Chapter 4. TENSION MEMBER DESIGN

4.1 INTRODUCTORY CONCEPTS

- Stress: The stress in an axially loaded tension member is given by Equation (4.1)

$$f = \frac{P}{A} \tag{4.1}$$

where, P is the magnitude of load, and

A is the cross-sectional area normal to the load

- The stress in a tension member is uniform throughout the cross-section except:
  - near the point of application of load, and
  - at the cross-section with holes for bolts or other discontinuities, etc.
- For example, consider an 8 x ½ in. bar connected to a gusset plate and loaded in tension as shown below in Figure 4.1

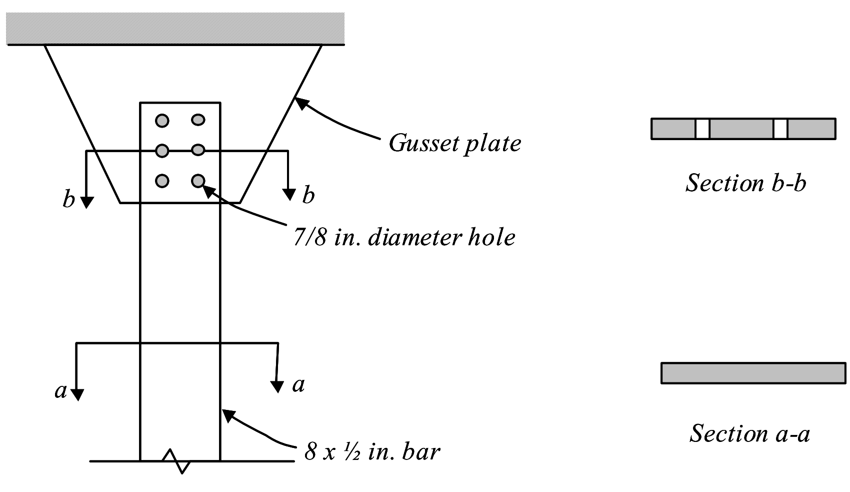


Figure 4.1 Example of tension member.

- Area of bar at section *a – a* = 8 x ½ = 4 in^2
- Area of bar at section *b – b* = (8 – 2 x 7/8) x ½ = 3.12 in^2

- Therefore, by definition (Equation 4.1) the reduced area of section *b – b* will be subjected to higher stresses
- However, the reduced area and therefore the higher stresses will be localized around section b – b.
- The unreduced area of the member is called its gross area = A_g
- The reduced area of the member is called its net area = A_n

4.2 STEEL STRESS-STRAIN BEHAVIOR

- The stress-strain behavior of steel is shown below in Figure 4.2

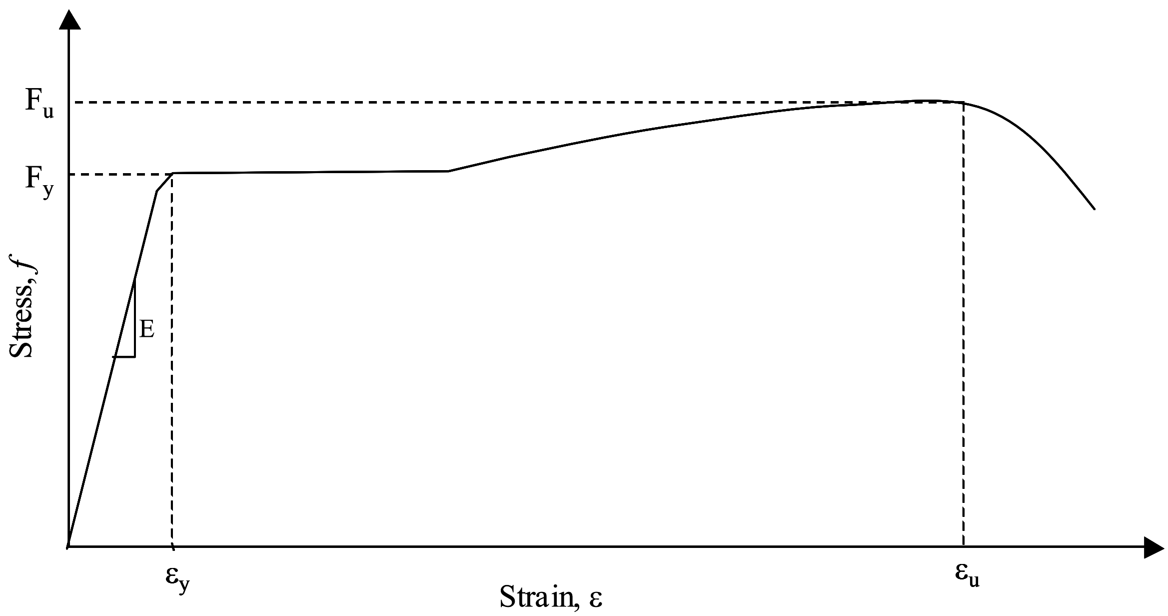


Figure 4.2 Stress-strain behavior of steel

- In Figure 4.2, E is the elastic modulus = 29000 ksi.

  F_y is the yield stress and F_u is the ultimate stress

  ε_y is the yield strain and ε_u is the ultimate strain

- Deformations are caused by the strain ε. Figure 4.2 indicates that the structural deflections will be small as long as the material is elastic ($f < F_y$)
- Deformations due to the strain ε will be large after the steel reaches its yield stress F_y.

4.3 DESIGN STRENGTH

- A tension member can fail by reaching one of two limit states:
  (1) excessive deformation; or (2) fracture
- Excessive deformation can occur due to the yielding of the gross section (for example section a-a from Figure 4.1) along the length of the member
- Fracture of the net section can occur if the stress at the net section (for example section b-b in Figure 4.1) reaches the ultimate stress F_u.
- The objective of design is to prevent these failure before reaching the ultimate loads on the structure (*Obvious*).
- This is also the load and resistance factor design approach recommended by AISC for designing steel structures

4.3.1 Load and Resistance Factor Design

The load and resistance factor design approach is recommended by AISC for designing steel structures. It can be understood as follows:

<u>Step I. Determine the ultimate loads acting on the structure</u>

- The values of D, L, W, etc. given by ASCE 7-98 are nominal loads (not maximum or ultimate)
- During its design life, a structure can be subjected to some maximum or ultimate loads caused by combinations of D, L, or W loading.

- The ultimate load on the structure can be calculated using factored load combinations, which are given by ASCE and AISC (see pages 2-10 and 2-11 of AISC manual). The most relevant of these load combinations are given below:

$$1.4\ D \tag{4.2 – 1}$$

$$1.2\ D + 1.6\ L + 0.5\ (L_r \text{ or } S) \tag{4.2 – 2}$$

$$1.2\ D + 1.6\ (L_r \text{ or } S) + (0.5\ L \text{ or } 0.8\ W) \tag{4.2 – 3}$$

$$1.2\ D + 1.6\ W + 0.5\ L + 0.5\ (L_r \text{ or } S) \tag{4.2 – 4}$$

$$0.9\ D + 1.6\ W \tag{4.2 – 5}$$

Step II. Conduct linear elastic structural analysis

- Determine the design forces (P_u, V_u, and M_u) for each structural member

Step III. Design the members

- The failure (design) strength of the designed member must be greater than the corresponding design forces calculated in Step II. See Equation (4.3) below:

$$\phi R_n > \sum \gamma_i\ Q_i \tag{4.3}$$

- Where, R_n is the calculated failure strength of the member
- ϕ is the resistance factor used to account for the reliability of the material behavior and equations for R_n
- Q_i is the nominal load
- γ_i is the load factor used to account for the variability in loading and to estimate the ultimate loading condition.

4.3.2 Design Strength of Tension Members

- Yielding of the gross section will occur when the stress f reaches F_y.

$$f = \frac{P}{A_g} = F_y$$

Therefore, nominal yield strength = $P_n = A_g F_y$ (4.4)

Factored yield strength = $\phi_t P_n$ (4.5)

where, $\phi_t = 0.9$ for tension yielding limit state

- See the AISC manual, section on <u>spec</u>ifications, Chapter D (page 16.1 –24)
- Facture of the net section will occur after the stress on the net section area reaches the ultimate stress F_u

$$f = \frac{P}{A_e} = F_u$$

Therefore, nominal fracture strength = $P_n = A_e F_u$

Where, A_e is the effective net area, which may be equal to the net area or smaller.

The topic of A_e will be addressed later.

Factored fracture strength = $\phi_t A_e F_u$ (4.6)

Where, $\phi_t = 0.75$ for tension fracture limit state (See page 16.1-24 of AISC manual)

4.3.3 Important notes

- <u>Note 1</u>. Why is fracture (& not yielding) the relevant limit state at the net section?

  Yielding will occur first in the net section. However, the deformations induced by yielding will be localized around the net section. These localized deformations will *not* cause excessive deformations in the complete tension member. Hence, yielding at the net section will *not* be a failure limit state.

- <u>Note 2.</u> Why is the resistance factor (ϕ_t) smaller for fracture than for yielding?

  The smaller resistance factor for fracture ($\phi_t = 0.75$ as compared to $\phi_t = 0.90$ for yielding) reflects the more serious nature and consequences of reaching the fracture limit state.

- <u>Note 3.</u> What is the design strength of the tension member?

The design strength of the tension member will be the lesser value of the strength for the two limit states (gross section yielding and net section fracture).

- Note 4. Where are the F_y and F_u values for different steel materials?

  The yield and ultimate stress values for different steel materials are noted in Table 2 in the *AISC* manual on pages *16.1–141* and *16.1–142*.

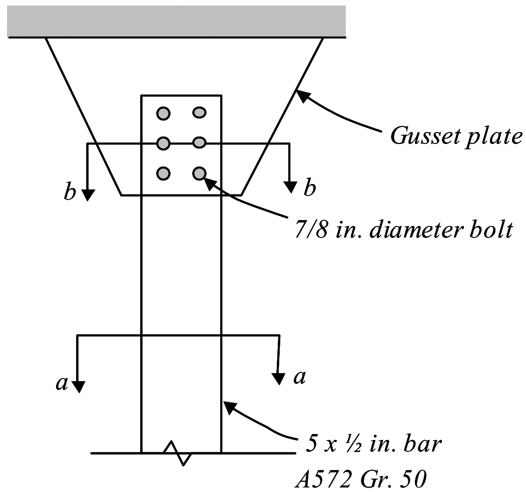
- Note 5. What are the most common steels for structural members?

  See Table 2-1 in the *AISC* manual on pages *2–24* and *2-25*. According to this Table: the preferred material for *W* shapes is *A992* (F_y = 50 ksi; F_u = 65 ksi); the preferred material for *C*, *L* , *M* and *S* shapes is *A36* (F_y = 36 ksi; F_u = 58 ksi). All these shapes are also available in *A572 Gr. 50* (F_y = 50 ksi; F_u = 65 ksi).

- Note 6. What is the amount of area to be deducted from the gross area to account for the presence of bolt-holes?

  - The *nominal* diameter of the hole (d_h) is equal to the bolt diameter (d_b) + 1/16 in.
  - However, the bolt-hole fabrication process damages additional material around the hole diameter.
  - Assume that the material damage extends 1/16 *in*. around the hole diameter.
  - Therefore, for calculating the net section area, assume that the gross area is *reduced by a hole diameter* equal to the nominal hole-diameter + 1/16 in.

Example 3.1 A 5 x ½ bar of A572 Gr. 50 steel is used as a tension member. It is connected to a gusset plate with six 7/8 in. diameter bolts as shown in below. Assume that the effective net area A_e equals the actual net area A_n and compute the tensile design strength of the member.



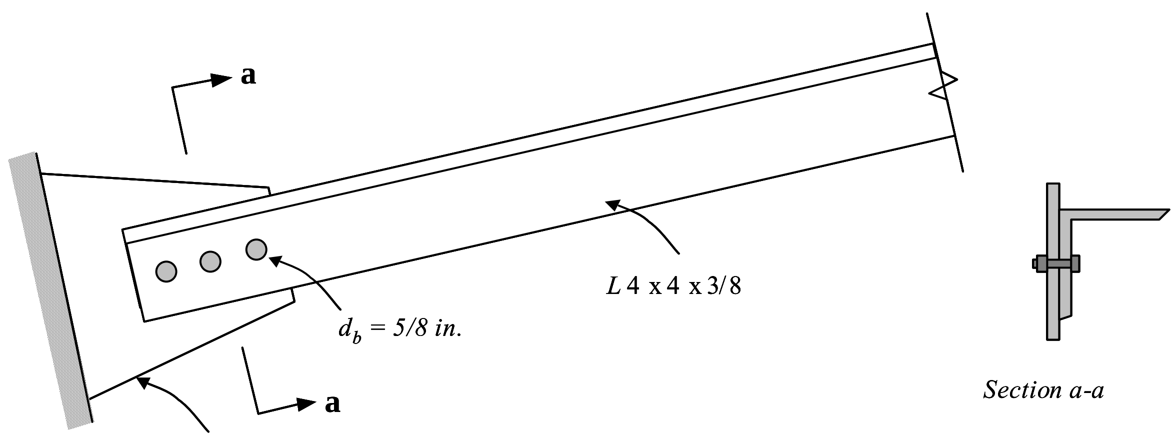
Solution

- Gross section area = A_g = 5 x ½ = 2.5 in^2
- Net section area (A_n)
  - Bolt diameter = d_b = 7/8 in.
  - Nominal hole diameter = d_h = 7/8 + 1/16 in. = 15/16 in.
  - Hole diameter for calculating net area = 15/16 + 1/16 in. = 1 in.
  - Net section area = A_n = (5 – 2 x (1)) x ½ = 1.5 in^2
- Gross yielding design strength = $\phi_t P_n = \phi_t F_y A_g$
  - Gross yielding design strength = 0.9 x 50 ksi x 2.5 in^2 = 112.5 kips
- Fracture design strength = $\phi_t P_n = \phi_t F_u A_e$
  - Assume $A_e = A_n$ (only for this problem)
  - Fracture design strength = 0.75 x 65 ksi x 1.5 in^2 = 73.125 kips
- Design strength of the member in tension = smaller of 73.125 kips and 112.5 kips

- Therefore, design strength = 73.125 kips (*net section fracture controls*).

<u>Example 3.2</u> A single angle tension member, *L* 4 x 4 x 3/8 in. made from A36 steel is connected to a gusset plate with 5/8 in. diameter bolts, as shown in Figure below. The service loads are 35 kips dead load and 15 kips live load. Determine the adequacy of this member using AISC specification. Assume that the effective net area is 85% of the computed net area. (*Calculating the effective net area will be taught in the next section*).

- Gross area of angle = A_g = 2.86 in^2 (*from Table 1-7 on page 1-36 of AISC*)



- Net section area = A_n
  - Bolt diameter = 5/8 in.
  - Nominal hole diameter = 5/8 + 1/16 = 11/16 in.
  - Hole diameter for calculating net area = 11/16 + 1/16 = 3/4 in.
  - Net section area = A_g – (3/4) x 3/8 = 2.86 – 3/4 x 3/8 = 2.579 in^2
- Effective net area = A_e = 0.85 x 2.579 in^2 = 2.192 in^2
- Gross yielding design strength = ϕ_t A_g F_y = 0.9 x 2.86 in^2 x 36 ksi = 92.664 kips
- Net section fracture = ϕ_t A_e F_u = 0.75 x 2.192 in^2 x 58 ksi = 95.352 kips

- Design strength = 92.664 kips (gross yielding governs)
- Ultimate (design) load acting for the tension member = P_u
  - The ultimate (design) load can be calculated using factored load combinations given on page *2-11* of the *AISC* manual, or Equations (*4.2-1 to 4.2-5*) of notes (see pg. 4)
  - According to these equations, two loading combinations are important for this problem. These are: (1) 1.4 *D*; and (2) 1.2 *D* + 1.6 *L*
  - The corresponding ultimate (design) loads are:

    1.4 x (P_D) = 1.4 (35) = 49 kips

    1.2 (P_D) + 1.6 (P_L) = 66 kips (controls)
  - The ultimate design load for the member is 66 kips, where the *factored* dead + live loading condition controls.
- Compare the design strength with the ultimate design load
  - The design strength of the member (92.664 kips) is greater than the ultimate design load (66 kips).
  - ϕ_t P_n (92.664 kips) > P_u (66 kips)
- The *L* 4 x 4 x 3/8 in. made from A36 steel is adequate for carrying the factored loads.

4.4 EFFECTIVE NET AREA

- The *connection* has a significant influence on the performance of a tension member. A connection almost always weakens the member, and a measure of its influence is called joint efficiency.
- Joint efficiency is a function of: (a) material ductility; (b) fastener spacing; (c) stress concentration at holes; (d) fabrication procedure; and (e) **shear lag**.

- All factors contribute to reducing the effectiveness but shear lag is the most important.
- Shear lag occurs when the tension force is not transferred *simultaneously* to all elements of the cross-section. This will occur when some elements of the cross-section are not connected.
- For example, see Figure 4.3 below, where only one leg of an angle is bolted to the gusset plate.

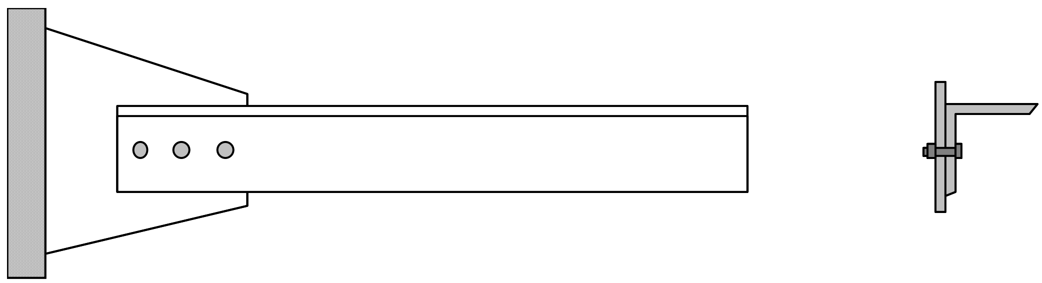


Figure 4.3 Single angle with bolted connection to only one leg.

- A consequence of this partial connection is that the connected element becomes overloaded and the unconnected part is not fully stressed.
- Lengthening the connection region will reduce this effect
- Research indicates that shear lag can be accounted for by using a reduced or effective net area A_e
- Shear lag affects both bolted and welded connections. Therefore, the effective net area concept applied to both types of connections.
  - For bolted connection, the effective net area is $A_e = U\, A_n$
  - For welded connection, the effective net area is $A_e = U\, A_g$
- Where, the reduction factor U is given by:

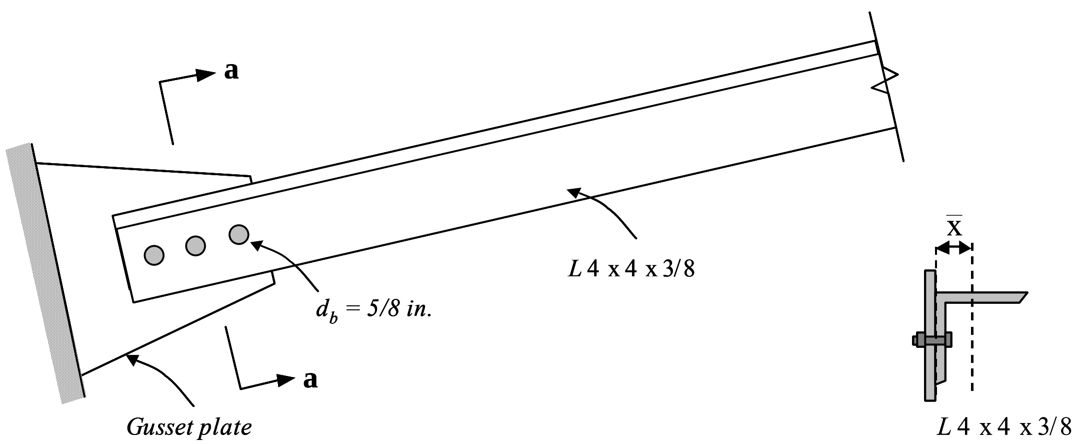
$$U = 1 - \frac{\bar{x}}{L} \leq 0.9 \qquad (4.7)$$

- Where, $\bar{x}$ is the distance from the centroid of the connected area to the plane of the connection, and L is the length of the connection.
  - If the member has two symmetrically located planes of connection, $\bar{x}$ is measured from the centroid of the nearest one – half of the area.
  - Additional approaches for calculating $\bar{x}$ for different connection types are shown in the *AISC* manual on page ***16.1-178.***
- The distance L is defined as the length of the connection in the direction of load.
  - For bolted connections, L is measured from the center of the bolt at one end to the center of the bolt at the other end.
  - For welded connections, it is measured from one end of the connection to other.
  - If there are weld segments of different length in the direction of load, L is the length of the longest segment.
  - Example pictures for calculating L are given on page ***16.1-179*** of *AISC.*

• The AISC manual also gives values of *U* that can be used instead of calculating $\bar{x}/L$.
- They are based on average values of $\bar{x}/L$ for various bolted connections.
- For W, M, and S shapes with width-to-depth ratio of at least 2/3 and for Tee shapes cut from them, if the connection is through the flanges with at least three fasteners per line in the direction of applied load ……………………………………………...$U = 0.90$
- For all other shapes with at least three fasteners per line …………………... $U = 0.85$
- For all members with only two fasteners per line …………………………… $U = 0.75$
- For better idea, see Figure 3.8 on page 41 of the Segui text-book.
- These values are acceptable but not the best estimate of U
- If used in the exam or homeworks, full points for calculating U will not be given

<u>Example 3.3</u> Determine the effective net area and the corresponding design strength for the single angle tension member of *Example 3.2*. The tension member is an *L* 4 x 4 x 3/8 in. made from A36 steel. It is connected to a gusset plate with 5/8 in. diameter bolts, as shown in Figure below. The spacing between the bolts is 3 in. center-to-center.

- Compare your results with those obtained for *Example 3.2*.



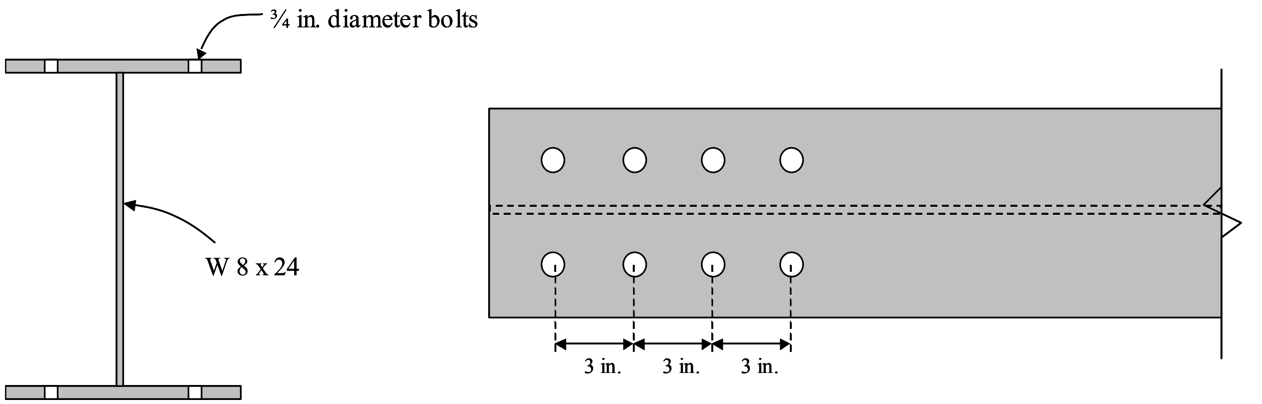
- Gross area of angle = A_g = 2.86 in^2 (*from Table 1-7 on page 1-36 of AISC*)
- Net section area = A_n
  - Bolt diameter = 5/8 in.
  - Hole diameter for calculating net area = 11/16 + 1/16 = 3/4 in.
  - Net section area = A_g – (3/4) x 3/8 = 2.86 – 3/4 x 3/8 = 2.579 in^2
- $\bar{x}$ is the distance from the centroid of the area connected to the plane of connection
  - For this case $\bar{x}$ is equal to the distance of centroid of the angle from the edge.
  - This value is given in the *Table 1-7 on page 1-36 of* the *AISC* manual.
  - $\bar{x}$ = 1.13 in.
- L is the length of the connection, which for this case will be equal to 2 x 3.0 in.

- L = 6.0 in.

- $U = 1 - \frac{\bar{x}}{L} = 1 - \frac{1.13}{6.0} = 0.8116$ in.
- Effective net area = A_e = 0.8116 x 2.579 in^2 = 2.093 in^2
- Gross yielding design strength = $\phi_t\, A_g\, F_y$ = 0.9 x 2.86 in^2 x 36 ksi = 92.664 kips
- Net section fracture = $\phi_t\, A_e\, F_u$ = 0.75 x 2.093 in^2 x 58 ksi = 91.045 kips
- Design strength = 91.045 kips (net section fracture governs)
- In Example 3.2
  - Factored load = P_u = 66.0 kips
  - Design strength = $\phi_t\, P_n$ = 92.66 kips (*gross section yielding governs*)
  - Net section fracture strength = $\phi_t\, P_n$ = 95.352 kips (*assuming* A_e = 0.85)
- Comparing Examples 3.2 and 3.3
  - Calculated value of *U* (0.8166) is less than the assumed value (0.85)
  - The assumed value was unconservative.
  - It is preferred that the *U* value be specifically calculated for the section.
  - After including the calculated value of *U*, net section fracture governs the design strength, but the member is still adequate from a design standpoint.

Example 3.4 Determine the design strength of an ASTM A992 W8 x 24 with four lines if ¾ in. diameter bolts in standard holes, two per flange, as shown in the Figure below.

Assume the holes are located at the member end and the connection length is 9.0 in. Also calculate at what length this tension member would cease to satisfy the slenderness limitation in LRFD specification B7



Holes in beam flange

Solution:

- For ASTM A992 material: F_y = 50 ksi; and F_u = 65 ksi
- For the W8 x 24 section:
  - A_g = 7.08 in^2 d = 7.93 in.
  - t_w = 0.285 in. b_f = 6.5 in.
  - t_f = 0.4 in. r_y = 1.61 in.
- Gross yielding design strength = $\phi_t P_n = \phi_t A_g F_y$ = 0.90 x 7.08 in^2 x 50 ksi = 319 kips
- Net section fracture strength = $\phi_t P_n = \phi_t A_e F_u$ = 0.75 x A_e x 65 ksi
  - $A_e = U A_n$ - for bolted connection
  - $A_n = A_g$ – (no. of holes) x (diameter of hole) x (thickness of flange)

    A_n = 7.08 – 4 x (diameter of bolt + 1/8 in.) x 0.4 in.

    A_n = 5.68 in^2

- $U = 1 - \dfrac{\bar{x}}{L} \leq 0.90$

- What is $\bar{x}$ for this situation?

  $\bar{x}$ *is the distance from the edge of the flange to the centroid of the half* (*T*) *section*

$$\bar{x} = \frac{(b_f \times t_f) \times \dfrac{t_f}{2} + (\dfrac{d - 2\,t_f}{2} \times t_w) \times (\dfrac{d + 2\,t_f}{4})}{b_f \times t_f + \dfrac{d}{2} \times t_w} = \frac{6.5 \times 0.4 \times 0.2 + 3.565 \times 0.285 \times 2.1825}{6.5 \times 0.4 + 3.565 \times 0.285} = 0.76$$

- $\bar{x}$ can be obtained from the dimension tables for Tee section *WT 4 x 12*. See page ***1-50*** and ***1-51*** of the AISC manual:

  $\bar{x}$ = 0.695 in.

- The calculated value is *not accurate* due to the deviations in the geometry

- $U = 1 - \dfrac{\bar{x}}{L} = 1 - \dfrac{0.695}{9.0} = 0.923$

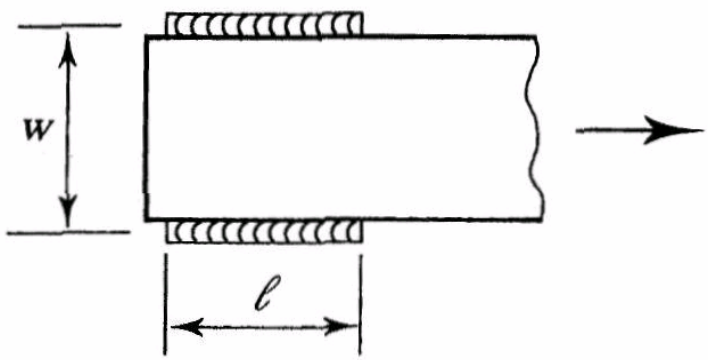
- But, $U \leq 0.90$. Therefore, assume $U = 0.90$

- Net section fracture strength = $\phi_t\, A_e\, F_u$ = 0.75 x 0.9 x 5.68 x 65 = 249.2 kips
- The design strength of the member is controlled by net section fracture = 249.2 kips
- According to LRFD specification B7, the maximum unsupported length of the member is limited to 300 r_y = 300 x 1.61 in. = 543 in. = 40.3 ft.

4.4.1 Special cases for welded connections

- If some elements of the cross-section are not connected, then A_e will be less than A_n
  - For a rectangular bar or plate A_e will be equal to A_n
  - However, if the connection is by longitudinal welds at the ends as shown in the figure below, then $A_e = UA_g$

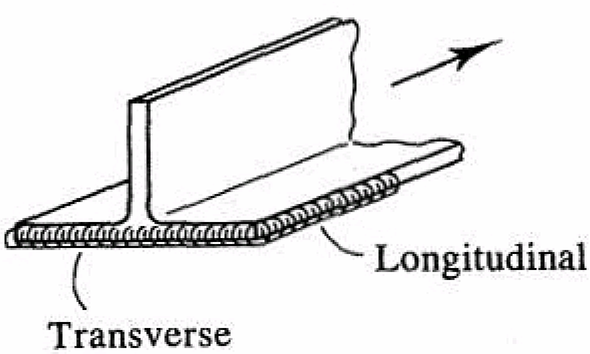
Where, $U = 1.0$ for $L \geq w$
$U = 0.87$ for $1.5\ w \leq L < 2\ w$
$U = 0.75$ for $w \leq L < 1.5\ w$
L = length of the pair of welds ≥ w
w = distance between the welds or width of plate/bar



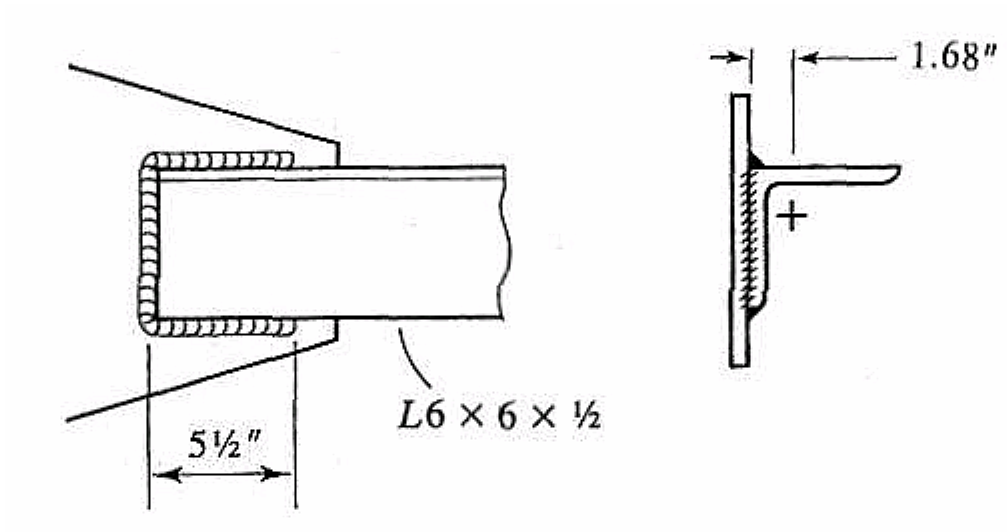
- AISC Specification B3 gives another special case for welded connections.

For any member connected by *transverse welds alone*,

A_e = area of the connected element of the cross-section

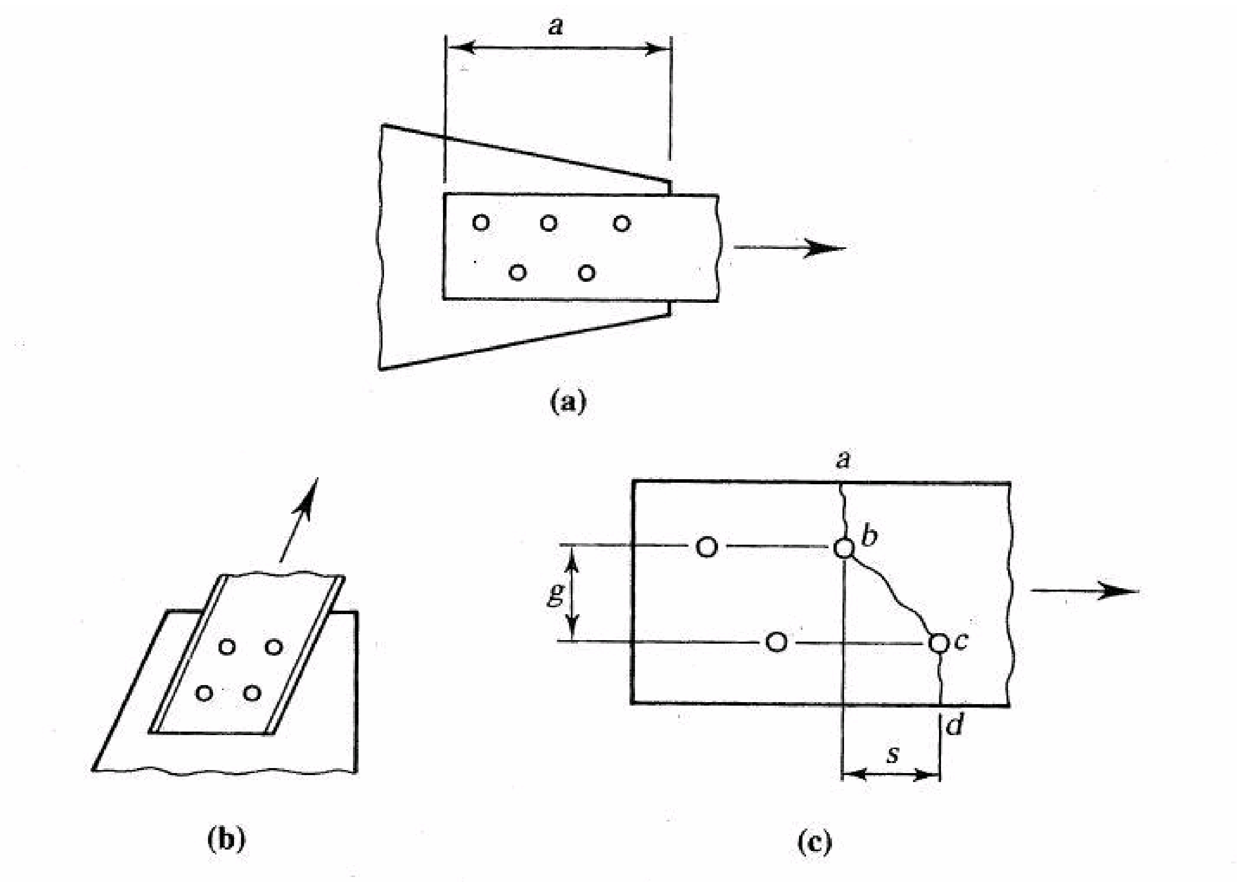


Example 3.5 Consider the welded single angle *L* 6x 6 x ½ tension member made from A36 steel shown below. Calculate the tension design strength.



Solution

- $A_g = 5.00 \text{ in}^2$
- $A_n = 5.00 \text{ in}^2$ - because it is a welded connection
- $A_e = U\,A_n$ - where, $U = 1 - \dfrac{\bar{x}}{L}$
  - $\bar{x}$ = 1.68 in. for this welded connection
  - L = 6.0 in. for this welded connection
  - $U = 1 - \dfrac{1.168}{6.0} = 0.72$
- Gross yielding design strength = $\phi_t\, F_y\, A_g$ = 0.9 x 36 x 5.00 = 162 kips
- Net section fracture strength = $\phi_t\, F_u\, A_e$ = 0.75 x 58 x 0.72 x 5.00 = 156.6 kips
- Design strength = 156.6 kips (*net section fracture governs*)



4.5 STAGGERED BOLTS

For a bolted tension member, the connecting bolts can be staggered for several reasons:

(1) To get more capacity by increasing the effective net area

(2) To achieve a smaller connection length

(3) To fit the geometry of the tension connection itself.

- For a tension member with staggered bolt holes (see example figure above), the relationship $f = P/A$ does not apply and the stresses are a combination of tensile and shearing stresses on the inclined portion *b-c*.

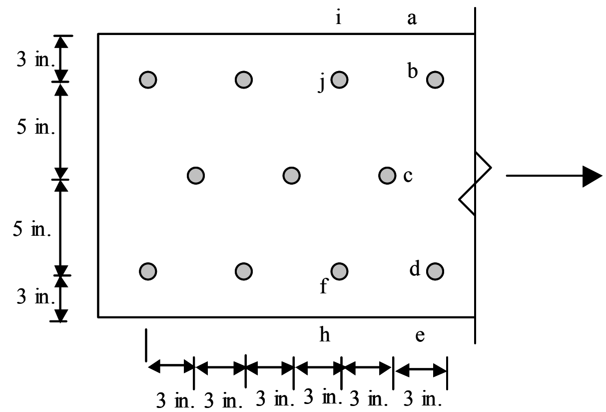
- Net section fracture can occur along any zig-zag or straight line. For example, fracture can occur along the inclined path *a-b-c-d* in the figure above. However, all possibilities must be examined.

- Empirical methods have been developed to calculate the net section fracture strength

  According to AISC Specification B2

  - net width = gross width $- \sum d + \sum \frac{s^2}{4g}$
  - where, d is the diameter of hole to be deducted (d_h + 1/16, or d_b + 1/8)
  - $s^2/4g$ is added for each gage space in the chain being considered
  - *s* is the longitudinal spacing (pitch) of the bolt holes in the direction of loading
  - *g* is the transverse spacing (gage) of the bolt holes perpendicular to loading dir.
  - net area (A_n) = net width x plate thickness
  - effective net area (A_e) = $U\,A_n$ where $U = 1 - \bar{x}/L$
  - net fracture design strength = $\phi_t\, A_e\, F_u$ ($\phi_t = 0.75$)

EXAMPLE 3.6 Compute the smallest net area for the plate shown below: The holes are for 1 in. diameter bolts.



- The effective hole diameter is 1 + 1/8 = 1.125 in.
- For line *a-b-d-e*

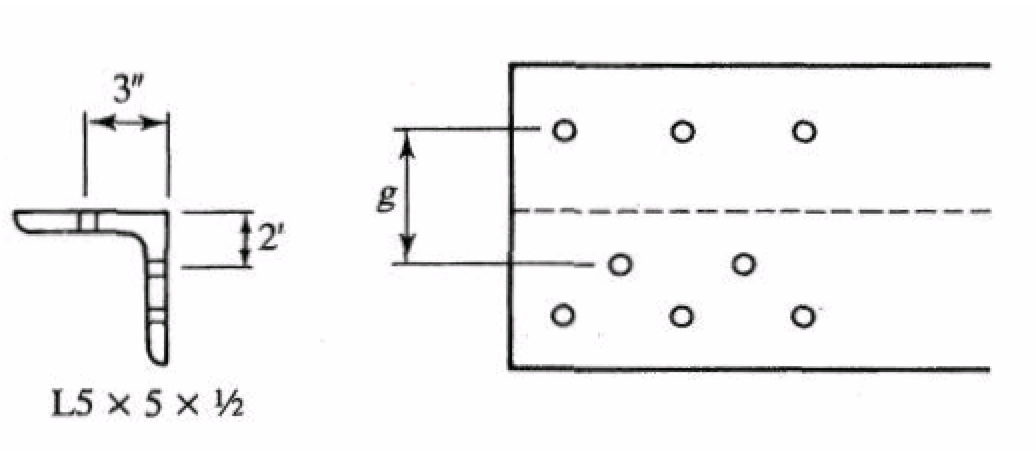
  $w_n = 16.0 – 2\ (1.125) = 13.75$ in.
- For line *a-b-c-d-e*

  $w_n = 16.0 – 3\ (1.125) + 2 \times 3^2/\ (4 \times 5) = 13.52$ in.
- The line *a-b-c-d-e* governs:
- $A_n = t\ w_n = 0.75\ (13.52) = 10.14\ in^2$

Note

- Each fastener resists an equal share of the load
- Therefore different potential failure lines may be subjected to different loads.
- For example, line *a-b-c-d-e* must resist the full load, whereas *i-j-f-h* will be subjected to 8/11 of the applied load. The reason is that 3/11 of the load is transferred from the member before *i-j-f-h* received any load.

- **<u>Staggered bolts in angles.</u>** If staggered lines of bolts are present in both legs of an angle, then the net area is found by first unfolding the angle to obtain an equivalent plate. This plate is then analyzed like shown above.
  - The unfolding is done at the middle surface to obtain a plate with gross width equal to the sum of the leg lengths minus the angle thickness.
  - AISC Specification B2 says that any gage line crossing the heel of the angle should be reduced by an amount equal to the angle thickness.
  - See Figure below. For this situation, the distance g will be = 3 + 2 – ½ in.



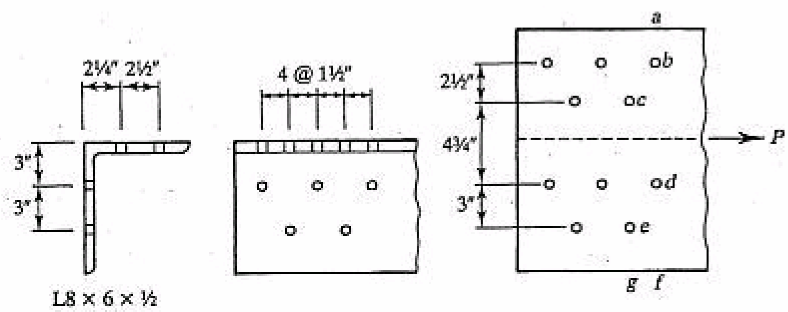
EXAMPLE 3.6

Find the design tensile strength of the angle shown in Figure 3.16. A36 steel is used, and holes are for ⅞-inch-diameter bolts.

SOLUTION Compute the net width:

$$w_g = 8 + 6 - \tfrac{1}{2} = 13.5 \text{ in.}$$

FIGURE 3.16



Effective hole diameter = ⅞ + ⅛ = 1 in.

For line *abdf*,

$$w_n = 13.5 - 2(1) = 11.5 \text{ in.}$$

For line *abceg*,

$$w_n = 13.5 - 3(1) + \frac{(1.5)^2}{4(2.5)} = 10.73 \text{ in.}$$

Because 1/10 of the load has been transferred from the member by the fastener at *d*, this potential failure line must resist only 9/10 of the load. Therefore the net width of 10.73 inch should be multiplied by 10/9 to obtain a net width that can be compared with those lines that resist the full load. Use $w_n = 10.73(10/9) = 11.92$ inch. For line *abcdeg*,

$$g_{cd} = 3 + 2.25 - 0.5 = 4.75 \text{ in.}$$

$$w_n = 13.5 - 4(1) + \frac{(1.5)^2}{4(2.5)} + \frac{(1.5)^2}{4(4.75)} + \frac{(1.5)^2}{4(3)} = 10.03 \text{ in.}$$

The last case controls:

$$A_n = t(w_n) = 0.5(10.03) = 5.015 \text{ in.}^2$$

Both legs of the angle are connected, so

$$A_e = A_n = 5.015 \text{ in.}^2$$

The design strength based on fracture is

$$\phi_t P_n = 0.75 F_u A_e = 0.75(58)(5.015) = 218 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90 F_y A_g = 0.90(36)(6.75) = 219 \text{ kips}$$

ANSWER Fracture controls; design strength = 218 kips.

4.6 BLOCK SHEAR

- For some connection configurations, the tension member can fail due to 'tear-out' of material at the connected end. This is called block shear.
- For example, the single angle tension member connected as shown in the Figure below is susceptible to the phenomenon of *block shear*.

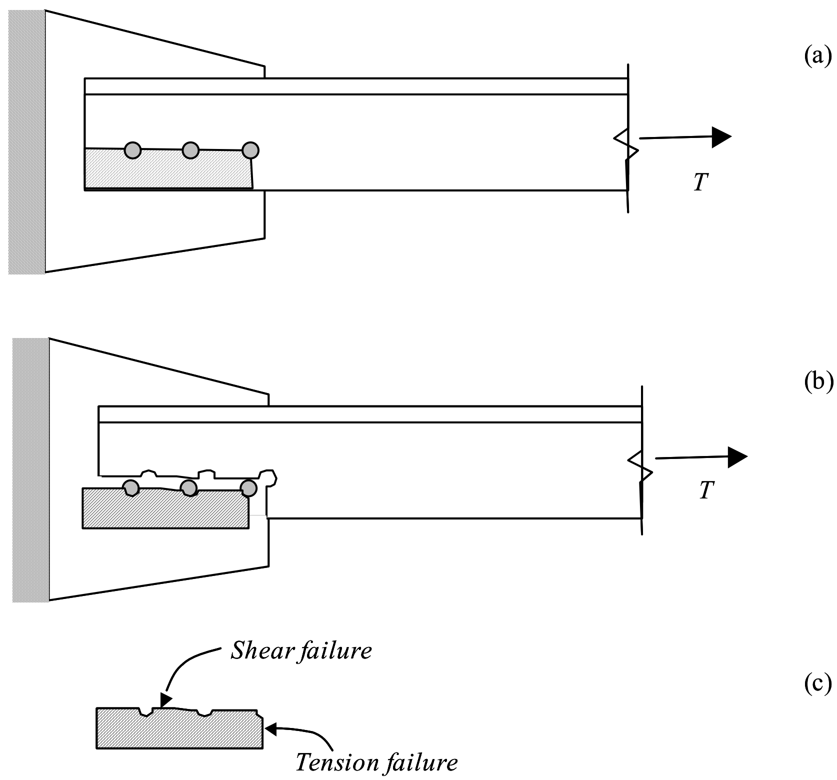


Figure 4.4 Block shear failure of single angle tension member

- For the case shown above, shear failure will occur along the longitudinal section a-b and tension failure will occur along the transverse section b-c
- AISC Specification (SPEC) Chapter D on tension members does not cover block shear failure explicitly. But, it directs the engineer to the Specification Section J4.3

- Block shear strength is determined as the <u>sum</u> of the shear strength on a failure path and the tensile strength on a perpendicular segment.
  - Block shear strength = net section fracture strength on shear path + gross yielding strength on the tension path
  - **<u>OR</u>**
  - Block shear strength = gross yielding strength of the shear path + net section fracture strength of the tension path
- Which of the two calculations above governs?
  - See page **16.1 – 67** (Section J4.3) of the AISC manual
  - When $F_u A_{nt} \geq 0.6 F_u A_{nv}$; $\phi_t R_n = \phi\,(0.6\ F_y\ A_{gv} + F_u\ A_{nt}) \leq \phi\,(0.6\ F_u A_{nv} + F_u\ A_{nt})$
  - When $F_u A_{nt} < 0.6 F_u A_{nv}$; $\phi_t R_n = \phi\,(0.6\ F_u\ A_{nv} + F_y\ A_{gt}) \leq \phi\,(0.6\ F_u A_{nv} + F_u\ A_{nt})$
  - Where, $\phi = 0.75$

    A_{gv} = gross area subject to shear

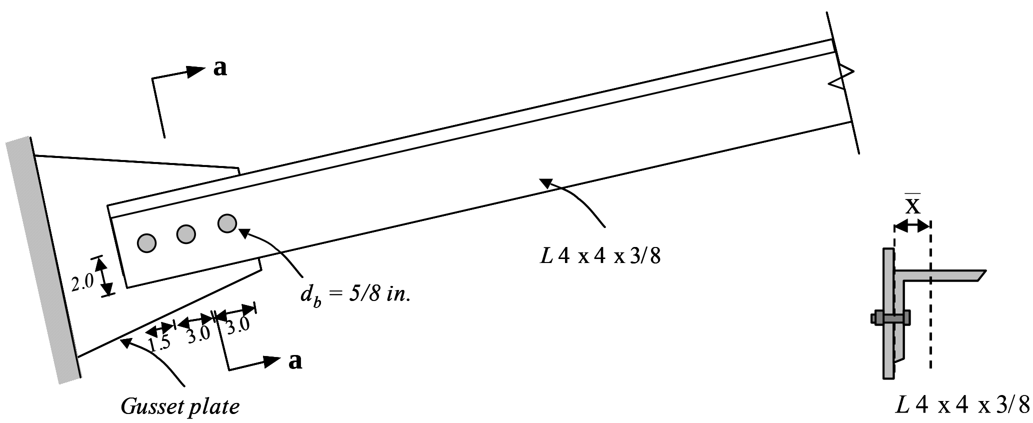
    A_{gt} = gross area subject to tension

    A_{nv} = net area subject to shear

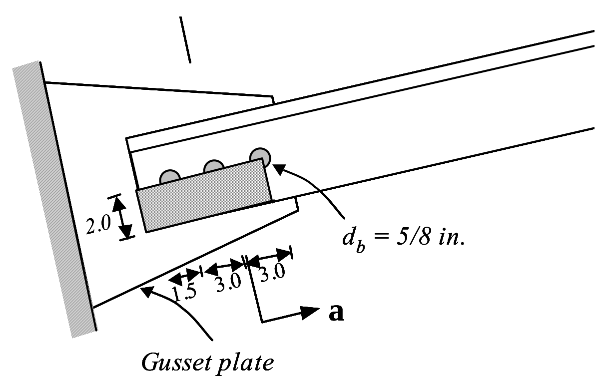
    A_{nt} = net area subject to tension

EXAMPLE 3.8 Calculate the block shear strength of the single angle tension member considered in Examples 3.2 and 3.3. The single angle *L 4 x 4 x 3/8* made from A36 steel is connected to the gusset plate with 5/8 in. diameter bolts as shown below. The bolt spacing is 3 in. center-to-center and the edge distances are 1.5 in and 2.0 in as shown in the Figure below.

Compare your results with those obtained in *Example 3.2 and 3.3*



- Step I. Assume a block shear path and calculate the required areas



- A_{gt} = gross tension area = 2.0 x 3/8 = 0.75 in^2
- A_{nt} = net tension area = 0.75 – **0.5** x (5/8 + 1/8) x 3/8 = 0.609 in^2
- A_{gv} = gross shear area = (3.0 + 3.0 +1.5) x 3/8 = 2.813 in^2
- A_{nv} = net tension area = 2.813 – **2.5** x (5/8 + 1/8) x 3/8 = 2.109 in^2

- Step II. Calculate which equation governs
  - $0.6\ F_u\ A_{nv} = 0.6 \times 58 \times 2.109 = 73.393$ kips
  - $F_u\ A_{nt} = 58 \times 0.609 = 35.322$ kips
  - $0.6\ F_u\ A_{nv} > F_u\ A_{nt}$
  - Therefore, equation with fracture of shear path governs
- Step III. Calculate block shear strength
  - $\phi_t\ R_n = 0.75\ (0.6\ F_u\ A_{nv} + F_y\ A_{gt})$
  - $\phi_t\ R_n = 0.75\ (73.393 + 36 \times 0.75) =$ **75.294 kips**
- Compare with results from previous examples

Example 3.2:

Ultimate factored load = P_u = 66 kips

Gross yielding design strength = $\phi_t\ P_n$ = 92.664 kips

Assume $A_e = 0.85\ A_n$

Net section fracture strength = 95.352 kips

Design strength = 92.664 kips (gross yielding governs)

Example 3.3

Calculate $A_e = 0.8166\ A_n$

Net section fracture strength = 91.045 kips

Design strength = 91.045 kips (net section fracture governs)

Member is still adequate to carry the factored load (P_u) = 66 kips

Example 3.8

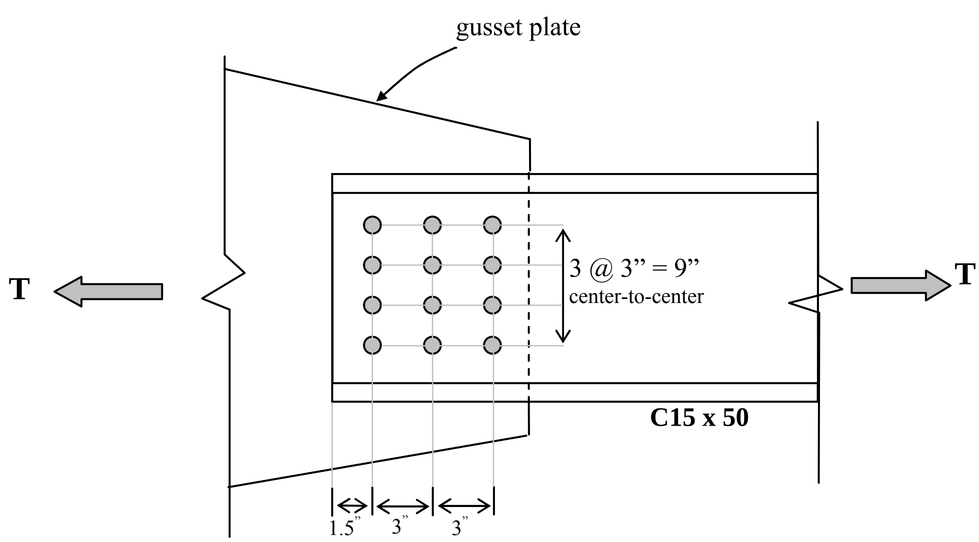
Block shear fracture strength = 75.294 kips

Design strength = 75.294 kips (block shear fracture governs)

Member is still adequate to carry the factored load (P_u) = 66 kips

- *Bottom line*:
  - Any of the three limit states (gross yielding, net section fracture, or block shear failure) can govern.
  - The design strength for all three limit states has to be calculated.
  - The member design strength will be the smallest of the three calculated values
  - The member design strength must be greater than the ultimate factored design load in tension.

Practice Example Determine the design tension strength for a single channel C15 x 50 connected to a 0.5 in. thick gusset plate as shown in Figure. Assume that the holes are for 3/4 in. diameter bolts and that the plate is made from structural steel with yield stress (F_y) equal to 50 ksi and ultimate stress (F_u) equal to 65 ksi.



- **Limit state of yielding due to tension:**

$$\phi T_n = 0.9 * 50 * 14.7 = 662 kips$$

- **Limit state of fracture due to tension:**

$$A_n = A_g - n d_e t = 14.7 - 4\left(\frac{7}{8}\right)(0.716) = 12.19 in^2$$

$$A_e = U A_n = \left(1 - \frac{x}{L}\right) A_n = \left(1 - \frac{0.798}{6}\right) * 12.19 = 10.57 in^2$$

Check: $U = 0.867 \leq 0.9$ OK.

Note: The connection eccentricity, x, for a C15X50 can be found on page 1-51 (LRFD).

$$\phi T_n = 0.75 * 65 * 10.57 = 515 kips$$

- **Limit state of block shear rupture:**

$$0.6 F_u A_{nv} = 0.6 * 65 * \left[2 * \left(7.5 - 2.5 * \frac{7}{8} \right) \right] * 0.716 = 296.6925$$

$$F_u A_{nt} = 65 * \left[9 - 3\left(\frac{7}{8} \right) \right] * 0.716 = 296.6925$$

$$F_u A_{nt} \geq 0.6 F_u A_{nv}$$

$$\therefore \phi R_n = \phi \left[0.6 F_y A_{gv} + F_u A_{nt} \right] = 0.75 \left[0.6 * 50 * 15 * 0.716 + 65 * \frac{296.6925}{65} \right] = 464 kips$$

<u>Block shear rupture is the critical limit state and the design tension strength is 464kips.</u>

4.7 Design of tension members

- The design of a tension member involves finding the lightest steel section (angle, wide-flange, or channel section) with design strength (ϕP_n) greater than or equal to the maximum factored design tension load (P_u) acting on it.
  - $\phi P_n \geq P_u$
  - P_u is determined by structural analysis for factored load combinations
  - ϕP_n is the design strength based on the *gross section yielding*, *net section fracture*, and *block shear rupture* limit states.
- For gross yielding limit state, $\phi P_n = 0.9 \text{ x } A_g \text{ x } F_y$
  - Therefore, $0.9 \text{ x } A_g \text{ x } F_y \geq P_u$
  - Therefore, $A_g \geq \dfrac{P_u}{0.9 \times F_y}$
- For net section fracture limit state, $\phi P_n = 0.75 \text{ x } A_e \text{ x } F_u$
  - Therefore, $0.75 \text{ x } A_e \text{ x } F_u \geq P_u$
  - Therefore, $A_e \geq \dfrac{P_u}{0.75 \times F_u}$
  - But, $A_e = U\, A_n$
  - Where, U and A_n depend on the end connection.
- Thus, designing the tension member goes hand-in-hand with designing the end connection, which we have not covered so far.
- Therefore, for this chapter of the course, the end connection details will be given in the examples and problems.
- The AISC manual tabulates the tension design strength of standard steel sections
  - Include: wide flange shapes, angles, tee sections, and double angle sections.

- The gross yielding design strength and the net section fracture strength of each section is tabulated.
- This provides a great *starting point* for selecting a section.

• **There is one serious limitation**

- The net section fracture strength is tabulated for an assumed value of *U* = 0.75, obviously because the precise connection details are not known
- For all W, Tee, angle and double-angle sections, A_e is assumed to be = 0.75 A_g
- The engineer can **first** select the tension member based on the tabulated gross yielding and net section fracture strengths, and then check the net section fracture strength and the block shear strength using the actual connection details.

• Additionally for each shape the manual tells the value of A_e below which net section fracture will control:

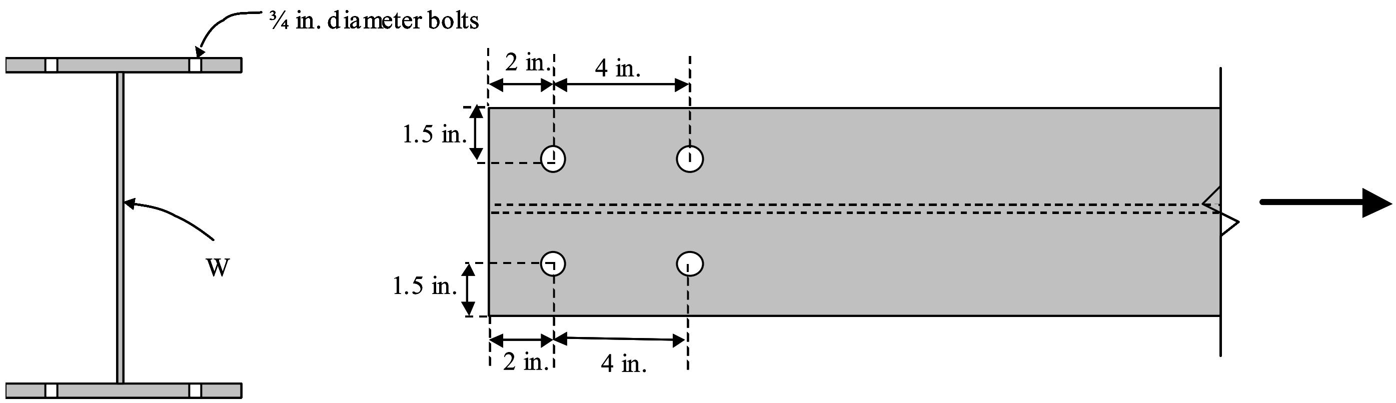
- Thus, for W shapes net section fracture will control if $A_e < 0.923\ A_g$
- For single angles, net section fracture will control if $A_e < 0.745\ A_g$
- For Tee shapes, net section fracture will control if $A_e < 0.923$
- For double angle shapes, net section fracture will control if $A_e < 0.745\ A_g$

• **Slenderness limits**

- Tension member slenderness l/r must preferably be limited to 300 as per LRFD specification B7

Example 3.10 Design a member to carry a factored maximum tension load of 100 kips.

(a) Assume that the member is a wide flange connected through the flanges using eight ¾ in. diameter bolts in two rows of four each as shown in the figure below. The center-to-center distance of the bolts in the direction of loading is 4 in. The edge distances are 1.5 in. and 2.0 in. as shown in the figure below. Steel material is A992



Holes in beam flange

SOLUTION

- **Step I. Select a section from the Tables**
  - Go to the **TEN** section of the AISC manual. See Table 3-1 on pages 3-17 to 3-19.
  - From this table, select W8x10 with $A_g = 2.96$ in^2, $A_e = 2.22$ in^2.
  - Gross yielding strength = 133 kips, and net section fracture strength=108 kips
  - This is the lightest section in the table.
  - Assumed $U = 0.75$. And, net section fracture will govern if $A_e < 0.923\ A_g$
- **Step II. Calculate the net section fracture strength for the actual connection**
  - According to the Figure above, $A_n = A_g - 4\ (d_b + 1/8) \times t_f$
  - $A_n = 2.96 - 4\ (3/4 + 1/8) \times 0.205 = 2.24$ in^2
  - The connection is only through the flanges. Therefore, the shear lag factor U will be the distance from the top of the flange to the centroid of a WT 4 x 5.

- See **DIM** section of the AISC manual. See Table 1-8, on pages 1-50, 1-51
- $\bar{x} = 0.953$
- $U = 1 - \bar{x}/L = 1 - 0.953 / 4 = 0.76$
- $A_e = 0.76\ A_n = 0.76 \times 2.24 = 1.70\ in^2$
- $\phi_t P_n = 0.75 \times F_u \times A_e = 0.75 \times 65 \times 1.70 = 82.9$ kips
- Unacceptable because $P_u = 100$ kips; **REDESIGN** required

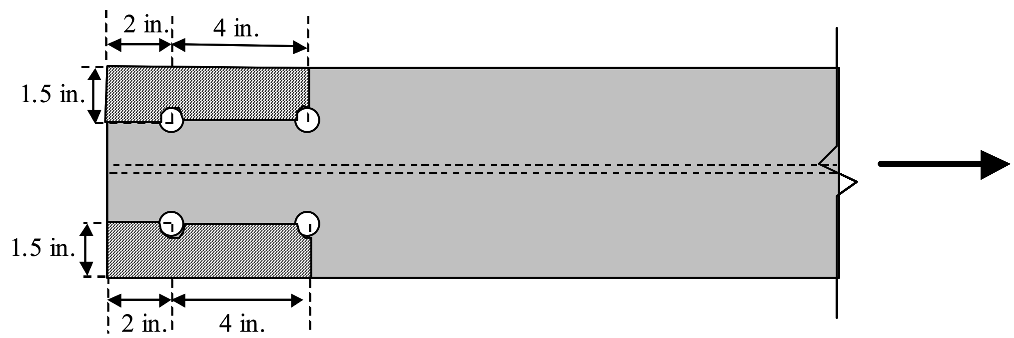
- **Step III. Redesign**

Many ways to redesign. One way is shown here:

- Assume $\phi_t\ P_n > 100$ kips
- Therefore, $0.75 \times 65 \times A_e > 100$ kips
- Therefore, $A_e > 2.051\ in^2$
- Assume, $A_e = 0.76\ A_n$ (based on previous calculations, step II)
- Therefore $A_n > 2.7\ in^2$
- But, $A_g = A_n + 4\ (d_b + 1/8) \times t_f$ (based on previous calculations, step II)
- Therefore $A_g > 2.7 + 3.5 \times t_f$
- Go to the section dimension table 1-1 on page 1-22 of the AISC manual. Select next highest section.
  - For W 8 x 13, $t_f = 0.255$ in.
  - Therefore, $A_g > 2.7 + 3.5 \times 0.255 = 3.59\ in^2$
  - From Table 1-1, W8 x 13 has $A_g = 3.84\ in^2 > 3.59\ in^2$
  - Therefore, W8 x 13 is acceptable and is chosen.

- **Step IV. Check selected section for net section fracture**
  - $A_g = 3.84$ in^2
  - $A_n = 3.84 - 3.5 \times 0.255 = 2.95$ in^2
  - From dimensions of WT4 x 6.5, $\bar{x} = 1.03$ in.
  - Therefore, $U = 1 - \bar{x}/L = 1 - 1.03/4 = 0.74$
  - Therefore, $A_e = U\, A_n = 0.74 \times 2.95 = 2.19$ in^2
  - Therefore, net section fracture strength = 0.75 x 65 x 2.19 = 106.7 kips
  - Which is greater than 100 kips (design load). Therefore, W 8 x 13 is acceptable.

- **Step V. Check the block shear rupture strength**
  - o Identify the block shear path



  - The block shear path is show above. **<u>Four blocks</u>** will separate from the tension member (two from each flange) as shown in the figure above.
  - $A_{gv} = [(4+2) \times t_f\,] \times 4 = 6 \times 0.255 \times 4 = 6.12$ in^2 - for four tabs
  - $A_{nv} = \{4+2 - 1.5 \times (d_b+1/8)\} \times t_f \times 4 = 4.78$ in^2
  - $A_{gt} = 1.5 \times t_f \times 4 = 1.53$ in^2
  - $A_{nt} = \{1.5 - 0.5 \times (d_b+1/8)\} \times t_f \times 4 = 1.084$ in^2
  - o Identify the governing equation:

- F_uA_{nt} = 65 x 1.084 = 70.4 kips
- $0.6F_uA_{nv}$ = 0.6 x 65 x 4.78 = 186.42 kips , which is > F_uA_{nt}

o Calculate block shear strength

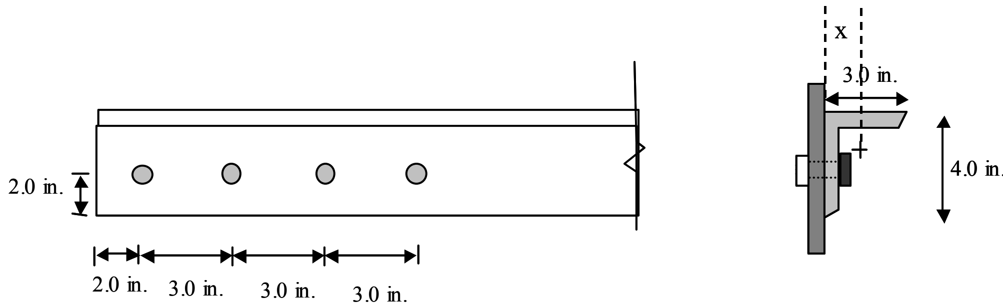
- $\phi_t R_n = 0.75\ (0.6F_uA_{nv} + F_yA_{gt})$ = 0.75 (186.42 + 50 x 1.53) = 197.2 kips
- Which is greater than P_u = 100 kips. Therefore W8 x 13 is still acceptable

- **Summary of solution**

Mem.	**Design load**	**A_g**	**A_n**	**U**	**A_e**	**Yield strength**	**Fracture strength**	**Block-shear strength**
W8x13	100 kips	3.84	2.95	0.74	2.19	173 kips	106.7 kips	197.2 kips
		Design strength = 106.7 kips (net section fracture governs) W8 x 13 is adequate for P_u = 100 kips and the given connection						

<u>EXAMPLE 3.11</u> Design a member to carry a factored maximum tension load of 100 kips.

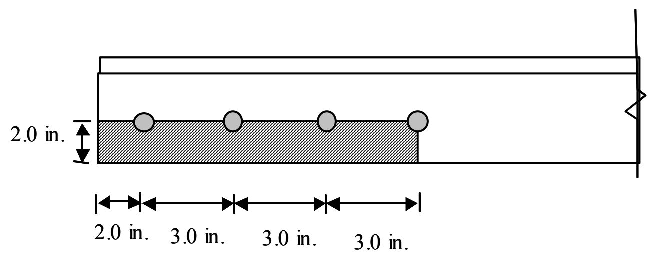
(b) The member is a single angle section connected through one leg using four 1 in. diameter bolts. The center-to-center distance of the bolts is 3 in. The edge distances are 2 in. Steel material is A36



- **Step I. Select a section from the Tables**
  - Go to the **TEN** section of the AISC manual. See Table 3-2 on pages 3-20 to 3-21.
  - From this table, select *L*4x3x1/2 with $A_g = 3.25$ in^2, $A_e = 2.44$ in^2.
  - Gross yielding strength = 105 kips, and net section fracture strength=106 kips
  - This is the lightest section in the table.
  - Assumed $U = 0.75$. And, net section fracture will govern if $A_e < 0.745\ A_g$
- **Step II. Calculate the net section fracture strength for the actual connection**
  - According to the Figure above, $A_n = A_g - 1\ (d_b + 1/8) \times t$
  - $A_n = 3.25 - 1(1 + 1/8) \times 0.5 = 2.6875$ in^2
  - The connection is only through the **<u>long leg</u>**. Therefore, the shear lag factor *U* will be the distance from the back of the long leg to the centroid of the angle.
  - See **DIM** section of the AISC manual. See Table 1-7, on pages 1-36, 1-37
  - $\bar{x} = 0.822$ in.
  - $U = 1 - \bar{x}/L = 1 - 0.822/9 = 0.908$
  - But *U* must be ≤ 0.90. Therefore, let $U = 0.90$

- $A_e = 0.90\ A_n = 0.90 \times 2.6875 = 2.41\ in^2$
- $\phi_t P_n = 0.75 \times F_u \times A_e = 0.75 \times 58 \times 2.41 = 104.8$ kips
- Acceptable because $P_u = 100$ kips.

- **Step V. Check the block shear rupture strength**
  - o Identify the block shear path



- $A_{gv} = (9+2) \times 0.5 = 5.5\ in^2$
- $A_{nv} = [11 - 3.5 \times (1+1/8)] \times 0.5 = 3.53\ in^2$
- $A_{gt} = 2.0 \times 0.5 = 1.0\ in^2$
- $A_{nt} = [2.0 - 0.5 \times (1 + 1/8)] \times 0.5 = 0.72\ in^2$

  - o Identify the governing equation:

- $F_u A_{nt} = 58 \times 0.72 = 41.76$ kips
- $0.6 F_u A_{nv} = 0.6 \times 58 \times 3.53 = 122.844\ in^2$, which is $> F_u A_{nt}$

  - o Calculate block shear strength

- $\phi_t R_n = 0.75\ (0.6 F_u A_{nv} + F_y A_{gt}) = 0.75\ (122.84 + 36 \times 1.0) = 119.133$ kips
- Which is greater than $P_u = 100$ kips. Therefore *L*4x3x1/2 is still acceptable

- **Summary of solution**

Mem.	**Design load**	**A_g**	**A_n**	**U**	**A_e**	**Yield strength**	**Fracture strength**	**Block-shear strength**
*L*4x3x1/2	100 kips	3.25	2.69	0.9	2.41	105 kips	104.8 kips	119.13 kips
		Design strength = 104.8 kips (net section fracture governs) *L*4x3x1/2 is adequate for P_u = 100 kips and the given connection						

- Note: For this problem A_e/A_g = 2.41/3.25 = 0.741, which is < 0.745. As predicted by the AISC manual, when A_e/A_g < 0.745, net section fracture governs.

CHAPTER 3. COMPRESSION MEMBER DESIGN

3.1 INTRODUCTORY CONCEPTS

- Compression Members: Structural elements that are subjected to axial compressive forces only are called *columns*. Columns are subjected to axial loads thru the centroid.
- Stress: The stress in the column cross-section can be calculated as

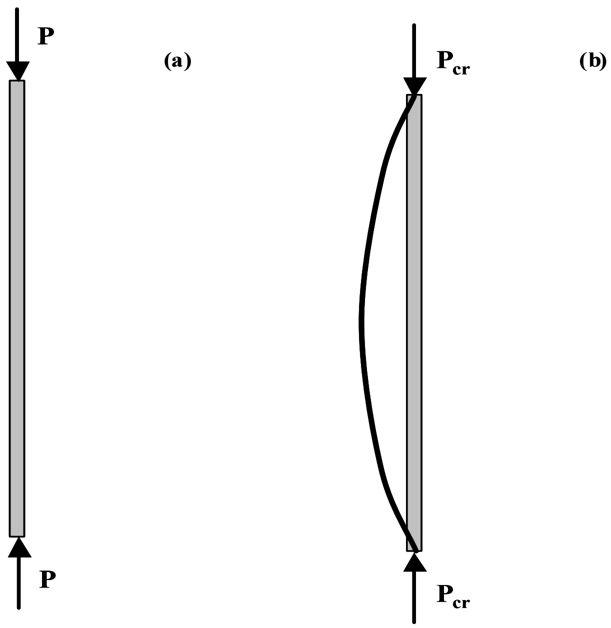
$$f = \frac{P}{A} \tag{2.1}$$

where, f is assumed to be uniform over the entire cross-section.

- This ideal state is never reached. The stress-state will be non-uniform due to:
  - Accidental eccentricity of loading with respect to the centroid
  - Member out-of –straightness (crookedness), or
  - Residual stresses in the member cross-section due to fabrication processes.
- Accidental eccentricity and member out-of-straightness can cause bending moments in the member. However, these are secondary and are usually ignored.
- Bending moments cannot be neglected if they are acting on the member. Members with axial compression and bending moment are called *beam-columns*.

3.2 COLUMN BUCKLING

- Consider a long slender compression member. If an axial load P is applied and increased slowly, it will ultimately reach a value P_{cr} that will cause buckling of the column. P_{cr} is called the critical buckling load of the column.



What is buckling?

Buckling occurs when a straight column subjected to axial compression suddenly undergoes bending as shown in the Figure 1(b). Buckling is identified as a failure limit-state for columns.

Figure 1. Buckling of axially loaded compression members

- The critical buckling load P_{cr} for columns is theoretically given by Equation (3.1)

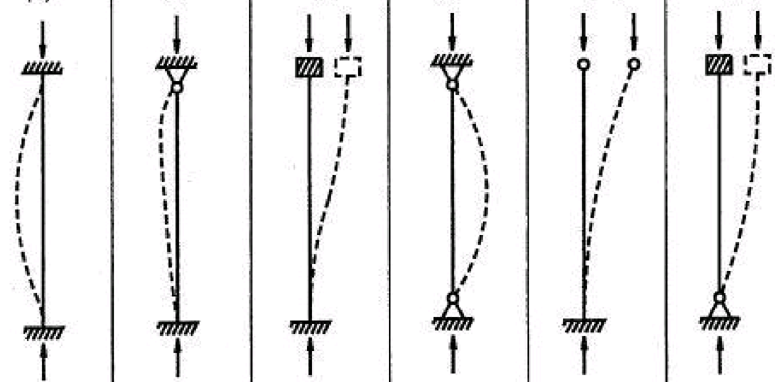
$$P_{cr} = \frac{\pi^2 E\,I}{(K\;L)^2} \tag{3.1}$$

where, I = moment of inertia about axis of buckling

K = effective length factor based on end boundary conditions

- Effective length factors are given on page 16.1-189 of the AISC manual.

Table C-C2.1
K Values for Columns

Buckled shape of column is shown by dashed line 	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K Value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal condition are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End Condition code	 Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free					

- In examples, homeworks, and exams please state clearly whether you are using the theoretical value of *K* or the recommended design values.

EXAMPLE 3.1 Determine the buckling strength of a W 12 x 50 column. Its length is 20 ft. For major axis buckling, it is pinned at both ends. For minor buckling, is it pinned at one end and fixed at the other end.

Solution

Step I. Visualize the problem

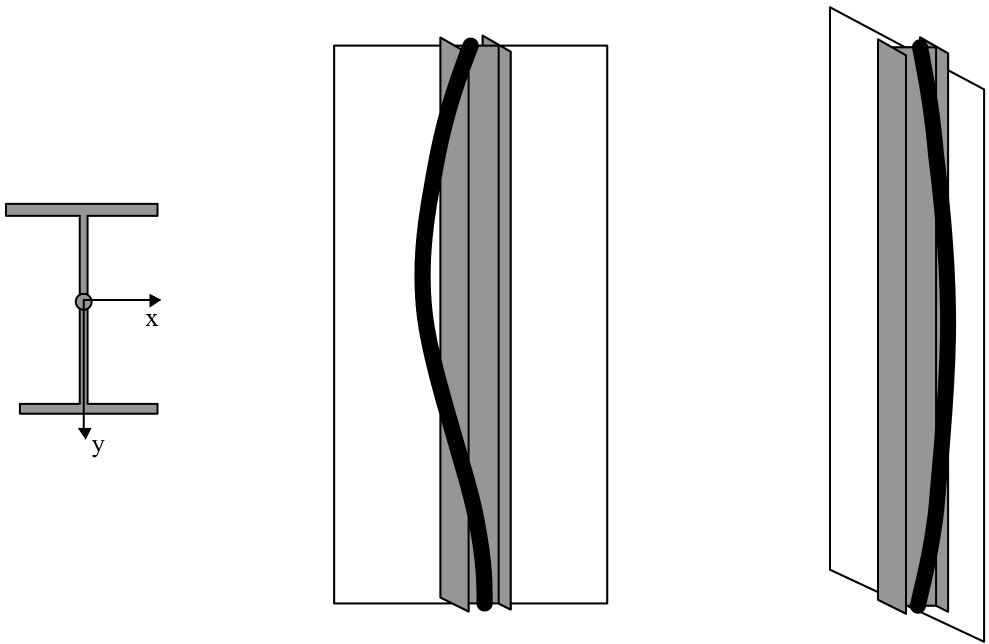


Figure 2. (a) Cross-section; (b) major-axis buckling; (c) minor-axis buckling

- For the W12 x 50 (or any wide flange section), x is the major axis and y is the minor axis. Major axis means axis about which it has greater moment of inertia ($I_x > I_y$)

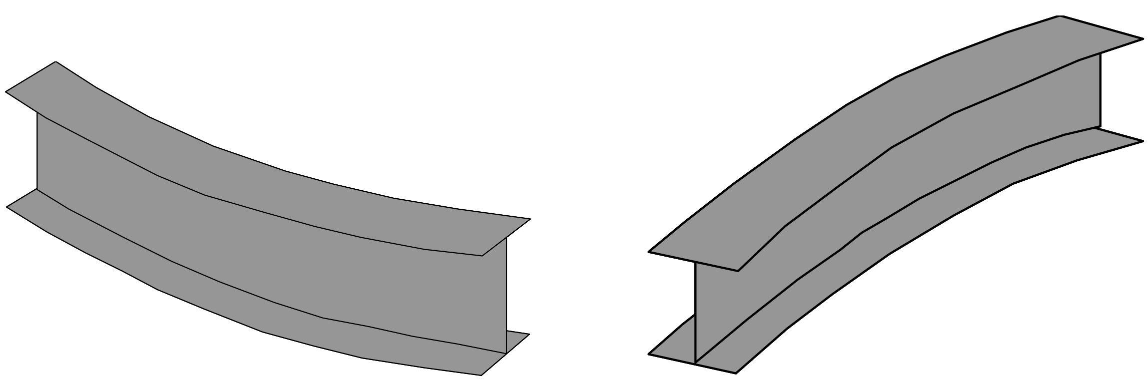


Figure 3. (a) Major axis buckling; (b) minor axis buckling

Step II. Determine the effective lengths

- According to Table C-C2.1 of the AISC Manual (see page 16.1 - 189):
  - For pin-pin end conditions about the minor axis

    K_y = 1.0 (theoretical value); and K_y = 1.0 (recommended design value)
  - For pin-fix end conditions about the major axis

    K_x = 0.7 (theoretical value); and K_x = 0.8 (recommended design value)
- According to the problem statement, the unsupported length for buckling about the major (x) axis = L_x = 20 ft.
- The unsupported length for buckling about the minor (y) axis = L_y = 20 ft.
- Effective length for major (x) axis buckling = K_x L_x = 0.8 x 20 = 16 ft. = 192 in.
- Effective length for minor (y) axis buckling = K_y L_y = 1.0 x 20 = 20 ft. = 240 in.

Step III. Determine the relevant section properties

- For W12 x 50: elastic modulus = E = 29000 ksi (constant for all steels)
- For W12 x 50: I_x = 391 in^4. I_y = 56.3 in^4 (see page 1-21 of the AISC manual)

Step IV. Calculate the buckling strength

- Critical load for buckling about x - axis = $P_{cr\text{-}x} = \dfrac{\pi^2 E I_x}{(K_x\ L_x)^2} = \dfrac{\pi^2 \times 29000 \times 391}{(192)^2}$

$$P_{cr\text{-}x} = 3035.8 \text{ kips}$$

- Critical load for buckling about y-axis = $P_{cr\text{-}y} = \dfrac{\pi^2 E I_y}{(K_y\ L_y)^2} = \dfrac{\pi^2 \times 29000 \times 56.3}{(240)^2}$

$$P_{cr\text{-}y} = 279.8 \text{ kips}$$

- Buckling strength of the column = smaller ($P_{cr\text{-}x}$, $P_{cr\text{-}y}$) = $\underline{P_{cr} = 279.8 \text{ kips}}$

  Minor (y) axis buckling governs.

- ***Notes:***

- *Minor axis buckling usually governs for all doubly symmetric cross-sections. However, for some cases, major (x) axis buckling can govern.*
- *Note that the steel yield stress was irrelevant for calculating this buckling strength.*

3.3 INELASTIC COLUMN BUCKLING

- Let us consider the previous example. According to our calculations P_{cr} = 279.8 kips. This P_{cr} will cause a uniform stress $f = P_{cr}/A$ in the cross-section
- For W12 x 50, A = 14.6 in^2. Therefore, for P_{cr} = 279.8 kips; f = 19.16 ksi

  The calculated value of f is within the elastic range for a 50 ksi yield stress material.

- However, if the unsupported length was only 10 ft., $P_{cr} = \dfrac{\pi^2 E\, I_y}{\left(K_y\ L_y\right)^2}$ would be calculated as

  1119 kips, and f = 76.6 kips.

- This value of f is ridiculous because the material will yield at 50 ksi and never develop f = 76.6 kips. <u>The member would yield before buckling.</u>
- **Equation (3.1) is valid only when the material everywhere in the cross-section is in the elastic region. If the material goes inelastic then Equation (3.1) becomes useless and cannot be used.**
- What happens in the inelastic range?

  Several other problems appear in the inelastic range.

  - The member out-of-straightness has a significant influence on the buckling strength in the inelastic region. It must be accounted for.

- The residual stresses in the member due to the fabrication process causes yielding in the cross-section much before the uniform stress *f* reaches the yield stress F_y.
- The shape of the cross-section (W, C, etc.) also influences the buckling strength.
- In the inelastic range, the steel material can undergo strain hardening.

All of these are very advanced concepts and beyond the scope of CE405. You are welcome to CE805 to develop a better understanding of these issues.

- So, what should we do? We will directly look at the AISC Specifications for the strength of compression members, i.e., Chapter E (page 16.1-27 of the AISC manual).

3.4 AISC SPECIFICATIONS FOR COLUMN STRENGTH

- The AISC specifications for column design are based on several years of research.
- These specifications account for the elastic and inelastic buckling of columns including all issues (member crookedness, residual stresses, accidental eccentricity etc.) mentioned above.
- The specification presented here (AISC Spec E2) will work for all doubly symmetric cross-sections and channel sections.
- The design strength of columns for the flexural buckling limit state is equal to $\boldsymbol{\phi_c P_n}$

Where, $\phi_c = 0.85$ (Resistance factor for compression members)

$$P_n = A_g F_{cr} \quad (3.2)$$

- For $\lambda_c \leq 1.5$

$$F_{cr} = \left(0.658^{\lambda_c^2}\right) F_y \quad (3.3)$$

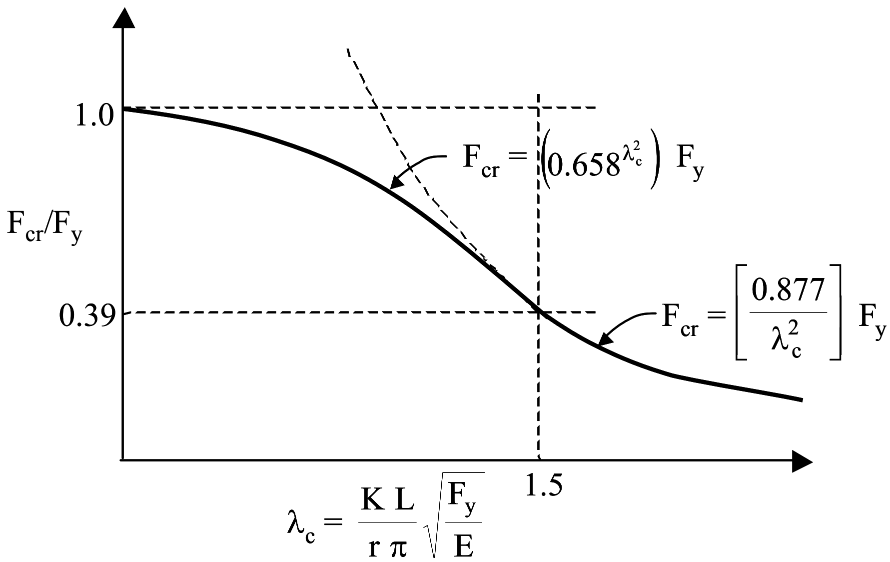
- For $\lambda_c > 1.5$

$$F_{cr} = \left[\frac{0.877}{\lambda_c^2}\right] F_y \quad (3.4)$$

Where, $\lambda_c = \dfrac{K\,L}{r\,\pi}\sqrt{\dfrac{F_y}{E}}$ (3.5)

A_g = gross member area; K = effective length factor

L = unbraced length of the member; r = governing radius of gyration



- Note that the original Euler buckling equation is $P_{cr} = \frac{\pi^2 E I}{(K\,L)^2}$

$$\therefore F_{cr} = \frac{P_{cr}}{A_g} = \frac{\pi^2 E}{(K\,L)^2} \times \frac{I}{A_g} = \frac{\pi^2 E}{(K\,L)^2} \times r^2 = \frac{\pi^2 E}{\left(\frac{K\,L}{r}\right)^2}$$

$$\therefore \frac{F_{cr}}{F_y} = \frac{\pi^2 E}{\left(\frac{K\,L}{r}\right)^2 \times F_y} = \frac{1}{\left(\frac{K\,L}{r\,\pi} \times \sqrt{\frac{F_y}{E}}\right)^2} = \frac{1}{\lambda_c^2}$$

$$\therefore F_{cr} = F_y \times \frac{1}{\lambda_c^2}$$

- Note that the AISC equation for $\boldsymbol{\lambda_c < 1.5}$ is $F_{cr} = F_y \times \frac{0.877}{\lambda_c^2}$
  - The 0.877 factor tries to account for initial crookedness.

- For a given column section:
  - Calculate I, A_g, r
  - Determine effective length *K L* based on end boundary conditions.
  - Calculate λ_c
  - If λ_c is greater than 1.5, *elastic buckling* occurs and use Equation (3.4)

- If λ_c is less than or equal to 1.5, *inelastic buckling* occurs and use Equation (3.3)

- Note that the column can develop its yield strength F_y as λ_c approaches zero.

3.5 COLUMN STRENGTH

- In order to simplify calculations, the AISC specification includes Tables.
  - Table 3-36 on page **16.1**-143 shows KL/r vs. $\phi_c F_{cr}$ for steels with F_y = 36 ksi.
  - You can calculate KL/r for the column, then read the value of $\phi_c F_{cr}$ from this table
  - The column strength will be equal to $\phi_c F_{cr}$ x A_g
  - Table 3-50 on page **16.1**-145 shows KL/r vs. $\phi_c F_{cr}$ for steels with F_y = 50 ksi.
- In order to simplify calculations, the AISC specification includes more Tables.
  - Table 4 on page **16.1**-147 shows λ_c vs. $\phi_c F_{cr}/F_y$ for all steels with any F_y.
  - You can calculate λ_c for the column, the read the value of $\phi_c F_{cr}/F_y$
  - The column strength will be equal to $\phi_c F_{cr}/F_y$ x (A_g x F_y)

EXAMPLE 3.2 Calculate the design strength of W14 x 74 with length of 20 ft. and pinned ends. A36 steel is used.

Solution

- Step I. Calculate the effective length and slenderness ratio for the problem

  $K_x = K_y = 1.0$

  $L_x = L_y = 240$ in.

  Major axis slenderness ratio = $K_x L_x / r_x = 240/6.04 = 39.735$

  Minor axis slenderness ratio = $K_y L_y / r_y = 240/2.48 = 96.77$

- Step II. Calculate the buckling strength for governing slenderness ratio

The governing slenderness ratio is the larger of (K_xL_x/r_x, K_yL_y/r_y)

K_yL_y/r_y is larger and the governing slenderness ratio; $\lambda_c = \frac{K_y\ L_y}{r_y\ \pi}\sqrt{\frac{F_y}{E}} = 1.085$

$\lambda_c < 1.5$; Therefore, $F_{cr} = \left(0.658^{\lambda_c^2}\right) F_y$

Therefore, F_{cr} = 21.99 ksi

Design column strength = $\phi_c P_n = 0.85\ (A_g\ F_{cr}) = 0.85\ (21.8\ in^2 \times 21.99\ ksi) = 408$ kips

Design strength of column = 408 kips

- Check calculated values with Table 3-36. For KL/r = 97, $\phi_c F_{cr}$ = 18.7 ksi
- Check calculated values with Table 4. For λ_c = 1.08, $\phi_c F_{cr}$ = 0.521

3.6 LOCAL BUCKLING LIMIT STATE

- The AISC specifications for column strength assume that column buckling is the governing limit state. However, if the column section is made of thin (slender) plate elements, then failure can occur due to *local buckling* of the flanges or the webs.

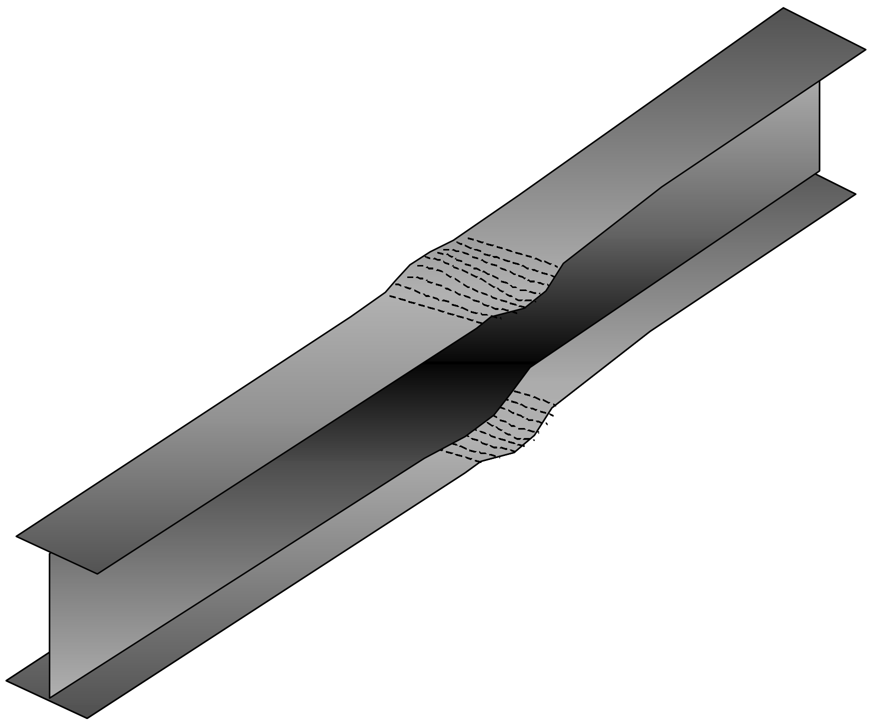


Figure 4. Local buckling of columns

- If *local buckling* of the individual plate elements occurs, then the column may not be able to develop its buckling strength.
- Therefore, the local buckling limit state must be prevented from controlling the column strength.
- Local buckling depends on the slenderness (width-to-thickness *b/t* ratio) of the plate element and the yield stress (F_y) of the material.
- Each plate element must be stocky enough, i.e., have a *b/t* ratio that prevents local buckling from governing the column strength.

- The AISC specification B5 provides the slenderness (b/t) limits that the individual plate elements must satisfy so that *local buckling* does not control.
- The AISC specification provides two slenderness limits (λ_p and λ_r) for the local buckling of plate elements.

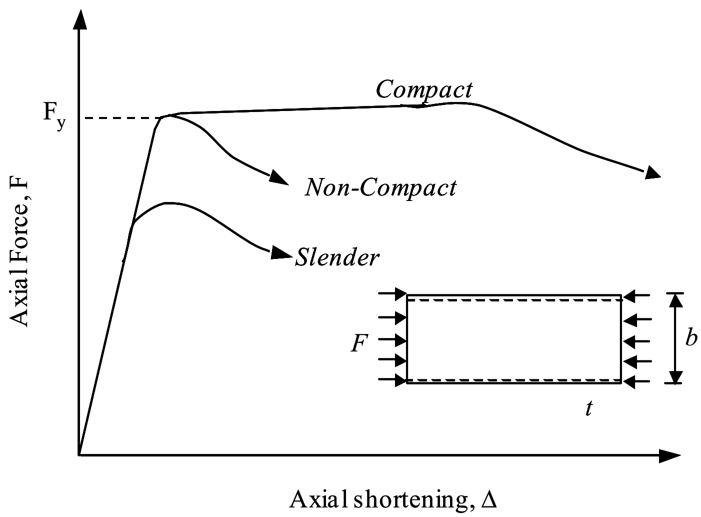


Figure 5. Local buckling behavior and classification of plate elements

- If the slenderness ratio (b/t) of the plate element is greater than λ_r then it *is slender*. It will locally buckle in the elastic range *before* reaching F_y
- If the slenderness ratio (b/t) of the plate element is less than λ_r but greater than λ_p, then it is *non-compact*. It will locally buckle *immediately* after reaching F_y
- If the slenderness ratio (b/t) of the plate element is less than λ_p, then the element is *compact*. It will locally buckle *much after* reaching F_y

- If all the plate elements of a cross-section are compact, then the section is *compact*.
  - If any one plate element is non-compact, then the cross-section is non-compact
  - If any one plate element is slender, then the cross-section is slender.
- The slenderness limits λ_p and λ_r for various plate elements with different boundary conditions are given in Table B5.1 on pages **16.1**-14 and **16.1**-15 of the AISC Spec.

- Note that the slenderness limits (λ_p and λ_r) and the definition of plate slenderness (b/t) ratio depend upon the boundary conditions for the plate.
  - If the plate is supported along *two edges* parallel to the direction of compression force, then it is a *stiffened* element. For example, the webs of W shapes
  - If the plate is supported along only *one edge* parallel to the direction of the compression force, then it is an *unstiffened* element. Ex., the flanges of W shapes.

- The local buckling limit state can be prevented from controlling the column strength by using sections that are non-compact
  - If all the elements of the cross-section have calculated slenderness (b/t) ratio less than λ_r, then the local buckling limit state will not control.
  - For the definitions of b/t, λ_p, λ_r for various situations see Table B5.1 and Spec B5.

EXAMPLE 3.3 Determine the local buckling slenderness limits and evaluate the W14 x 74 section used in Example 3.2. Does local buckling limit the column strength?

Solution

- Step I. Calculate the slenderness limits

  See Table B5.1 on page 16.1 – 14.
  - For the flanges of I-shape sections in pure compression

$$\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 0.56 \times \sqrt{\frac{29000}{36}} = 15.9$$

  - For the webs of I-shapes section in pure compression

$$\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 0.56 \times \sqrt{\frac{29000}{36}} = 15.9$$

$$\lambda_r = 1.49 \text{ x } \sqrt{\frac{E}{F_y}} = 1.49 \text{ x } \sqrt{\frac{29000}{36}} = 42.3$$

- Step II. Calculate the slenderness ratios for the flanges and webs of W14 x 74
  - For the flanges of I-shape member, $b = b_f/2$ = flange width / 2

    Therefore, $b/t = b_f/2t_f$. (See pg. **16.1**-12 of AISC)

    For W 14 x 74, $b_f/2t_f = 6.41$ (See Page 1-19 in AISC)
  - For the webs of I shaped member, b = h

    h is the clear distance between flanges less the fillet / corner radius of each flange

    For W14 x 74, $h/t_w = 25.4$ (See Page 1-19 in AISC)
- Step III. Make the comparisons and comment

  For the flanges, $b/t < \lambda_r$. Therefore, the flange is non-compact

  For the webs, $h/t_w < \lambda_r$. Therefore the web is non-compact

  Therefore, the section is <u>compact</u>

  Therefore, local buckling will not limit the column strength.

3.7 COLUMN DESIGN

- The AISC manual has tables for column strength. See page **4**-21 onwards.
- For wide flange sections, *the column buckling strength* ($\phi_c P_n$) *is tabulated with respect to the effective length about the minor axis $K_y L_y$* in Table 4-2.
  - The table takes the $K_y L_y$ value for a section, and <u>internally</u> calculates the $K_y L_y/r_y$, then $\lambda_c = \frac{K_y\ L_y}{r_y\ \pi}\sqrt{\frac{F_y}{E}}$; and then the *tabulated* column strength using either Equation E2-2 or E2-3 of the specification.

- If you want to use the Table 4-2 for calculating the column strength for buckling about *the major axis*, then do the following:
  - Take the major axis K_xL_x value. Calculate an equivalent $(KL)_{eq} = \dfrac{K_x L_x}{r_x / r_y}$
  - Use the calculated $(KL)_{eq}$ value to find ($\phi_c P_n$) the column strength for buckling about the *major axis* from Table (4-2)
- For example, consider a W14 x 74 column with K_yL_y = 20 ft. and K_xL_x = 25 ft.
  - Material has yield stress = 50 ksi (<u>always</u> in Table 4-2).
  - See Table 4-2, for K_yL_y = 20 ft., $\phi_c P_n$ = 467 kips (minor axis buckling strength)
  - r_x/r_y for W14x74 = 2.44 from Table 4-2 (see page 4-23 of AISC).
  - For K_xL_x = 25 ft., $(KL)_{eq}$ = 25/2.44 = 10.25 ft.
  - For $(KL)_{eq}$ = 10.25 ft., $\phi_c P_n$ = 774 kips (major axis buckling strength)
  - If calculated value of $(KL)_{eq} < K_yL_y$ then minor axis buckling will govern.

<u>EXAMPLE 3.4</u> Determine the design strength of an ASTM A992 W14 x 132 that is part of a braced frame. Assume that the physical length L = 30 ft., the ends are pinned and the column is braced at the ends only for the X-X axis and braced at the ends and mid-height for the Y-Y axis.

<u>Solution</u>

- <u>Step I.</u> Calculate the *effective lengths*.

  For W14 x 132: r_x = 6.28 in; r_y = 3.76 in; A_g =38.8 in^2

  K_x = 1.0 and K_y = 1.0

  L_x = 30 ft. and L_y = 15 ft.

  K_xL_x = 30 ft. and K_yL_y = 15 ft.

- <u>Step II.</u> Determine the governing slenderness ratio

K_xL_x/r_x = 30 x 12 in./6.28 in.= 57.32

K_yL_y/r_y = 15 x 12 in./3.76 in. = 47.87

The larger slenderness ratio, therefore, buckling about the major axis will govern the column strength.

- <u>Step III.</u> Calculate the column strength

K_xL_x = 30 ft. Therefore, $(KL)_{eq} = \dfrac{K_xL_x}{r_x/r_y} = \dfrac{30}{6.28/3.76}$ = 17.96 ft.

From Table 4-2, for $(KL)_{eq}$ = 18.0 ft. $\phi_c P_n$ = 1300 kips (design column strength)

- <u>Step IV.</u> Check the local buckling limits

For the flanges, $b_f/2t_f = 7.15 < \lambda_r = 0.56 \times \sqrt{\dfrac{E}{F_y}} = 13.5$

For the web, $h/t_w = 17.7 < \lambda_r = 1.49 \times \sqrt{\dfrac{E}{F_y}} = 35.9$

Therefore, the section is non-compact. OK.

<u>EXAMPLE 3.5</u> A compression member is subjected to service loads of 165 kips dead load and 535 kips of live load. The member is 26 ft. long and pinned at each end. Use A992 (50 ksi) steel and select a W shape

<u>Solution</u>

- Calculate the factored design load P_u

$P_u = 1.2\ P_D + 1.6\ P_L$ = 1.2 x 165 + 1.6 x 535 = 1054 kips

- Select a W shape from the AISC manual Tables

For K_yL_y = 26 ft. and required strength = 1054 kips

- Select W14 x 145 from page 4-22. It has $\phi_c P_n$ = 1160 kips

- Select W12 x 170 from page 4-24. It has $\phi_c P_n$ = 1070 kips
- No no W10 will work. See Page 4-26
- W14 x 145 is the lightest.

- Note that column sections are usually W12 or W14. Usually sections bigger than W14 are usually not used as columns.

3.8 EFFECTIVE LENGTH OF COLUMNS IN FRAMES

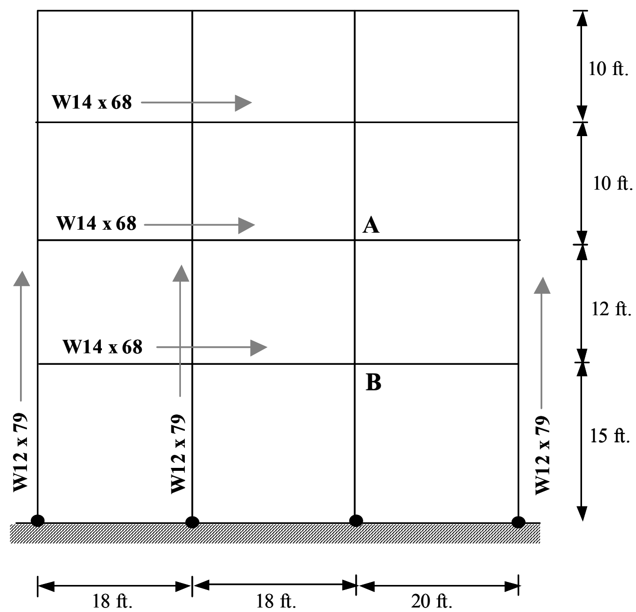
- So far, we have looked at the buckling strength of individual columns. These columns had various boundary conditions at the ends, but they were not connected to other members with moment (fix) connections.
- The effective length factor K for the buckling of an individual column can be obtained for the appropriate end conditions from Table C-C2.1 of the AISC Manual .
- However, when these individual columns are part of a frame, their ends are connected to other members (beams etc.).
  - Their effective length factor K will depend on the restraint offered by the other members connected at the ends.
  - Therefore, the effective length factor K will depend on the relative rigidity (stiffness) of the members connected at the ends.

The effective length factor for columns in frames must be calculated as follows:

- First, you have to determine whether the column is part of a braced frame or an unbraced (moment resisting) frame.
  - If the column is part of a braced frame then its effective length factor $0 < K \leq 1$
  - If the column is part of an unbraced frame then $1 < K \leq \infty$

- Then, you have to determine the relative rigidity factor G for both ends of the column
  - G is defined as the ratio of the summation of the rigidity (EI/L) of all columns coming together at an end to the summation of the rigidity (EI/L) of all beams coming together at the same end.
  - $G = \dfrac{\sum \dfrac{E\,I_c}{L_c}}{\sum \dfrac{E\,I_b}{L_b}}$ - It must be calculated for both ends of the column.
- Then, you can determine the effective length factor K for the column using the calculated value of G at both ends, i.e., G_A and G_B and the appropriate alignment chart
- There are two alignment charts provided by the AISC manual,
  - One is for columns in braced (sidesway inhibited) frames. See Figure C-C2.2a on page 16.1-191 of the AISC manual. $0 < K \leq 1$
  - The second is for columns in unbraced (sidesway uninhibited) frames. See Figure C-C2.2b on page 16.1-192 of the AISC manual. $1 < K \leq \infty$
  - The procedure for calculating G is the same for both cases.

<u>EXAMPLE 3.6</u> Calculate the effective length factor for the **W12 x 53** column AB of the frame shown below. Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame. Assume that the columns are braced at each story level for out-of-plane buckling. Assume that the same column section is used for the stories above and below.



Step I. Identify the frame type and calculate L_x, L_y, K_x, and K_y if possible.

- It is an unbraced (sidesway uninhibited) frame.
- $L_x = L_y = 12$ ft.
- $K_y = 1.0$
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.

Step II - Calculate K_x

- I_{xx} of W 12 x 53 = 425 in^4 $\quad\quad$ I_{xx} of W14x68 = 753

- $$G_A = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{425}{10\times12}+\frac{425}{12\times12}}{\frac{723}{18\times12}+\frac{723}{20\times12}} = \frac{6.493}{6.360} = 1.021$$

- $$G_B = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{425}{12\times12}+\frac{425}{15\times12}}{\frac{723}{18\times12}+\frac{723}{20\times12}} = \frac{5.3125}{6.360} = 0.835$$

- Using G_A and G_B: $K_x = 1.3$ - from Alignment Chart on Page 3-6

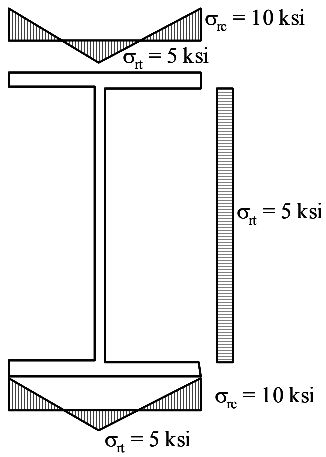
Step III – Design strength of the column

- $K_yL_y = 1.0 \times 12 = 12$ ft.
- $K_x L_x = 1.3 \times 12 = 15.6$ ft.
  - r_x / r_y for W12x53 = 2.11
  - $(KL)_{eq} = 15.6 / 2.11 = 7.4$ ft.
- $K_yL_y > (KL)_{eq}$
- Therefore, y-axis buckling governs. Therefore $\phi_c P_n = 518$ kips

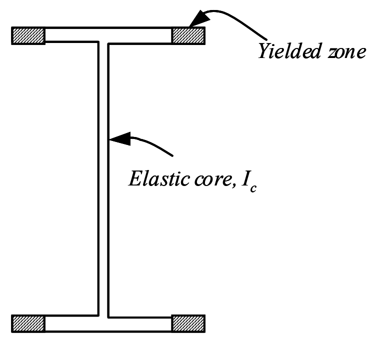
3.8.1 Inelastic Stiffness Reduction Factor – Modification

- This concept for calculating the effective length of columns in frames was widely accepted for many years.
- Over the past few years, a lot of modifications have been proposed to this method due to its several assumptions and limitation. Most of these modifications have not yet been accepted in to the AISC provisions.
- One of the accepted modifications is the inelastic stiffness reduction factor. As presented earlier, G is a measure of the *relative flexural rigidity* of the columns (EI_c/L_c) with respect to the beams (EI_b/L_b)

- However, if column buckling were to occur in the inelastic range (λ_c < 1.5), then the flexural rigidity of the column will be reduced because I_c will be the moment of inertia of only the elastic core of the entire cross-section. See figure below



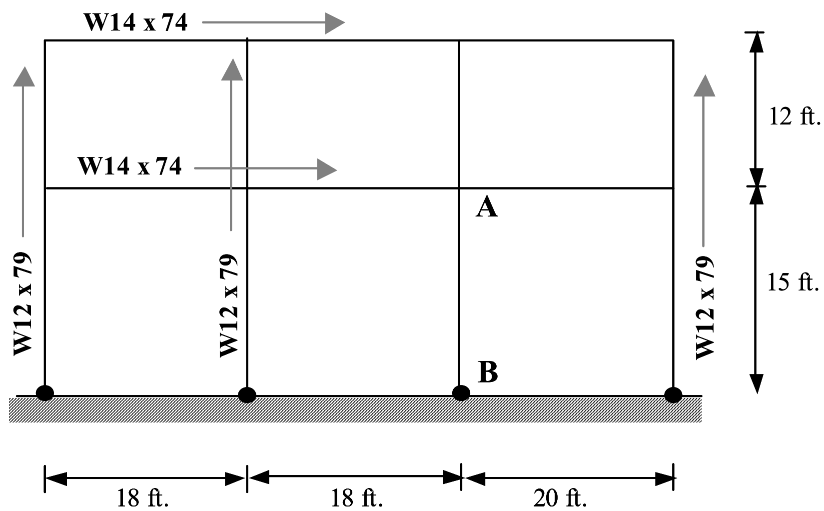
(a) Initial state – residual stress



(b) Partially yielded state at buckling

- The beams will have greater flexural rigidity when compared with the reduced rigidity (EI_c) of the inelastic columns. As a result, the beams will be able to restrain the columns better, which is good for column design.
- This effect is incorporated in to the AISC column design method through the use of Table 4-1 given on page 4-20 of the AISC manual.
- Table 4-1 gives the stiffness reduction factor (τ) as a function of the yield stress F_y and the stress P_u/A_g in the column, where P_u is factored design load (analysis)

EXAMPLE 3.7 Calculate the effective length factor for a W10 x 60 column AB made from 50 ksi steel in the unbraced frame shown below. Column AB has a design factor load P_u = 450 kips. The columns are oriented such that major axis bending occurs in the plane of the frame. The columns are braced *continuously along the length* for out-of-plane buckling. Assume that the same column section is used for the story above



Solution

Step I. Identify the frame type and calculate L_x, L_y, K_x, and K_y if possible.

- It is an unbraced (*sidesway uninhibited*) frame.
- L_y = 0 ft.
- K_y has no meaning because out-of-plane buckling is not possible.
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.

Step II (a) - Calculate K_x

- I_{xx} of W 14 x 74 = 796 in^4 I_{xx} of W 10 x 60 = 341 in^4

- $$G_A = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{341}{12\times 12} + \frac{341}{15\times 12}}{\frac{796}{18\times 12} + \frac{796}{20\times 12}} = \frac{4.2625}{7.002} = 0.609$$
- $G_B = 10$ - for pin support, see note on Page 16.1-191
- Using G_A and G_B: $K_x = 1.8$ - from Alignment Chart on Page 16.1-192
- Note, K_x is greater than 1.0 because it is an unbraced frame.

Step II (b) - Calculate $K_{x-inelastic}$ using stiffness reduction factor method

- Reduction in the flexural rigidity of the column due to residual stress effects
  - First calculate, $P_u / A_g = 450 / 17.6 = 25.57$ ksi
  - Then go to Table 4-1 on page 4-20 of the manual, and read the value of stiffness reduction factor for $F_y = 50$ ksi and $P_u/A_g = 25.57$ ksi.
  - Stiffness reduction factor = $\tau = 0.833$
- $G_{A\text{-inelastic}} = \tau \times G_A = 0.833 \times 0.609 = 0.507$
- $G_B = 10$ - for pin support, see note on Page **16.1**-191
- Using $G_{A\text{-inelastic}}$ and G_B, $K_{x\text{-inelastic}} = 1.75$ - alignment chart on Page **16.1**-192
- Note: *You can combine Steps II (a) and (b) to calculate the $K_{x\text{-inelastic}}$ directly.* You don't need to calculate elastic K_x first. It was done here for demonstration purposes.
- Note that $K_{x\text{-inelastic}} < K_x$. This is in agreement with the fact that the beams offer better resistance to the *inelastic* column AB because it has reduced flexural rigidity.

Step III – Design strength of the column

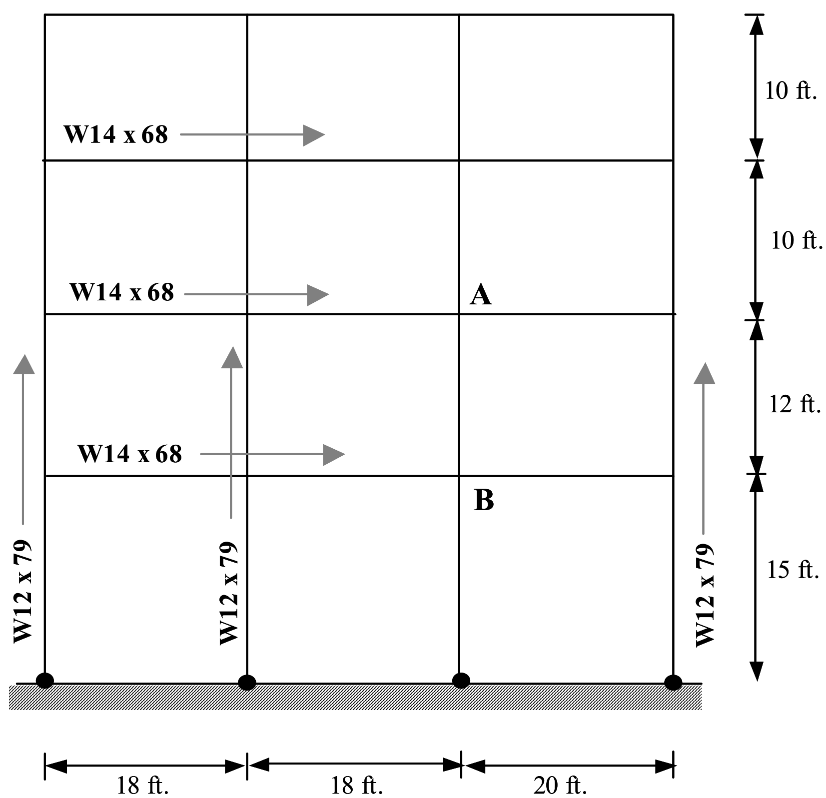
- $K_x L_x = 1.75 \times 15 = 26.25$ ft.
  - r_x / r_y for W10x60 = 1.71 - from Table **4**-2, see page **4**-26
  - $(KL)_{eq} = 26.25/1.71 = 15.35$ ft.

- $\phi_c P_n$ for X-axis buckling = 513.9 kips - from Table **4**-2, see page **4**-26
- Section slightly over-designed for P_u = 450 kips.

Column design strength = $\phi_c P_n$ = 513.9 kips

<u>EXAMPLE 3.8:</u>

- Design Column AB of the frame shown below for a design load of 500 kips.
- Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame.
- Assume that the columns are braced at each story level for out-of-plane buckling.
- Assume that the same column section is used for the stories above and below.



Step I - Determine the design load and assume the steel material.

- Design Load = P_u = 500 kips
- Steel yield stress = 50 ksi (A992 material)

Step II. Identify the frame type and calculate L_x, L_y, K_x, and K_y if possible.

- It is an unbraced (sidesway uninhibited) frame.

- $L_x = L_y = 12$ ft.
- $K_y = 1.0$
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.
- Need to select a section to calculate K_x

Step III - Select a column section

- *Assume minor axis buckling governs.*
- $K_y L_y = 12$ ft.
- See Column Tables in AISC-LRFD manual
  Select section W12x53
- $\phi_c P_n$ for y-axis buckling = 518 kips

Step IV - Calculate $K_{x\text{-inelastic}}$

- I_{xx} of W 12 x 53 =425 in^4 I_{xx} of W14x68 = 753 in^4
- Account for the reduced flexural rigidity of the column due to residual stress effects
  - $P_u/A_g = 500 / 15.6 = 32.05$ ksi
  - Stiffness reduction factor = $\tau = 0.58$
- $$G_A = \frac{\tau \times \sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{0.58 \times \left(\frac{425}{10 \times 12} + \frac{425}{12 \times 12}\right)}{\frac{723}{18 \times 12} + \frac{723}{20 \times 12}} = \frac{3.766}{6.360} = 0.592$$
- $$G_B = \frac{\tau \times \sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{0.58 \times \left(\frac{425}{12 \times 12} + \frac{425}{15 \times 12}\right)}{\frac{723}{18 \times 12} + \frac{723}{20 \times 12}} = \frac{3.0812}{6.360} = 0.484$$
- Using G_A and G_B: $K_{x\text{-inelastic}} = 1.2$ - from Alignment Chart

Step V - Check the selected section for X-axis buckling

- $K_x L_x = 1.2 \times 12 = 14.4$ ft.
- r_x / r_y for W12x53 = 2.11

- Calculate $(KL)_{eq}$ to determine strength ($\phi_c P_n$) for X-axis buckling

  $(KL)_{eq}$ = 14.4 / 2.11 = 6.825 ft.
- From the column design tables, $\phi_c P_n$ for X-axis buckling = 612.3 kips

Step VI. Check the local buckling limits

For the flanges, $b_f/2t_f = 8.69 \quad < \quad \lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$

For the web, $h/t_w = 28.1 \quad < \quad \lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 35.9$

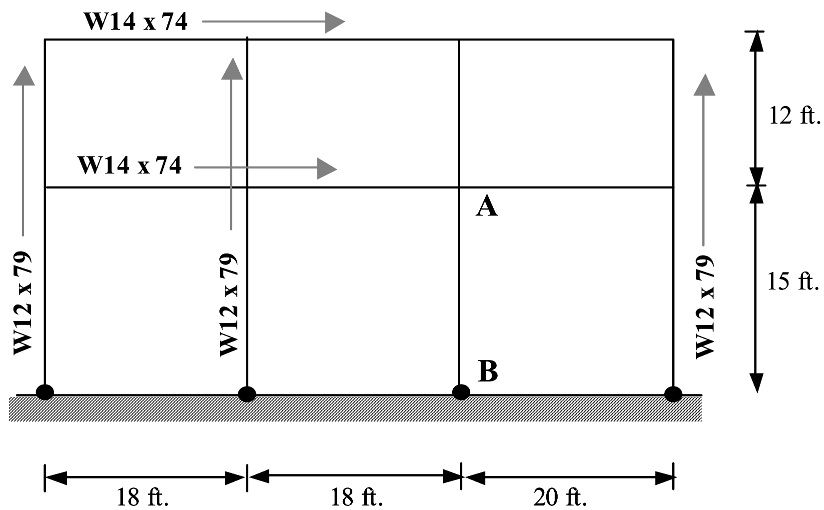
Therefore, the section is non-compact. OK, local buckling is not a problem

Step VII - Summarize the solution

$L_x = L_y = 12$ ft.	*$K_y = 1.0$*
$K_x = 1.2$ (inelastic buckling - sway frame-alignment chart method)	
$\phi_c P_n$ for Y-axis buckling = 518 kips	
$\phi_c P_n$ for X-axis buckling = 612.3 kips	
Y-axis buckling governs the design.	
Selected Section is W12 x 53 made from 50 ksi steel.	

EXAMPLE 3.9

- Design Column AB of the frame shown below for a design load of 450 kips.
- Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame.
- Assume that the columns are braced continuously along the length for out-of-plane buckling.
- Assume that the same column section is used for the story above.



Step I - Determine the design load and assume the steel material.

- Design Load = P_u = 450 kips
- Steel yield stress = 50 ksi

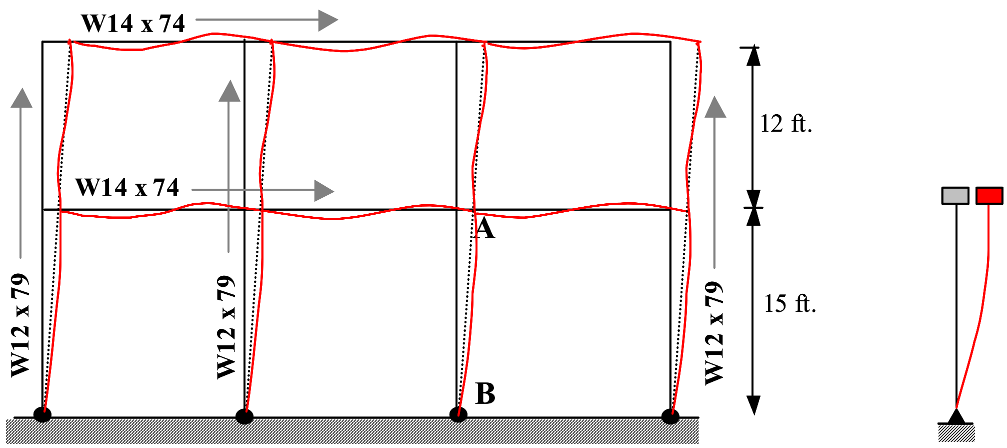
Step II. Identify the frame type and calculate L_x, L_y, K_x, and K_y if possible.

- It is an unbraced (*sidesway uninhibited*) frame.
- L_y = 0 ft.
- K_y has no meaning because out-of-plane buckling is not possible.
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.
- Need to select a section to calculate K_x

Step III. Select a section

- There is no help from the minor axis to select a section
- Need to assume K_x to select a section.

  See Figure below:



$K_x = 2.0$

Best Case Scenario

- The best case scenario for K_x is when the beams connected at joint A have infinite flexural stiffness (rigid). In that case $K_x = 2.0$ from Table C-C2.1
- Actually, the beams don't have infinite flexural stiffness. Therefore, calculated K_x should be greater than 2.0.
- To select a section, assume $K_x = 2.0$
  - $K_xL_x = 2.0 \times 15.0$ ft. = 30.0 ft.
- Need to be able to calculate $(KL)_{eq}$ to be able to use the column design tables to select a section. Therefore, need to assume a value of r_x/r_y to select a section.
  - See the W10 column tables on page 4-26.
  - Assume $r_x/r_y = 1.71$, which is valid for W10 x 49 to W10 x 68.
- $(KL)_{eq} = 30.0/1.71 = 17.54$ ft.
  - Obviously from the Tables, for $(KL)_{eq} = 17.5$ ft., W10 x 60 is the first section that will have $\phi_c P_n > 450$ kips
- Select W10x60 with $\phi_c P_n = 457.7$ kips for $(KL)_{eq} = 17.5$ ft.

Step IV - Calculate $K_{x\text{-inelastic}}$ using selected section

- I_{xx} of W 14 x 74 = 796 in^4 $\quad$ I_{xx} of W 10 x 60 = 341 in^4
- Account for the reduced flexural rigidity of the column due to residual stress effects
  - P_u/A_g = 450 / 17.6 = 25.57 ksi
  - Stiffness reduction factor = τ = 0.833
- $$G_A = \frac{\tau \times \sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{0.833 \times \left(\frac{341}{12 \times 12} + \frac{341}{15 \times 12}\right)}{\frac{796}{18 \times 12} + \frac{796}{20 \times 12}} = \frac{3.550}{7.002} = 0.507$$
- $G_B = 10$ $\quad$ **-** for pin support
- Using G_A and G_B: $K_{x\text{-inelastic}}$ = 1.75 $\quad$ - from Alignment Chart on Page 3-6
- Calculate value of $K_{x\text{-inelastic}}$ is less than 2.0 (the assumed value) because G_B was assumed to be equal to 10 instead of ∞

Step V - Check the selected section for X-axis buckling

- $K_x L_x$ = 1.75 x 15 = 26.25 ft.
  - r_x / r_y for W10x60 = 1.71
  - $(KL)_{eq}$ = 26.25/1.71 = 15.35 ft.
  - $(\phi_c P_n)$ for X-axis buckling = 513.9 kips
- Section slightly over-designed for P_u = 450 kips.
- *W10 x 54 will probably be adequate, Student should check by calculating K_x inelastic and $\phi_c P_n$ for that section.*

Step VI. Check the local buckling limits

For the flanges, $b_f/2t_f$ = 7.41 $\quad < \quad$ $\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$

For the web, h/t_w = 18.7 $\quad < \quad$ $\lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 35.9$

Therefore, the section is non-compact. OK, local buckling is not a problem

- **Step VII - Summarize the solution**

$L_y = 0$ *ft.* K_y = *no buckling*
$K_x = 1.75$ *(inelastic buckling - sway frame - alignment chart method)*
$\phi_c P_n$ *for X-axis buckling = 513.9 kips*
X-axis buckling governs the design.
Selected section is W10 x 60 *(W10 x 54 will probably be adequate).*

3.9 DESIGN OF SINGLY SYMMETRIC CROSS-SECTIONS

- So far, we have been talking about doubly symmetric wide-flange (I-shaped) sections and channel sections. These rolled shapes always fail by *flexural* buckling.
- Singly symmetric (Tees and double angle) sections fail either by *flexural* buckling about the axis of non-symmetry or by *flexural-torsional* buckling about the axis of symmetry and the longitudinal axis.

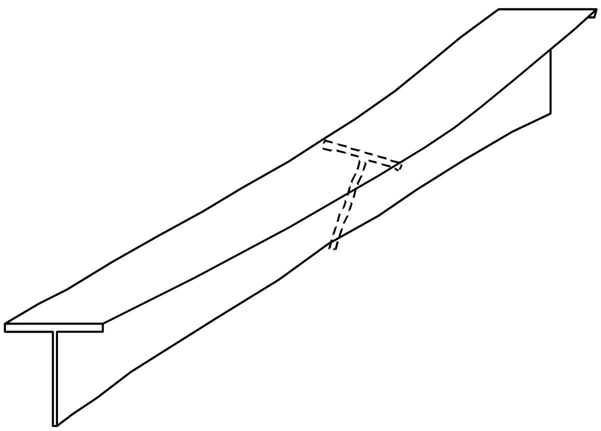


Figure 6(a). Flexural buckling

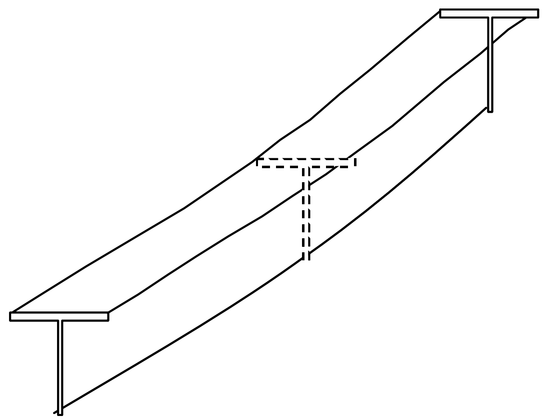
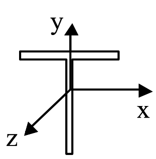


Figure 6(b). Flexural-torsional buckling



Flexural buckling will occur about the x-axis

Flexural-torsional buckling will occur about the y and z-axis

Smaller of the two will govern the design strength

Figure 6(c). Singly symmetric cross-section

- The AISC specification for flexural-torsional buckling is given by Spec. E3.

$$\text{Design strength} = \phi_c P_n = 0.85\ A_g\ F_{crft} \qquad (1)$$

$$\text{Where, } F_{crft} = \left(\frac{F_{cry} + F_{crz}}{2\ H}\right)\left[1 - \sqrt{1 - \frac{4\ F_{cry}\ F_{crz}\ H}{(F_{cry} + F_{crz})^2}}\right] \qquad (2)$$

F_{cry} = critical stress for buckling about the y-axis, see Spec. E2. (3)

$$F_{crz} = \frac{GJ}{A\,\bar{r}_o^{\,2}} \tag{4}$$

$$\bar{r}_o^2 = \text{polar radius of gyration about shear center (in.)} = y_o^2 + \frac{I_x + I_y}{A} \tag{5}$$

$$H = 1 - \frac{y_o^2}{\bar{r}_o^2} \tag{6}$$

y_o = distance between shear center and centroid (in.) (7)

- The section properties for calculating the flexural-torsional buckling strength F_{crft} are given as follows:

  - $G = \dfrac{E}{2(1+\upsilon)}$
  - J, $\bar{r}_o^2$, H are given for WT shapes in Table 1-32 on page 1-101 to page 1-105
  - $\bar{r}_o^2$, H are given for double-angle shapes in Table 1-35 on page 1-108 to 1-110
  - J for single-angle shape in Table 1-31 on page 1-98 to 1-100. ($J_{2L} = 2 \times J_L$)

- The design tables for WT shapes given in Table 4-5 on page 4-35 to 4-47. These design tables include the axial compressive strength for flexural buckling about the x axis and flexural-torsional buckling about the y and z axis.

<u>EXAMPLE 3.10</u> Calculate the design compressive strength of a WT10.5 x 66. The effective length with respect to x-axis is 25ft. 6in. The effective length with respect to the y-axis is 20 ft. and the effective length with respect to z-axis is 20ft. A992 steel is used.

<u>Solution</u>

- **Step I.** Buckling strength about x-axis

$$\lambda_{c\text{-}x} = \frac{K_x\ L_x}{r_x\ \pi}\sqrt{\frac{F_y}{E}} = \frac{306}{3.06 \times 3.1416}\sqrt{\frac{50}{29000}} = 1.321$$

$\phi_c P_n = 0.85 \text{ x } (0.658)^{1.321^2} \text{ x } 50 \text{ x } 19.4 = 397.2$ kips

Values for A_g and r_x from page 4-41 of the manual. Compare with tabulated design strength for buckling about x-axis in Table 4-5

- **Step II.** Flexural-torsional buckling about the y and z axes

- Calculate F_{cry} and F_{crz} then calculate F_{crft} and $\phi_c P_n$

- $$\lambda_{c\text{-}y} = \frac{K_y\ Ly}{r_y\ \pi}\sqrt{\frac{F_y}{E}} = \frac{240}{2.93 \times 3.1416}\sqrt{\frac{50}{29000}} = 1.083$$

- $F_{cry} = (0.658)^{1.083^2} \text{ x } 50 = 30.6$ ksi

- $F_{crz} = GJ/A\,\bar{r}_o^2 = 11{,}153 \text{ x } 5.62/(4.60^2 \text{ x } 19.4) = 152.69$

- $$F_{crft} = \left(\frac{F_{cry} + F_{crz}}{2\ H}\right)\left[1 - \sqrt{1 - \frac{4\ F_{cry}\ F_{crz}\ H}{(F_{cry} + F_{crz})^2}}\right] = \left(\frac{30.6 + 152.7}{2 \times 0.844}\right)\left[1 - \sqrt{1 - \frac{4 \times 30.6 \times 152.7 \times 0.844}{(30.6 + 152.7)^2}}\right]$$

  $F_{crft} = 108.58 \text{ x } 0.272 = 29.534$ ksi

- $\phi_c P_n = 0.85 \text{ x } F_{crft} \text{ x } A_g = 0.85 \text{ x } 29.534 \text{ x } 19.4 = 487$ kips

Values for J, $\bar{r}_o^2$, and H were obtained from flexural-torsional properties given in Table 1-32 on page 1-102. Compare the $\phi_c P_n$ value with the value reported in Table 4-5 (page 4-41) of the AISC manual.

- **Step III.** Design strength and check local buckling

  Flanges: $b_f/2t_f$ = 12.4/(2 x 1.03) = 6.02 , which is < $\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$

  Stem of Tee: d/t_w = 10.9/0.65 = 16.77, which is < $\lambda_r = 0.75 \times \sqrt{\frac{E}{F_y}} = 18.08$

  Local buckling is not a problem. Design strength = 397.2 kips. X-axis flexural buckling governs.

3.10 DESIGN OF DOUBLE ANGLE SECTIONS

- Double-angle sections are very popular as compression members in trusses and bracing members in frames.
  - These sections consist of two angles placed back-to-back and connected together using bolts or welds.
  - You have to make sure that the two single angle sections are connected such that they do not buckle (individually) between the connections along the length.
  - The AISC specification E4.2 requires that $\mathbf{Ka/r_z}$ of the individual single angles < ¾ of the *governing* **KL/r** of the double angle.
    - where, a is the distance between connections and r_z is the smallest radius of gyration of the single angle (see dimensions in Table 1-7)
- Double-angle sections can fail by flexural buckling about the x-axis or flexural torsional buckling about the y and z axes.

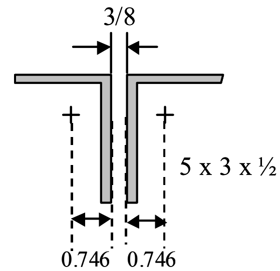
- For flexural buckling about the x-axis, the moment of inertia $I_{x\text{-}2L}$ of the double angle will be equal to two times the moment of inertia $I_{x\text{-}L}$ of each single angle.
- For flexural torsional buckling, there is a slight problem. The double angle section will have some *additional flexibility* due to the intermittent connectors. This added flexibility will depend on the connection parameters.

- According to AISC Specification E4.1, a modified $(KL/r)_m$ must be calculated for the double angle section for buckling about the y-axis to account for this added flexibility
  - Intermediate connectors that are snug-tight bolted $\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_z}\right)^2}$
  - Intermediate connectors that are welded or fully tensioned bolted:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82\frac{\alpha^2}{1+\alpha^2}\left(\frac{a}{r_y}\right)^2}$$

where, α = separation ratio = $h/2r_y$

h = distance between component centroids in the y direction



EXAMPLE 3.11 Calculate the design strength of the compression member shown in the figure. Two angles, 5 x 3 x ½ are oriented with the long legs back-to-back and separated by 3/8 in. The effective length KL is 16 ft. A36 steel is used. Assume three welded intermediate connectors

Solution

Step I. Determine the relevant properties from the AISC manual

Property	Single angle	Double angle
A_g	3.75 in^2	7.5 in^2
r_x	1.58 in.	1.58 in.
r_y	0.824 in.	1.24 in.
r_z	0.642 in.	-----
J	0.322 in^4	0.644 in^4
$\bar{r}_o^2$		2.51 in.
H		0.646
AISC Page no.	**1**-36, **1**-37, **1**-99	**1**-75, **1**-109

Step II. Calculate the x-axis buckling strength

- KL/r_x = 16 x 12 /1.58 = 120.8
- $\lambda_{c\text{-}x} = \dfrac{K_x\ L_x}{r_x\ \pi}\sqrt{\dfrac{F_y}{E}} = \dfrac{120.8}{3.1416}\sqrt{\dfrac{36}{29000}} = 1.355$
- $\phi_c P_n = 0.85 \times (0.658)^{1.355^2} \times 36 \times (2 \times 3.75) = 106$ kips

Step III. Calculate $(KL/r)_m$ for y-axis buckling

- (KL/r) = 16 x 12/1.24 = 154.8

- $a/r_z = 48/0.648 = 74.07$

  $a/r_z = 74.07 < 0.75 \times KL/r = 0.75 \times 154.8 = 115.2$ (OK!)

- $\alpha = h/2r_y = (2 \times 0.75 + 0.375)/(2 \times 0.829) = 1.131$

- $$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82\frac{\alpha^2}{1+\alpha^2}\left(\frac{a}{r_y}\right)^2}$$

  $$= \sqrt{(154.8)_o^2 + 0.82\frac{1.131^2}{1+1.131^2}\left(\frac{48}{0.829}\right)^2} = 158.5$$

Step IV. Calculate flexural torsional buckling strength.

- $$\lambda_{c\text{-}y} = \left(\frac{KL}{r}\right)_m \times \frac{1}{\pi} \times \sqrt{\frac{F_y}{E}} = 1.778$$

- $$F_{cry} = \frac{0.877}{\lambda_{c-y}^2} \times F_y = \frac{0.877}{1.778^2} \times 36 = 9.987 \text{ ksi}$$

- $$F_{crz} = \frac{GJ}{A\bar{r}_o^2} = \frac{11{,}200 \times 0.644}{7.5 \times 2.51^2} = 151.4 \text{ ksi}$$

- $$F_{crft} = \left(\frac{F_{cry} + F_{crz}}{2\,H}\right)\left[1 - \sqrt{1 - \frac{4\,F_{cry}\,F_{crz}\,H}{(F_{cry} + F_{crz})^2}}\right] = \left(\frac{9.987 + 151.4}{2 \times 0.646}\right)\left[1 - \sqrt{1 - \frac{4 \times 9.987 \times 151.4 \times 0.646}{(9.987 + 151.4)^2}}\right]$$

  $F_{crft} = 9.748$ ksi

- $\phi_c P_n = 0.85 \times F_{crft} \times A_g = 0.85 \times 9.748 \times 7.50 = 62.1$ kips

Flexural torsional buckling strength controls. The design strength of the double angle member is 62.1 kips.

Step V. Compare with design strengths in Table 4-10 (page **4-**84) of the AISC manual

- $\phi_c P_n$ for x-axis buckling with unsupported length = 16 ft. = 106 kips

- $\phi_c P_n$ for y-z axis buckling with unsupported length = 16 ft. = 61.3 kips

These results make indicate excellent correlation between the calculations in steps II to IV and the tabulated values.

Design tables for double angle compression members are given in the AISC manual. See Tables 4-9, 4-10, and 4-11 on pages 4-78 to **4**-93

- In these Tables F_y = 36 ksi
- Back to back distance = 3/8 in.
- Design strength for buckling about x axis
- Design strength for flexural torsional buckling accounting for the *modified* slenderness ratio depending on the number of intermediate connectors.
- These design Tables can be used to design compression members as double angle sections.