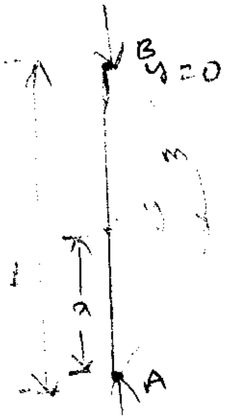
Unit - II

1) Derive an expression for Euler's buckling load for a column with

(a) Both the ends hinged



From bending moment equation

$\Rightarrow \quad M = EI \dfrac{d^2y}{dx^2}$

$\Rightarrow \quad M = -Py$

$\Rightarrow \quad EI \dfrac{d^2y}{dx^2} = -Py$ $\qquad \left(\alpha = \sqrt{P/EI}\right)$

$\Rightarrow EI \dfrac{d^2y}{dx^2} + Py = 0$

$\Rightarrow \dfrac{d^2y}{dx^2} + \left(P/EI\right) y = 0$

$y = C_1 \cos(\alpha x) + C_2 \sin(\alpha x)$

$y = C_1 \cos(x\sqrt{P/EI}) + C_2 \sin(x\sqrt{P/EI}) \quad - (1)$

(i) At 'A'

$x=0, \quad y=0;$

eauation (1) substituting we get

$= C_1 \cos(0) + C_2 \sin(0)$

$= C_1 = 0$

(ii) At 'B'

$x = L. \quad y=0$

eauat(1) substituting x.y we get

$\Rightarrow C_2 \sin(L\sqrt{P/EI})$

$C_2 = 0$

$\sin(L\sqrt{(P/EI)}) = 0$

$\sin(l\sqrt{P/EI}) = 0$

$L \cdot (\sqrt{P/EI}) = 0 \cdot \pi, \; 2\pi \cdots n\pi$

$L\sqrt{P/EI} = \pi$

$$= \boxed{P = \frac{\pi^2 EI}{L^2}}$$

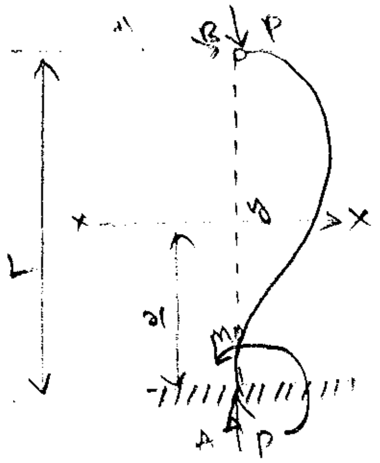
E = Young's modulus

I = moment of inertia

L = length of the beam

② Derive the expression for

b) One end is fixed and the other is pinned (or) hinged ?



H = Horizontal reaction at B due to 'm₀' at A

From bending moment equation

$$M = EI \frac{d^2y}{dx^2}$$

$$M = -Py + H(L-x)$$

$$-Py + H(l-x) = EI \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = \frac{H(l-x)}{EI}$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = \left(\frac{P}{EI}\right)\left(\frac{H(l-x)}{P}\right)$$

Solution of the Differential equation

$$y = C_1 \cos(P/EI)x + C_2 \sin((P/EI)x) + \frac{H(L-x)}{P} \quad -①$$

(i) At 'A'

$x=0, \quad y=0$

equation (1) substituting x and y we get

$$c_1 = -\frac{H(l)}{P}$$

$$c_1 = -\frac{Hl}{P}$$

(ii) At

$x=0 \quad \frac{dy}{dx}=0$

substituting in equation (1) we get

$$\frac{dy}{dx} = c_1 \sin(\sqrt{P/EI})(\sqrt{P/EI}) + c_2 \cos(x\sqrt{P/EI}) - (\sqrt{P/EI}) - \frac{H}{P} \quad (2)$$

$$\Rightarrow c_2(\sqrt{P/EI}) - \frac{H}{P}$$

$$c_2 = \frac{H}{P}\left(\sqrt{\frac{EI}{P}}\right)$$

equation (1) substituting above values

$$(1) \Rightarrow y = \frac{-Hl}{P}\cos(x\sqrt{P/EI}) + \frac{H}{P}\left(\sqrt{\frac{EI}{P}}\right)\sin(x\sqrt{P/EI}) + \frac{H(l-x)}{P} \quad (3)$$

iii) at 'B'

$x=L \quad y=0$

substituting in equation (1) x, y values we get

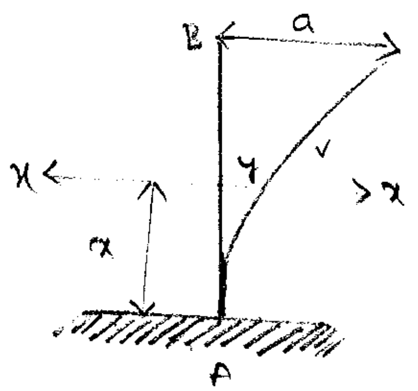
$$L\sqrt{P/EI} = 4.5^c = \sqrt{2}\pi$$

$$\tan(L\sqrt{P/EI}) = L\sqrt{P/EI}$$

$$\boxed{P = \frac{2\pi^2 EI}{L^2}}$$

(3) Derive the equations for

(C) One end is fixed and the other is free ?



From Bending moment equation

$$M = EI \frac{d^2y}{dx^2}$$

$$M = (a - y) P$$

$$EI \frac{d^2y}{dx^2} = P(a-y)$$

$$\Rightarrow EI \frac{d^2y}{dx^2} - P(a-y) = 0$$

$$\Rightarrow EI \frac{d^2y}{dx^2} + Py = Pa$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right) y = (P/EI)\, a$$

$$\therefore \frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 a$$

$$y = C_1 \cos(\alpha x) + C_2 \sin(\alpha x) + a$$

$$\Rightarrow y = C_1 \cos(\sqrt{P/EI}\, x) + C_2 \sin(\sqrt{P/EI}\, x) + a \quad \text{--- (1)}$$

(i) At A

$x=0,\ y=0$

substituting in equation (1) x and y values.

we get

$① = 0$

$\Rightarrow C_1 + 0 + a = 0$

$\Rightarrow C_1 = -a$

(ii) At A $x=0$ $\frac{dy}{dx} = 0$

substituting in equation (1)

we get

$① \Rightarrow \frac{dy}{dx} = -C_1 (\sin(\sqrt{P/EI}\, x))\sqrt{P/EI} + C_2 \cos(\sqrt{P/EI}\, x)\sqrt{P/EI}$ —— ②

$② \Rightarrow 0$

$= C_2 \sqrt{P/EI}$

$C_2 = 0$

$P/EI \neq 0$

$\therefore y = -a\cos(\sqrt{P/EI}\, x) + a$ —— ③

iii) At 'B'

$x = L$ $y = a$

$\Rightarrow (3) = 0$

$= -a\cos(L\sqrt{P/EI}) + a$

$\therefore \cos(L\sqrt{P/EI}) = 0$

$L\sqrt{P/EI} = \frac{\pi}{2}, \frac{3\pi}{2}$

$L\sqrt{P/EI} = \frac{\pi}{2}$

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

(8)

Problem: A solid round bar 3 m long and 5 cm in diameter is used as a strut with both ends ~~being~~ hinged. Determine the crippling (or collapsing) load. Take $E = 2.0 \times 10^5 \ N/mm^2$ and also determine the crippling load, when the given strut is used with the following conditions:

i) One end of the strut is fixed and the other end is free

ii) Both the ends of strut are fixed

iii) One end is fixed and other is hinged

Sol: Given: The data from problem is

Length of bar, $l = 3m = 3000 \ mm$

Diameter of bar, $d = 5cm = 50 \ mm$

Young's modulus $E = 2.0 \times 10^5 \ N/mm^2$

moment of inertia $I = \frac{\pi}{64} \times 5^4 = 30.68 \ cm^4$

$= 30.68 \times 10^4 \ mm^4$

Let P = crippling load.

Let i) Crippling load when One End Is fixed and other End is free

Using equation $p = \frac{\pi^2 EI}{4 l^2}$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{4 \times 3000^2}$$

$$= 16822 \text{ N}.$$

Alternate Method :

The crippling load for any type of End Condition is given by equation

$$P = \frac{\pi^2 EI}{L_e^2} \qquad \text{——— (1)}$$

where L_e = Effective length

The effective length (L_e) when One End Is fixed and other End is free

$$L_e = 2l = 2 \times 3000 = 6000 \text{ mm}$$

Substituting the Value of L in equation (1) we get,

$$P = \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{6000^2} = 16822 \text{ N}$$

ii) Crippling load when both ends are fixed:

Useing equation $P = \frac{4\pi^2 EI}{l^2}$

$$= \frac{4\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{3000^2}$$

$$= 269152\ N = 269.152\ kN.$$

Alternate method:

Using equation $P = \frac{\pi^2 EI}{Le^2}$

Le = Effective length

$$= l/2$$

$$= \frac{3000}{2}$$

$$= 1500\ mm$$

$$P = \frac{\pi^2 \times 2.0 \times 10^5 \times 30.68 \times 10^4}{1500^2}$$

$$P = 269152\ N.$$

iii) Crippling load when One End is fixed and the other is hinged :

Useing eqn. $P = \frac{2\pi^2 E I}{l^2}$

$= \frac{2 \times \pi^2 \times 2.0 \times 10^5 \times 30.68 \times 10^4}{3000^2}$

$= 134576 \text{ N}$

Alternate Method :

Using eqn. $P = \frac{\pi^2 E I}{Le^2}$

Le = EFFECTIVE LENGTH.

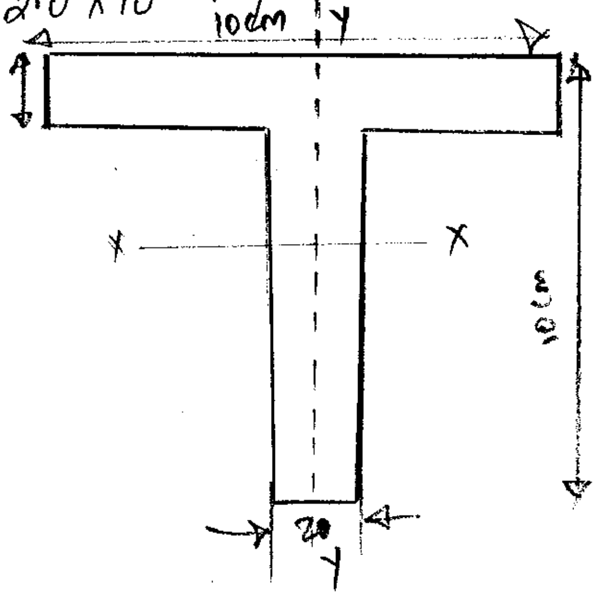
$= l/\sqrt{2}$ (when One End is fixed and the Other is hinged)

$P = \frac{\pi^2 \times 2.0 \times 10^5 \times 30.68 \times 10^4}{\left[\frac{3000}{\sqrt{2}}\right]^2}$

$P = 134576 \text{ N}.$

⑥ problem: Determine the crippling load for a T-section of dimensions 10cm × 10cm × 2cm and of length 5m when it is used as strut with both of its ends hinged. Take Young's modulus $E = 2.0 \times 10^5$ N/mm²

Sol:



Given: Dimensions of T-section = 10cm × 10cm × 2cm

Actual length, $l = 5m = 5000$ mm

Young's modulus, $E = 2.0 \times 10^5$ N/mm²

y_1 = Distance of C.G of area a_1 from the bottom end $= 8 + 1 = 9$ cm

For the web, we have $a_2 = 8 \times 2 = 16$ cm²

y_2 = Distance of C.G of area a_2 from bottom end $= 8/2 = 4$ cm.

Using the relation, $\bar{y} = \dfrac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$

$$= \frac{20 \times 9 + 16 \times 4}{20 + 16}$$

$$= \frac{180 + 64}{36}$$

$$= 6.777 \text{ cm}$$

Moment of inertia of the section about the axis X-X,

$$I_{xx} = \left(\frac{10 \times 8^3}{12} + 20 \times 2.223^2\right) + \left(\frac{2 \times 8^3}{12} + 16 \times 2.777^2\right)$$

$$= (6.667 + 98.834) + (85.333 + 123.387) = 314.221 \text{ cm}^4$$

Moment of inertia of the section about the axis Y-Y

$$I_{yy} = \frac{2 \times 10^3}{12} + \frac{8 \times 2^3}{12}$$

$$= 166.67 + 5.33$$

Least value of moment of inertia is about Y-Y axis.

$$= 172 \text{ cm}^2.$$

$$I = 172 \times 10^4 \text{ mm}^4$$

Since the strouct is hinged at both of its End

$\therefore$ Effective length, $L_e = l = 5000$ mm

Let P = crippling load.

Using equation we get

$$P = \frac{\pi^2 E I}{L_e^2}$$

$$= \frac{\pi^2 \times 2.0 \times 10^5 \times 172 \times 10^4}{5000^2}$$

$$= 135805.7 \text{ N}.$$

(7)

problem: If the given Column of ~~problem~~ Circular Section is Subjected to a load of 120 kN The load is parallel to the axis but Eccentric by an amount of 2.5 mm. The External and internal diameters of Columns are 60 mm and 50 mm respectively. If both the Ends of the Column are hinged. And Column is 2.1 m long. Is Subjected to an Eccentric load of 100 kN and Maximum permissible stress is limited to 320 MN/m² then determine the maximum Eccentricity of the load.

Sol: Given : Data from above

Load $P = 120\ kN = 120 \times 10^3\ N$

Eccentricity $e = 2.5\ mm = 2.5 \times 10^{-3}\ m$

$D = 60\ mm = 0.06\ m$, $d = 50\ mm = 0.05\ m$, $l = 2.1\ m$

Both Ends are hinged, $L_e = l = 2.1\ m$

Value of $E = 200\ GN/m^2 = 200 \times 10^9\ N/m^2$

The max. stress is given by eqn.

$$\sigma_{max} = \frac{P}{A} + \frac{P \times e \times \sec\left(\frac{L_e}{2} \times \sqrt{\frac{P}{EI}}\right)}{2}$$

Let us first find the value of $\sec\left(\frac{L_e}{2} \times \sqrt{\frac{P}{EI}}\right)$

$$\sec\left(\frac{L_e}{2} \times \sqrt{\frac{P}{EI}}\right) = \sec\left[\frac{2.1}{2} \times \sqrt{\frac{100 \times 10^3}{200 \times 10^9 \times 0.0329 \times 10^{-5}}}\right]$$

$$= \sec\left[1.294\ rads\right] = \sec\left[1.294 \times \frac{180^\circ}{\pi}\right]$$

$$= \sec(74.16^\circ) = 3.665$$

Substituting the known values in equation (1)

we get

$$320 \times 10^6 = \frac{100 \times 10^3}{8.639 \times 10^{-4}} + \frac{(100 \times 10^3) \times e \times 3.665}{1.0975 \times 10^{-5}}$$

$$= 115.754 \times 10^6 + 33394 e \times 10^6$$

$$320 = 115.754 + 33394 \times e$$

$$e = \frac{320 - 115.754}{33394} \ m$$

$$= 6.119 \times 10^{-3} \ m$$

$$e = 6.116 \ mm.$$

(8)

Problem : Determine the maximum stress induced in a cylindrical steel of length 1.2m and diameter 30 mm. The strut is hinged at both its ends and subjected to an axial thrust of 20 kN at its ends and a transverse point load of 1.8 kN at the centre. Take $E = 208 \ GN/m^2$

Sol?

Given: $l = 1.2 m$; $d = 30 mm = 0.03 m$

axial thrust, $P = 20 \ kN = 20 \times 10^3 N$

transverse point load, $W = 1.8\,kN = 1.8 \times 10^3\,N$

$E = 208\,GN/m^2 = 208 \times 10^9\,N/m^2$

Area $A = \pi/4\, d^2 = \pi/4\,(0.03)^2 = 7.068 \times 10^{-4}\,m^2$

M.O.I $I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(0.03)^4 = 3.976 \times 10^{-8}\,m^4$

Direct Stress is due to axial thrust

$$\sigma_0 = \frac{P}{A} = \frac{20 \times 10^3}{7.068 \times 10^{-4}} = 28.29 \times 10^6\,N/m^2$$

$$= 28.29\,MN/m^2$$

Max. bending stress is given by

$$\sigma_b = \frac{M_{max} \times y_c}{I} \quad \text{—— (1)}$$

Max. bending moment is given by eqn.

$$M_{max} = \frac{W}{2} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right). \quad \text{—— (2)}$$

Let us find $\sqrt{\frac{P}{EI}}$

Now,

$$\sqrt{\frac{P}{EI}} = \sqrt{\frac{20\times10^3}{(208\times10^9)\times(3.976\times10^{-8})}} = \sqrt{2.4183} = 1.555$$

$$\sqrt{\frac{EI}{P}} = \frac{1}{1.555} = 0.643$$

and Also $\frac{l}{2}\times\sqrt{\frac{P}{EI}} = \frac{1.2}{2}\times 1.555 = 0.933$ rad

$$= 0.933\times\frac{180^\circ}{\pi}$$

$$= 53.45^\circ$$

$$\therefore \tan\left(\frac{l}{2}\times\sqrt{\frac{P}{EI}}\right) = \tan(0.933\text{ rad})$$

$$= \tan(53.45^\circ) = 1.349$$

Substituting known values in equation (2) we get

$$M_{max} = \frac{1.8\times10^3}{2}\times0.643\times1.349$$

$$= 780.66 \text{ Nm}$$

Substituting the above value in equation (1)

$$\sigma_b = \frac{780.66 \times y_c}{I}$$

$$= \frac{780.66 \times 0.015}{3.976 \times 10^{-8}}$$

$\Big(\because y_c = \frac{d}{2} = \frac{30}{2} = 15mm = 0.015m$

and $I = 3.976 \times 10^{-8} m^4$ $= 294.51 \times 10^6 N/m^2 = 294.51 MN/m^2 \Big)$

$\therefore$ Maximum stress induced is given by

$$\sigma_{max} = \sigma_o + \sigma_b$$

$$= 28.29\ MN/m^2 + 294.51\ MN/m^2$$

$$= 322.8\ MN/m^2.$$