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Trusses (Pin jointed frames)

I. Basic concepts.

The truss is a frame structure which will continue to perform as a geometrically unchangeable system, even when all of its rigid joints are conventionally replaced by perfect hinges. The trusses are used for the same purposes as beams and girders, but the spans they cover are much larger.

In trusses all the members are subjected either to direct extension (tension) or compression. That ensures better utilization of the materials, the stress diagram for each of these members is constant.

The trusses can be space framed structures in which the elements are situated in several planes. However, the design of the three dimensional trusses can be reduced to the case of several plane systems.

Here we are going to examine two dimensional (plane) trusses for which all the elements and loads are situated in one plane.

The span of a truss is the distance between its supports. The lower and upper longitudinal members form the upper and lower chords of the truss. The members connecting both the chords are called the web members. They are subdivided into verticals and diagonals. The distance between two adjacent joints measured along the horizontal is usually called a panel (Fig. 1).

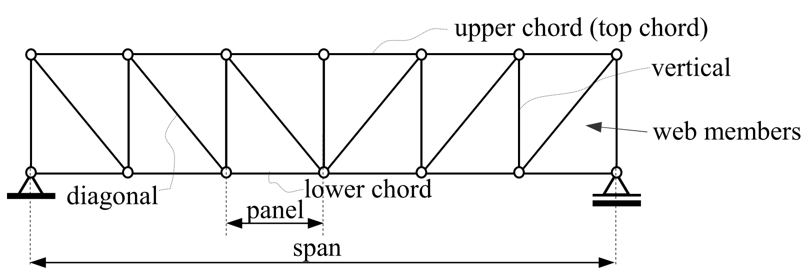


Figure 1 A truss with parallel chords

The following five criterions may serve as a basis for the classification of trusses:

1) The shape of the upper and lower chords;
2) The type of the web;
3) The conditions of the supports;
4) The purpose of the structure;
5) The level of the floor (lane, road)

According to the first criterion, the trusses can be classified into trusses with parallel chords (Fig. 1), polygonal and triangular trusses or trusses with inclined chords (Fig. 2).

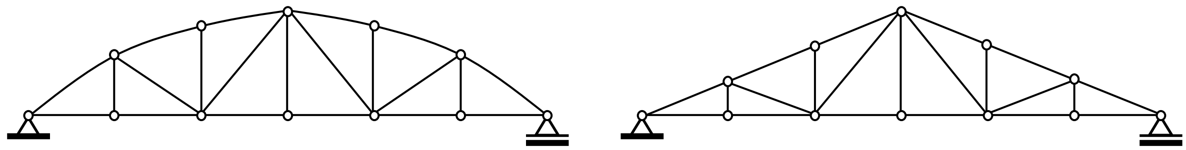


Figure 2 Polygonal and triangular trusses

The second criterion (type of the web) permits to subdivide the trusses into those with triangular patterns (Fig. 3a), those with quadrangular patterns (Fig. 3b) formed by vertical and diagonals, those with the web members form a letter K (Fig. 3c). Finally, trusses formed by superposition of two or more simple grids (Fig. 3d,e).

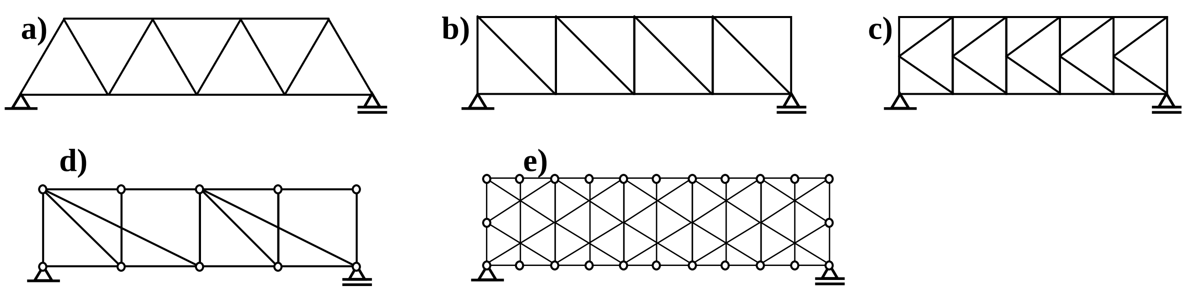


Figure 3 Trusses according to the type of the web

The third criterion permits to distinguish between the ordinary end-supported trusses (Fig. 4a), the cantilever trusses (Fig. 4b), the trusses cantilevering over one or both supports (Fig. 4c), and finally crescent or arched trusses (Fig. d,e).

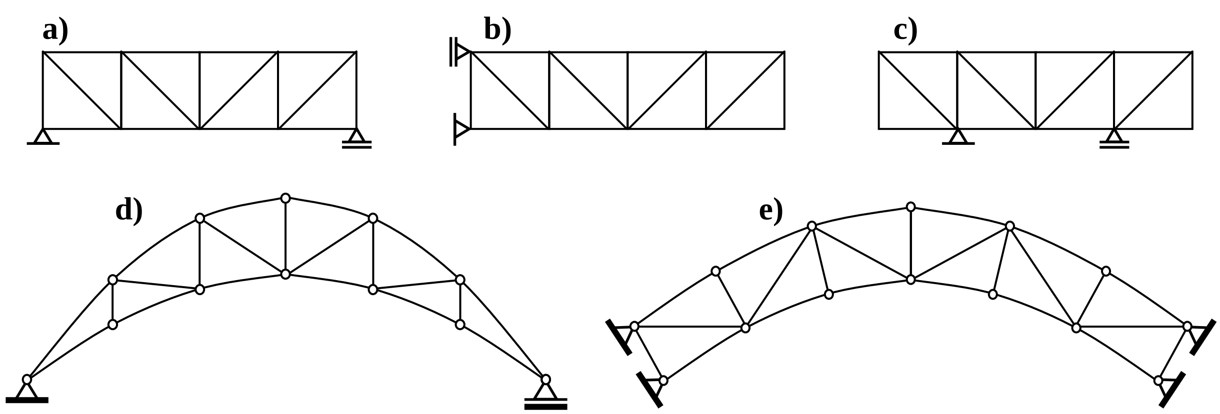


Figure 4 Trusses depending on the type of supports

According to their purpose the trusses may be classified as roof trusses, bridge trusses, those used in crane construction.

Regarding the level of the road the trusses can be constructed so that the road is carried by the bottom chord joints (Fig. 5a), or the upper chord joints (Fig. 5b). Sometimes the road (lane) is carried at some intermediate level (Fig. 5c).

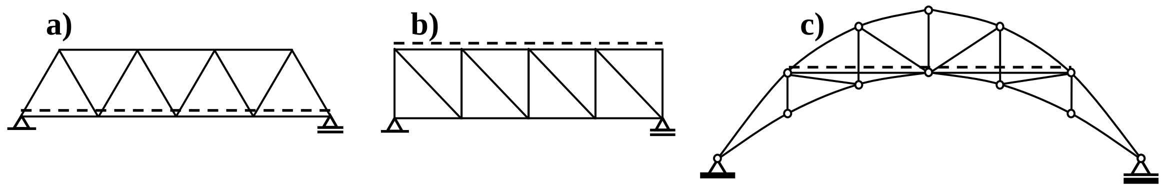


Figure 5 Trusses depending on the level of the floor

II. Statical determinacy

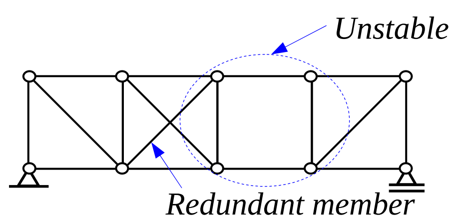
A statically determinate truss is one in which all member forces and external reactions may be determined by applying the equilibrium equations only.

In a simple truss with k nodes (hinges), including the supports, $2k$ equilibrium equations can be generated, since at each joint the equilibrium conditions $\Sigma H = 0$ and $\Sigma V = 0$ hold. Each member of the truss is subjected to an unknown axial force. If the truss has d members and a external restraints the number of unknowns is $d+a$. Thus, a simple truss is statically determinate when the number of unknowns equals the number of equilibrium equations or:

$w = 2 \cdot k - d - a = 0$.

A truss is statically indeterminate when $w<0$, and the truss is kinematically unstable when $w>0$.

However, a situation can occur in which a truss is deficient, from kinematical point of view, even when $w=0$.



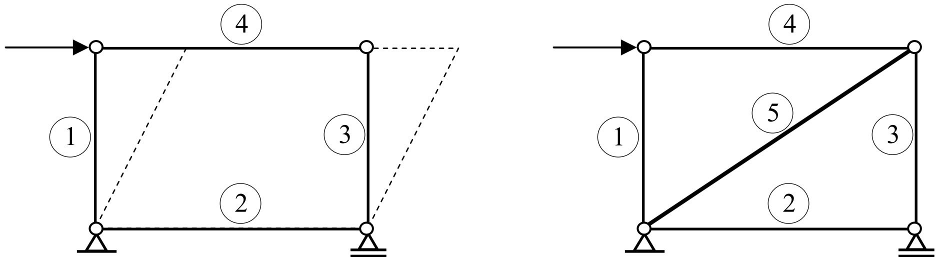
The left hand side of the truss has a redundant member, while the right hand side is unstable.

Thus, number of degrees of freedom is necessary condition for statical determinacy but insufficient condition for kinematical stability. The structure can be kinematical unstable even when $w=0$. In any case kinematical analysis using basic kinematical elements (dyads) needs to be performed.

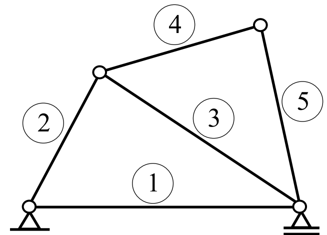
1. Internal stability

The internal stability of a truss can often be checked by careful inspection of the arrangement of its members (by kinematical analysis). If each joint is fixed so that it cannot move in a rigid body sense with respect to the other joints, then the truss will be internally stable.

The four members structure below will collapse unless a diagonal member is added for support, such as bar 5.

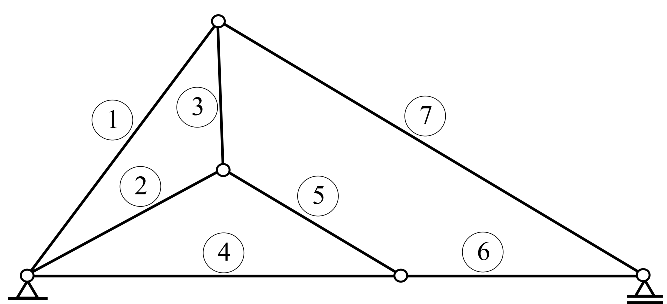


Therefore, a simple truss is geometrically stable when is constructed by starting with a basic triangular element such as (1+2.3), and connecting two members (4.5) to form an additional element.



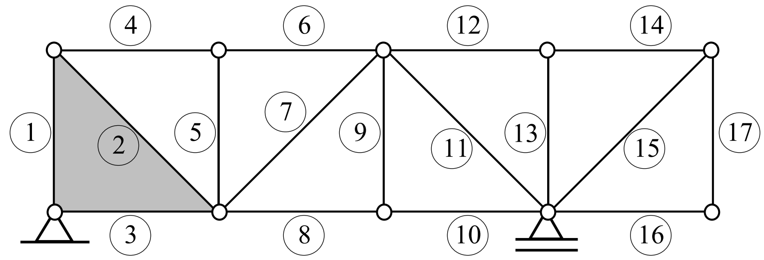
An example of a simple truss is shown bellow, where the basic triangle is (1+2.3). Bars 4 and 5 (which form a dyad) are added to the triangle, finally members 6 and 7 are added to form the structure.

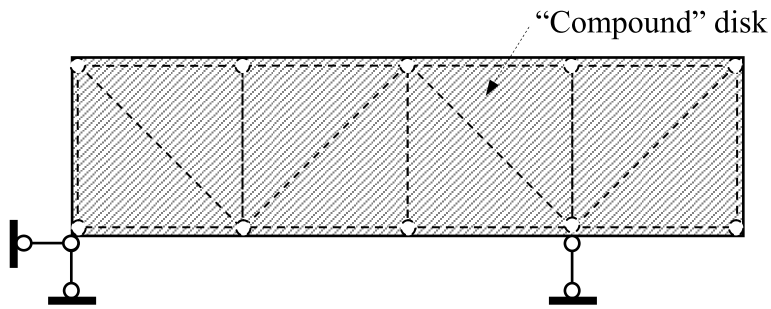
The simple trusses do not have to consist entirely of unchangeable triangles they could be constructed by consecutive adding of simple dyads, starting from a basic, primary bar.



In line with the above it can be concluded that a simple truss will always be internally stable if starting from a basic member, the truss is formed by consecutive adding of two additional pin jointed members (dyad), which results in additional joint of the truss.

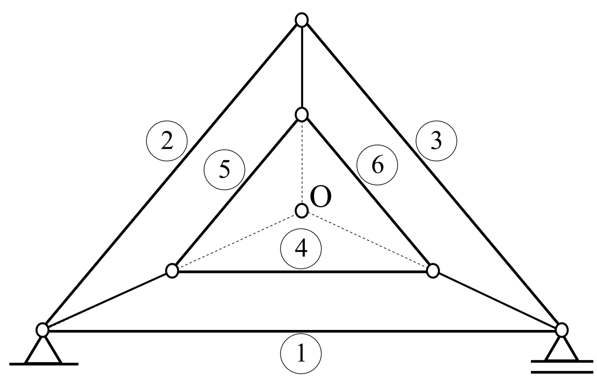
The truss depicted below exemplifies this statement. Starting with the shaded triangle (1+2.3), the successive dyads 4.5, 6.7, 8.9, 10.11, 12.13, 14.15 and 16.17 have been added. The summation of unchangeable triangles form the so called “compound” disk, which is fixed to the ground with three single supports, in order to form geometrically unchangeable system.





If a truss is formed so that it does not hold its joints in a fixed position, it will be unstable, which is the case of the first truss considered in this section.

To determine the internal stability of a compound truss, it is necessary to identify the way in which the simple trusses are connected together.

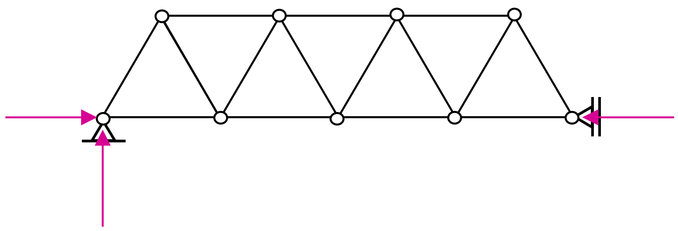


The presented compound truss is unstable since the inner simple truss (4+5.6) is connected to the outer simple truss (1+2.3) using three bars which are concurrent at point O.

If a truss is identified as complex (a complex truss is the one that cannot be classified as being either simple or compound), it may not be possible to tell by kinematic analysis if it is stable. The kinematical instability of any truss type, be it simple, compound or complex, may be determined by examination of the system of equations formed by the two equilibrium equations for each joint. If this system of equations is singular the truss is geometrically changeable. That is the mathematical criterion for the structure instability.

2. External stability

A structure is externally unstable if all of its reactions are concurrent or parallel. For example the trusses below are externally unstable because the support reactions cannot be derived.



III. Axial force determination in the members of simple trusses

1. Sign convention

A tensile force in a member is considered positive, and a compressive force is considered negative. Forces are depicted as acting from the member on the nodes as shown in figure 6.

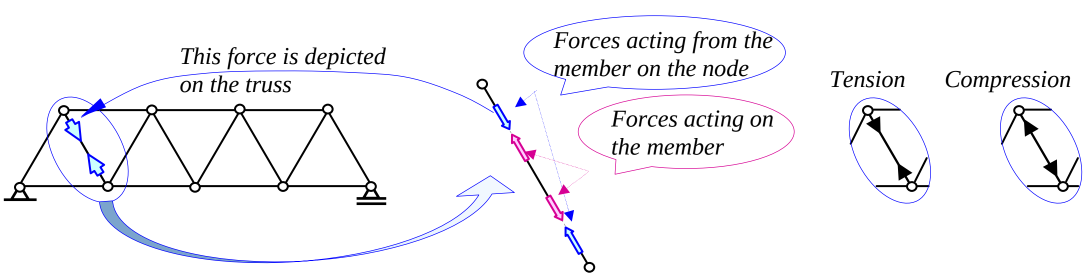


Figure 6 Sign convention

2. The method of joints for determination of axial forces

In this method the equilibrium of each joint is considered separately, by cutting the joint out from the whole truss. The influence of the rest of the truss is replaced by the forces acting in the cut bars. This method permits the successive determination of the forces acting in all the members starting with a joint formed by meeting of two bars only (two members joint).

Before the basic analysis of the truss it is instructive to find the zero force members.

Zero force members

1) Unloaded two members joint (Fig. 7a).

It follows from the equilibrium equations. From $\Sigma V = 0$ follows that $S_{2v}=0$ so $S_2=0$, from $\Sigma H = 0$ follows that $S_1=0$.

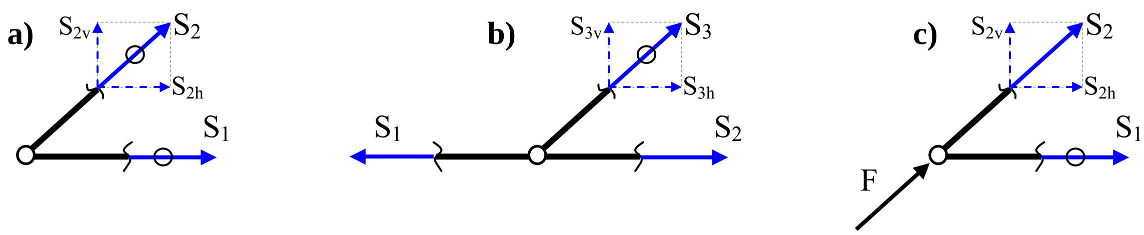


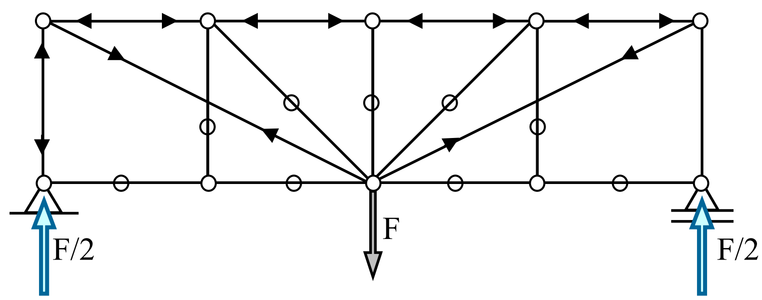
Figure 7 Zero force members

2) Unloaded three members joint (Fig. 7b), when two of the three members are collinear (two members have the same direction).

The force in the third member is zero. From $\Sigma V = 0$ follows that $S_{3v}=0$ from where $S_3=0$.

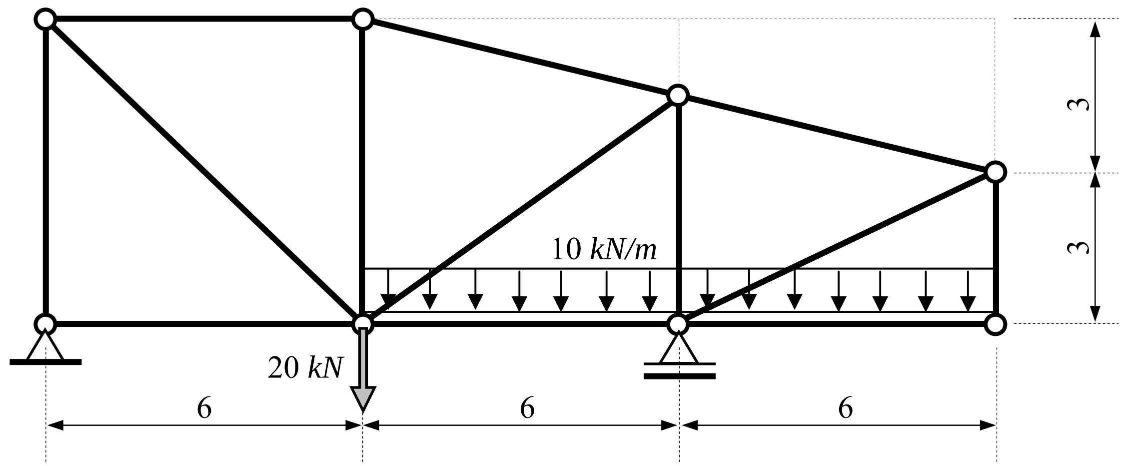
3) Two members joint when the external concentrated force is in the direction of one of the members, the force in the second member is zero – $S_1=0$.

Example for zero force members:



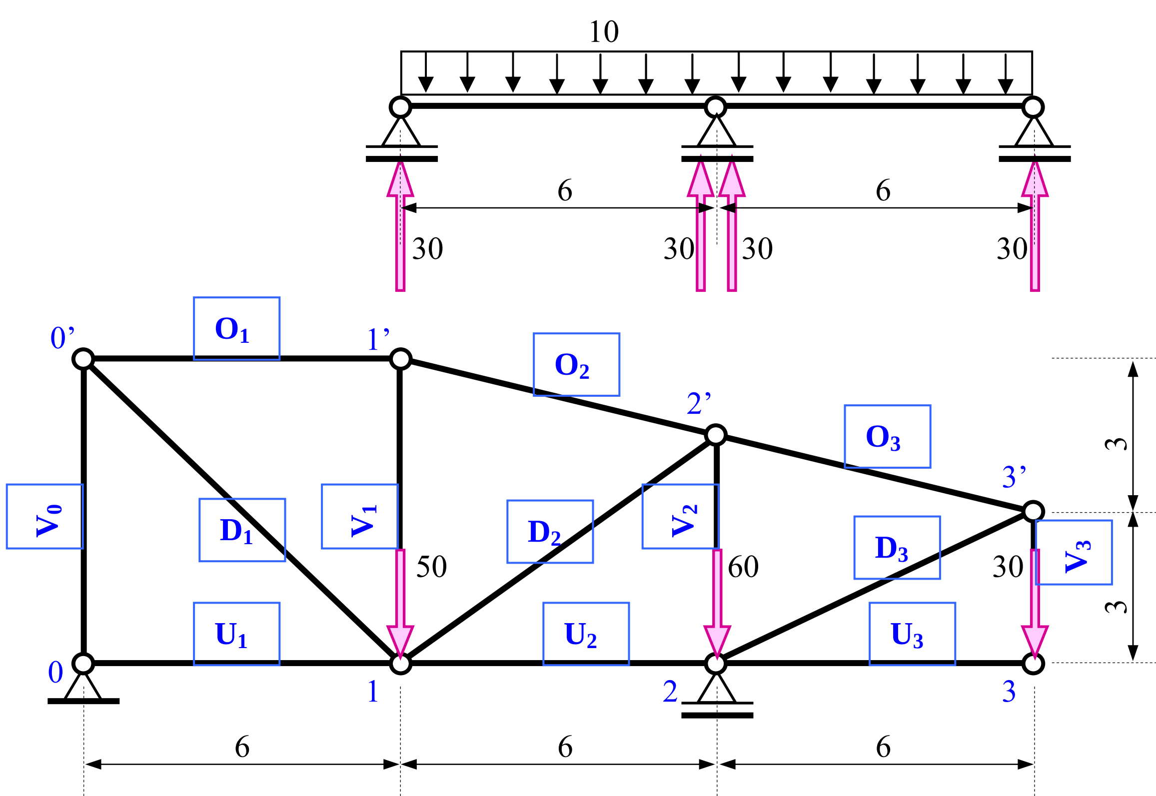
Numerical example

Calculate the member forces of the truss given below



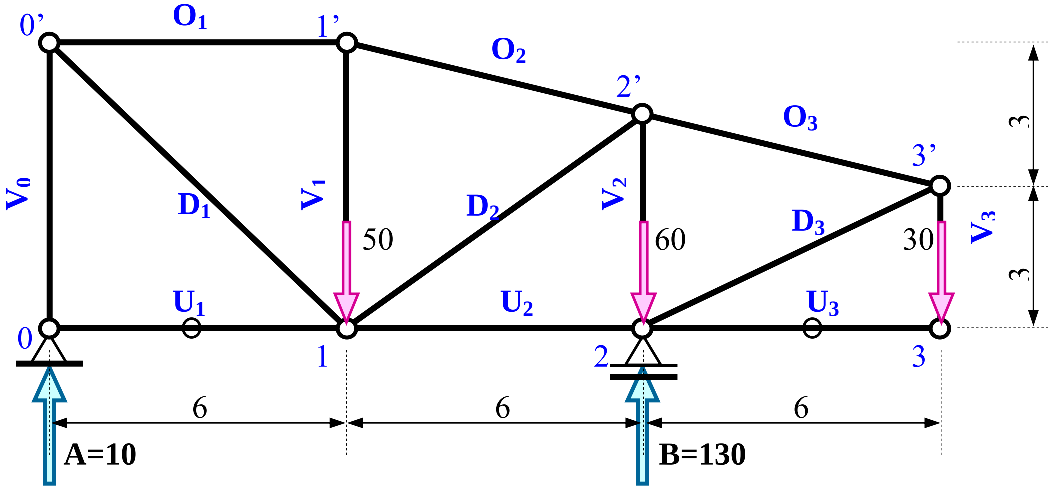
The loads must be applied as concentrated forces at the joints of the truss, because the truss members are subjected to tension or compression only (not to flexure). Thus, the first step is the modification of out of joint loading into equivalent joint forces. This modification will be realized by series of simply supported beams, restrained to the truss joints (restraints must be always joints belonging to the road chord). The support reactions taken as actions are the equivalent joint forces.

2.1 Modification of out of joint loads into equivalent joint forces.



At first the support reaction acting on the truss should be determined. Then, each node, at which not more than two unknown member forces are presented, is systematically selected in turn and the equilibrium equations are applied to solve the unknown forces.

2.2 Support reactions.

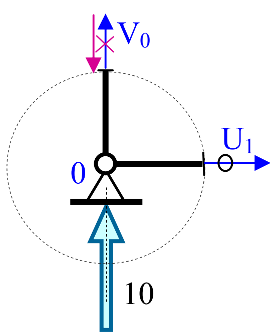


$\Sigma M_A = 0 \quad 50 \cdot 6 + 60 \cdot 12 + 30 \cdot 18 - B \cdot 12 = 0 \rightarrow \mathbf{B = 130}$

$\Sigma M_B = 0 \quad -50 \cdot 6 + 30 \cdot 6 + A \cdot 12 = 0 \rightarrow A = \mathbf{10}$

Verification: $\Sigma V = 0 \quad 10 + 130 - 50 - 60 - 30 = \mathbf{140 - 140}$

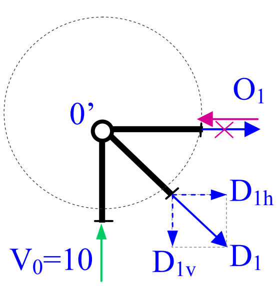
2.3 Member forces.



Joint 0

$\Sigma H = 0 \quad U_1 = 0$

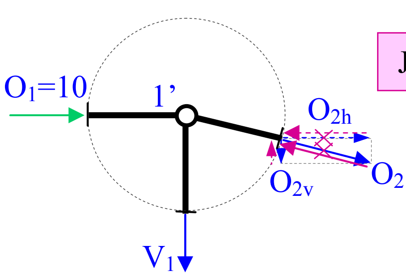
$\Sigma V = 0 \quad V_0 + 10 = 0, V_0 = -10$



Joint 0'

$\Sigma V = 0 \quad D_{1v} = 10, D_{1h} = (D_{1v}/6) \cdot 6 = 10$

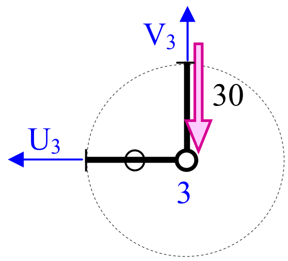
$\Sigma H = 0 \quad 10 + O_1 = 0, O_1 = -10$



Joint 1'

$\Sigma H = 0 \quad O_{2h} = -10, O_{2v} = O_{2h}/6 \cdot 1.5 = -2.5$

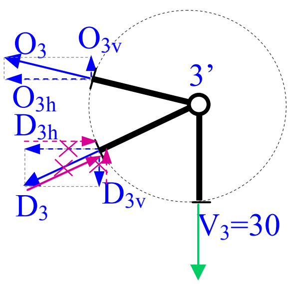
$\Sigma V = 0 \quad V_1 = 2.5$



Joint 3

$\Sigma H = 0 \quad U_3=0$

$\Sigma V = 0 \quad V_3=30$

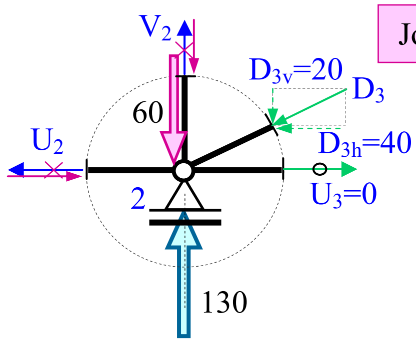


Joint 3'

$D_{3h}=D_{3v}/3\cdot 6=2\,D_{3v}$, $O_{3h}=O_{3v}/1.5\cdot 6=4O_{3v}$

$$\begin{array}{l|l} \Sigma H = 0 & O_{3v}-D_{3v}=30 \\ \Sigma V = 0 & -O_{3h}-D_{3h}=0 \end{array}$$

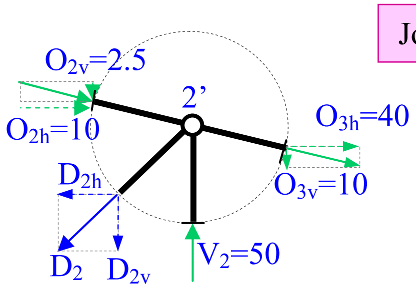
$$\left| \begin{array}{l} O_{3v}-D_{3v}=30 \\ -4O_{3v}-2\,D_{3v}=0 \end{array} \right. \Rightarrow \begin{array}{l} D_{3v}=-20 \text{ therefore } D_{3h}=-40 \\ O_{3v}=10 \text{ therefore } O_{3h}=40 \end{array}$$



Joint 2

$\Sigma H = 0 \quad U_2=-40$

$\Sigma V = 0 \quad V_2-60+130-20=0,\ V_2=-50$

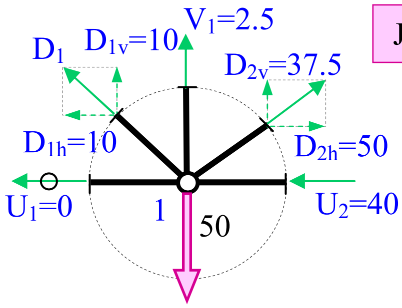


Joint 2'

$\Sigma H = 0 \quad -D_{2h}+10+40=0,\ D_{2v}=50,\ D_{2v}=D_{2h}/6\cdot 4.5=37.5$

$\Sigma V = 0 \quad$ - verification!

$50-37.5-2.5-10=50-50=0$



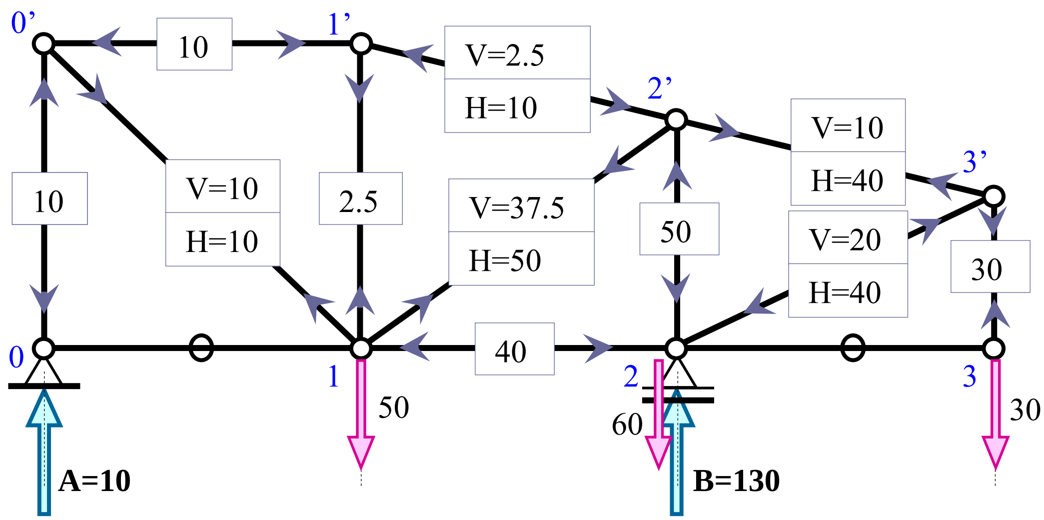
Joint 1

Verification!

$\Sigma H = 0 \quad -10+50-40=50-50=0$

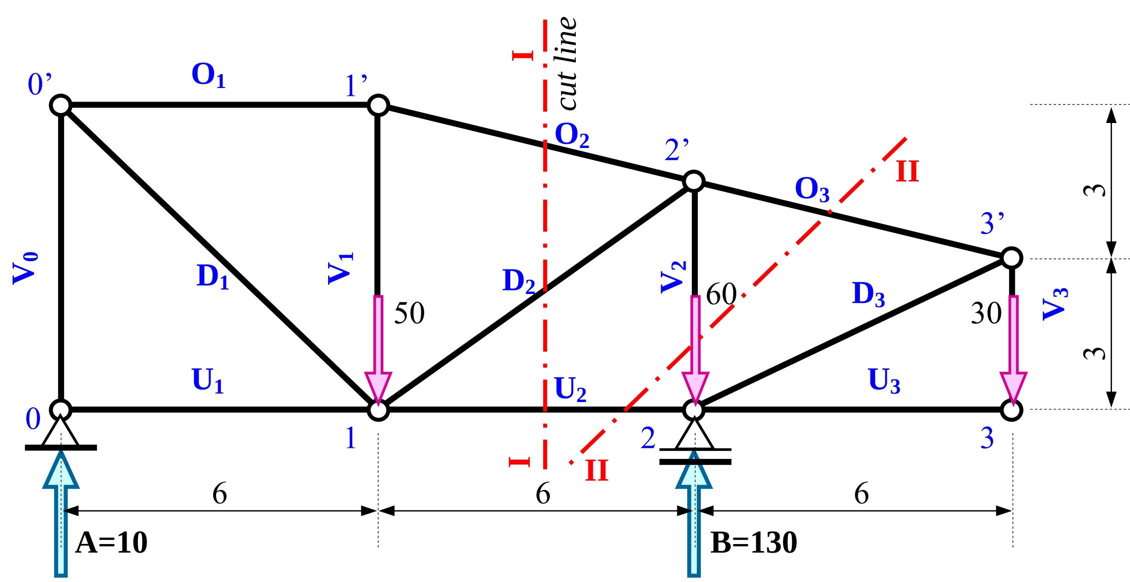
$\Sigma V = 0 \quad -50+10+2.5+37.5=-50+50=0$

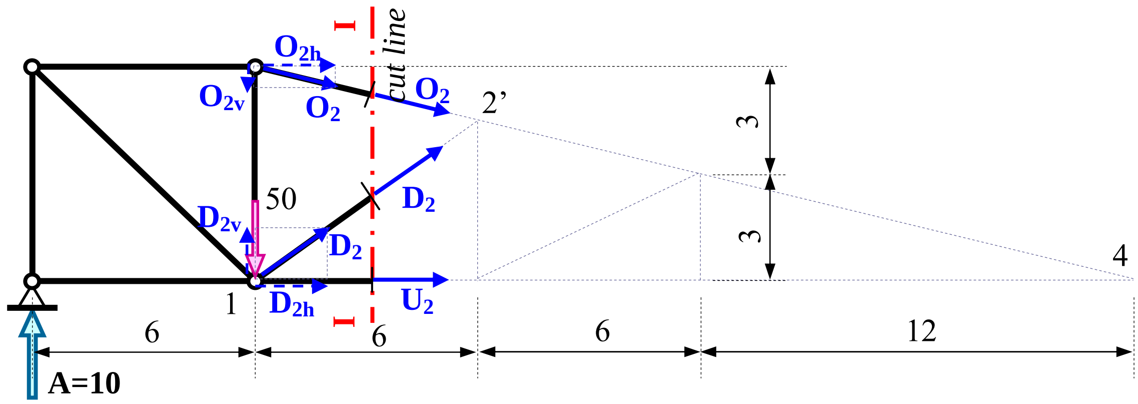
Finally, all member forces should be depicted on the whole truss.



3. Method of moments

This method uses the concept of a free body diagram to determine the member forces in the member of a specific panel of a truss. The section must passes in such a way as to cut three members with no common cross point. The axes of such members will intersect by pairs at three different points. The equilibrium equations of the moments of all forces (both internal and external) acting on the cut off portion taken for the intersection point will give one equation with one unknown. This unknown is the internal force acting in the bar not passing through the moment point.
The point of intersection of two members about which the moments are taken is usually called the origin of moments.
When two of the cut members are collinear, their intersection point is at the infinity, then the equilibrium equation is projection, usually $\Sigma V = 0$.





$\Sigma M_1 = 0$ $O_{2h} \cdot 6 + 10 \cdot 6 = 0 \rightarrow O_{2h} = -10$, $O_{2v} = O_{2h}/6 \cdot 1.5 = -10/6 \cdot 1.5 = -2.5$

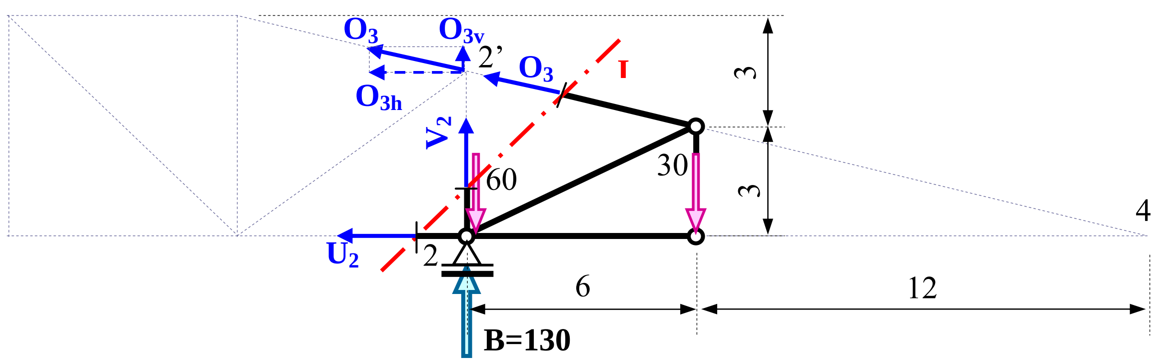
$\Sigma M_{2'} = 0$ $U_2 \cdot 4.5 - 10 \cdot 12 + 50 \cdot 6 = 0 \rightarrow U_2 = -40$

$\Sigma M_4 = 0$ $D_{2v} \cdot 24 - 50 \cdot 24 + 10 \cdot 30 = 0 \rightarrow D_{2v} = 37.5$, $D_{2h} = D_{2v}/4.5 \cdot 6 = 37.5/4.5 \cdot 6 = 50$

Verification:

$\Sigma V = 0$ $A + D_{2v} - O_{2v} - 50 = 10 + 37.5 + 2.5 - 50 = 50 - 50$

$\Sigma H = 0$ $D_{2h} + U_2 + O_{2h} = 50 - 40 - 10 = 50 - 50$



$\Sigma M_4 = 0$ $V_2 \cdot 18 + 130 \cdot 18 - 60 \cdot 18 - 30 \cdot 12 = 0 \rightarrow V_2 = -50$

$\Sigma M_2 = 0$ $O_{3h} \cdot 4.5 - 30 \cdot 6 = 0 \rightarrow O_{3h} = 40$, $O_{3v} = O_{3h}/6 \cdot 1.5 = 10$

Verification:

$\Sigma V = 0$ $V_2 + O_{3v} - 60 - 30 + 130 = -50 + 10 - 60 - 30 + 130 = -140 + 140$

$\Sigma H = 0$ $-U_2 - O_{3h} = 40 - 40 = 0$

Trusses with subdivided panels

The subdivision of the panels of main trusses, with the introduction of secondary members, forming auxiliary trusses, is meant to transmit the loads within the panel to the joints of the main truss. These auxiliary systems lead to the considerable reduction of the weight of the floor elements.

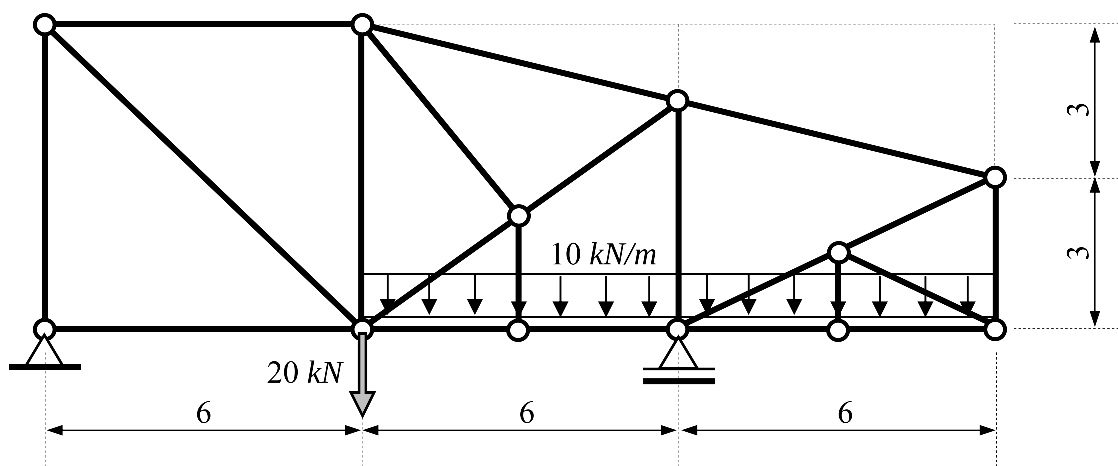
These systems will remain idle as long as the load is outside of the panel which they reinforce, and will become stressed (loaded) only while the load is within the limits of that panel.

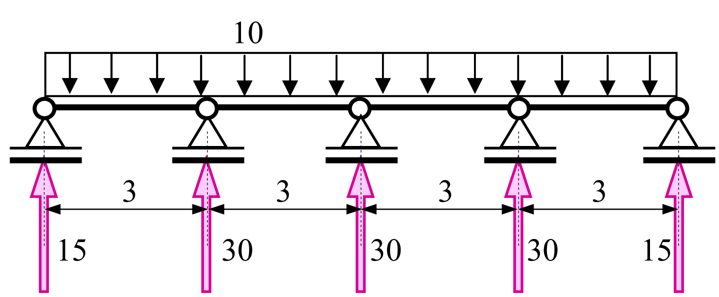
The members of trusses with subdivided panels can be classified into three groups:

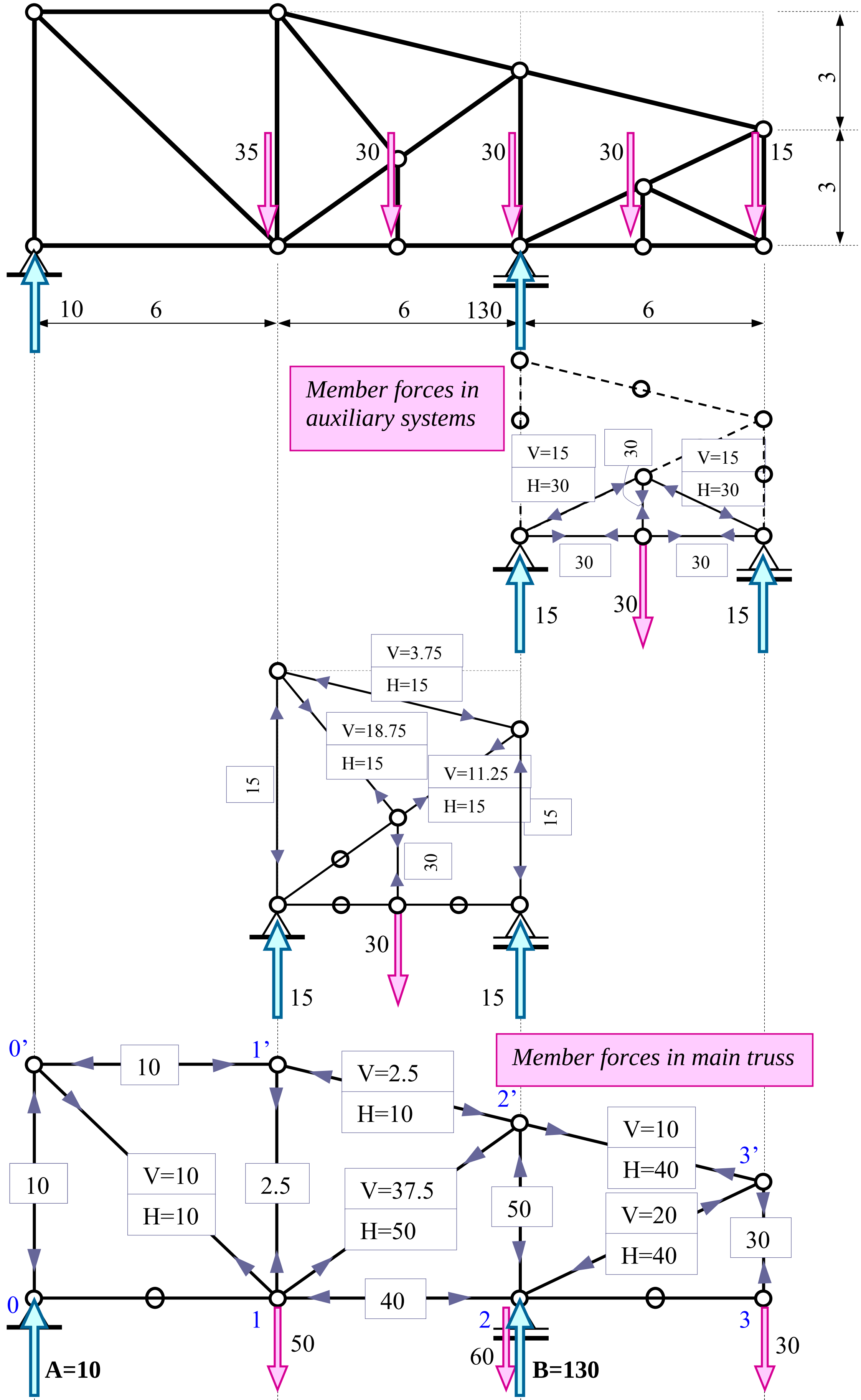
1) Members belonging to the main truss only, which are not influenced by the presence of auxiliary systems;
2) Members belonging entirely to the auxiliary systems. Their axial forces can be obtained in the same manner as for an isolated end-supported beam;
3) Members belonging simultaneously to the main and to the auxiliary systems. These member forces should be obtained by the summation of forces pertaining to the main and to the auxiliary trusses considered separately.

The member forces of the truss with subdivided panels should be obtained as follows:

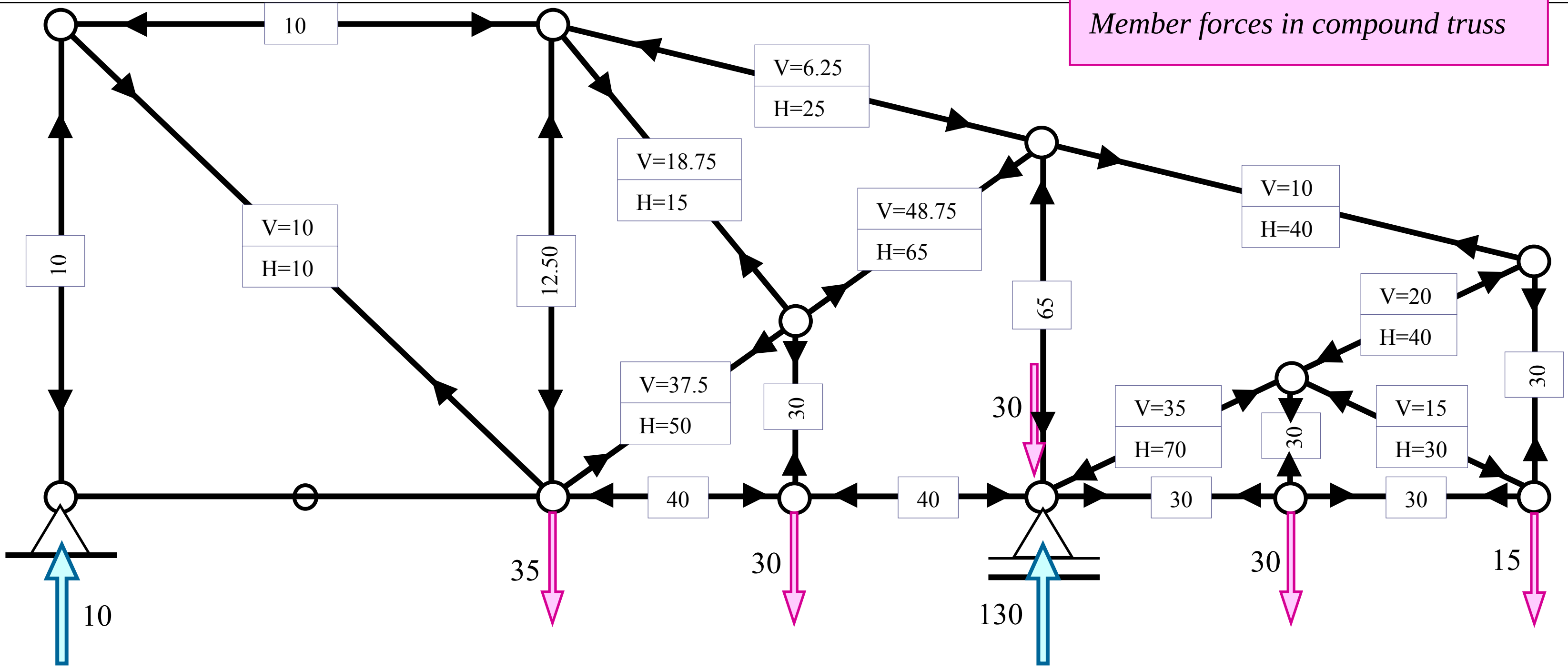
1) Calculation of the forces of the auxiliary systems isolated from the main truss, simply supported in the nodes of the road of the main truss;
2) What follows is the loading of the main truss with nodal concentrated forces along with the reactions of the auxiliary systems taken as actions. Next the calculation of the main truss, disregarding the presence of the secondary elements;
3) Finally summation of the member forces calculated for the main and auxiliary trusses.

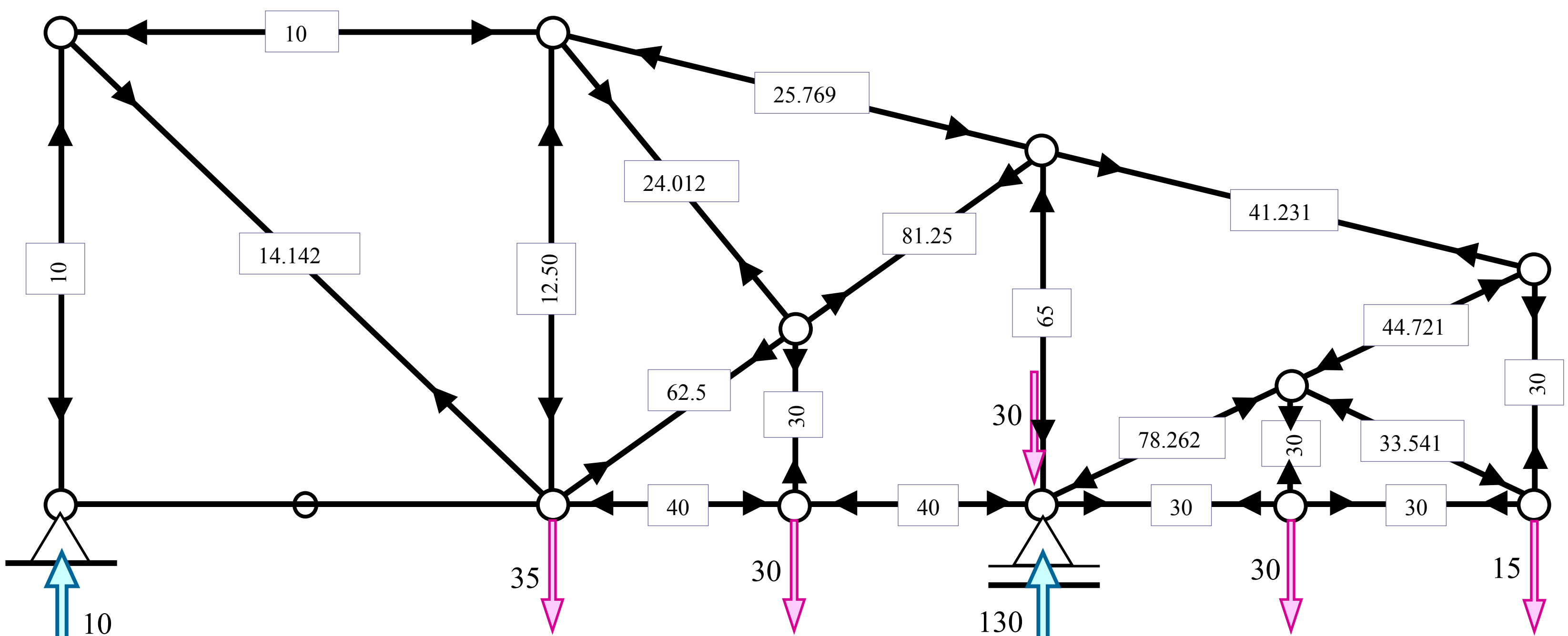





35
30
30
30
15
3
3
10
6
6
130
6
Member forces in auxiliary systems
V=15
H=30
30
V=15
H=30
30
30
15
30
15
V=3.75
H=15
V=18.75
H=15
V=11.25
H=15
15
15
30
30
15
15
Member forces in main truss
0'
1'
2'
3'
10
V=2.5
H=10
V=10
H=40
10
V=10
H=10
2.5
V=37.5
H=50
50
V=20
H=40
30
0
1
2
3
40
50
60
30
A=10
B=130

Member forces in compound truss





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