C:-

$$\sigma = \gamma_1 H_1' + (\gamma_{sat})_1 H_1'' + (\gamma_{sat})_2 H_2$$

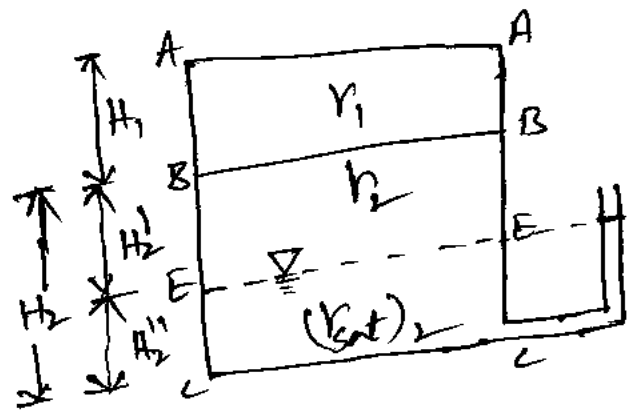
$$u = \gamma_w H_1'' + \gamma_w H_2$$

$$\begin{aligned} \therefore \bar{\sigma} &= \sigma - u = \gamma_1 H_1' + (\gamma_{sat})_1 H_1'' + (\gamma_{sat})_2 H_2 - \gamma_w H_1'' \\ &= \gamma_1 H_1' + [(\gamma_{sat})_1 - \gamma_w] H_1'' + [(\gamma_{sat})_2 - \gamma_w] H_2 \end{aligned}$$

$$\boxed{\bar{\sigma} = \gamma_1 H_1' + \gamma_1' H_1'' + \gamma_2' H_2} \rightarrow (10)$$

(e) Water Table in Soil-2:-

The fig. shows the condition when the water table is at EE in soil-2 at depth  $H_2'$ . The effective stresses at various sections are as under.



(a) Section A-A:-  $\sigma = 0, u = 0$

$$\therefore \boxed{\bar{\sigma} = 0} \rightarrow (11)$$

(b) At section B-B:-

$$\sigma = \gamma_1 H_1, u = 0$$

$$\therefore \bar{\sigma} = \sigma - u = \gamma_1 H_1 - 0$$

$$\boxed{\bar{\sigma} = \gamma_1 H_1} \rightarrow (12)$$



③ At section E-E:-

$$\sigma = \gamma_1 H_1 + \gamma_2 H_2'$$

$$u = 0$$

$$\bar{\sigma} = \sigma - u = \gamma_1 H_1 + \gamma_2 H_2' - 0$$

$$\therefore \boxed{\bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2'} \longrightarrow (13)$$

④ At section C-C:-

$$\sigma = \gamma_1 H_1 + \gamma_2 H_2' + (\gamma_{sat})_2 H_2''$$

$$u = \gamma_w H_2''$$

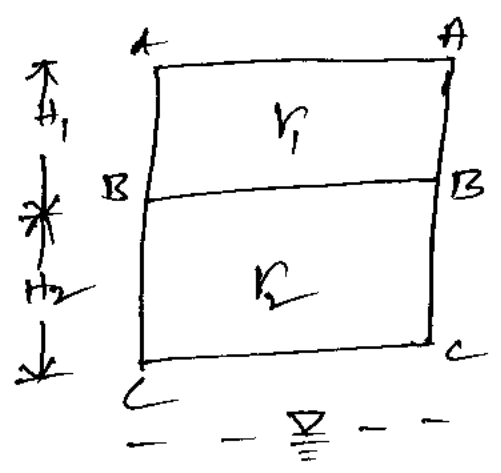
$$\bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2' + (\gamma_{sat})_2 H_2'' - \gamma_w H_2''$$

$$\bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2' + [(\gamma_{sat})_2 - \gamma_w] H_2''$$

$$\therefore \boxed{\bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2' + \gamma_2' H_2''} \longrightarrow (14)$$

⑤ Water Table below C-C :-

The fig shows the condition when the water table is below C-C. As the pore water pressure is zero everywhere, the effective stresses are also equal to the total stresses.



(a) section B-B:-  $\sigma = \bar{\sigma} = \gamma_1 H_1$   $\longrightarrow (15)$

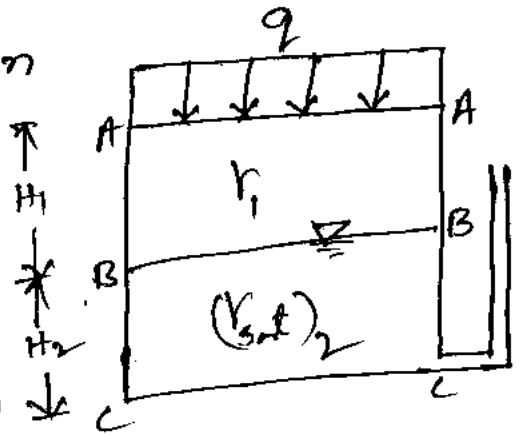
(b) section C-C:-  $\sigma = \bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2$   $\longrightarrow (16)$

## Increase in Effective Stresses due to Surcharge :-

Let us consider the case when

the soil surface is subjected to a surcharge load of intensity 'q' per unit area. Let us assume

that the water table is at level B-B. The stresses at various sections are determined as under.



Section A-A :-  $\sigma = q, u = 0$

$$\therefore \bar{\sigma} = q$$

i.e. All the points on the soil surface are subjected to an effective stress equal to q.

Section B-B :-

$$\sigma = q + \gamma_1 H_1, u = 0$$

$$\therefore \bar{\sigma} = q + \gamma_1 H_1$$

Section C-C :-

$$\sigma = q + \gamma_1 H_1 + (\gamma_{sat})_2 H_2$$

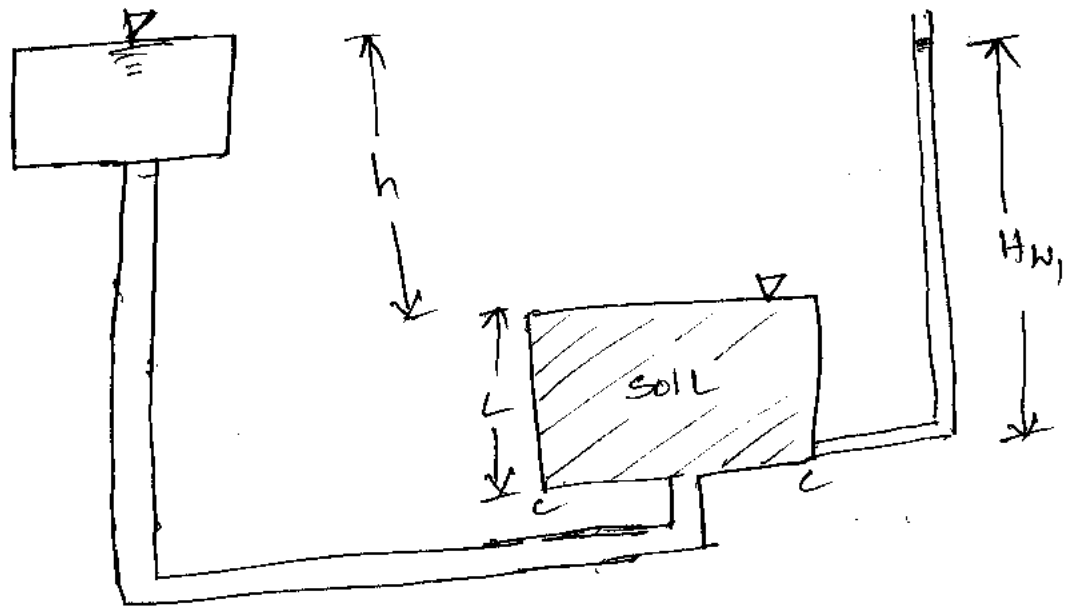
$$u = \gamma_w H_2$$

$$\bar{\sigma} = q + \gamma_1 H_1 + \gamma_2' H_2$$

From the above illustrations, it is clear that the effective stress throughout the depth is greater than the case with no surcharge discussed in the preceding section.

## QUICK SAND CONDITIONS:

We know the effective stress is reduced due to upward flow of water. When the head causing upward flow is increased, a stage is eventually reached when the effective stress is reduced to zero. The condition so developed is known as quick sand condition.



The above fig. shows a soil specimen of length  $L$  subjected to an upward pressure. Let us consider the stress developed at section  $c-c$ .

$$\sigma = \gamma_{\text{sat}} L = (\gamma' + \gamma_w) L$$

$$u = \gamma_w H_w = \gamma_w (L + h)$$

$$\bar{\sigma} = (\gamma' + \gamma_w) L - \gamma_w (L + h)$$

$$\bar{\sigma} = \gamma' L - \gamma_w h$$

The second term can be written in terms of the hydraulic gradient as under.

$$k_w h = k_w \cdot \left(\frac{h}{L}\right) \times L$$

$$k_w h = k_w i L$$

$$\therefore \sigma = r' L - k_w i L$$

The effective stress becomes zero if

$$r' L = k_w i L$$

$$i = \frac{r'}{k_w}$$

~~substituting~~ The hydraulic gradient at which the effective stress becomes zero is known as the critical gradient ( $i_c$ ). Thus

$$i_c = \frac{r'}{k_w}$$

~~substituting~~ substituting the value of the submerged unit weight in terms of void ratio from the following equation.

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$$

$$r' = \gamma_{sat} - \gamma_w$$

$$\therefore i_c = \frac{\gamma_{sat} - \gamma_w}{k_w} = \frac{\frac{(G+e)\gamma_w}{1+e} - \gamma_w}{k_w}$$

$$= \frac{(G+e-1-e)\gamma_w}{1+e} \Rightarrow$$

$$i_c = \frac{(G-1)\gamma_w}{1+e}$$

(8)

Taking the specific gravity of solids ( $G_s$ ) as 2.67 and the void ratio ( $e$ ) as 0.67

$$i_c = \frac{2.67 - 1}{1 + 0.67} = 1.0$$

Thus the effective stress becomes zero for the soil with above values of  $G_s$  and  $e$  when the hydraulic gradient is unity, i.e. the head causing flow is equal to the length of the specimen.

Alternative method:-

The above expression for the critical gradient can also be obtained from the equilibrium of forces. When the quick sand condition develops, the upward force is equal to the downward weight.

Thus, 
$$k_{sat}(L \times A) = (h + L) A k_w$$

$$(k_{sat} - k_w) LA = Ah k_w$$

$$L k' = h k_w$$

$$\frac{h}{L} = \frac{k'}{k_w}$$

$$\boxed{i = \frac{k'}{k_w}}$$

The shear strength of a cohesionless soil depends upon the effective stress. The shear strength is given by

$$s = \sigma \tan \phi$$

The shear strength of cohesive soils is given by

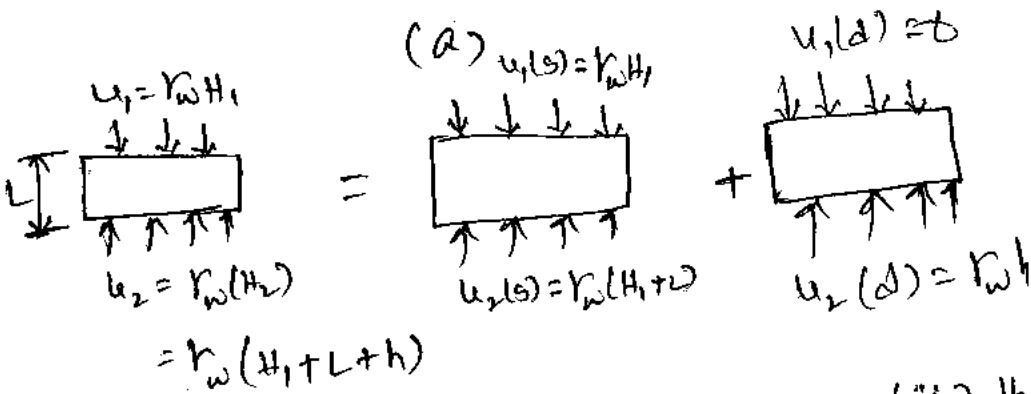
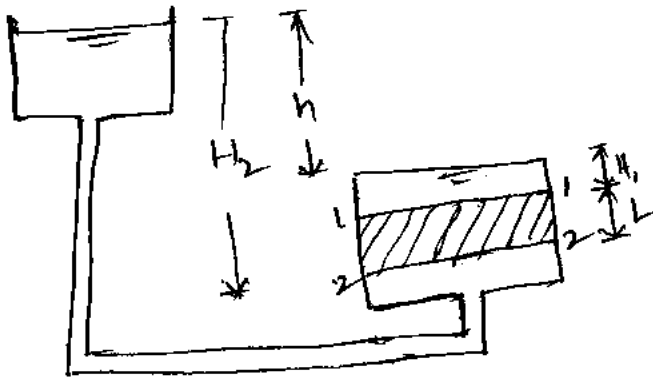
$$S = c + \bar{\sigma} \tan \phi$$

The quick sand conditions may be summarised as under.

1. Quick sand is not a special type of soil. It is a hydraulic condition.
2. A cohesionless soil becomes quick when the effective stress is equal to zero.
3. The ' $\bar{\sigma}_c$ ' at which a cohesionless soil becomes quick is about unity.
4. The discharge required to maintain a quick condition in a soil increases as the permeability of the soil increases.
5. A quick condition is most likely to occur in silt and fine sand.

Seepage Pressure :-

As the water flows through a soil, it exerts a force on the soil. This force acts in the direction of flow in the case of isotropic soils. The force is known as the drag force or Seepage force. The pressure induced in the soil is termed Seepage Pressure.



- (i) Boundary pressure
- (ii) Hydrostatic pressure
- (iii) Hydrodynamic pressure

Let us consider the upward flow of water in a soil sample of length L and c/s area A under a hydraulic head of h. The expression

For seepage force and seepage pressure can be derived considering the boundary water pressure  $u_1$  and  $u_2$  acting on the top and bottom of the soil sample, as shown in fig (b) (i). The boundary water pressure as shown in fig (b) (i), (ii), can be resolved into two components, namely, the hydrostatic pressure and the hydrodynamic pressure as shown in fig (b) (ii, iii).

- 1) The hydrostatic pressures  $u_1(s)$  and  $u_2(s)$  are the components which would occur if there were no flow. If the sample were submerged under water to a depth of  $h_1$ , these pressures would have occurred.
- 2) The hydrodynamic pressure  $u_1(d)$  and  $u_2(d)$  are the components which are responsible for flow of water. This pressure is spent as the water flows through the soil. These components cause the seepage pressure.

At the top of the sample  $u_1 = u_1(s) + u_1(d)$

$$h_1 h_1 = \gamma_w H_1 + 0$$

At the bottom of the sample  $u_2 = u_2(s) + u_2(d)$

$$\gamma_w (H_1 + L + h) = \gamma_w (H_1 + L) + \gamma_w h$$



The hydrodynamic pressure is due to hydraulic head the seepage force (J) acts on the soil skeleton due to flowing water through frictional drag. It is given by.

$$J = k_w h A$$

The seepage pressure (P<sub>s</sub>) is the seepage force per unit area,

$$P_s = \frac{J}{A} = k_w h.$$

The seepage pressure (P<sub>s</sub>) can be expressed in terms of the hydraulic gradient.

$$P_s = k_w h = k_w (h/L) \cdot L$$

$$P_s = i k_w L$$

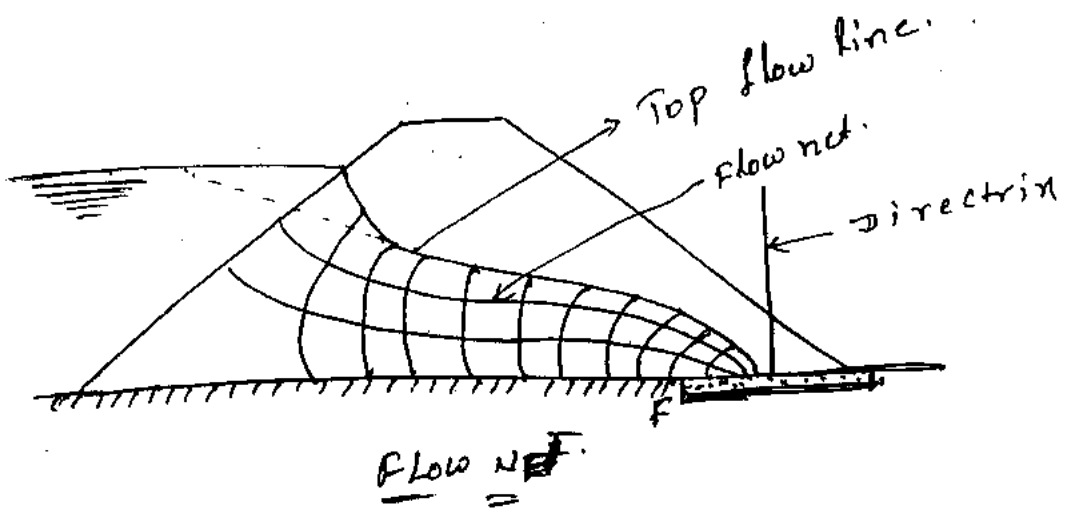
The seepage force (J) can be expressed as the force per unit volume (I), as

$$i = \frac{J}{A \cdot L} = \frac{k_w h A}{A L} = k_w \frac{h}{L}$$

$$i = i k_w$$

Thus, the seepage force per unit volume is equal to the product of the hydraulic gradient (i) and the unit weight of water.

FLOW NET:-



Properties of flow nets:-

1. The flow lines and equipotential lines meet at right angles to each other.
2. The fields are approximately squares, so that a cube can be drawn touching all the four sides of square.
3. The quantity flowing through each flow channel is the same, similarly, the same potential drop occurs between two successive equipotential lines.
4. Smaller the dimensions of the field, greater will be the hydraulic gradient and velocity of flow through it.
5. In a homogeneous soil, every transition in the shape of curves is smooth, being either elliptical or parabolic in shape.

The following Points should be kept in mind while sketching the flow net.

1. Too many flow channels distract the attention from the essential features. Normally three to five flow channels are sufficient. (The space b/w two flow lines is called flow channel)
2. The appearance of the entire flow net ~~has been~~ should be watched and not that of a part of it. Small details can be adjusted after the entire flow net has been roughly drawn.
3. The curves should be roughly elliptical (or) parabolic in shape.
4. All transitions should be smooth.
5. The flow lines and equipotential lines should be orthogonal and form approximate squares.
6. The size of the square in a flow channel should change gradually from the upstream to the downstream.

## Uses of Flow net:-

9

A flow net can be utilized for the following

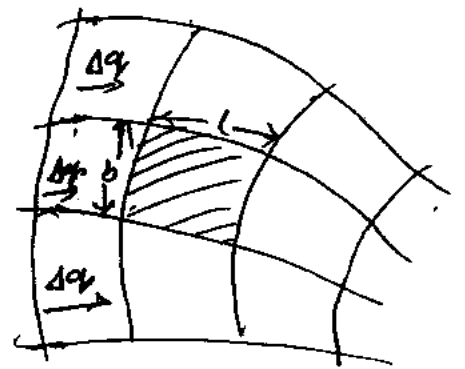
Purpose.

- 1) Determination of seepage. ~~2) Determination of hydrostatic pressure~~
- 3) Determination of seepage pressure.
- 4) Determination of exit gradient.

### Determination of seepage:-

Fig shows a portion of flow net.

The portion between any two successive flow lines is known as a flow channel.



The portion enclosed between two successive equipotential lines and successive flow lines is known as 'field' such as that shown hatched.

Let  $b$  and  $l$  be the width and length of the field.

$\Delta h$  = head drop through the field;  $dq$  = discharge passing through the flow channel.

$H$  = total hydraulic head causing flow = difference between u/s & D/s heads.

Then, from Darcy's law of flow through soils...

$$\Delta q = k \cdot \frac{\Delta h}{l} (b \times 1) \quad \left\{ \because \text{considering unit thickness} \right\}$$

2.) Determination of hydrostatic pressure:-

The hydrostatic pressure at any point within the soil mass is given by  $u = h_w \gamma_w$

$u$  = hydrostatic pressure ;  $h_w$  = piezometric head.

The hydrostatic pressure in terms of piezometric head  $h_w$  is calculated from the following relation.

$$h_w = h - z$$

$h$  = hydraulic potential of point under consider

$z$  = position head of the point above datum,

consider positive upward.

All the quantities  $h_w$ ,  $h$  and  $z$  can be expressed as the percentage of the total hydraulic head  $H$ .

eg:- If we want to plot the line of equal pressure corresponding to  $h_w = 20\%$  (say)

$$h_w = 20\% = h - z \quad h_w = 20\% \text{ on } h = 30\% H$$

3) Determination of seepage pressure:-

The hydraulic potential  $h$  at any point located after  $n$  potential drops, each of value  $\Delta h$  is given by

$$h = H - n\Delta h$$

The seepage pressure at any point equals the hydraulic potential or balance hydraulic head multiplied by the unit weight of water and hence, it is given by

$$P_s = h\gamma_w = (H - n\Delta h)\gamma_w$$

The pressure acts in the direction of flow.

4) Determination of exit gradient:- The exit gradient

is the hydraulic gradient at the downstream end of the flow line where the percolating water leaves the soil mass and emerges into the free water at the downstream. The exit gradient can be calculated

from the following expression, in which  $\Delta h$  represents the potential drop and  $l$  the average length of last field in the flow net at exit end.

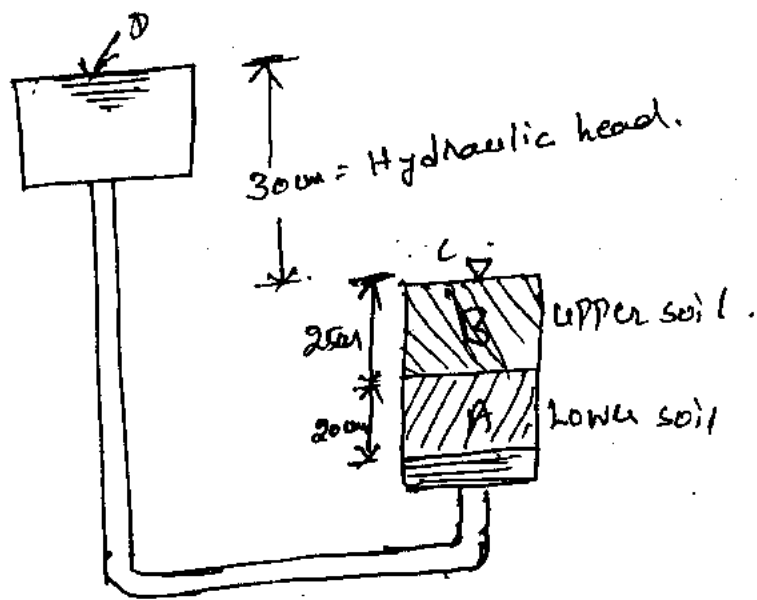
$$i_c = \frac{\Delta h}{l}$$

## Problems

- 1) A coarse-grained soil has a voids ratio of 0.78 and specific gravity as 2.67. calculate the critical gradient at which quick sand condition will occur.

$$\begin{aligned} \text{Sol)} \quad i_c &= \frac{i'}{i_w} = \frac{G-1}{1+e} \\ &= \frac{2.67-1}{1+0.78} = 0.94 \end{aligned}$$

- 2) In the test set-up shown in figure two different granular soils are placed in permeameter and flow is allowed to take place under a constant total head of 30 cm.
- Determine the total head and pressure head at Point A.
  - If 30% of the total head is lost as water flows upward through lower soil layer, what is the total head and pressure head at B?
  - If the Permeability of layer is  $3 \times 10^{-2}$  cm/sec calculate the quantity of water per second flowing through unit area of the soil.
  - What is the co-ef of permeability of the upper soil level?



sol) let the water level at C be the datum. The hydraulic head  $h = 30\text{m}$ .

a) Total head at D =  $h_w + Z$

where  $h_w$  = piezometric head (or) pressure head at D = 0  
 $Z$  = position head at ~~A~~ at D = 30m.

$\therefore$  Total head at D =  $0 + 30 = 30\text{m}$

Total head at A =  $h_w + Z$

$h_w$  = piezometric (or) pressure head at A  
 $= 30 + 25 + 20 = 75\text{m}$

$Z$  = position head at A =  $-45\text{m}$

Total head at A =  $75 - 45 = 30\text{m} = 100\%h$



b) loss of head from A to B = 30% of  $h$ .

$$= 0.3 \times 30 = 9 \text{ cm.}$$

$\therefore$  Total head at B = total head at A - head lost in AB

$$= 30 - 9 = 21 \text{ cm.}$$

But total head at B =  $h_w + z$  where  $z = -25 \text{ cm}$

$$21 = h_w - 25$$

$$h_w = 21 + 25 = 46 \text{ cm} = \text{PI head at I}$$

c) Head lost between A and B = 9 cm.

$$\text{Now, } q = k i A = k \times \frac{h}{z} \times A.$$

Taking  $A = 1 \text{ cm}^2$ ;  $h = 9 \text{ cm}$ ,  $z = 20 \text{ cm}$  we get

$$q = 3 \times 10^{-2} \times \frac{9}{20} \times 1 = 1.35 \times 10^{-2} \text{ cm}^3/\text{sec}$$

d) Same flow takes place through the upper soil

$$q = 1.35 \times 10^{-2} = k i A.$$

Now, total head at B = 21 cm Total head at C = 0

Head lost between B and C = 21 cm.

(Alternatively, head lost in upper soil 70% of  $h = 0.7 \times 21$ )

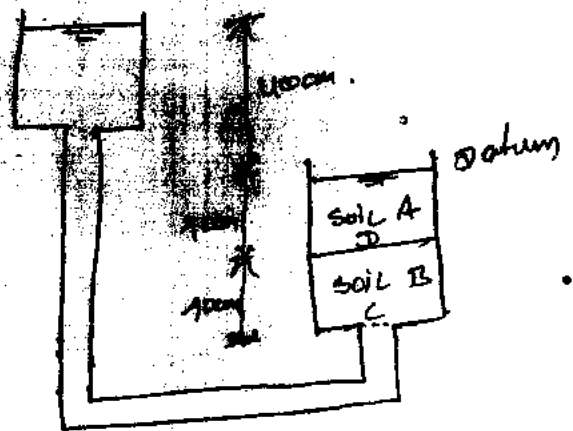
$$\therefore h \text{ for the upper soil} = \frac{21}{25}$$

$$\therefore k = \frac{q_v}{PA} = \frac{1.35 \times 10^{-2}}{\frac{21}{25} \times 1} = 1.6 \times 10^{-2} \text{ cm/sec}$$

3) In the experimental set up shown in the fig, flow takes place under a constant head through the soil A and B

(i) Determine the piezometric head at Potent C.

(ii) If 40% of the excess hydrostatic pressure is lost in flowing through soil B, what are the hydraulic head and piezometric head at Potent D.



(iii) If the coefficient of permeability of soil B is 0.05 cm/sec, determine the same for soil A.

(iv) what is the discharge per unit area?

Ans: (i) 120 cm, (ii) 24 cm, 64 cm

(iii) 0.033 cm/sec (iv) 0.02 ml/sec.